# QCD thermodynamics with continuum extrapolated Wilson fermions

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in collaboration with

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## Motivation

Let's do continuum QCD thermodynamics with a fermion formulation which is known to be correct!

- Rooting procedure not fully understood $\longrightarrow$ Steve Sharp Lattice 2006
- Domain wall fermions $\longrightarrow$ expensive
- Overlap fermions $\longrightarrow$ even more so (but: Stefan Krieg today 3:50)
- Wilson fermions $\longrightarrow$ theoretically sound, correct and straightforward continuum limit (even in practice)

## Motivation 2

Let’s check the staggered thermodynamics results!

- Even though ugly, probably correct
- Successful check for some quantities lends support for unchecked other quantities
- Check only meaningful in continuum limit for fully renormalized finite quantities
- For the check only, physical quark masses not essential (heavier quarks also universal)

## Outline

- Staggered strategy vs Wilson strategy for thermodynamics
- Renormalization of $\bar{\psi}\psi$, $\chi_s$ and $L$
- Action and simulation parameters
- Results and comparison with staggered results in continuum limit

| Staggered | | Wilson |
| --- | --- | --- |
| change $\beta$ ($N_t$ fix) | change $T = 1/aN_t$ | change $N_t$ ($\beta$ fix) |
| continuous | steps in $T$ | finite steps |
| low temperature | large discr. errors | high temperature |
| many times | tuning masses (LCP) | only few times |
| $N_t \to \infty$ | continuum limit | $\beta \to \infty$ |
| yes | chiral symmetry | no |
| simple | renormalization | more difficult |

Our quantities

- quark number susceptibility: $\chi_s/T^2$ , finite in continuum limit (as in staggered)

- chiral condensate: $\bar{\psi}\psi$ , needs renormalization (more tricky than staggered)

- Polyakov loop: L , needs renormalization

## Renormalization of chiral condensate

### Additive

$\Delta_{\bar{\psi}\psi}(T) = \langle\bar{\psi}_0\psi_0\rangle(T) - \langle\bar{\psi}_0\psi_0\rangle(T=0)$ Cancellation $O(a^{-3})$

$\Delta_{PP}(T) = \int d^4x \langle P_0(x) P_0(0)\rangle(T) - \int d^4x \langle P_0(x) P_0(0)\rangle(T=0)$

$P_0(x)$: bare pseudo-scalar condensate, cancellation $O(a^{-2})$

From axial Ward identity: $2 m_{PCAC} Z_A \Delta_{PP}(T) = \Delta_{\bar{\psi}\psi}(T) + O(a)$

## Renormalization of chiral condensate

### Multiplicative

$$m_R\langle\bar{\psi}\psi\rangle_R(T) = m_{PCAC} Z_A \Delta_{\bar{\psi}\psi}(T)$$

Last year we didn't have $Z_A$ so used Ward identity and

$$m_R\langle\bar{\psi}\psi\rangle_R(T) = \frac{\Delta^2_{\bar{\psi}\psi}(T)}{2\Delta_{PP}(T)}$$

Now we have $Z_A$ (and also $m_{PCAC}$) so can use

$$m_R\langle\bar{\psi}\psi\rangle_R(T) = 2N_f m^2_{PCAC} Z^2_A \Delta_{PP}(T)$$

Best scaling among 3 choices!

## Renormalization of chiral condensate, measurement of $Z_A$

$Z_A$ is finite and $Z_A \to 1$ in the continuum limit

- $Z_A$ is defined in the chiral limit
- $N_f = 3$ simulations at four quark masses, $m_s/3 < m_q < m_s$
- fixed volume $V \sim (2\,fm)^4$
- RI-MOM: compute $Z_V$ then $Z_A = Z_V \, \Gamma_V(p)/\Gamma_A(p)$
- dependence on $p$ very small (systematic error)
- extrapolate to $m_q \to 0$ (very smooth)

| $\beta$ | 3.30 | 3.57 | 3.70 | 3.85 |
|---|---|---|---|---|
| $Z_A$ | 0.892(7) | 0.951(2) | 0.966(2) | 0.976(5) |

## Renormalization of Polyakov loop

Additive divergence in free energy

Get rid of it by the scheme $L_R(T_0) = L_*$, for some $T_0 > T_c$

For Wilson: $L_R(T) = \left(\frac{L_*}{L_0(T_0)}\right)^{T_0/T} L_0(T)$ at each $\beta$

For staggered a bit more tricky: first usual renormalization via static potential, then finite scheme change to above scheme

## Action and simulation parameters

$2+1$ flavor, tree level Symanzik gauge action, 6 steps stout and tree level clover improved fermion action

$m_\pi/m_\Omega = 0.326(4)$, $m_K/m_\Omega = 0.366(4)$ for both staggered and Wilson

Quark mass ratios $(2m_K^2 - m_\pi^2)/m_\pi^2 = 1.530(7)$ are tuned very precisely, $m_s$ physical

$m_\Omega = 1672\,MeV$ sets scale $\rightarrow m_\pi \approx 545\,MeV$, $m_K \approx 612\,MeV$

Large volumes $m_\pi L \gtrsim 8$

4 lattice spacings:

| $\beta$ | 3.30 | 3.57 | 3.70 | 3.85 |
|---|---|---|---|---|
| a [fm] | 0.139(1) | 0.093(1) | 0.070(1) | 0.057(1) |
| V | $32^3 \times 6 - 16$ | $32^3 \times 8 - 16$ | $48^3 \times 8 - 28$ | $64^3 \times 12 - 28$ |

## Continuum limit

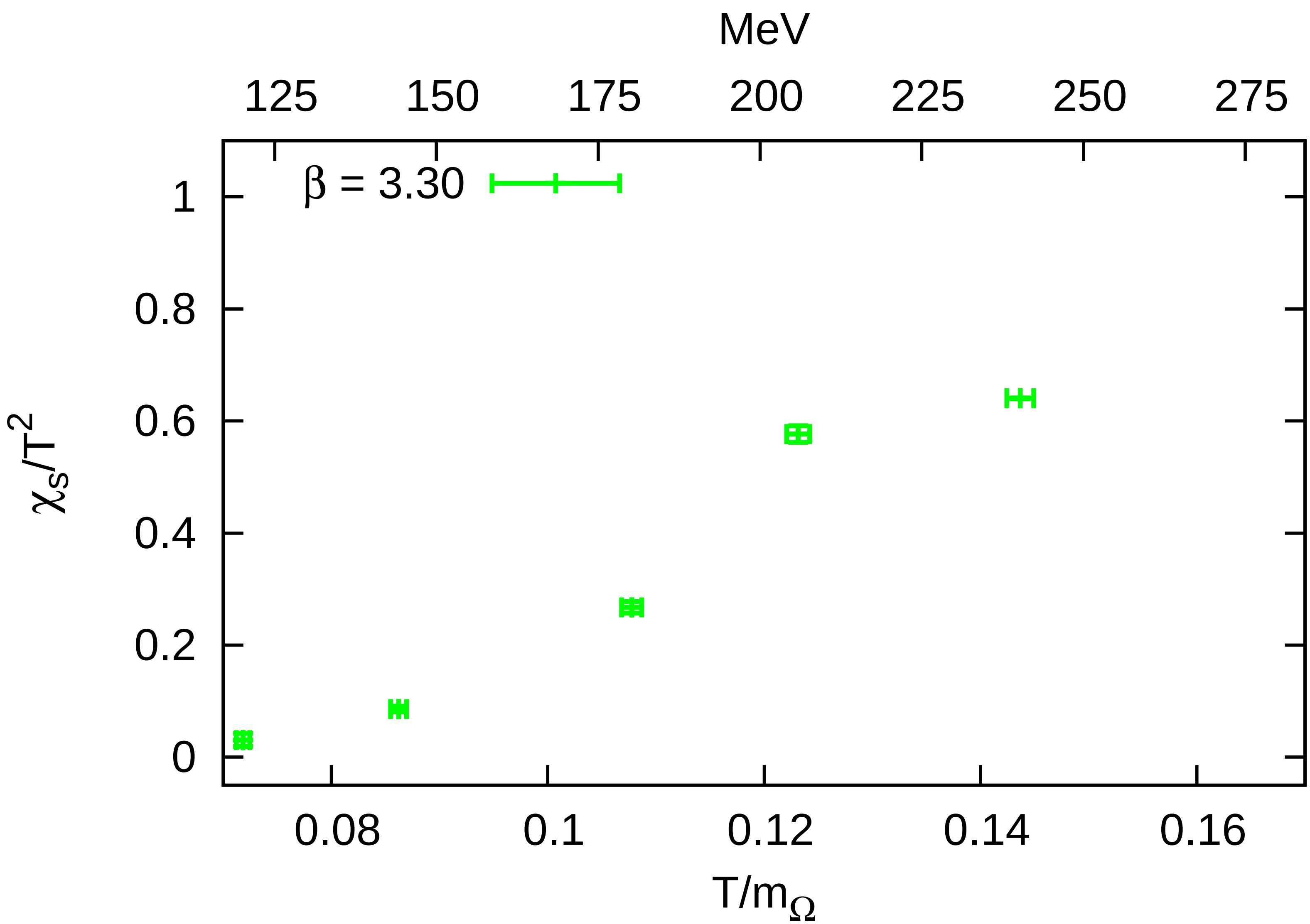
Wilson simulation: discrete $N_t \longrightarrow$ discrete $T$

Cubic spline interpolation in $T$

Continuum extrapolation of cubic spline coefficients using $O(a^2)$ and $O(a\alpha)$

Always have at least 3 lattice spacings

## Results, $\chi_s/T^2$ Wilson




Trick for disconnected part used from Ejiri et al. arXiv:0909.2121

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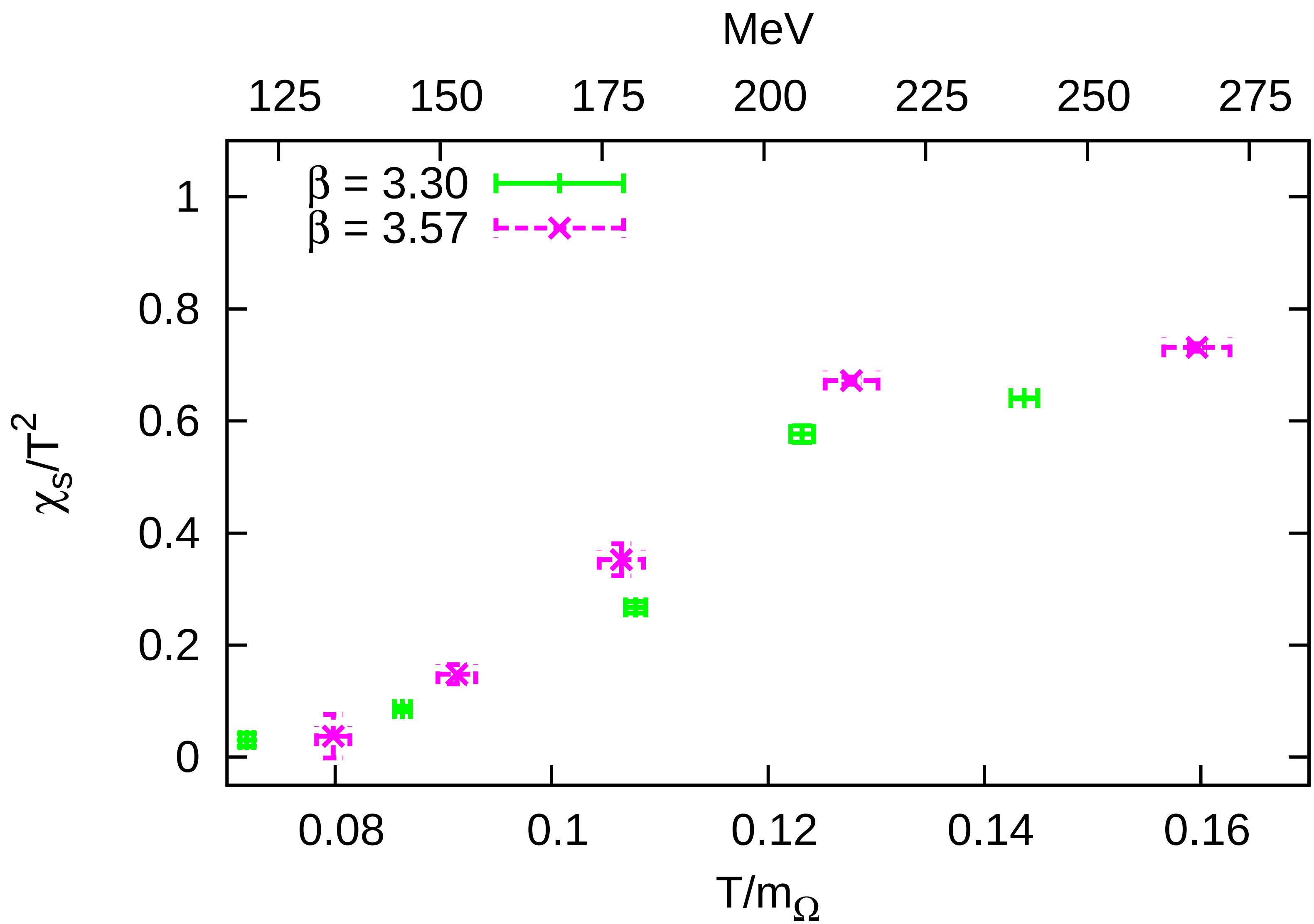


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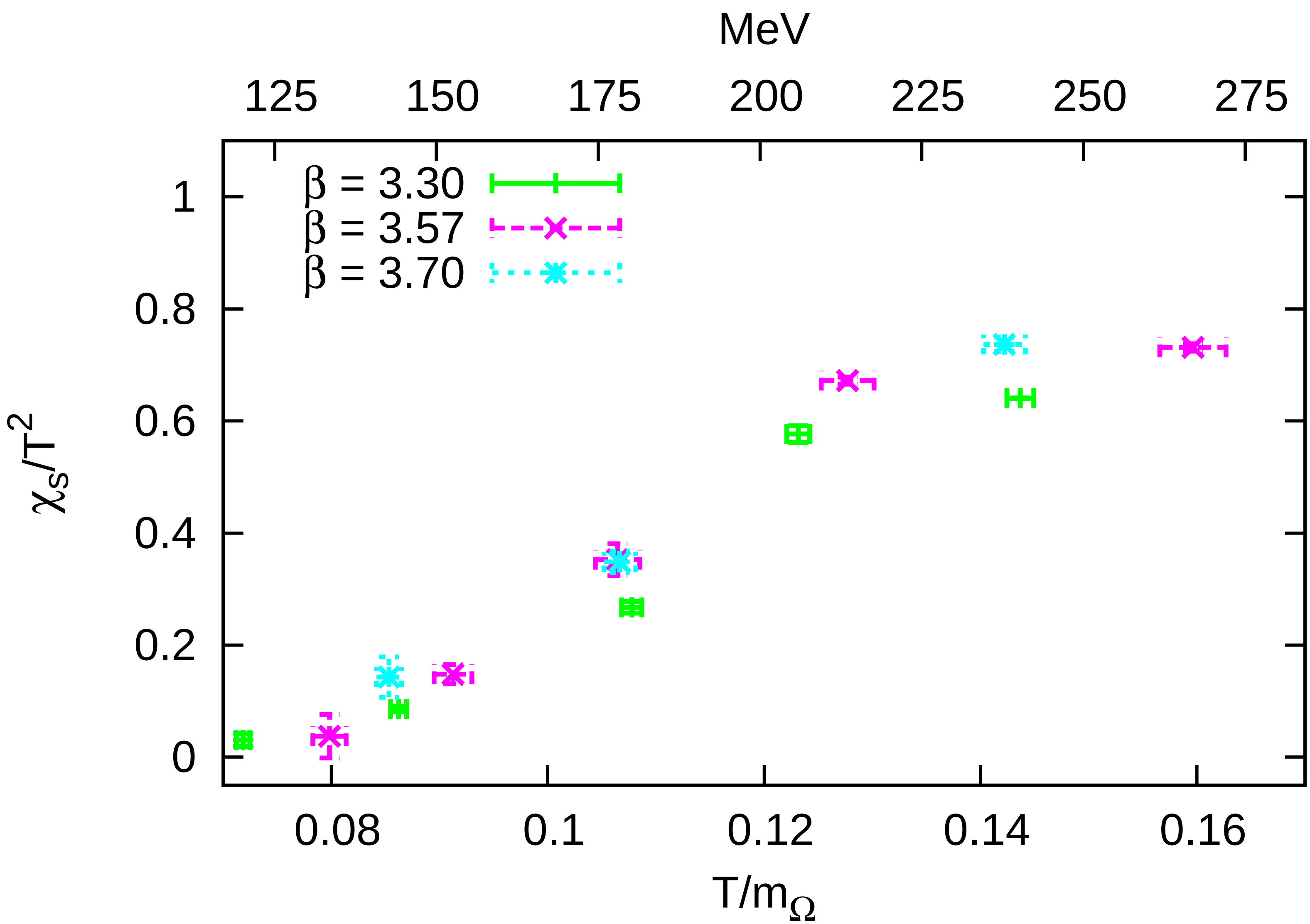


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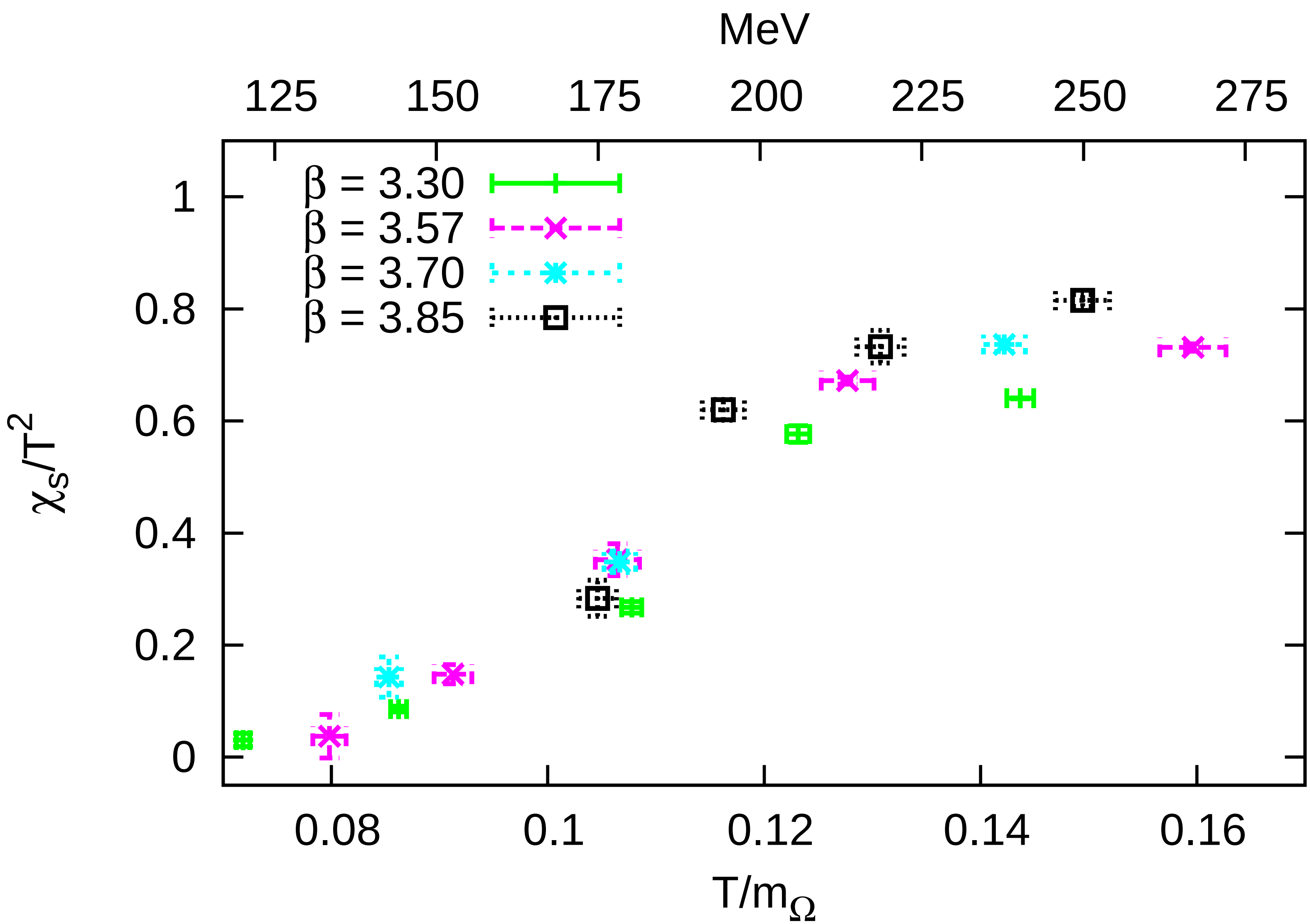


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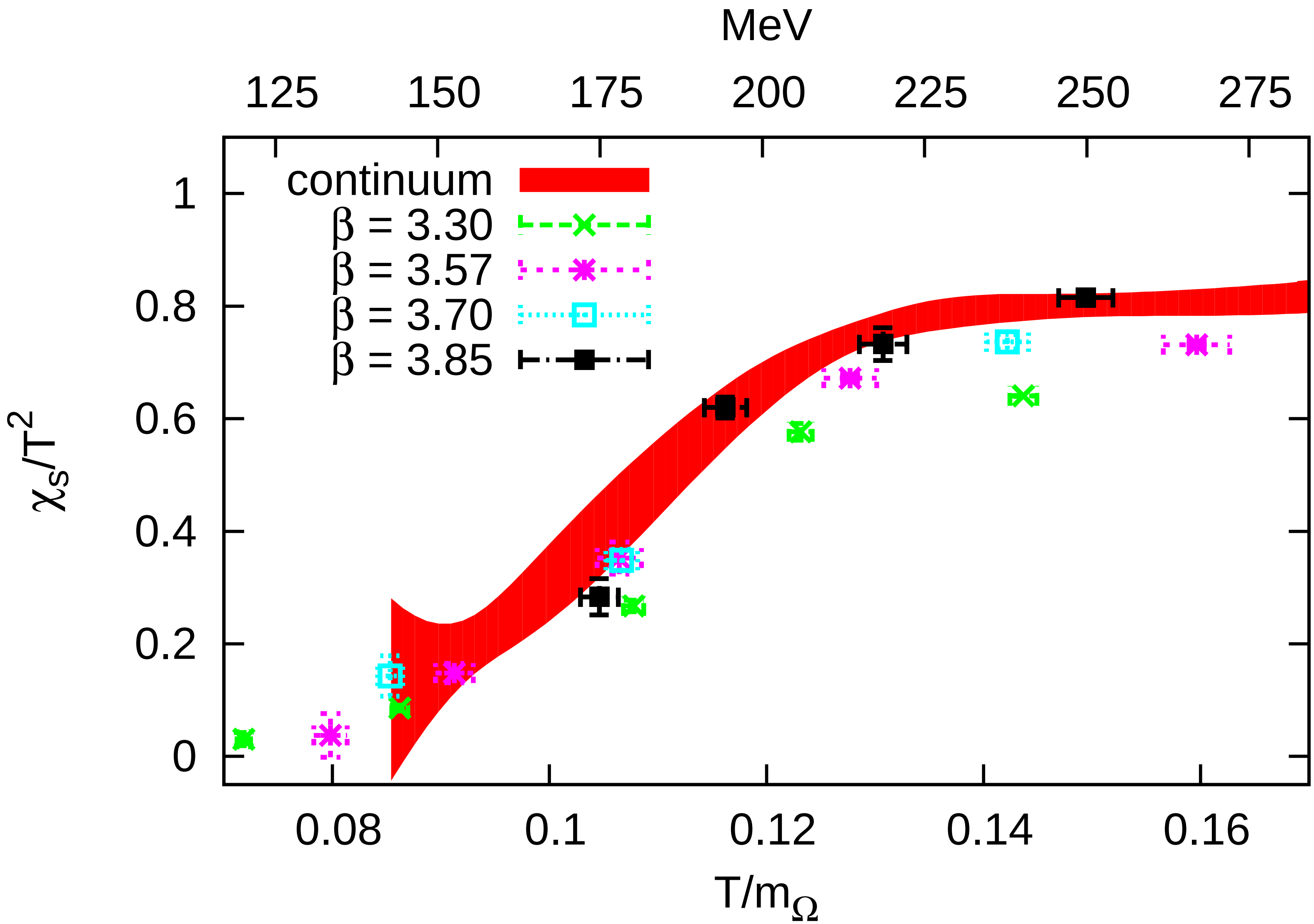


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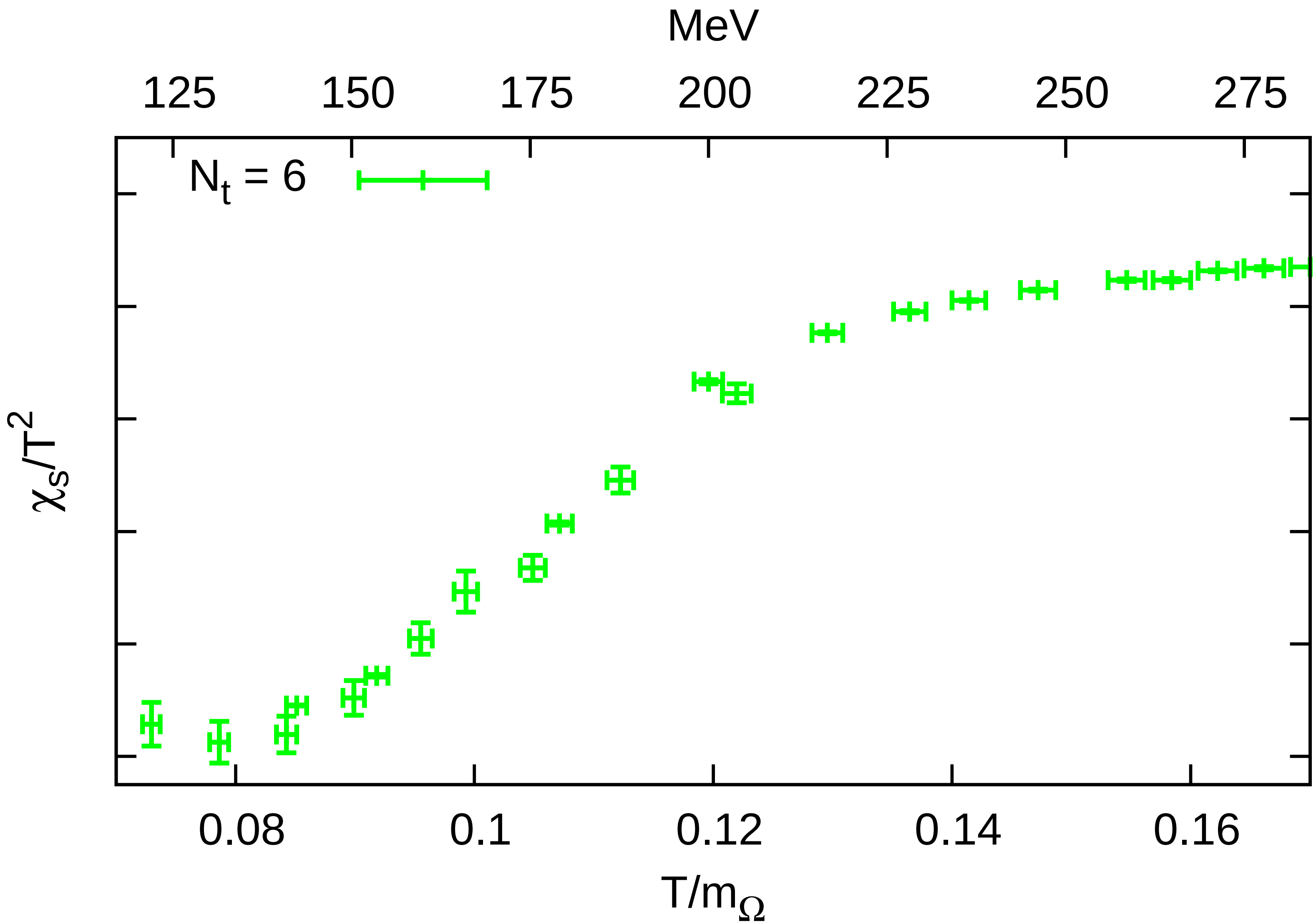
Results, $\chi_s/T^2$ staggered



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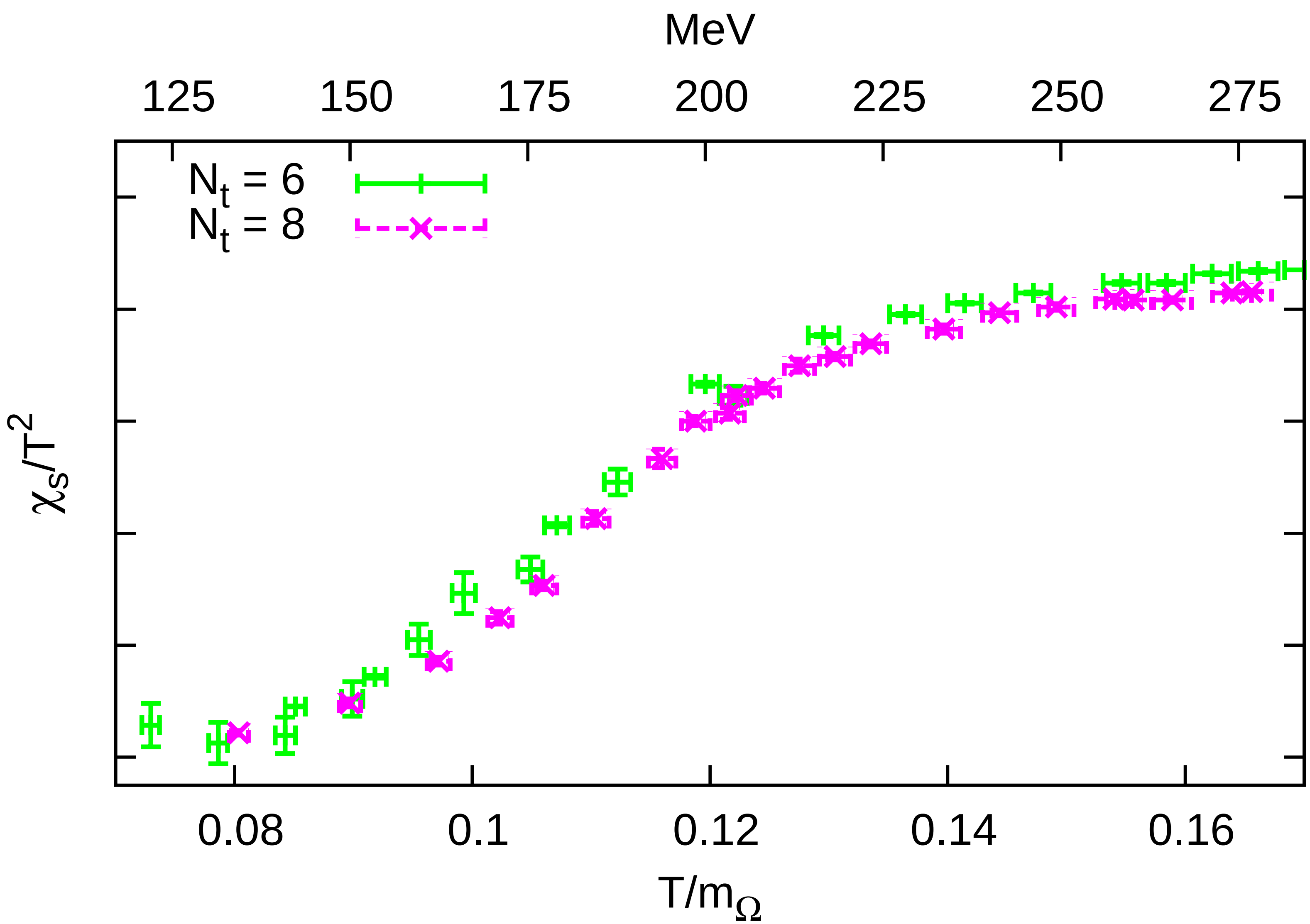

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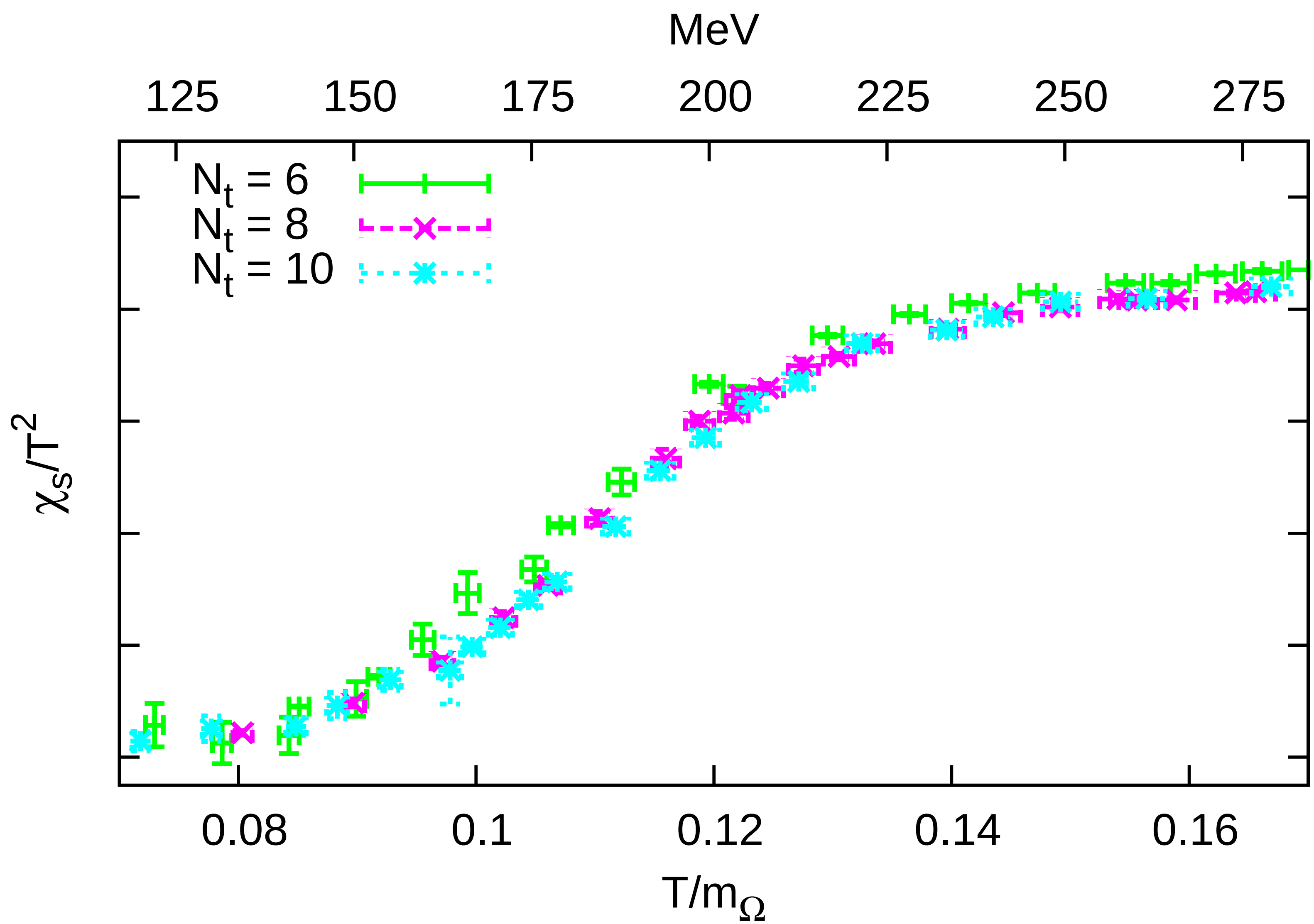

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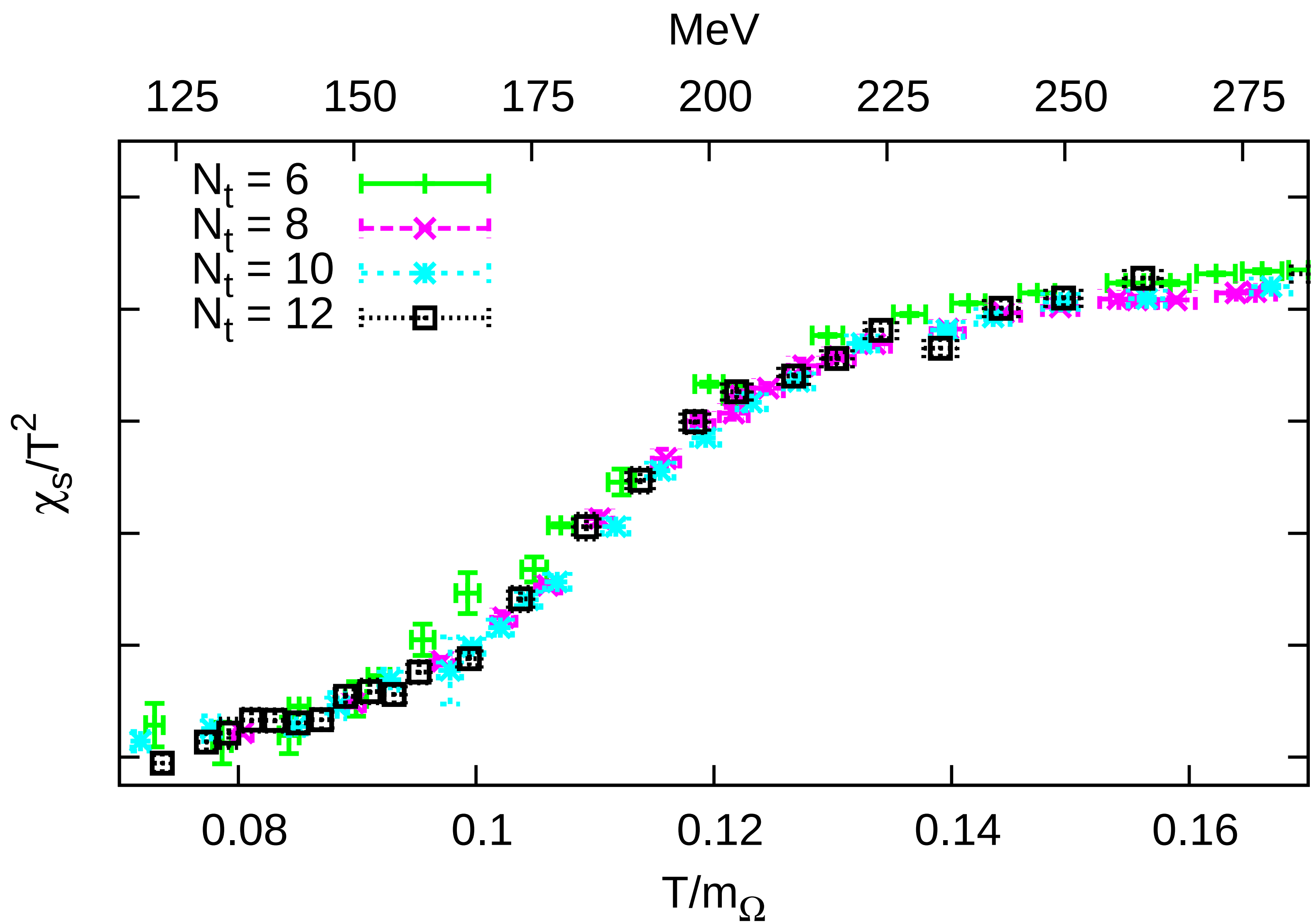

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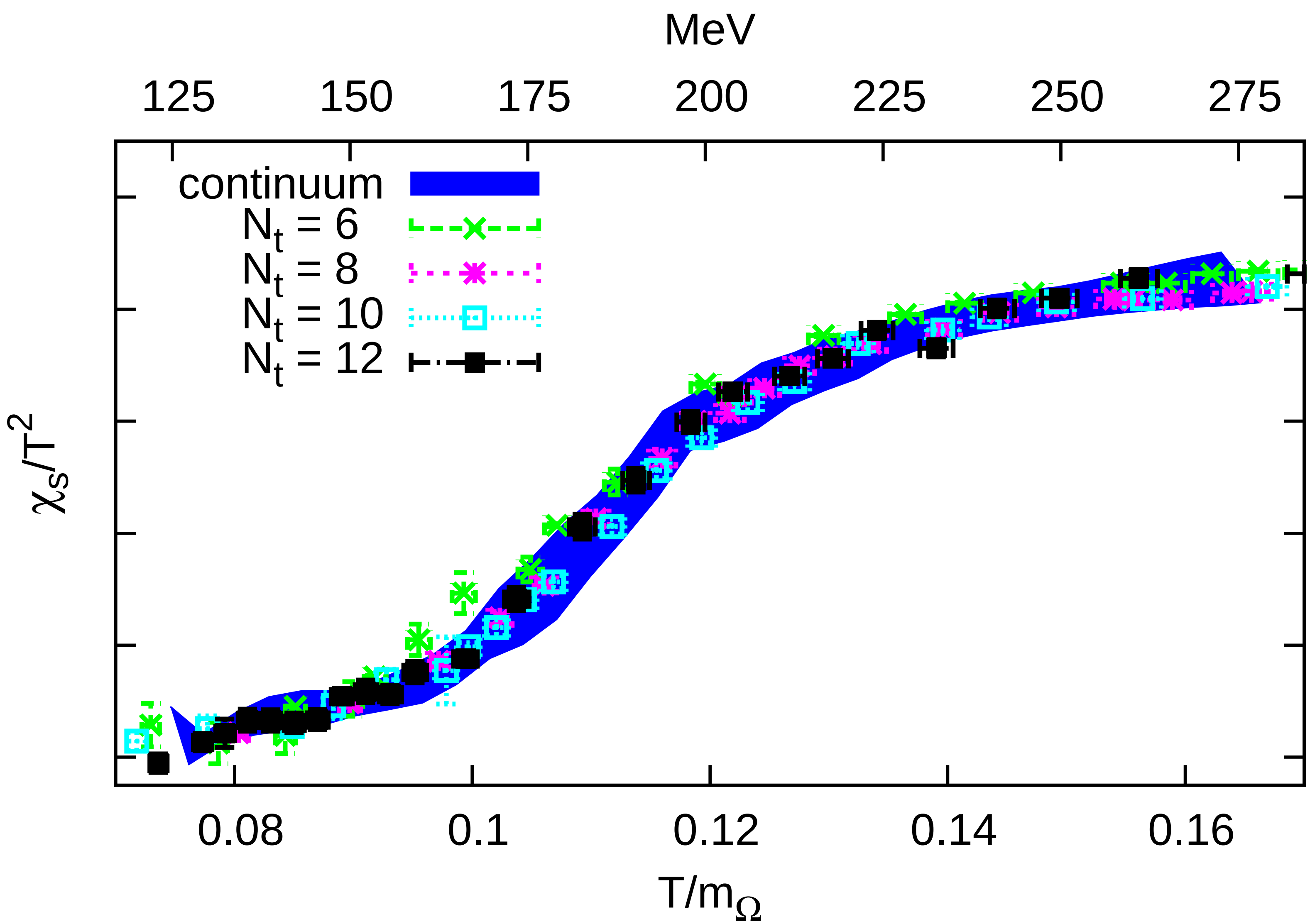
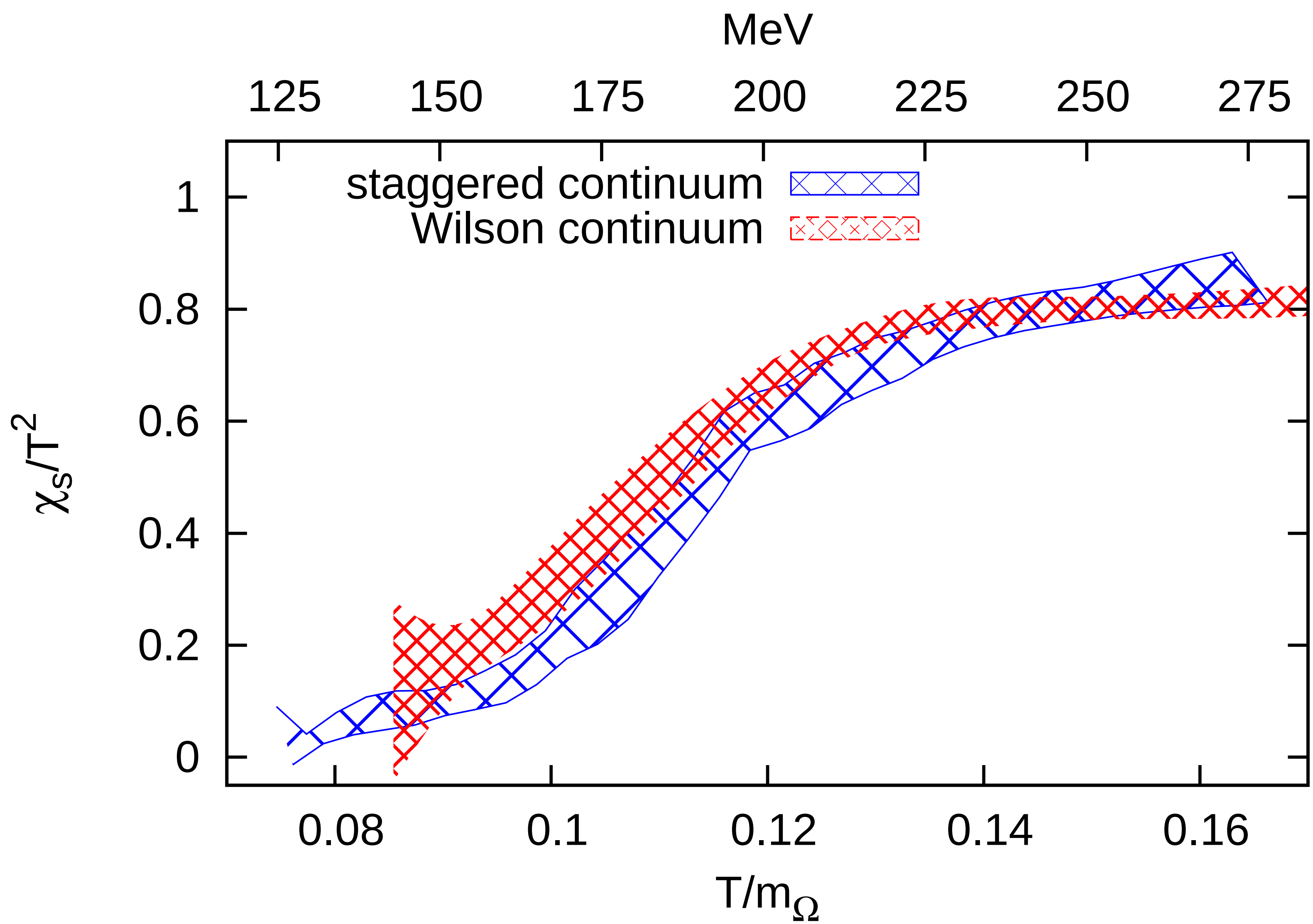
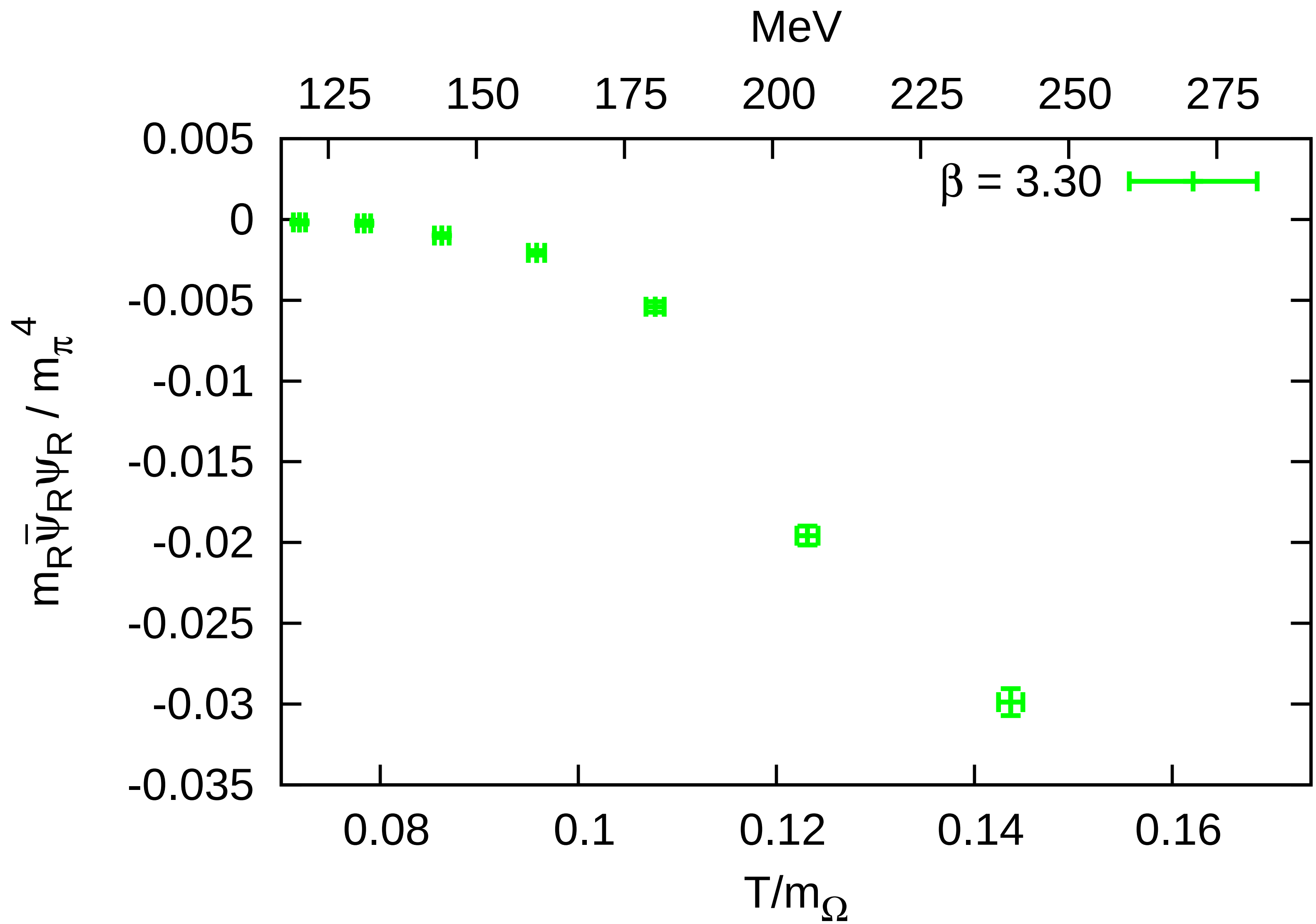


# Results, $\chi_s/T^2$ Wilson vs. staggered

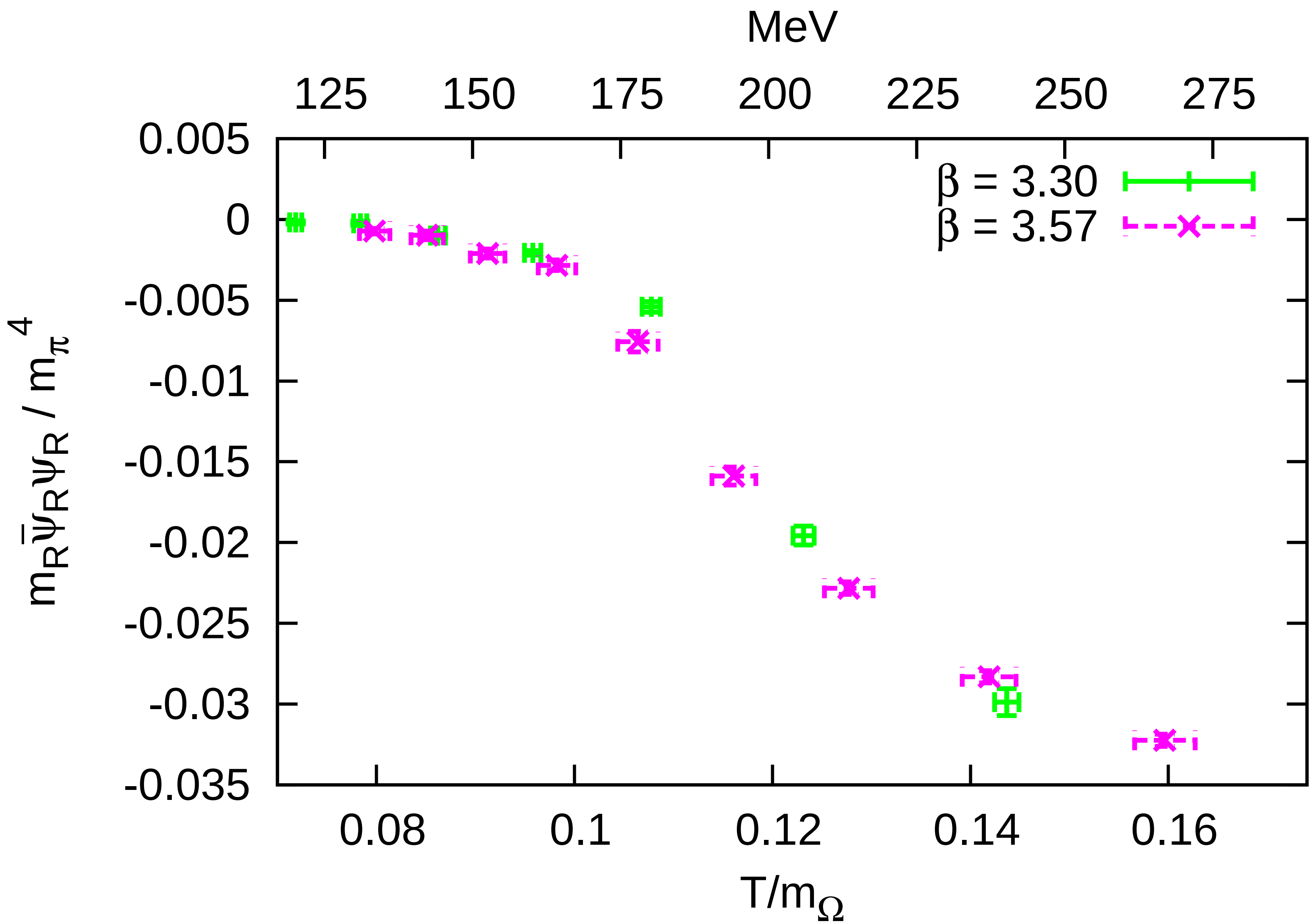


## Results, $\bar{\psi}\psi$ Wilson




$$m_R\bar{\psi}_R\psi_R(T) = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0))$$

## Results, $\bar{\psi}\psi$ Wilson

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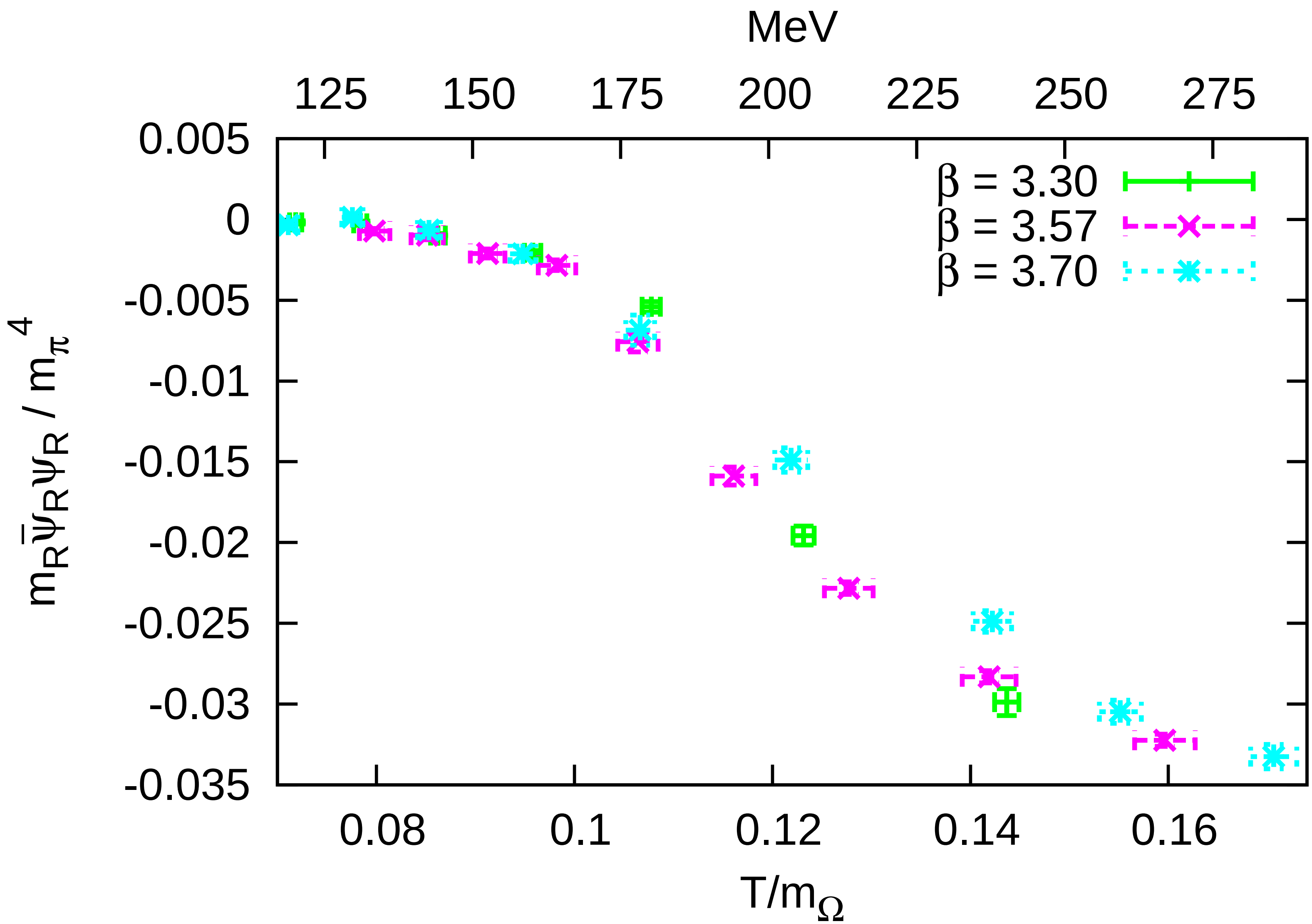


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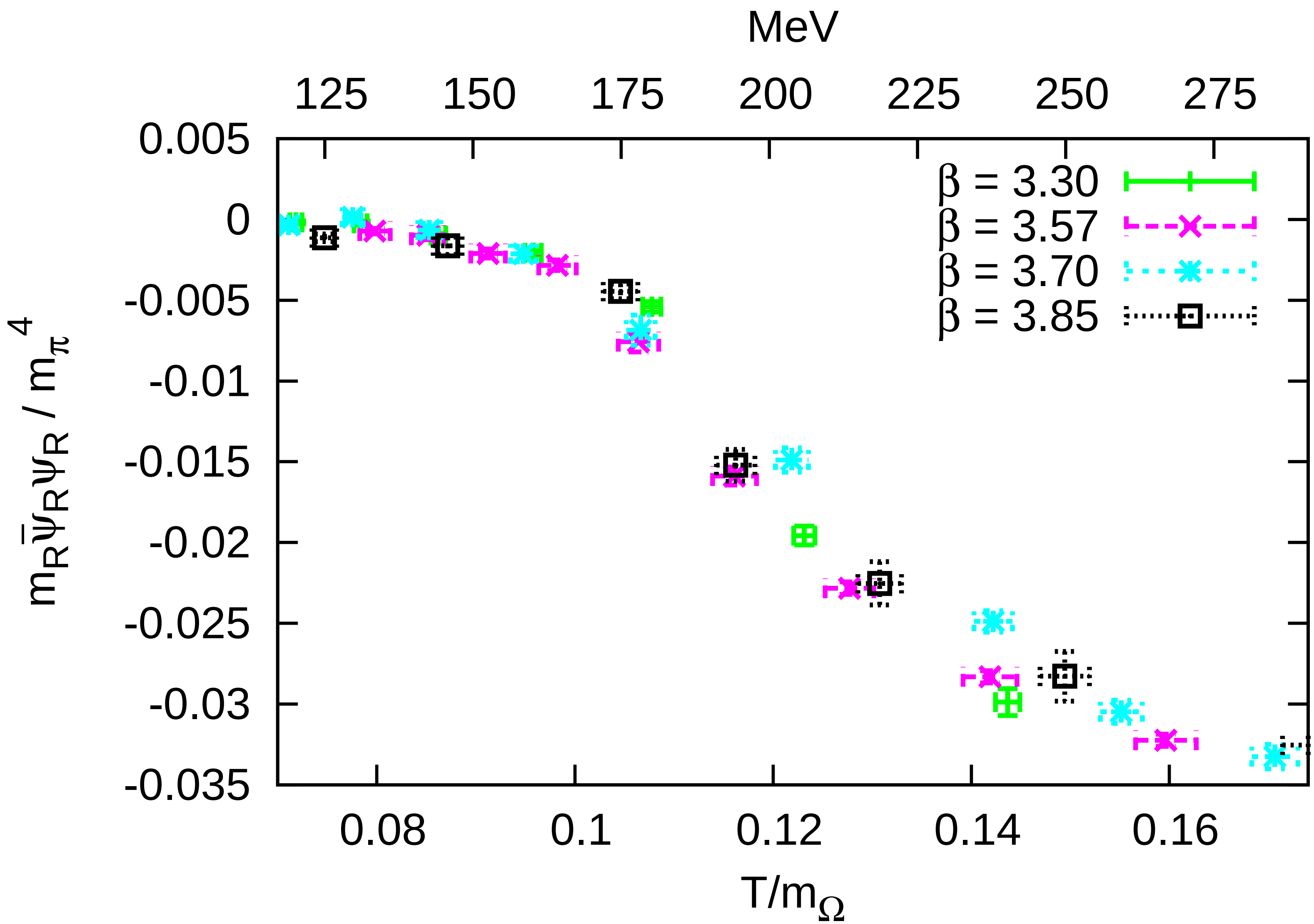


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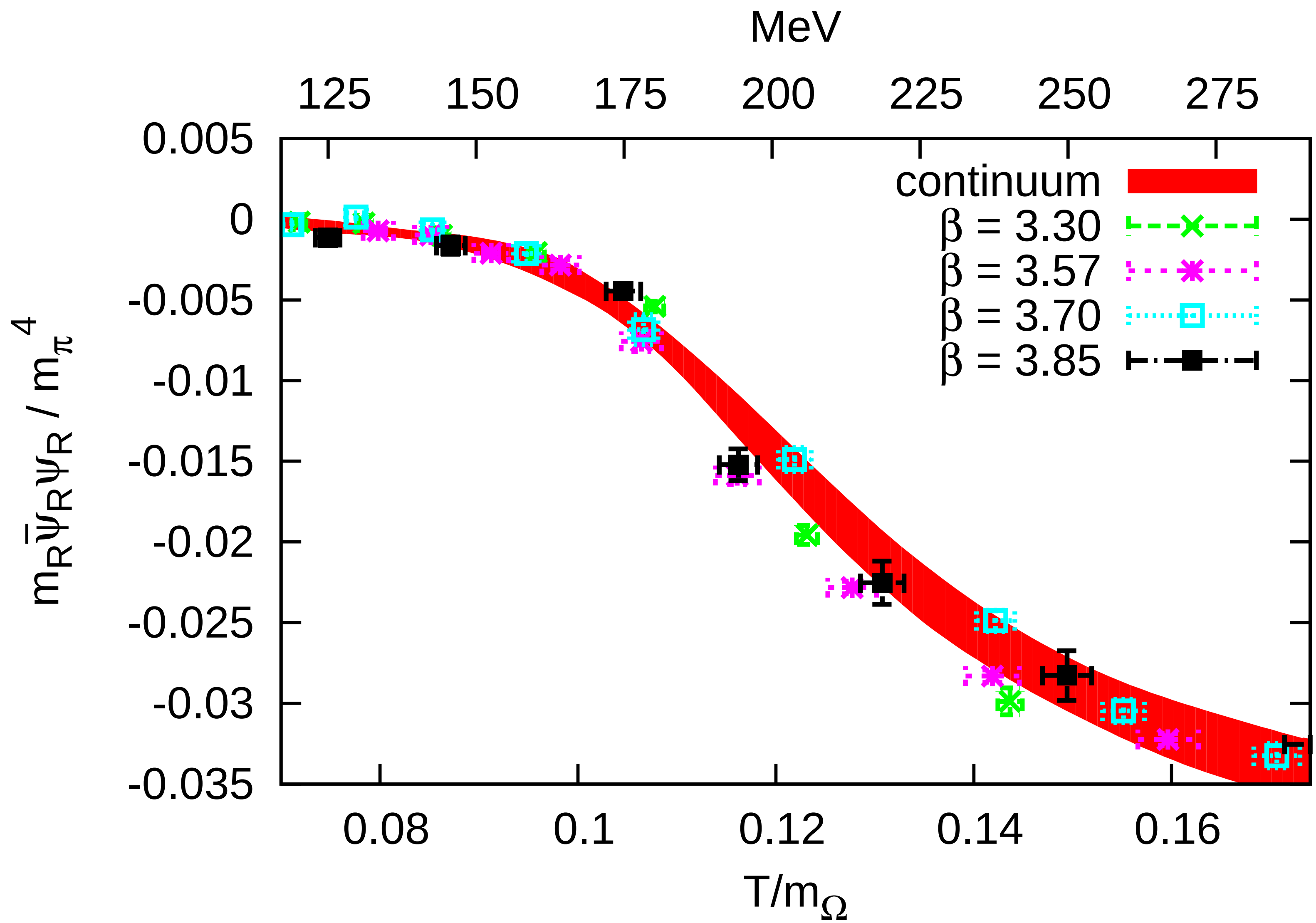
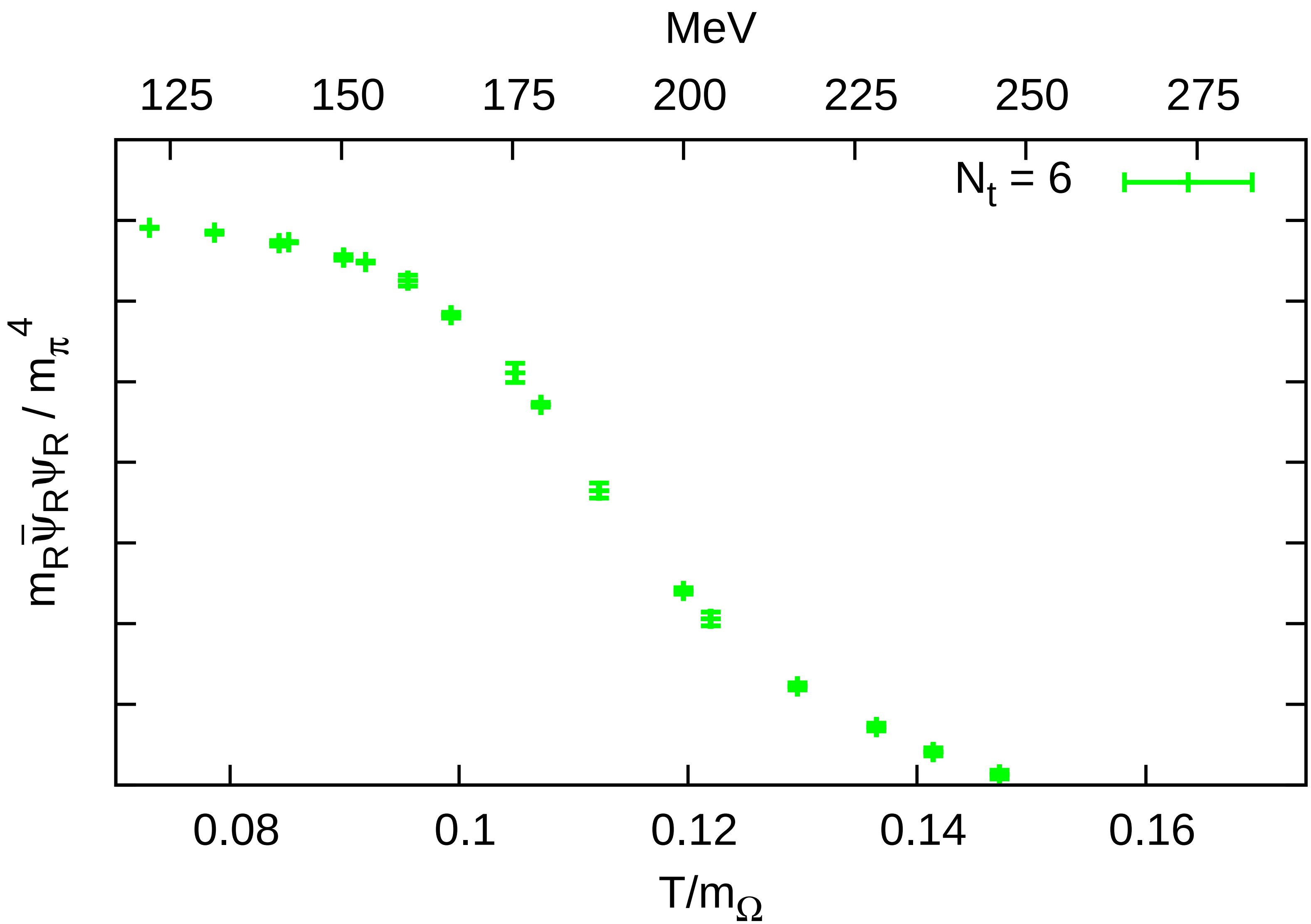



$$m_R\bar{\psi}_R\psi_R(T) = 2N_f m_{PCAC}^2 Z_A^2 (PP(T) - PP(0))$$

Results, $\bar{\psi}\psi$ staggered



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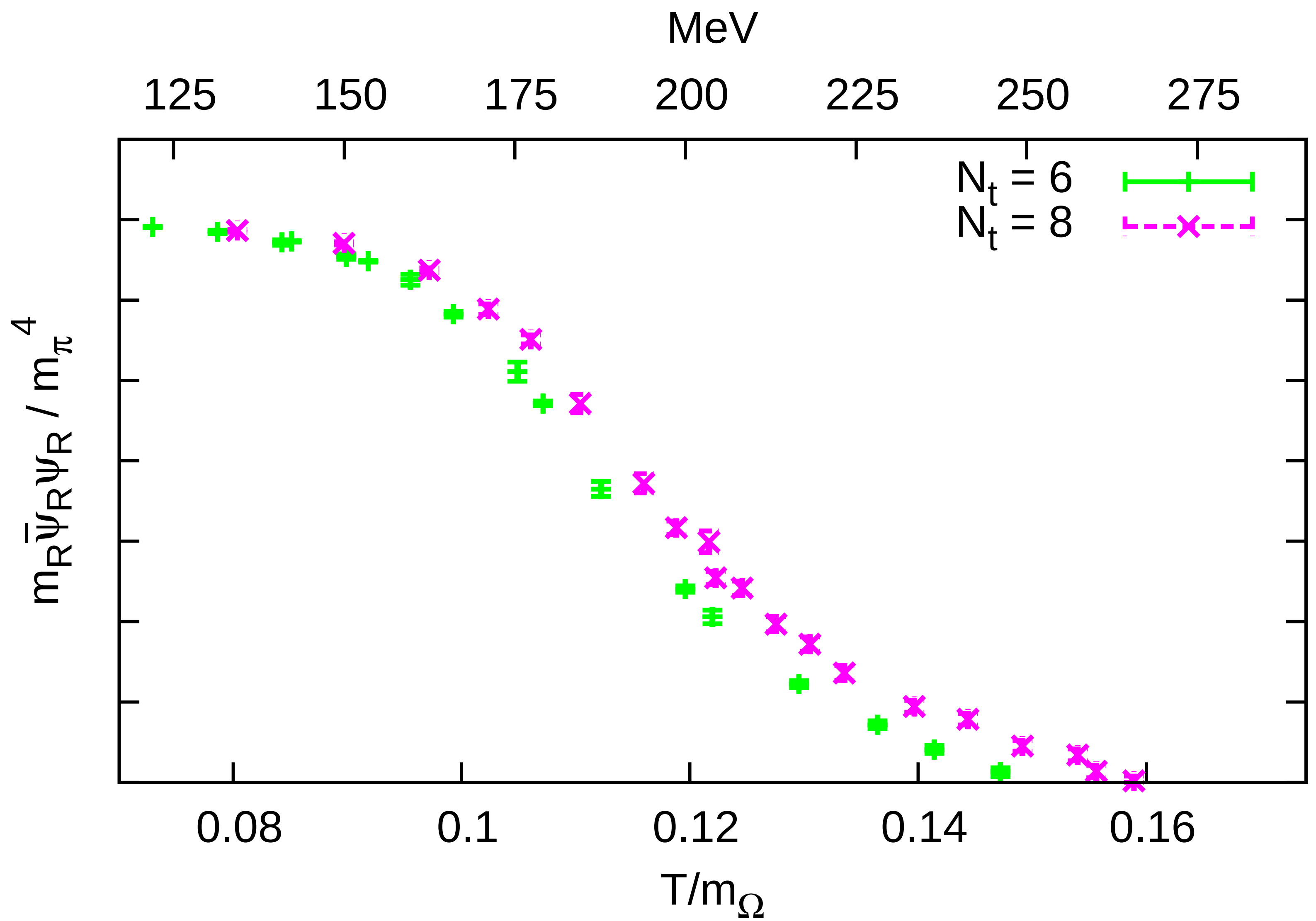

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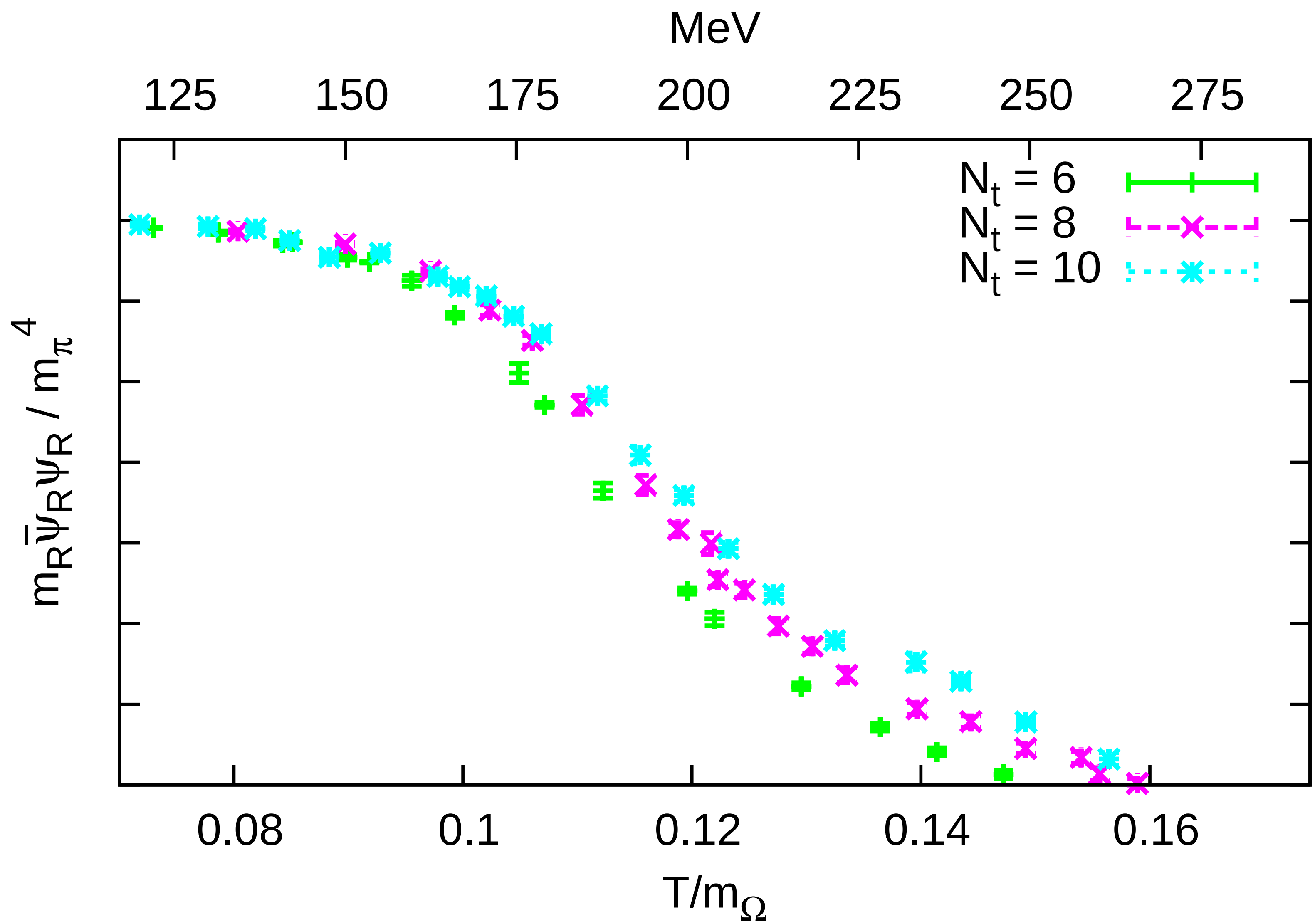

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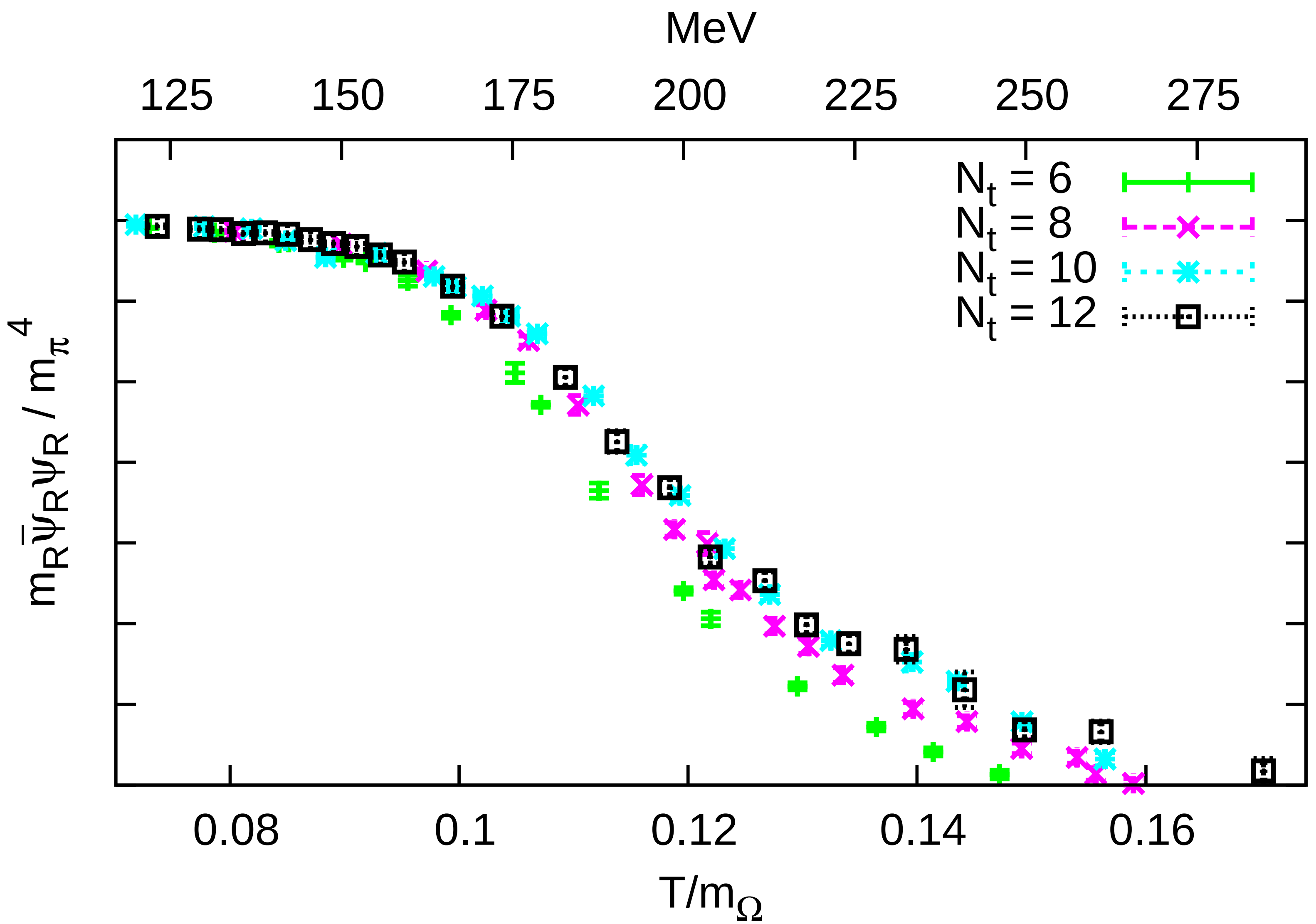

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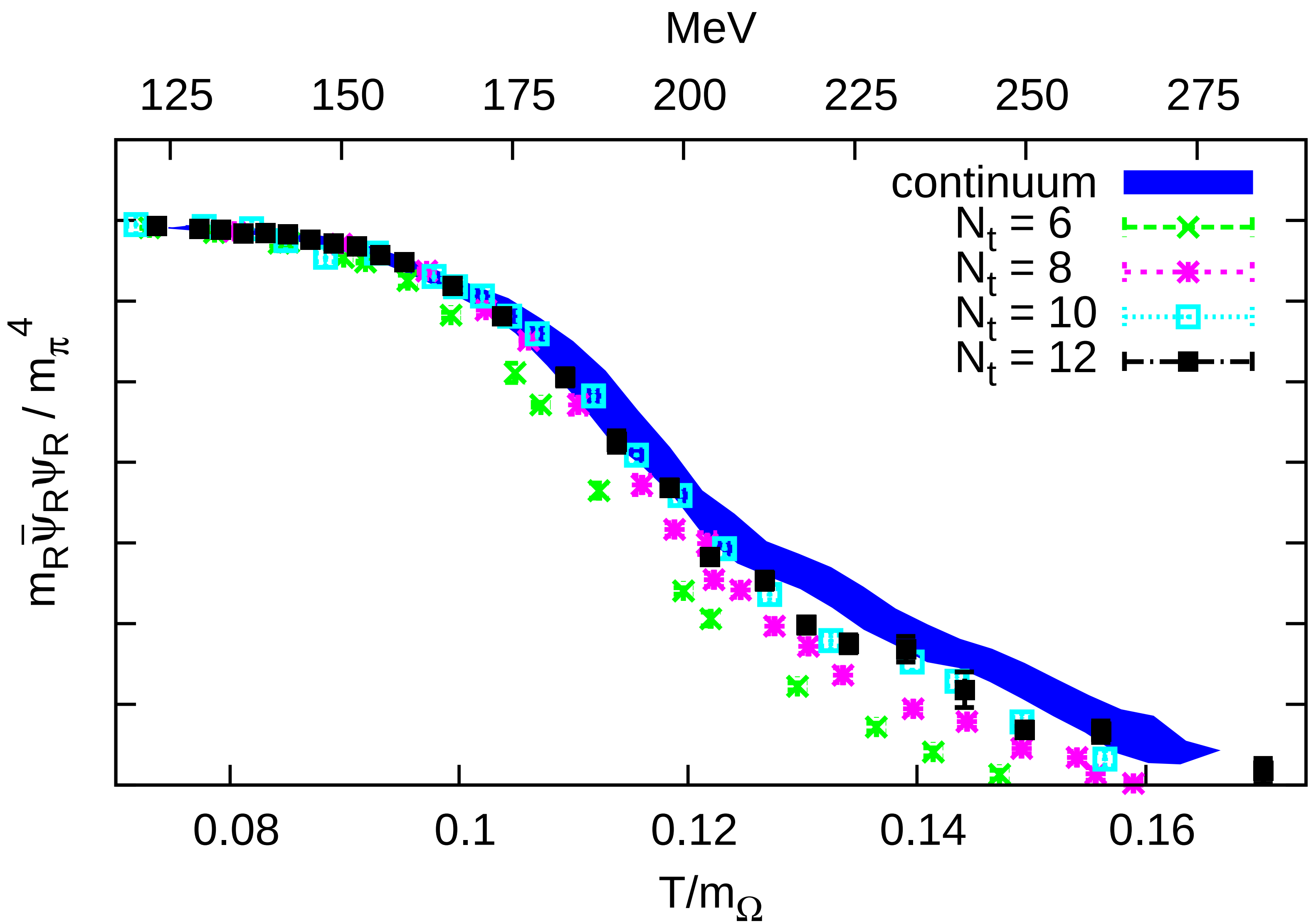
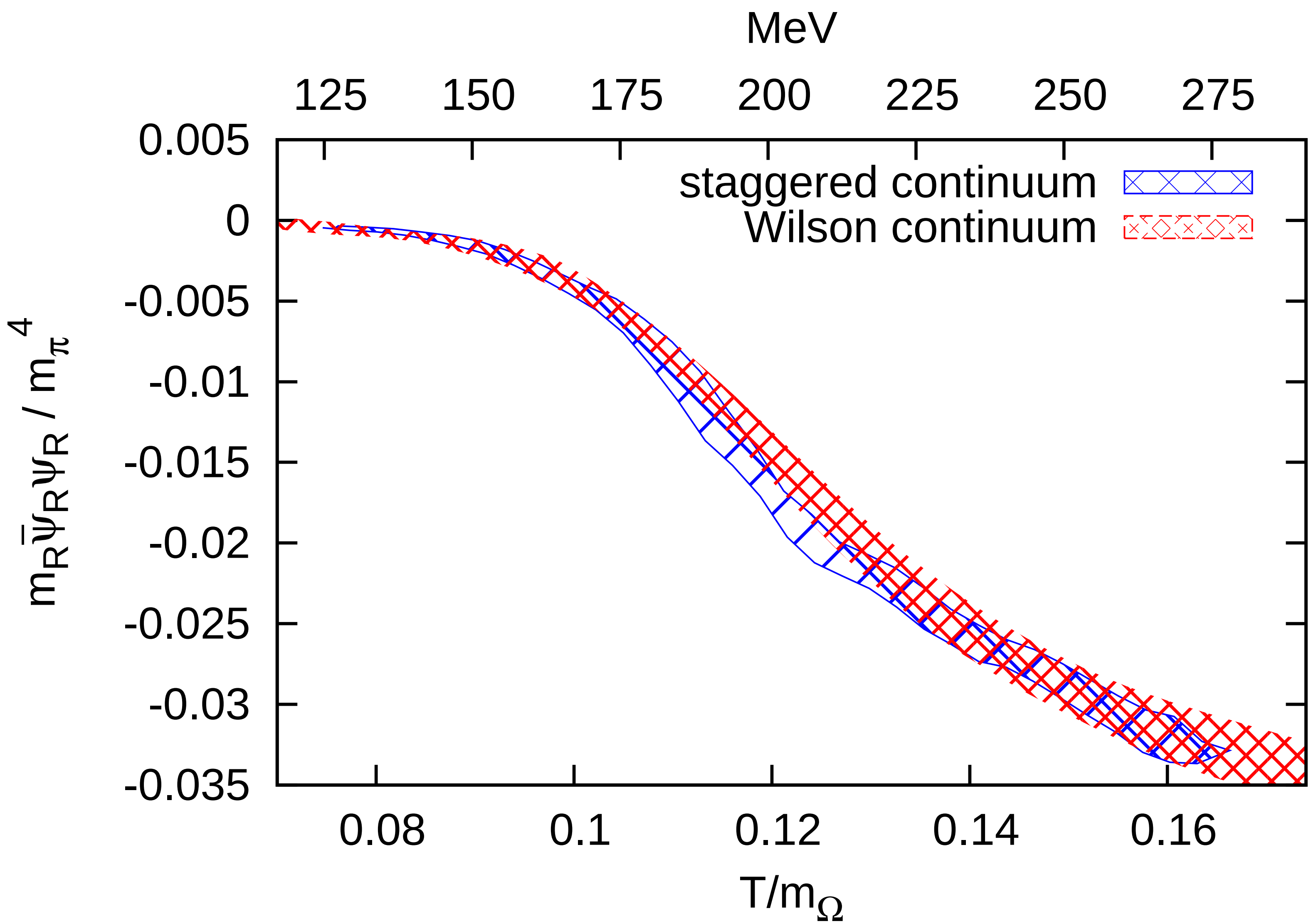
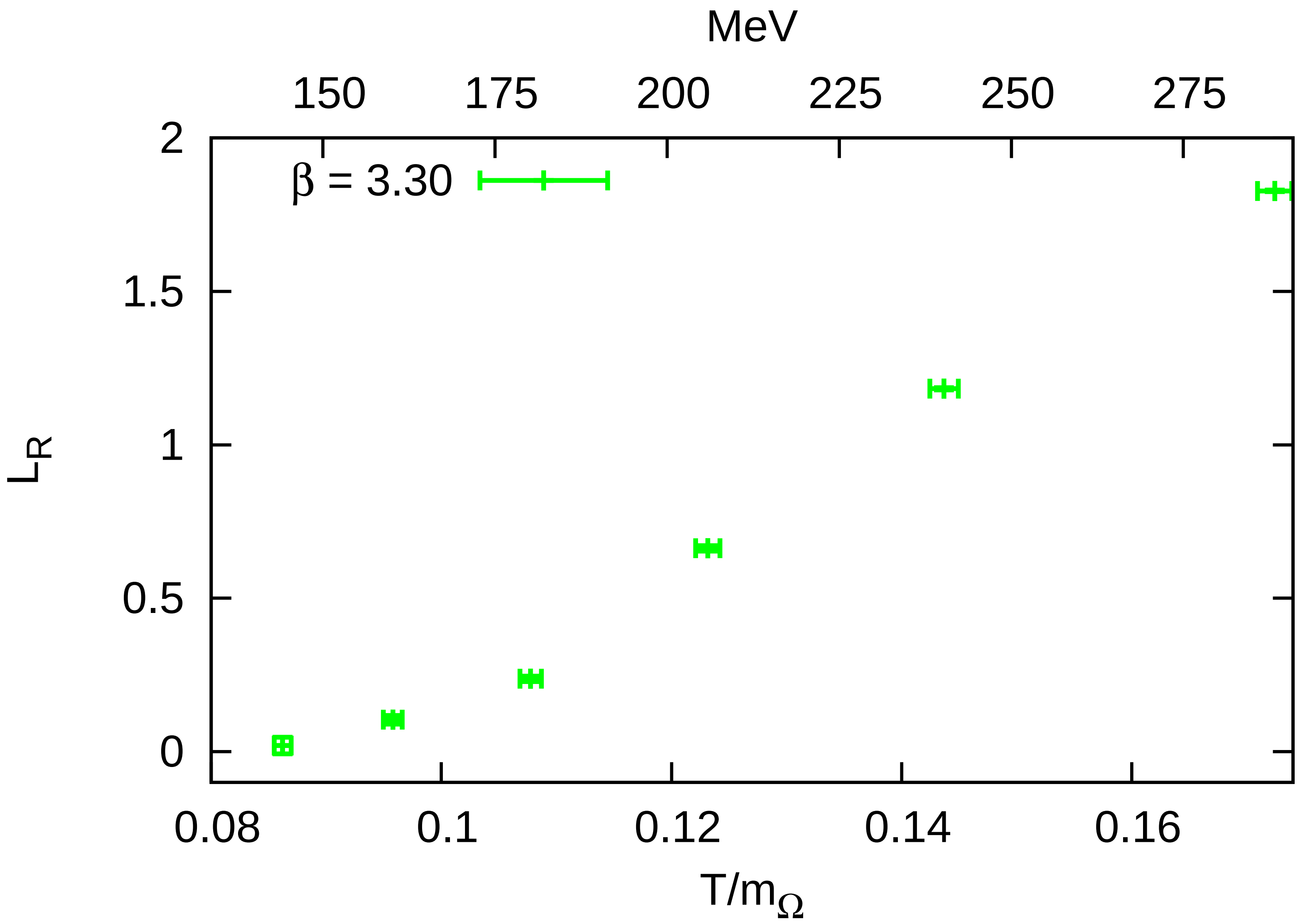


## Results, $\bar{\psi}\psi$ Wilson vs staggered



Results, $L$ Wilson



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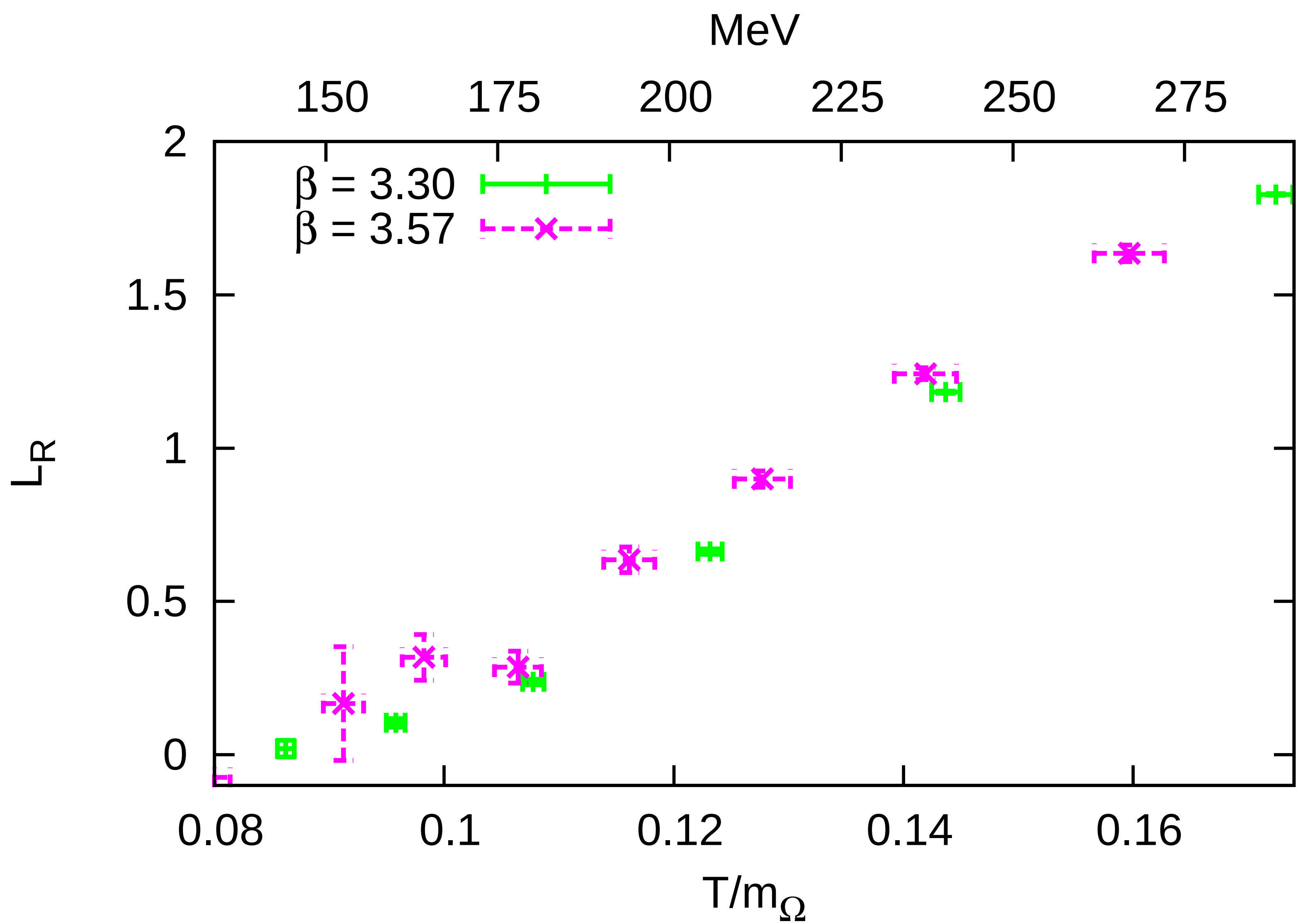
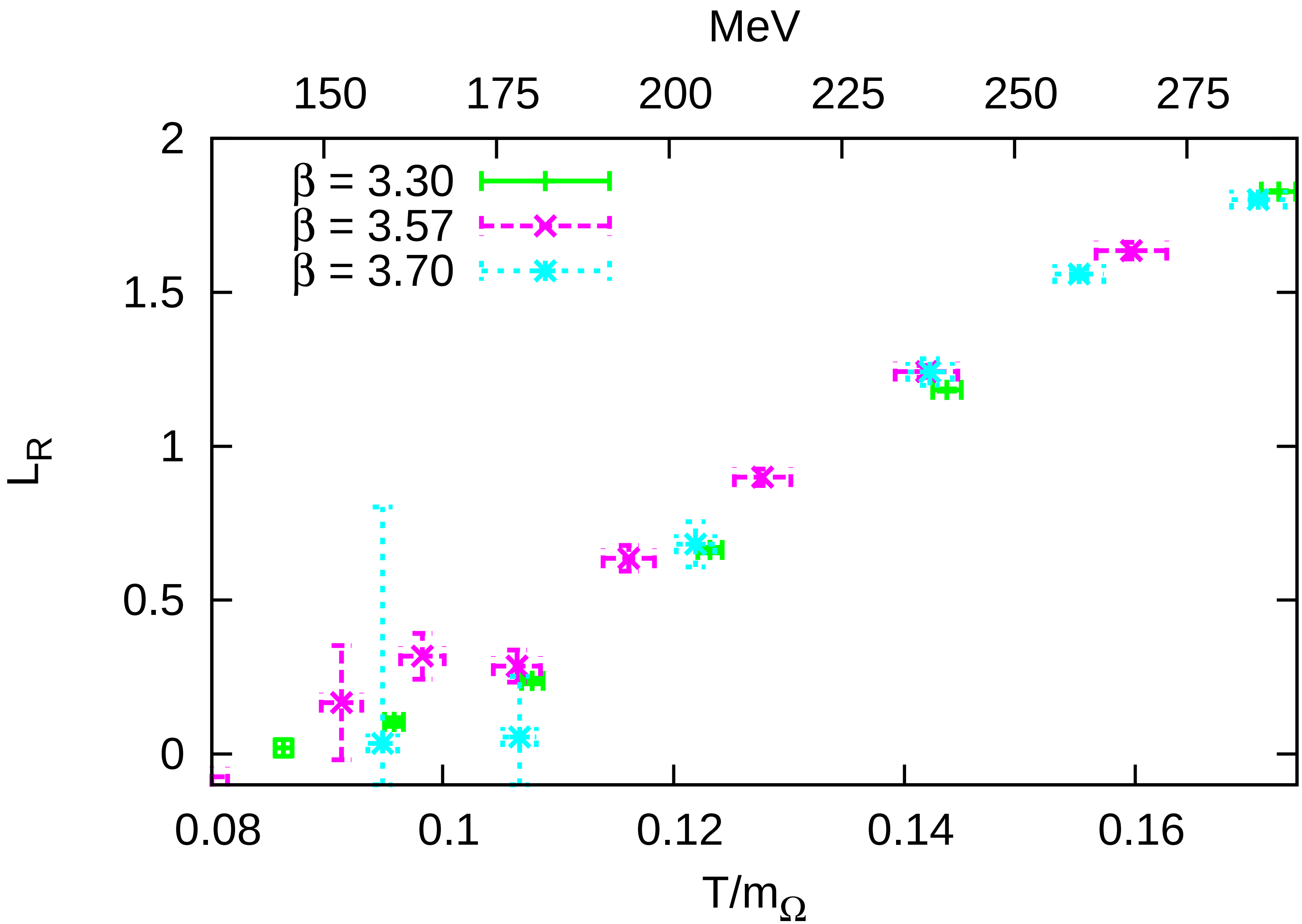
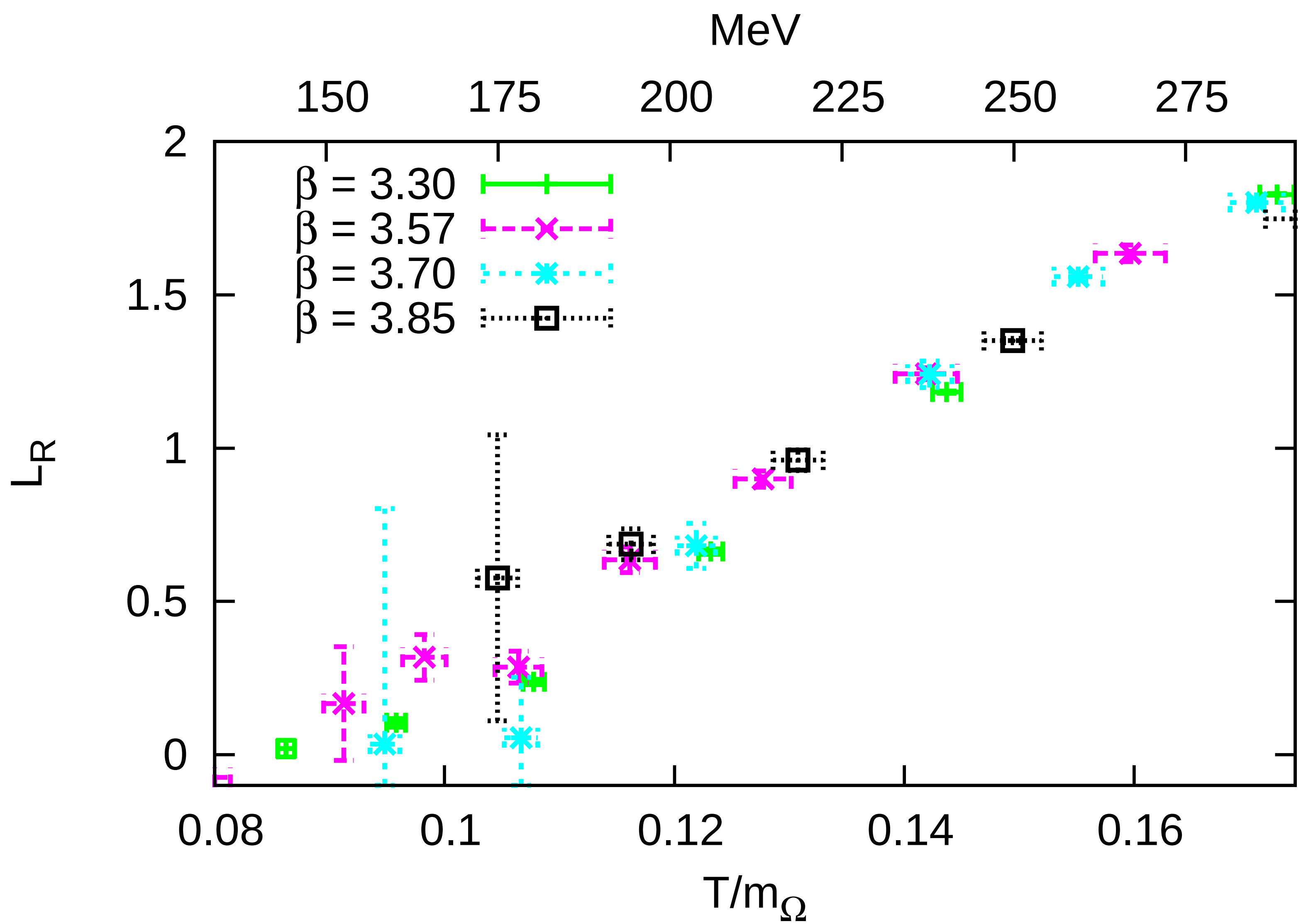


Results, $L$ Wilson



Results, $L$ Wilson



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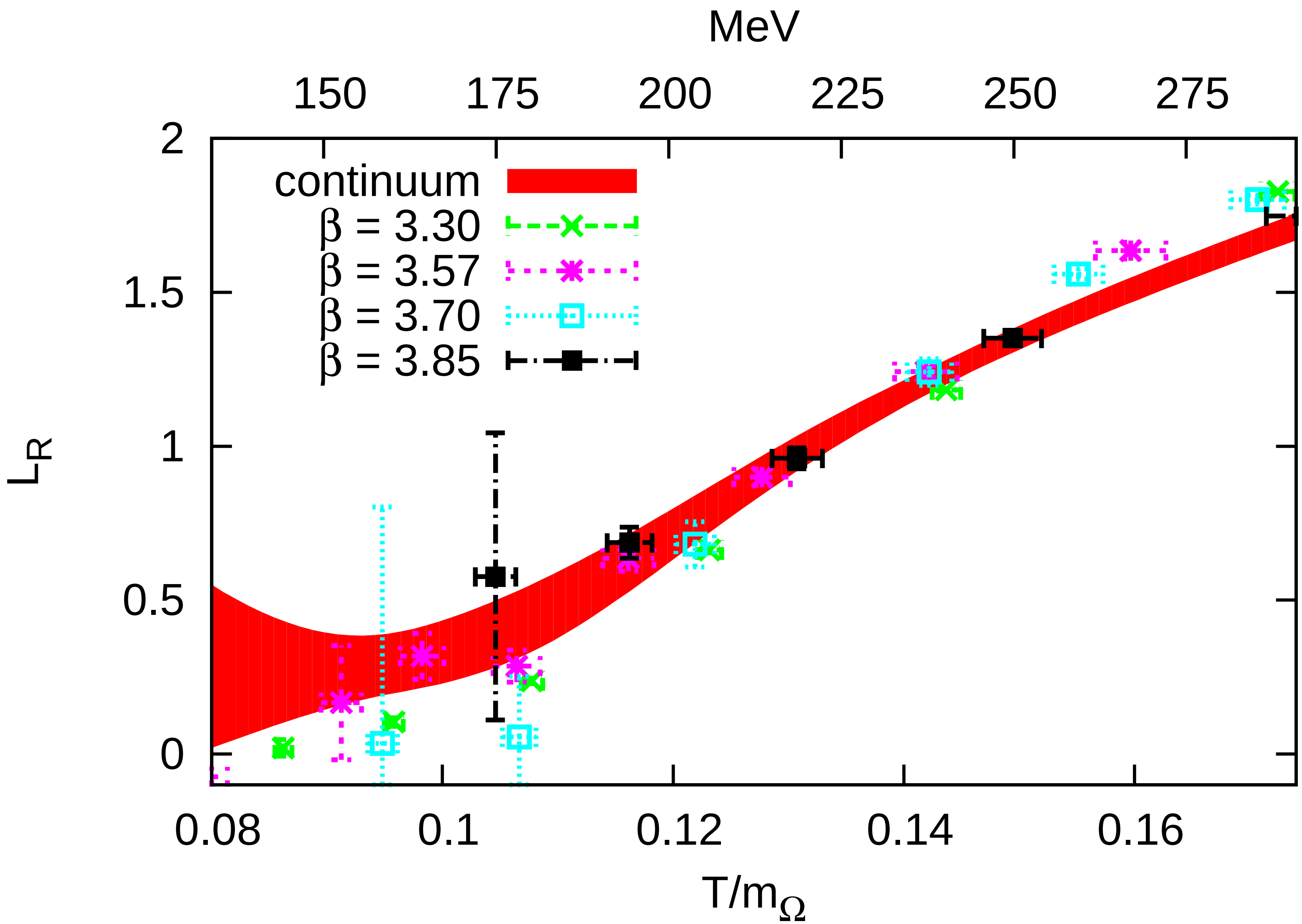

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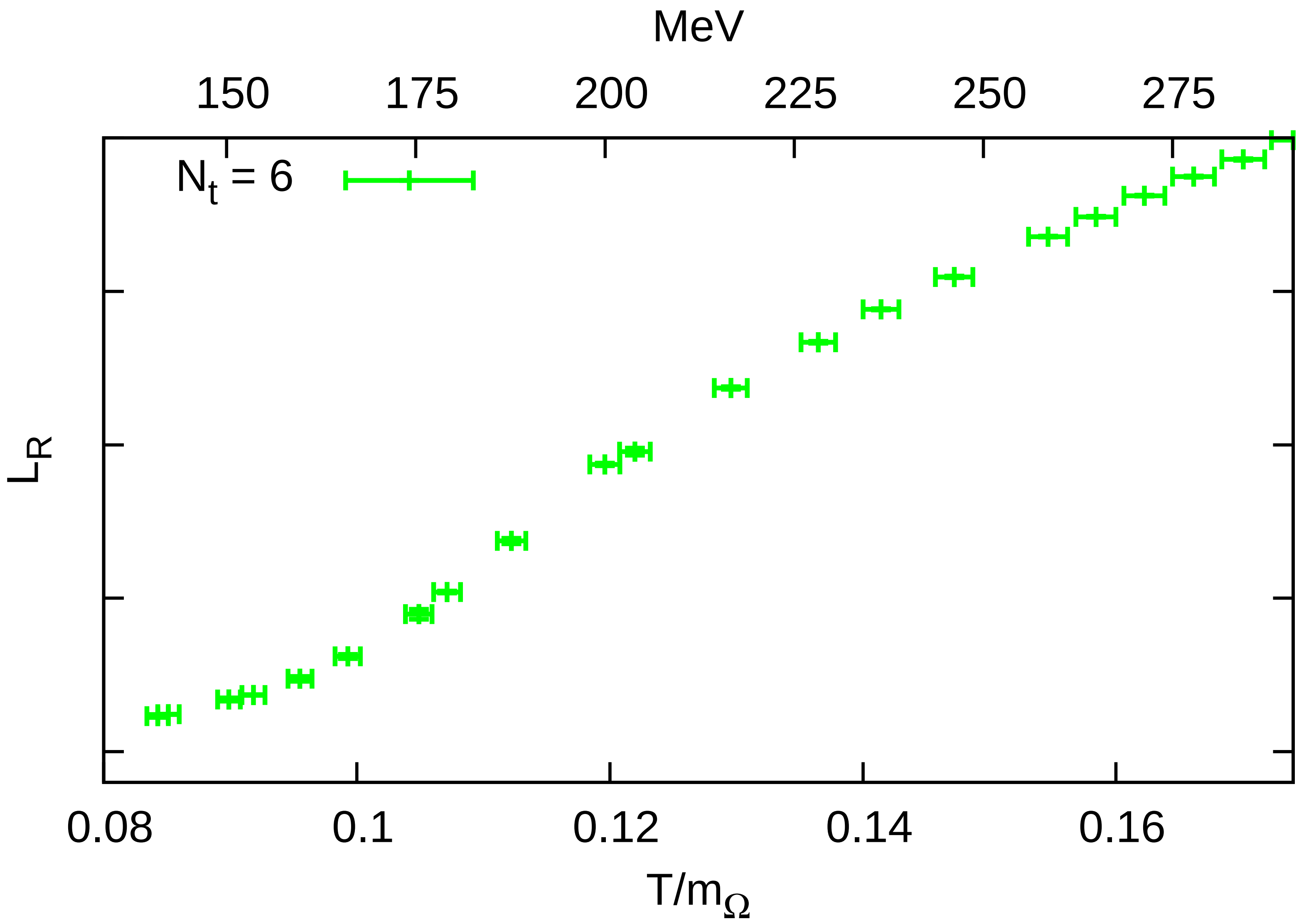

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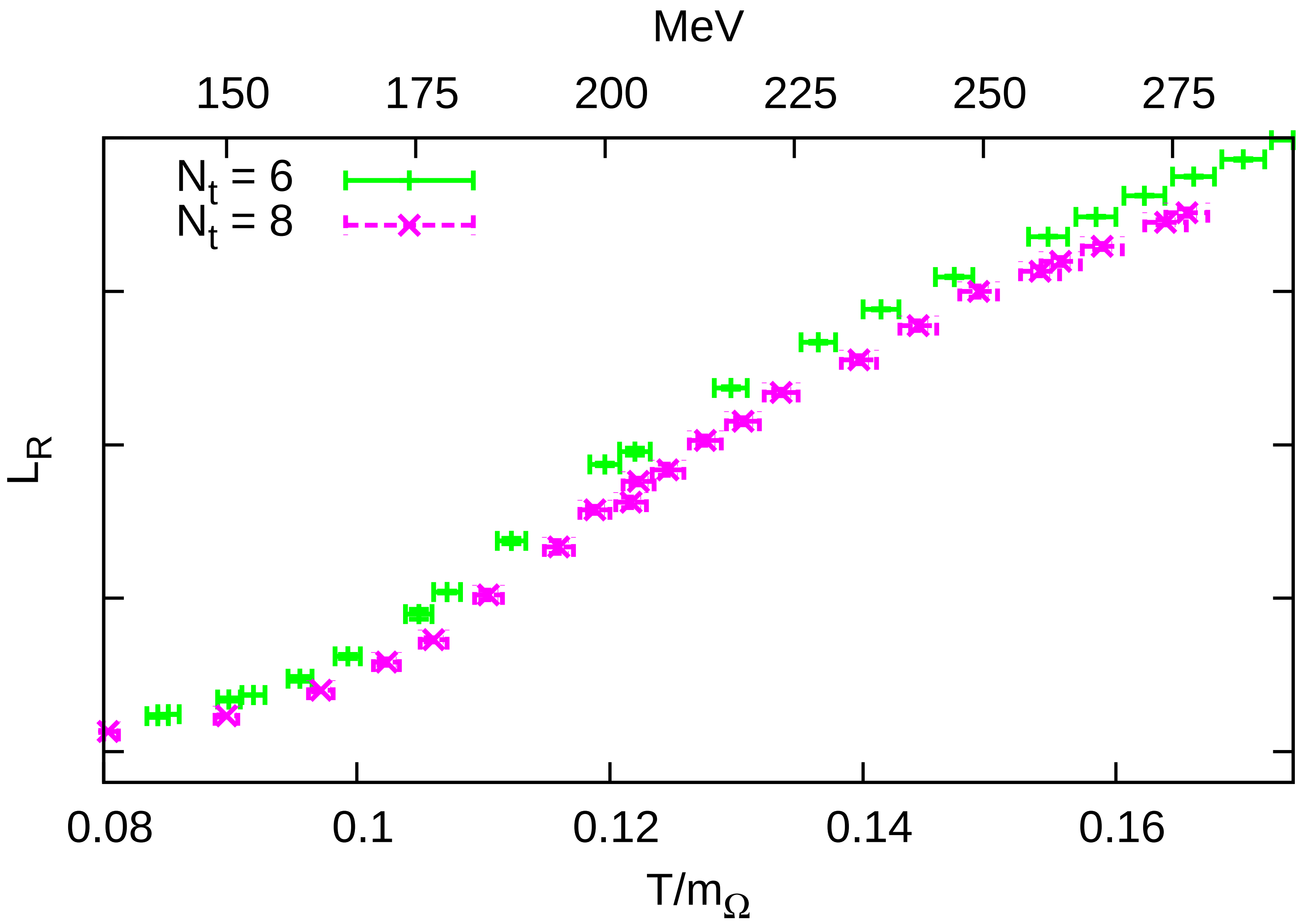

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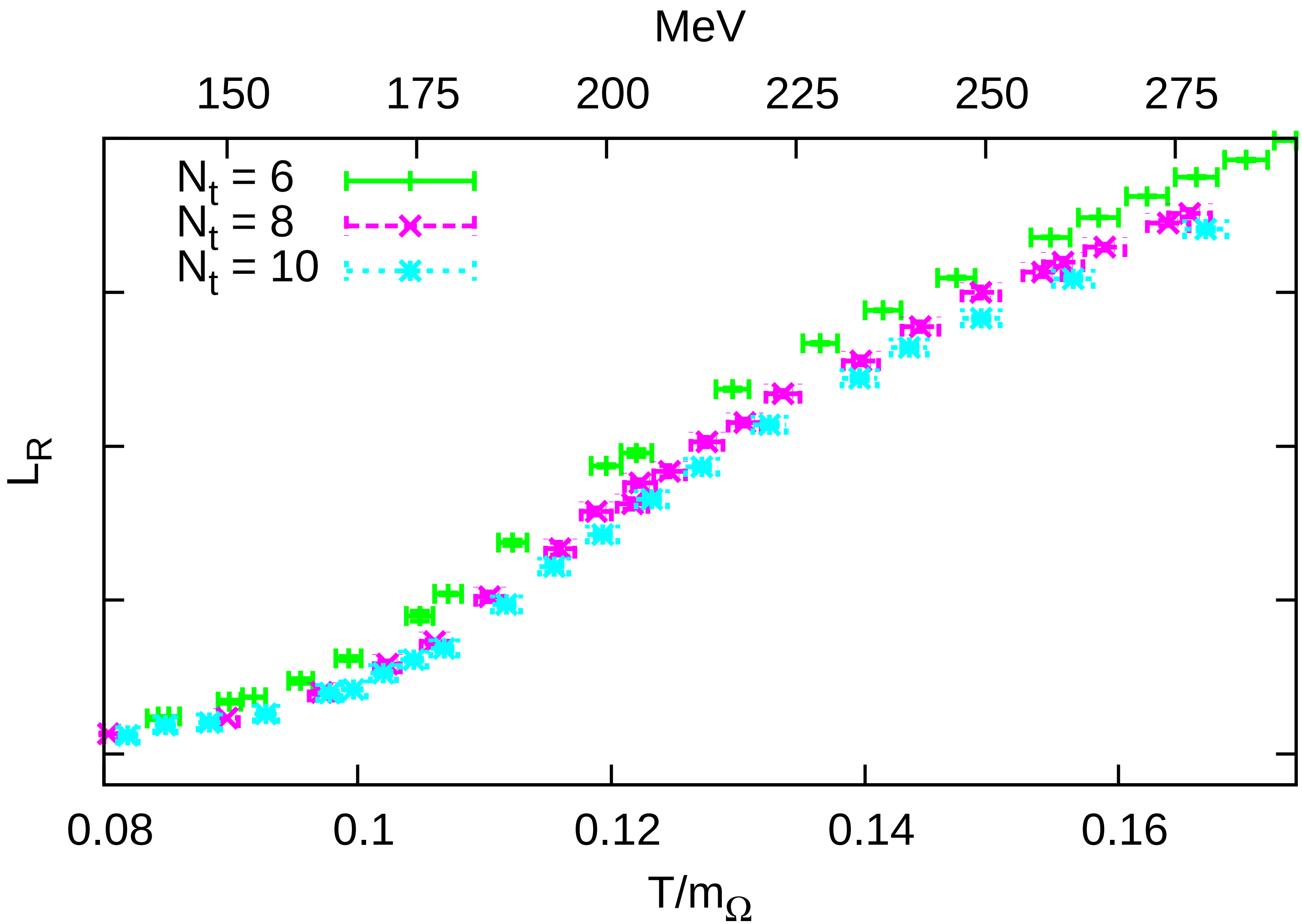

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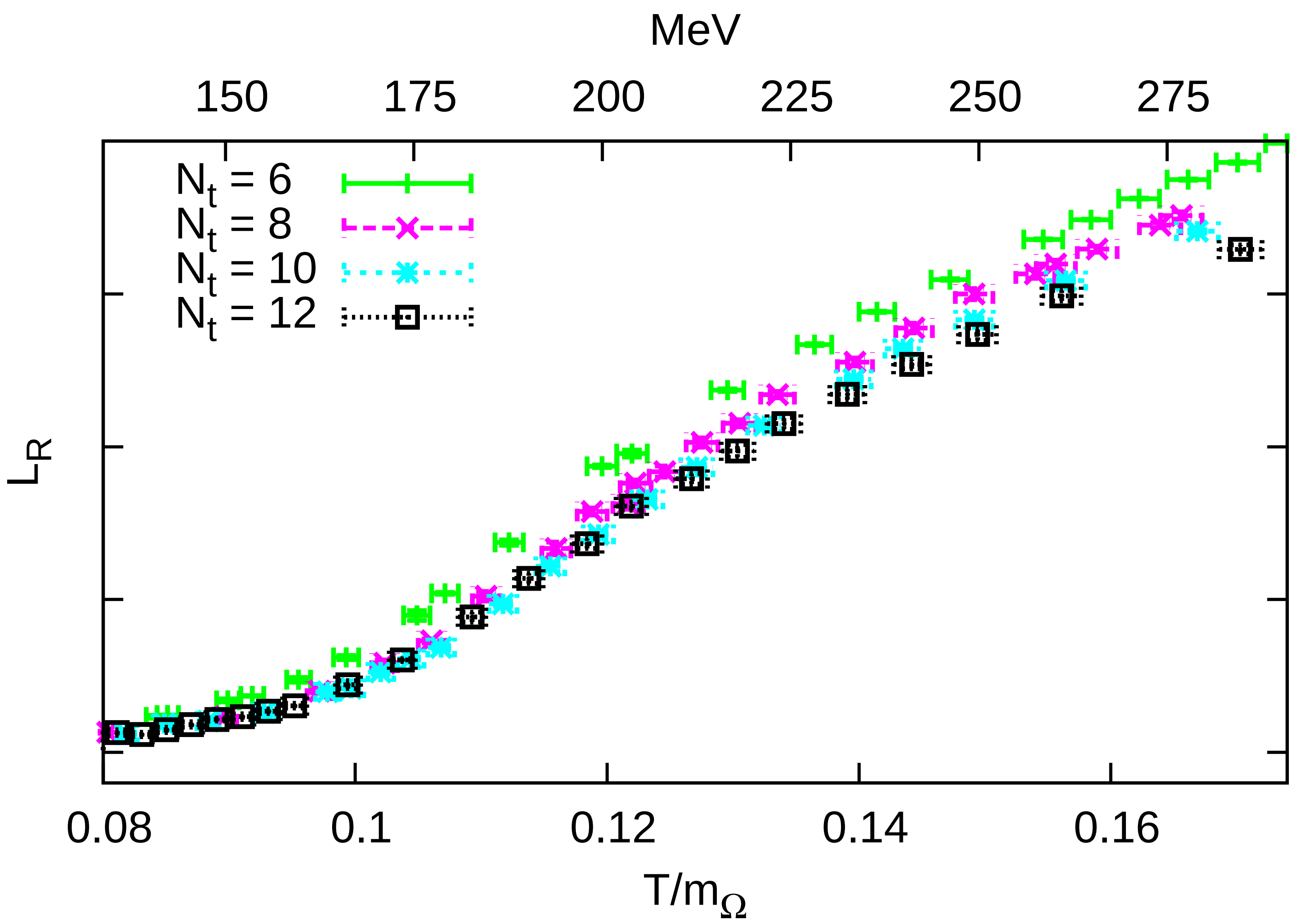

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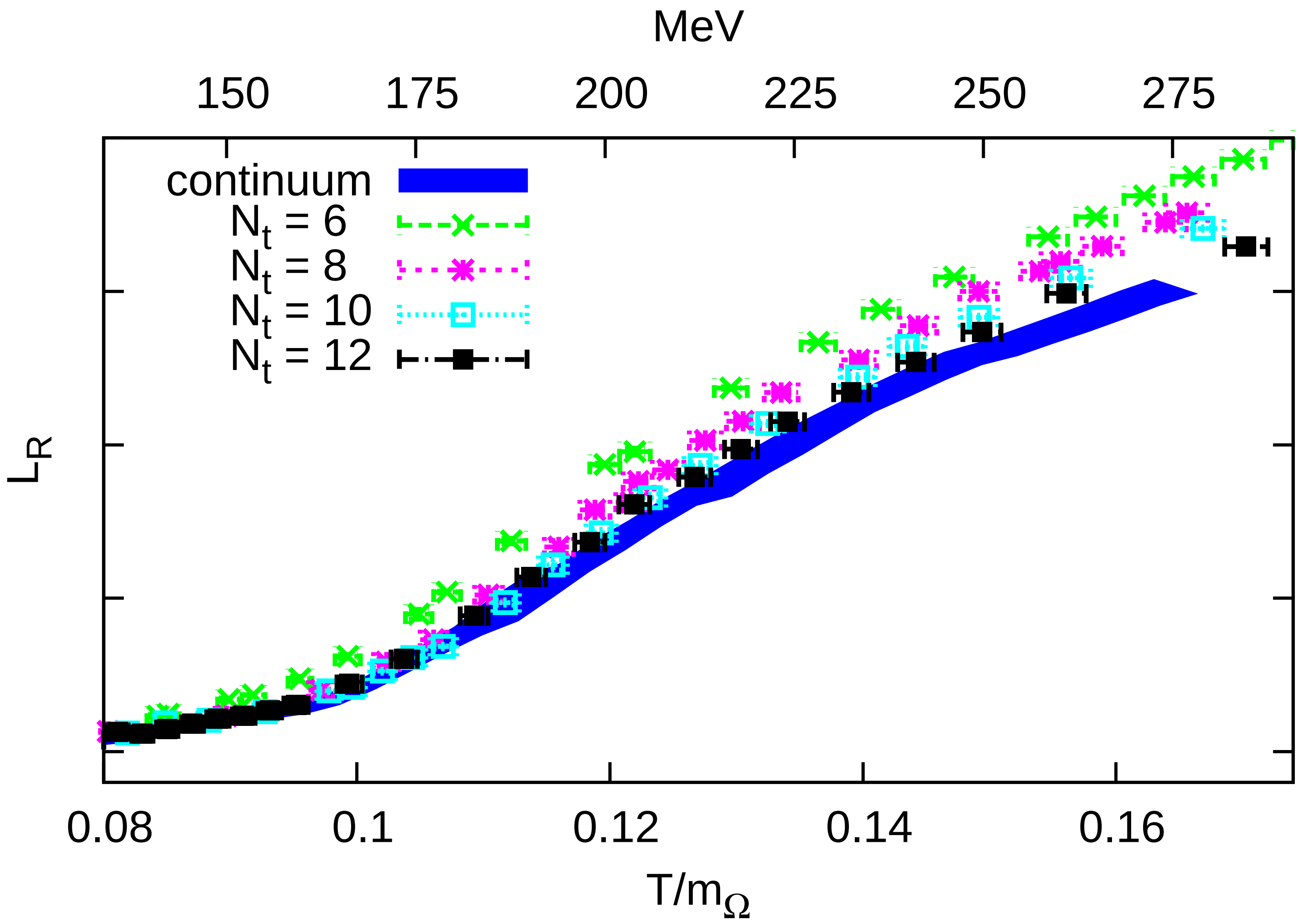
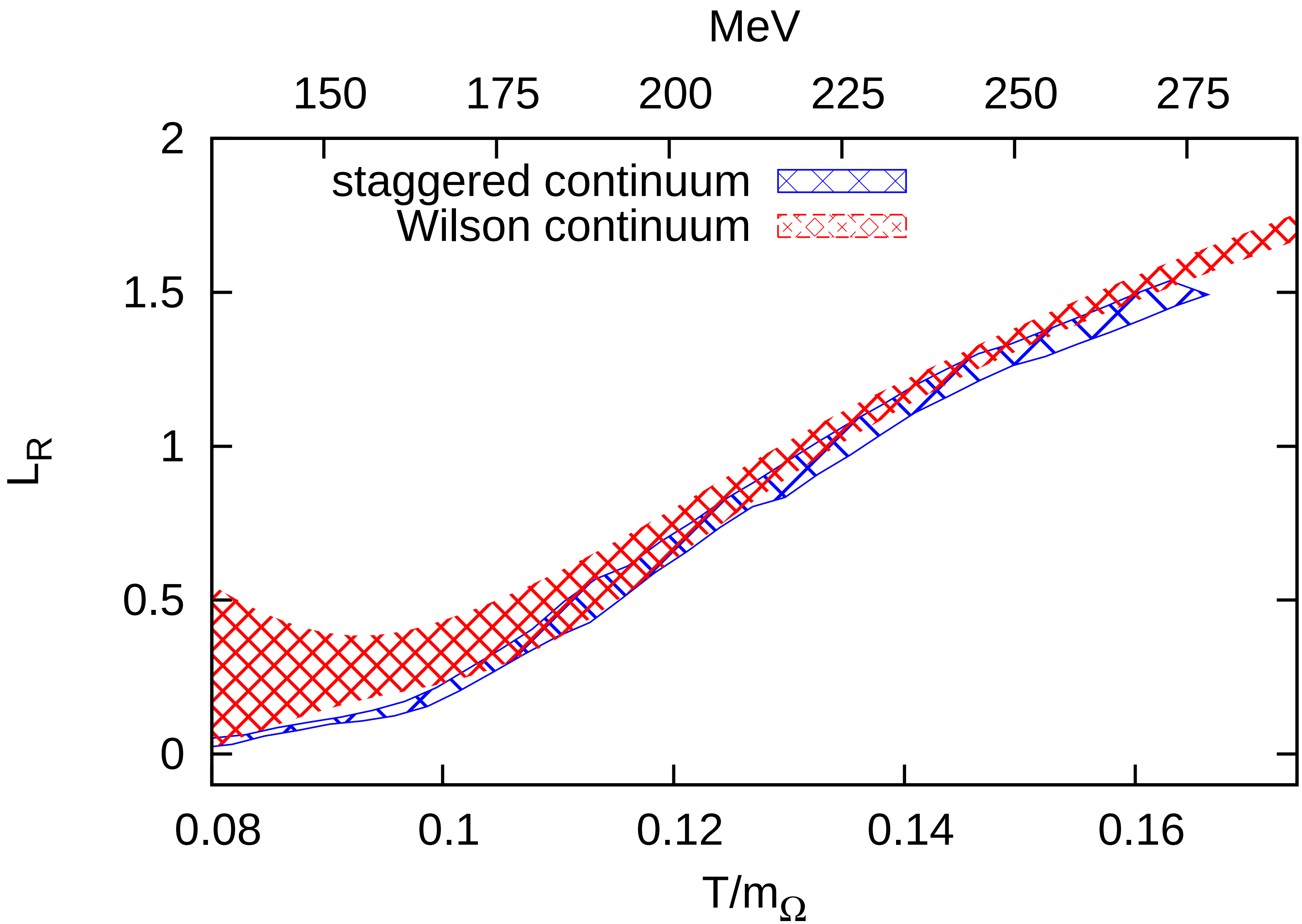


## Results, $L$ Wilson vs staggered



## Summary and outlook

- Continuum thermodynamics with Wilson fermions is feasible
- Agreement between continuum staggered and continuum Wilson results
- Lighter pions are currently ongoing (look feasible)

# Backup slides

Taste breaking in staggered simulations