Phase quenching in finite-density QCD: models, holography, and lattice

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with Y. Matsuo (KEK) and N. Yamamoto (INT, Seattle)

based on 1205.1030[hep-lat]

(based also on M.H.-Hoyos-Karch-Yaffe, 1201.3718[hep-th])

Parallel session @ LATTICE 2012

A historical remark

Cherman-M.H.-Robles, PRL 106, 091603(2011) (1009.1623[hep-th])
M.H.-Yamamoto, JHEP 1202, 138 (2012) (1103.5480[hep-ph])
M.H.-Hoyos-Karch-Yaffe, 1201.3718[hep-th], submitted to JHEP

Related good old works

T.D. Cohen, Phys.Rev. D70 (2004) 116009. hep-ph/0410156.
D. Toublan, Phys.Lett. B621 (2005) 145-150. hep-th/0501069.

Recently (1204.2405[hep-th]), Armoni (Swansea) and Patella (CERN) claim ***THEY have discovered*** many of the results of these papers.
Although we informed them all 'their' results had been known already, they still claim they have the priority.

Curiously:

(1)Actually they cite our papers and claime they add new findings, without explaining known results. Then they introduce known statements as their own 'new results'.

(2)Their proof is **wrong**. (Stehpahov, hep-lat/9604003; M.H.-Matsuo-Yamamoto, 1205.1030[hep-lat].)

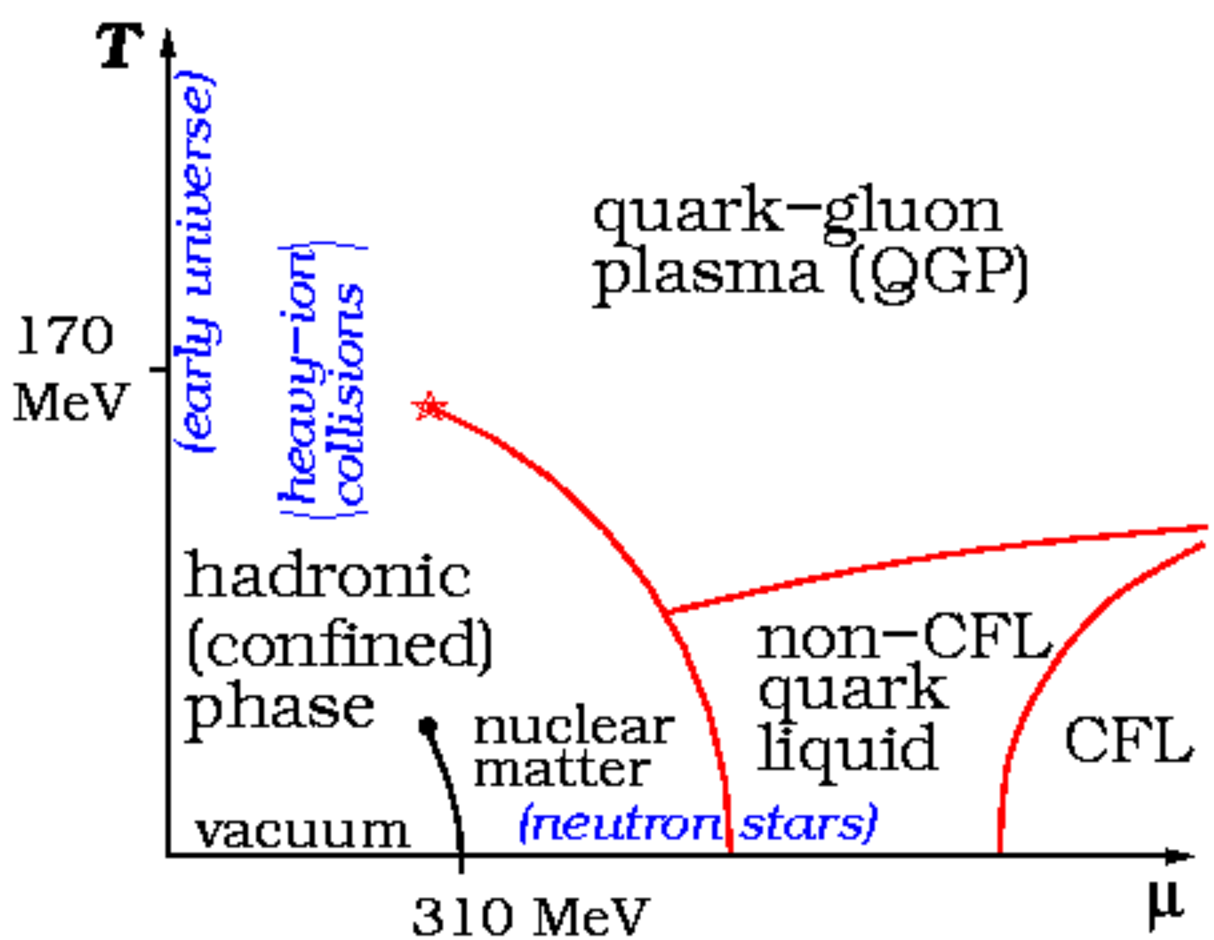
(3) **A part of 'their' results cannot follow from their argument in principle, even with their wrong 'proof'**. How could they find it??

'Their' claim and original references will be shown later.

Let’s move on to physics.

Motivation : QCD phase diagram

- Important for QGP, neutron star, ..
- QCD at finite baryon chemical potential cannot be studied tby Monte Carlo because of the '**sign problem**'. Nobody knows if the phase diagram below is correct.



$\mu_B = \mu \times N_c = 3\mu$

(from wikipedia)

Our solution

- Let's consider SU(Nc) instead of SU(3). Then...

QCD with the baryon chemical potential μ_B "=" QCD with the isospin chemical potential μ_I

↑ large-Nc (g^2Nc fix)

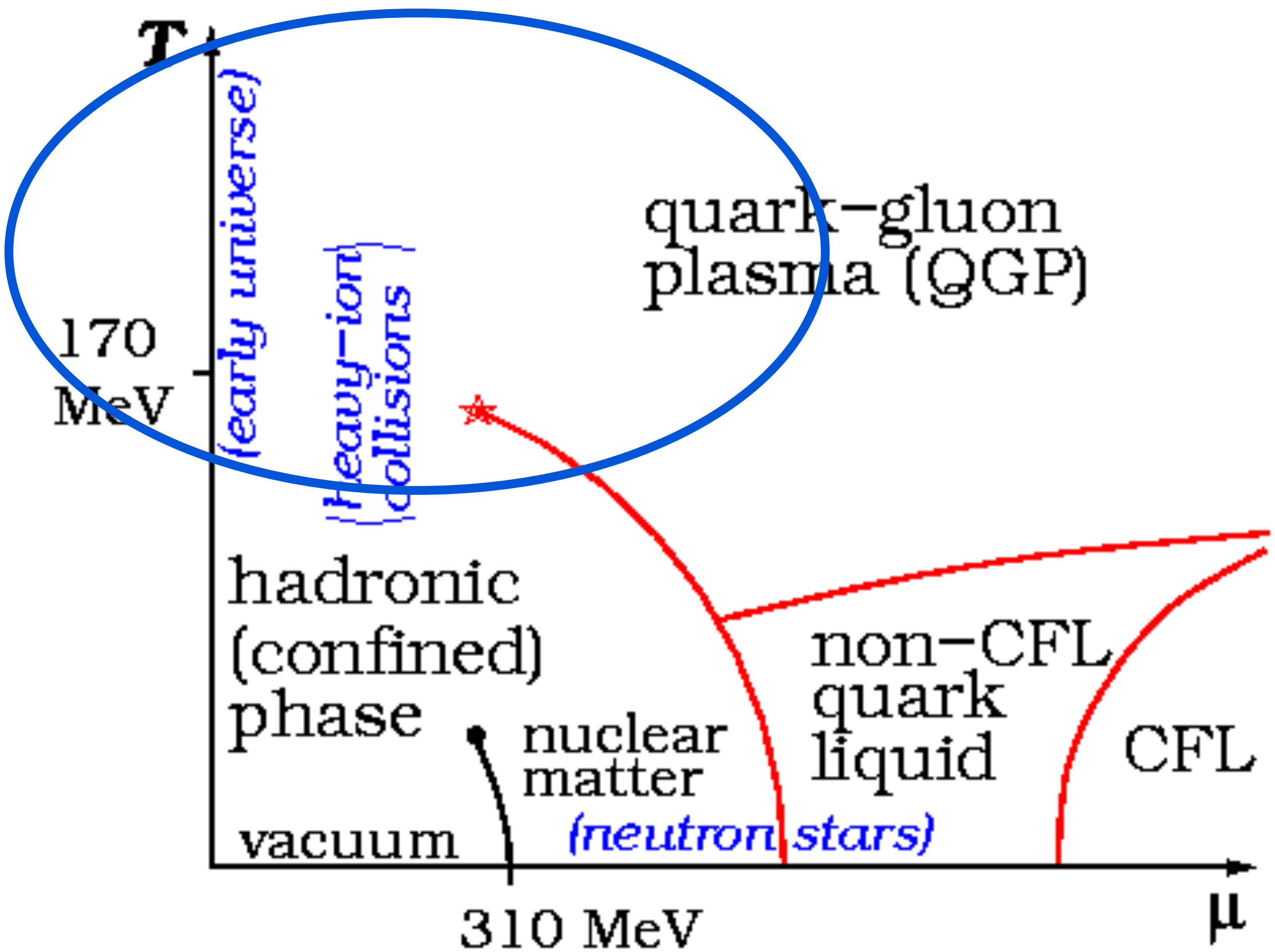
(Cherman-M.H.-Robles 2010, M.H.-Yamamoto 2011)

Certain operators (e.g. chiral condensate, Polyakov loop) take the same values in a certain parameter region. Equivalence holds up to fermion two-loop corrections.

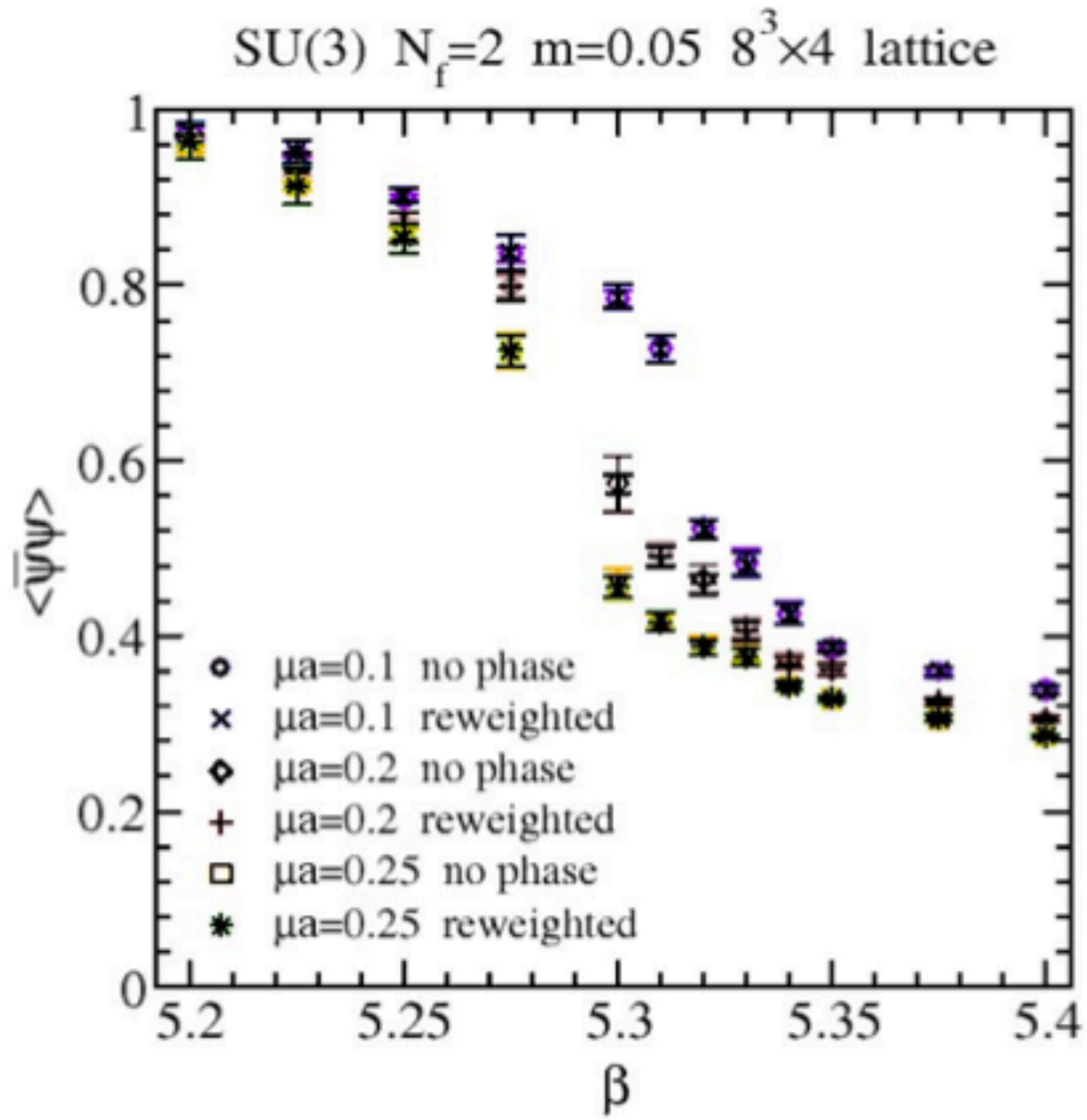
Effect of the phase is only a 1/Nc effect
→ no overlapping problem
in the phase reweighting method.

Equivalence holds here.

→ useful for heavy ion collision experiments!



QCD_B vs QCD_I

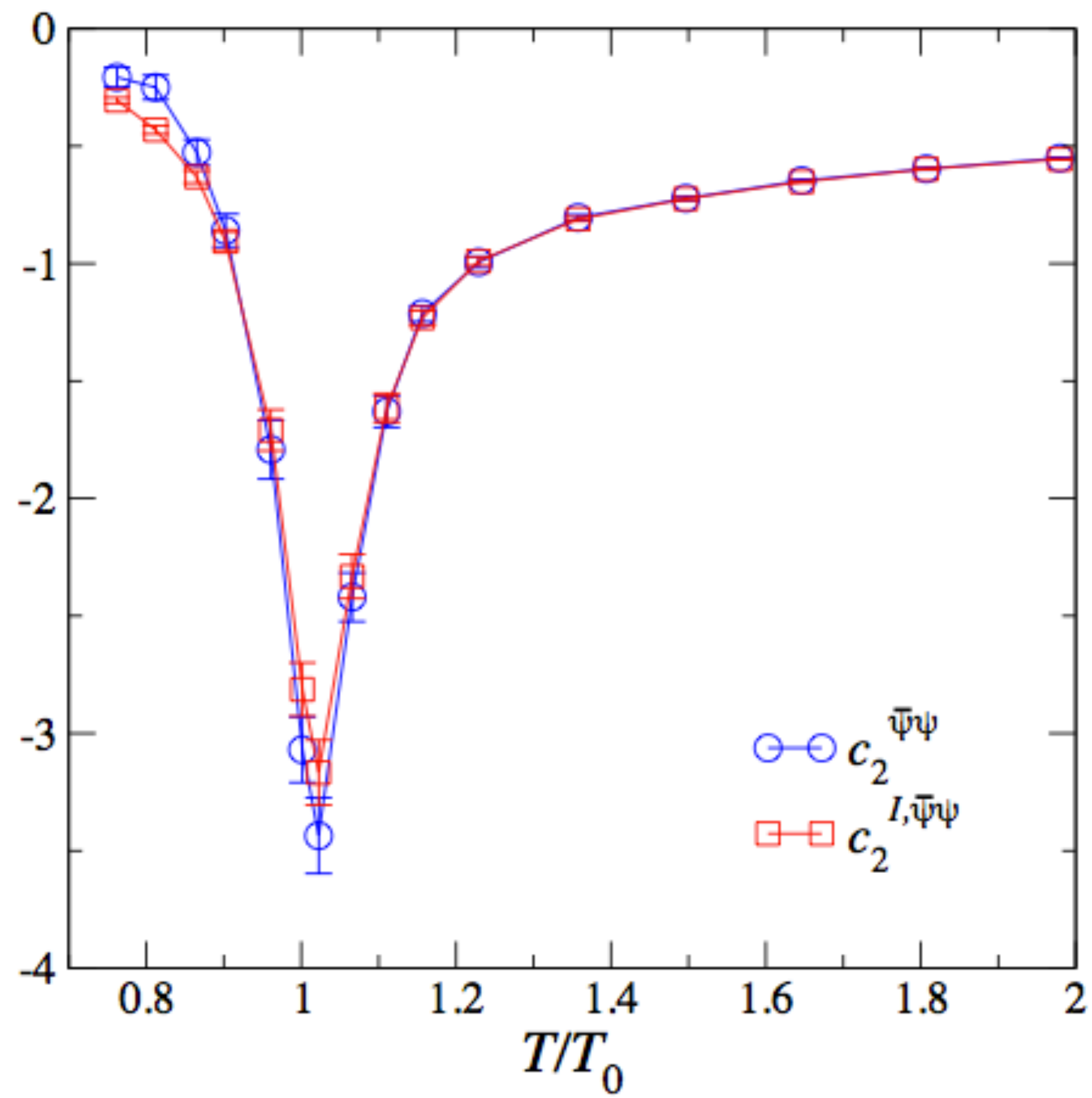


no difference can be seen already at Nc=3

Nakamura-Sasai-Takahashi 2005

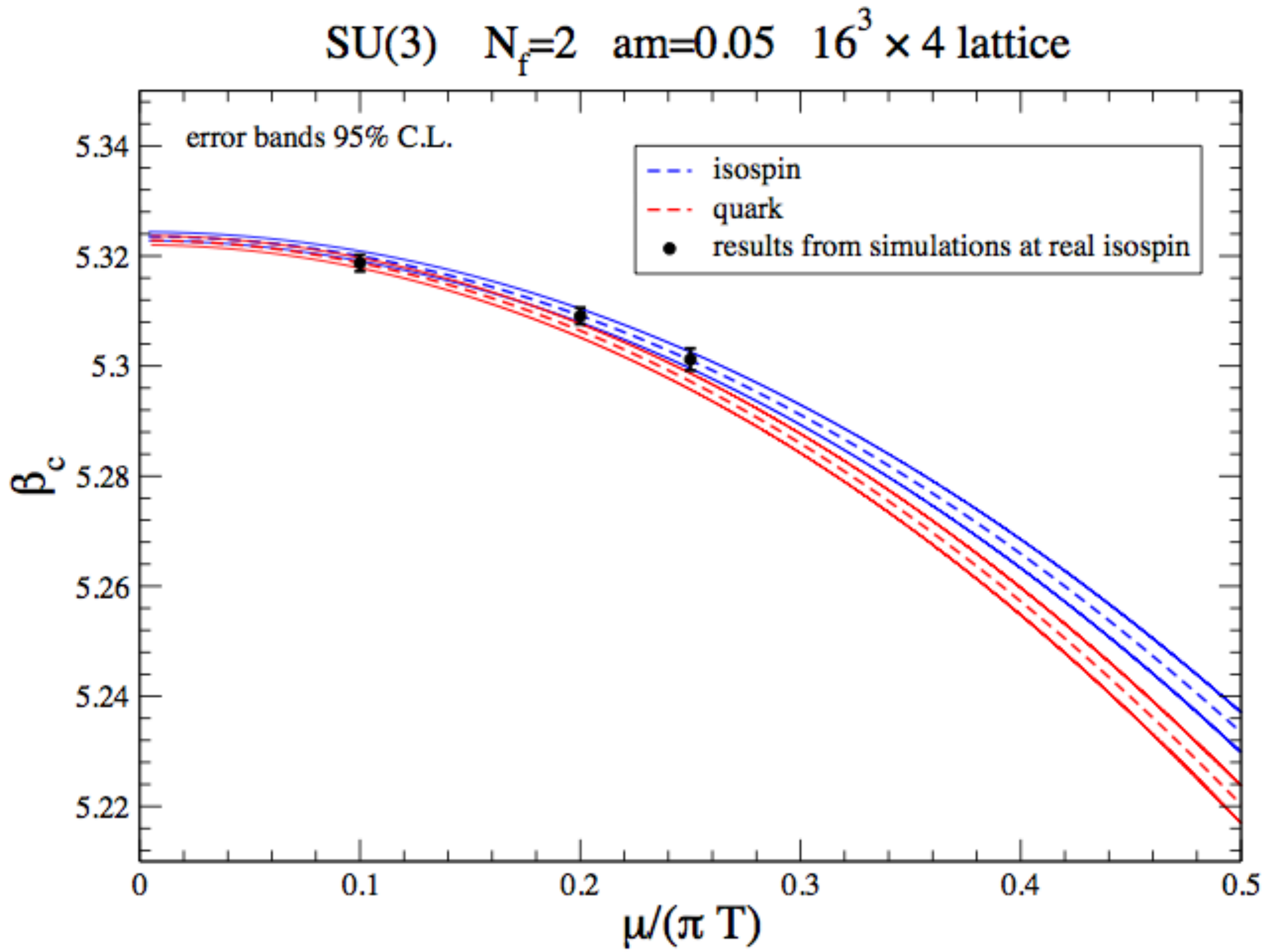
Chiral condensate

(First coefficients of the Taylor expansion)



Allton et al, 2005

Chiral transition temperature

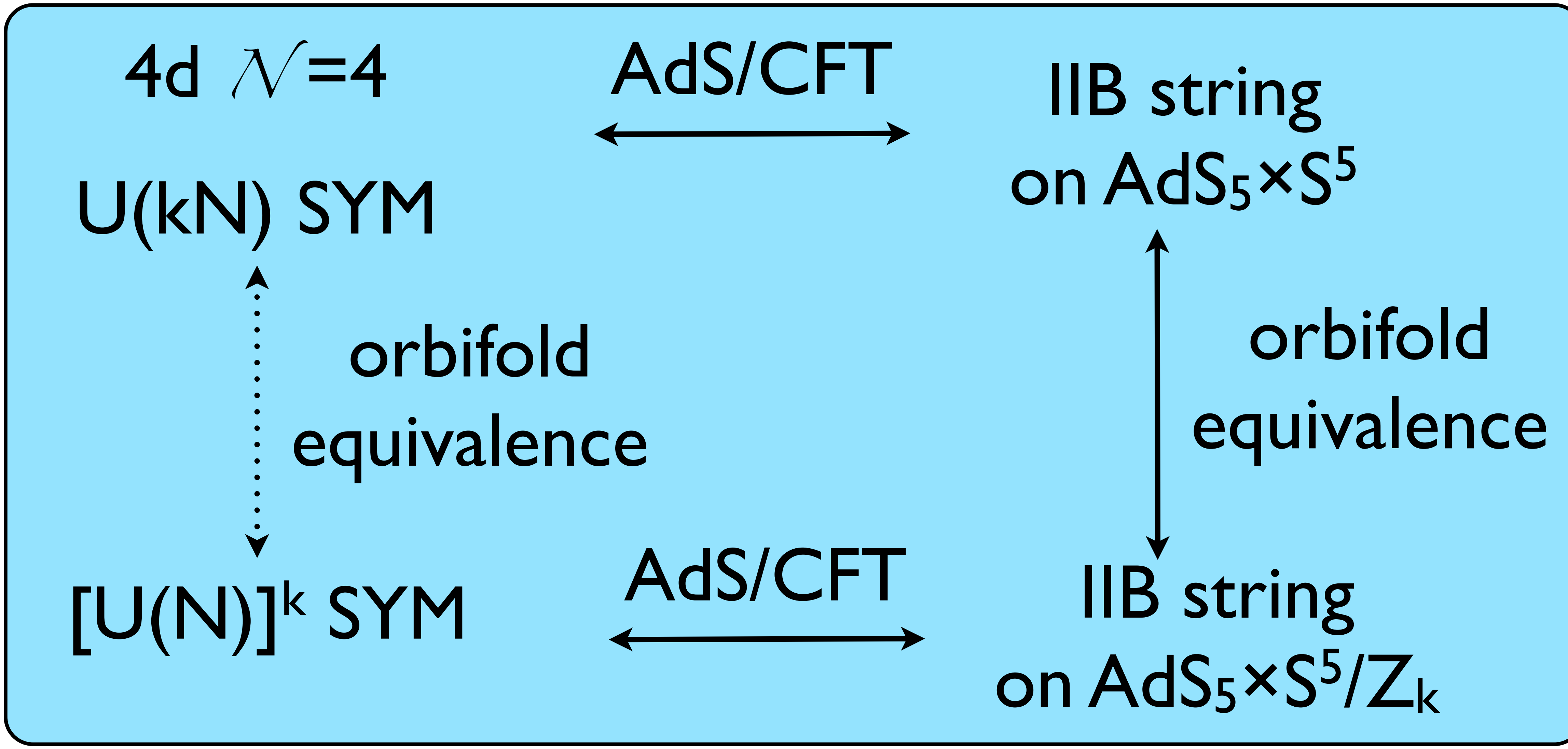


Cea et al, arXiv:1110.3910

(analytic contibuation from imaginary chemical potential)

Large-Nc orbifold equivalence

Kachru-Silverstein ’98, Bershadsky-Kakushadze-Vafa ’98, Bershadsky-Johansen ’98, ...

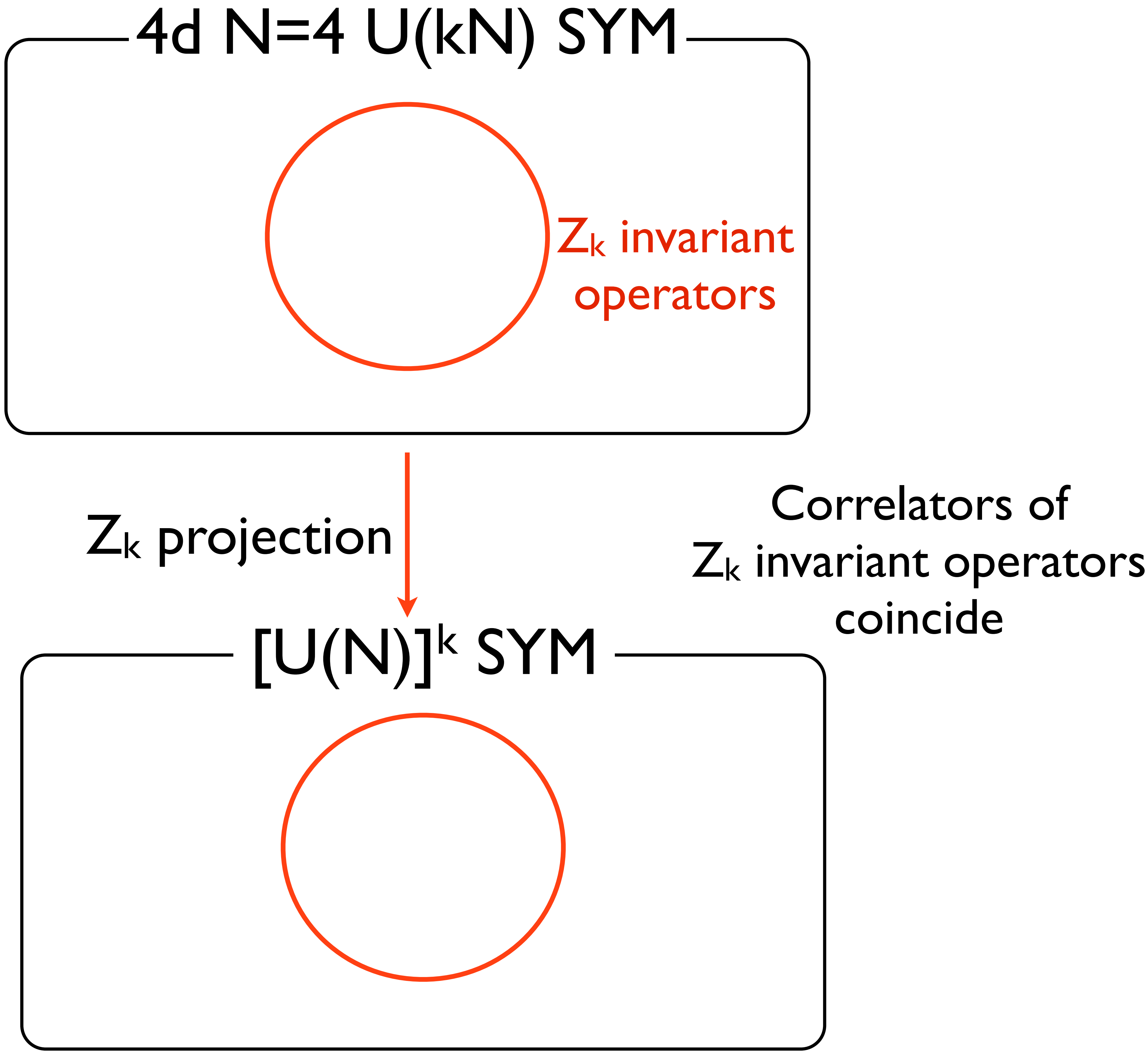


It can also be proven in the field theory language.

(Bershadsky-Johansen ’98, Kovtun-Unsal-Yaffe ’06,...)

It is applicable to usual large-Nc QCD. (‘t Hooft limit)

(Cherman-M.H.-Robles 2010, M.H.-Yamamoto 2011)


4d N=4 U(kN) SYM
Z_k invariant operators
Z_k projection
Correlators of Z_k invariant operators coincide
[U(N)]^k SYM

The large-N_c equivalence

$SU(N_c)$ + fund + μ_B ⟷ $SU(N_c)$ + fund + μ_I

sign problem

sign-free

The large-N_c equivalence

$SO(2N_c)$ + fund + μ_B

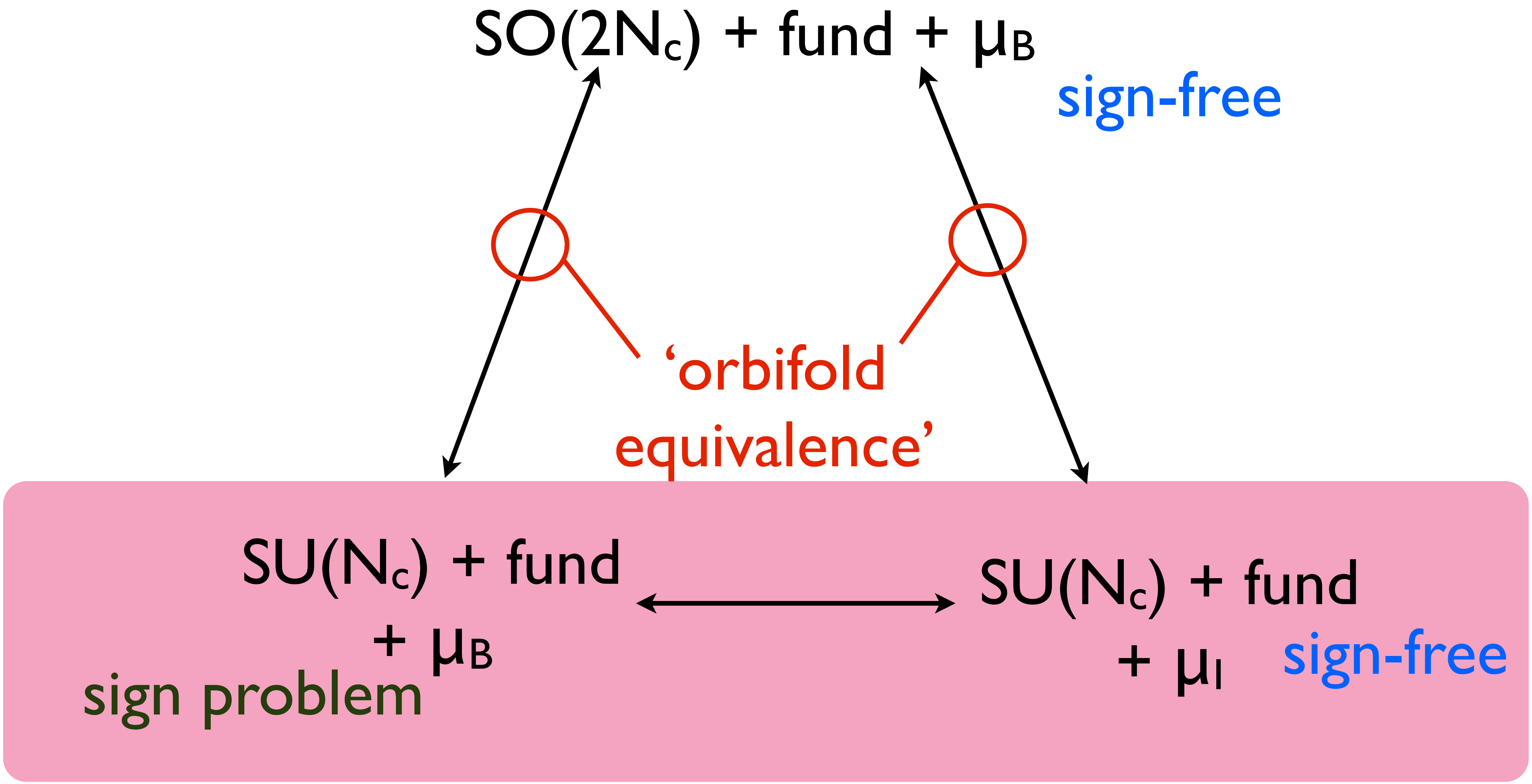
sign-free

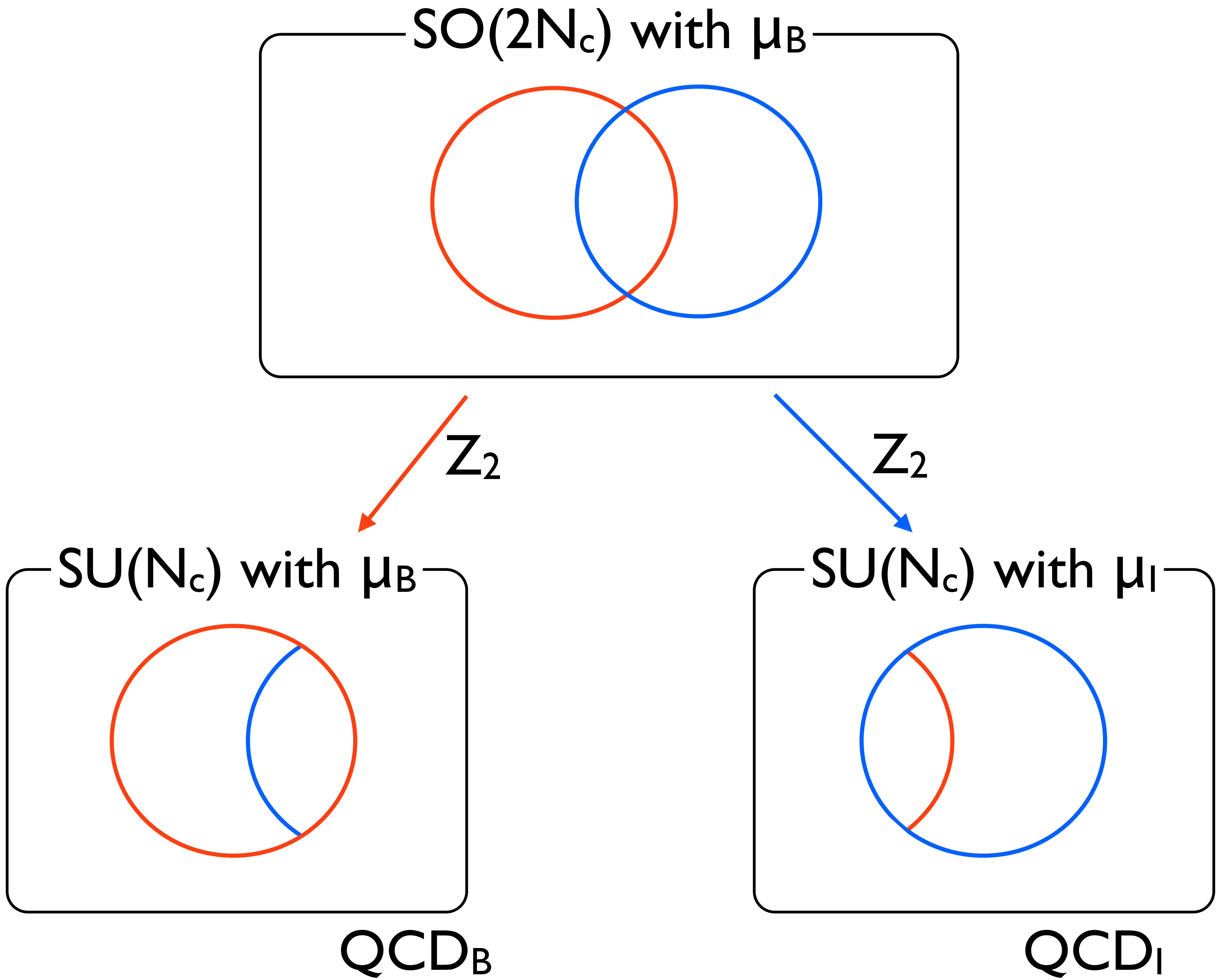
$SU(N_c)$ + fund + μ_B ⟷ $SU(N_c)$ + fund + μ_I

sign problem

sign-free

The large-N_c equivalence



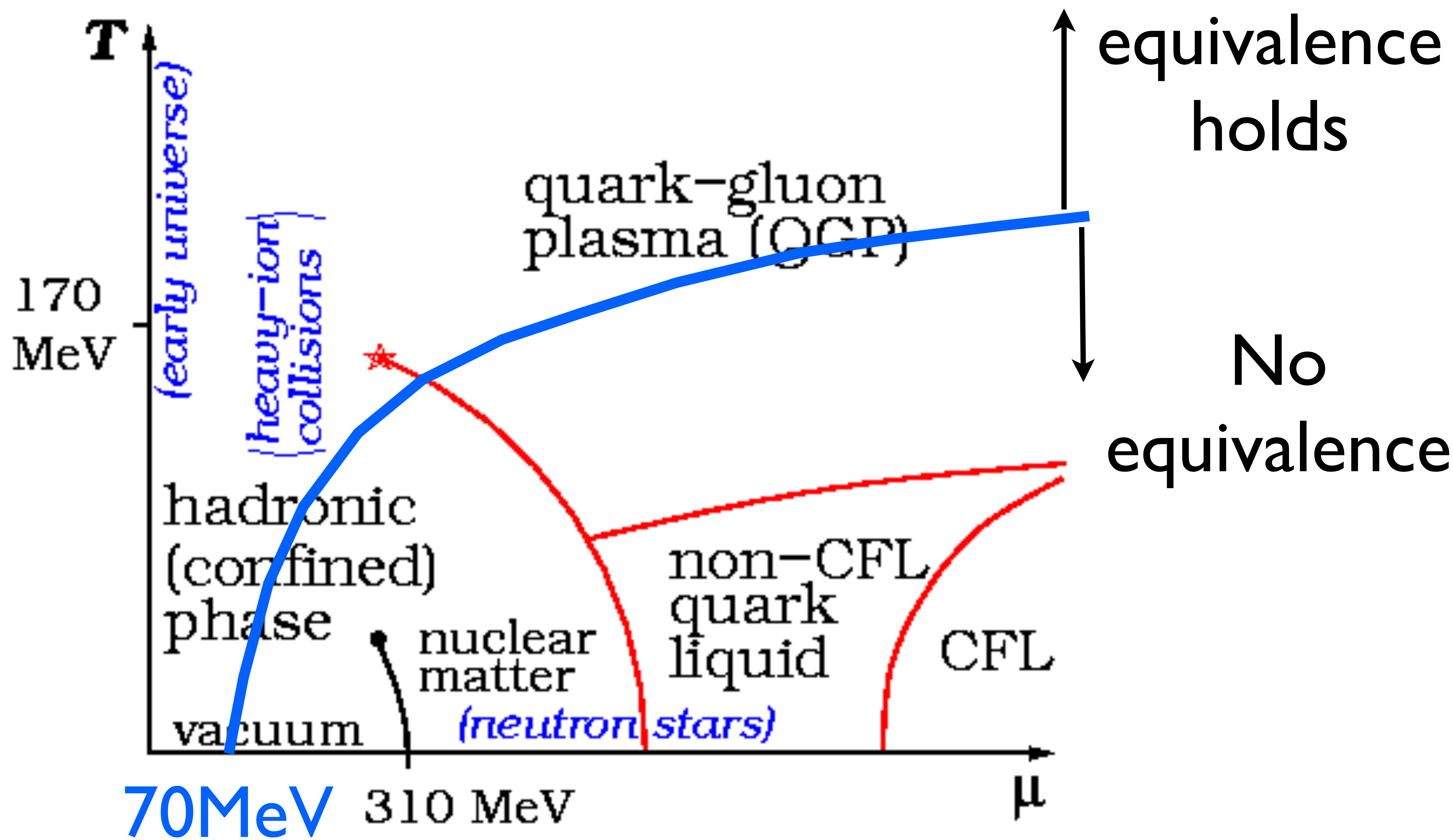

SO(2N_c) with μ_B
Z_2
Z_2
SU(N_c) with μ_B
SU(N_c) with μ_I
QCD_B
QCD_I

- The equivalence is gone once the projection symmetry breaks down spontaneously. (Kovtun-Unsal-Yaffe 2003)

- Pion condensation kills the projection symmetry. Hence there is no equivalence in the pion condensation region of QCD_I.

- The equivalence holds in high-T, small-μ region

Useful for heavy ison collision experiments!


T
170
MeV
(early universe)
(heavy-ion collisions)
quark-gluon
plasma (QGP)
hadronic
(confined)
phase
nuclear
matter
non-CFL
quark
liquid
CFL
vacuum
(neutron stars)
70MeV
310 MeV
μ
equivalence
holds
No
equivalence

- It can also be applied to various models, like NJL, holographic models, and chiral RMM.

large-Nc $\doteq$ mean field

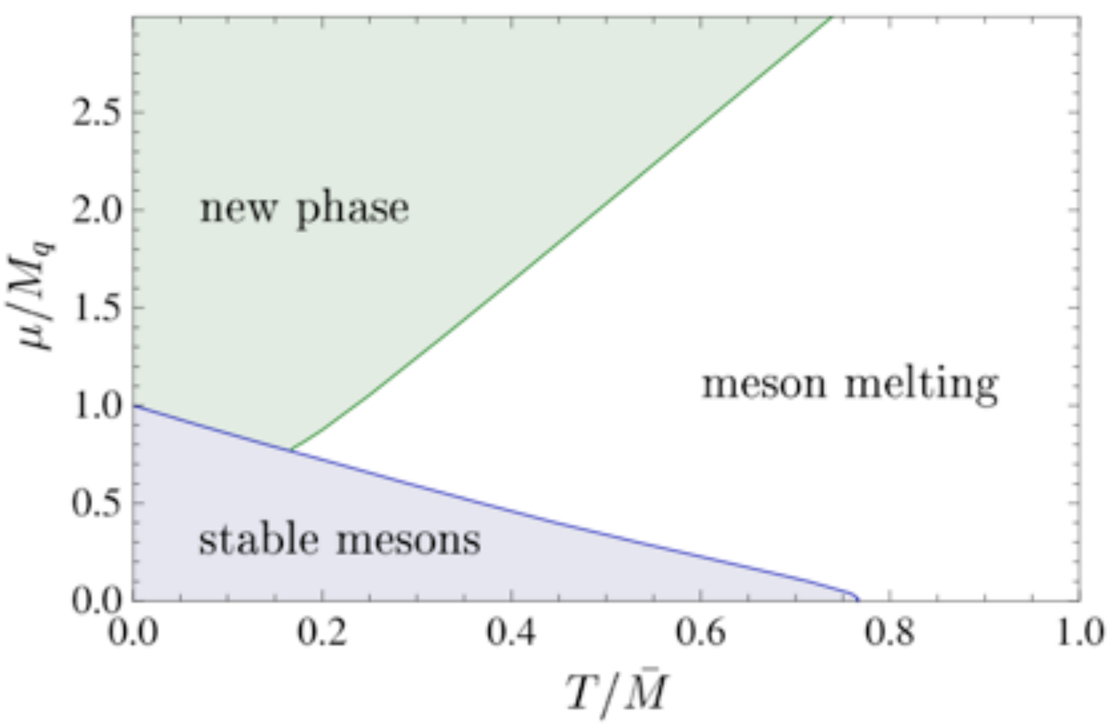
(M.H.-Yamamoto 2011,
M.H.-Hoyos-Karch-Yaffe 2012,
M.H.-Matsuo-Yamamoto 2012)

→ The equivalence in the mean field approximation (M.H.-Matsuo-Yamamoto 2012)

It has been known empirically.
(e.g. Kogut-Toublan 2003)

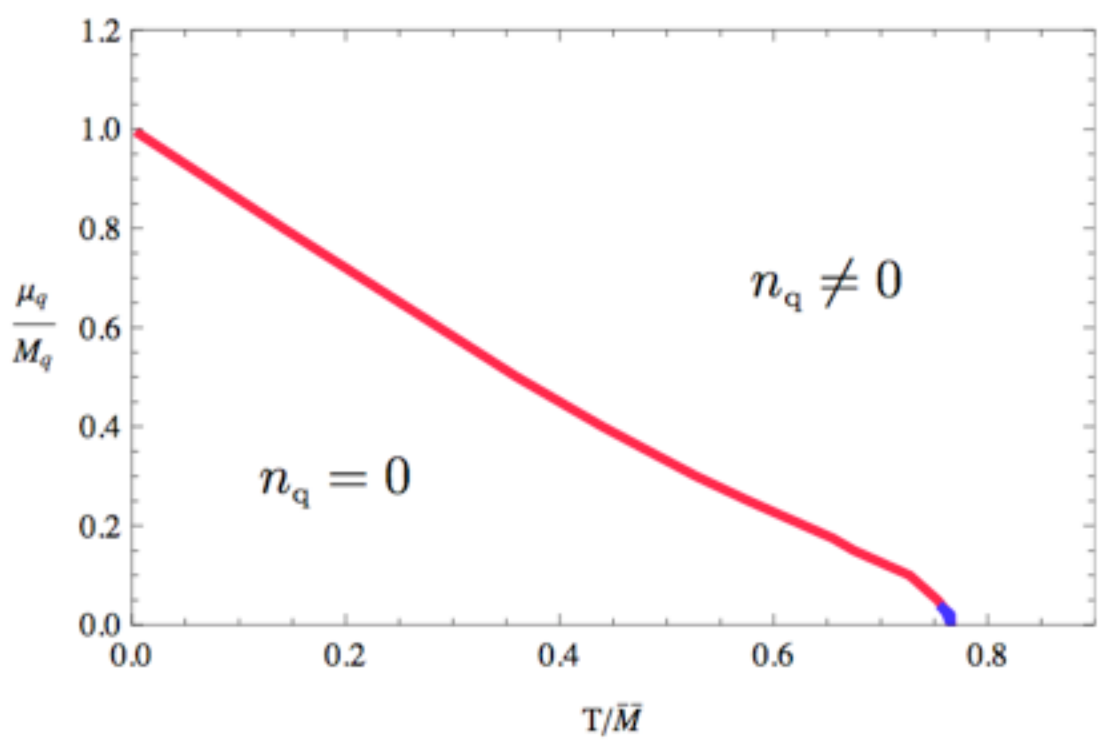
A solvable example: holographic model (D3/D7 system)

(M.H.-Hoyos-Karch-Yaffe 2012)



Ammon-Erdmenger-Kaminski-Kerner, **0903.1864** [hep-th]

(isospin chemical potential)



Mateos-Matsuura-Myers-Thomson, **0709.1225** [hep-th]

(baryon chemical potential)

So what is wrong with Armoni-Patella?

List of '***new***' results of Armoni-Patella

(according their paper and private communications)

- THEY are the first to give a field theory treatment! (Already given in [Cohen],[Toublan], [Cherman-M.H.-Robles],[M.H.-Yamamoto], and reviewed in [M.H.-Hoyos-Karch-Yaffe].)
- THEY are the first to consider the 1/N correction! (Already explained in [Cohen],[Toublan], [M.H.-Yamamoto],[M.H.-Hoyos-Karch-Yaffe].)
- THEY are the first to argue the nonperturbative equivalence, *although they don't even use the word 'nonperturbative' in their paper!* (Their 'proof' is nothing more than a _wrong_ rewritement of Toublan's perturbative argument. Evidence for the nonperturbative equivalence had been found in [Cherman-Tiburzi],[M.H.-Yamamoto],[M.H.-Hoyos-Karch-Yaffe].)
- THEY found the condition for the large-N equivalence, *although that cannot follow from their argument IN PRINCIPLE, even with their wrong assumption!* So it is very curious -- how could they find it??? (The condition had been found in [Cherman-M.H.-Robles] and discussed in detail in [Cherman-Tiburzi],[M.H.-Yamamoto],[M.H.-Hoyos-Karch-Yaffe], with nonperturbative considerations.)

All 'their' statements are taken from previous works.

Physicswise, why are they wrong?

"The main disadvantage of this approach is that it is perturbative in 1/N and hence the expansion is around the Yang-Mills vacuum (without quarks). As a result the discussion will be restricted to the phases of the theory with small μ where there is no breaking of baryon (or isospin) number."

(From Armoni-Patella)

They say fermion condensate does not appear at large-N... :o

(It's wrong, of course. Chiral symmetry cannot break otherwise.
Actually they carefully avoid the word "chiral symmetry" ... :o)

With this wrong assumption, they claim the quench approximation is exact at large-N. But quenched QCD is nonperturbatively different from real QCD. The equivalence in the quenched QCD is trivial and known for 20 years or so. And the equivalence holds everywhere in this wrong setup, as opposed to their claim. (see e.g. Stehpahov, hep-lat/9604003)

Trivial equivalence between wrong theories.

(※ Now they agree they just reproduced this trivial equivalence.
But they say it's nontrivial and new, and refuse to cite Stephanov's paper :o)

All the papers are in the arxiv.
Please read, compare and let us know
if you can find anything original
from Armoni-Patella.

We hope to see their counterargument
in the arxiv,
in case they don't agree with our criticism
expressed in 1205.1030[hep-lat].

Summary

- sign problem/overlapping problem can be avoided by using a string-inspired technique!
- *NO OVERLAPPING PROBLEM IN THE PHASE REWEIGHTING METHOD .*
- Various models（Holographic model, RMT, NJL,..)

  exhibit the large-Nc equivalence.
- Also they have the equivalence in the mean-field approximation.
- SU(3) looks rather large-Nc.

Backup slides

$$\langle \mathcal{O}_1^{(p)} \mathcal{O}_2^{(p)} \cdots \rangle_p = \langle \mathcal{O}_1^{(d)} \mathcal{O}_2^{(d)} \cdots \rangle_d$$

↑ parent (SO) — operators invariant under the projection symmetry

↑ daughter (SU) — operators made of projected fields

<u>SO → SU + μ_B</u>

- planar & at most one-fermion-loop → agree
- nonplanar and/or more than one-fermion loop → disagree

Equivalent in the 't Hooft large-N_c limit ($N_f/N_c \to 0$)

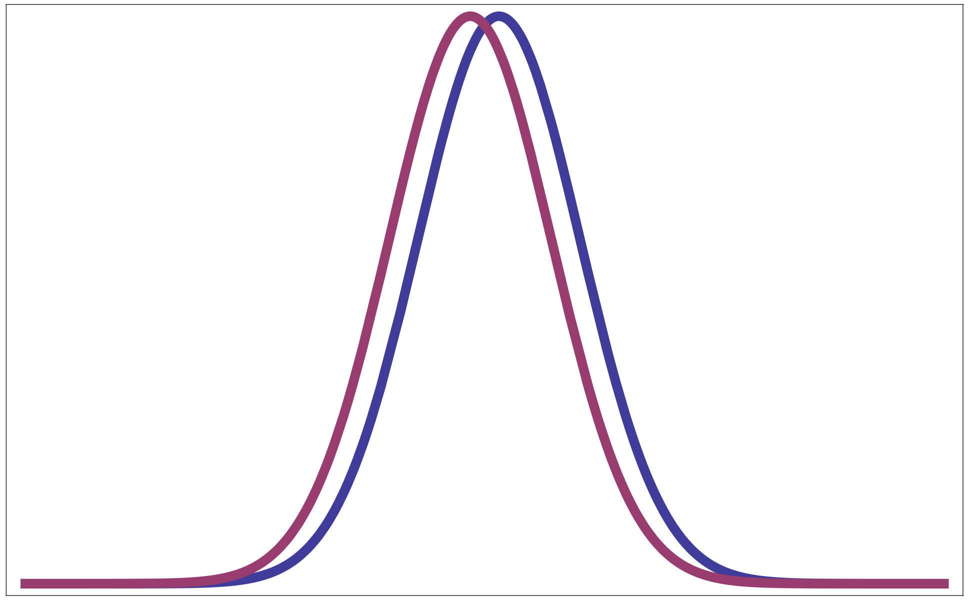
<u>SO → SU + μ_I</u>

Equivalent also at $N_f/N_c > 0$

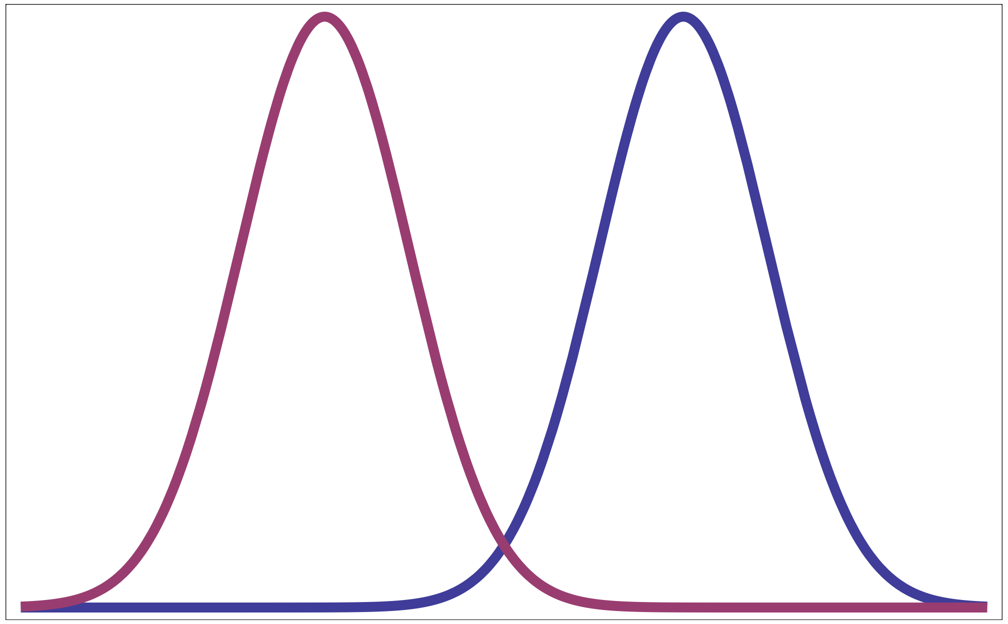
Reweighting method

$$\langle \mathcal{O} \rangle_{full\ theory} = \frac{\langle \mathcal{O} \cdot phase \rangle_{phase\ quench}}{\langle phase \rangle_{phase\ quench}}$$

- R.H.S. is calculable *in principle*
- Difficult in practice -- often <phase> becomes very small, so that the R.H.S. is essentially 0/0.
- "overlapping problem" appears in general.



no overlapping
problem



severe
overlapping problem