Finite size scaling for 4-flavor QCD with finite chemical potential

Shinji Takeda

Kanazawa University

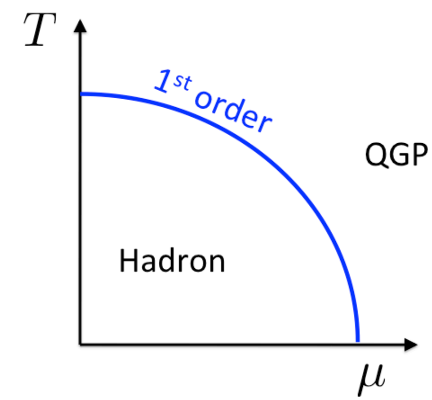
in collaboration with

X-Y. Jin, Y. Kuramashi, Y. Nakamura & A. Ukawa

Why 4-flavor ?

- Good testing ground before 3-flavor
- Expected to have

  1st order phase transition line
- Lattice study so far
  - Multi-parameter reweighting Fodor & Katz 01
  - Imaginary chemical potential D'Elia & Lombardo 02
  - Canonical approach de Forcrand 06, Kentucky 10



It is likely to be 1st order, but is not well investigated by finite size scaling study!

What we do here

- Performing finite size scaling
- Grand canonical approach with Wilson type fermions

$$\mathcal{Z}_{\mathrm{QCD}}(T,\mu) = \int [dU] e^{-S_{\mathrm{g}}[U]} \det D(\mu;U)$$ ← Complex

- Phase reweighting

$$\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{iN_{\mathrm{f}}\theta} \rangle_{||}}{\langle e^{iN_{\mathrm{f}}\theta} \rangle_{||}}$$

Phase can be controlled for larger temporal size

ST, Kuramashi & Ukawa (2011)

$$\mathcal{Z}_{||}(T,\mu) = \int [dU] e^{-S_{\mathrm{g}}[U]} |\det D(\mu;U)|$$

- Reduction technique Danzer & Gattringer (2008)
    - exact phase & quark number
    - Many-core & GPU machine

Details of simulation

- Clover fermions and Iwasaki gauge
- Parameters:
  - a=0.33fm (a^{-1}=610MeV)
  - pion mass=830MeV
  - T=150MeV
  - Chemical potential

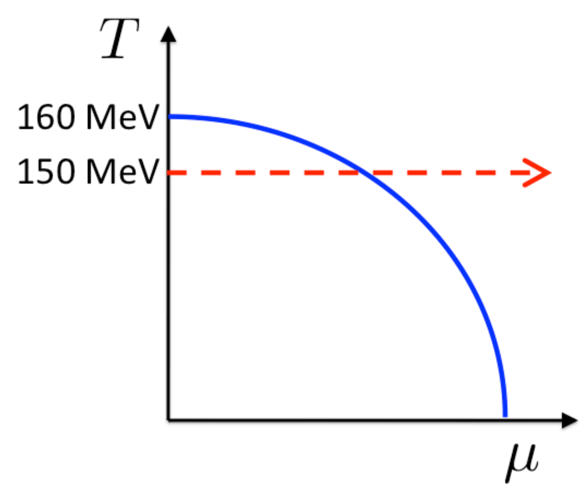
$$a\mu = 0.1 - 0.35$$

  - Spatial Volume

$$6^3,\ 668,\ 688,\ 8^3$$

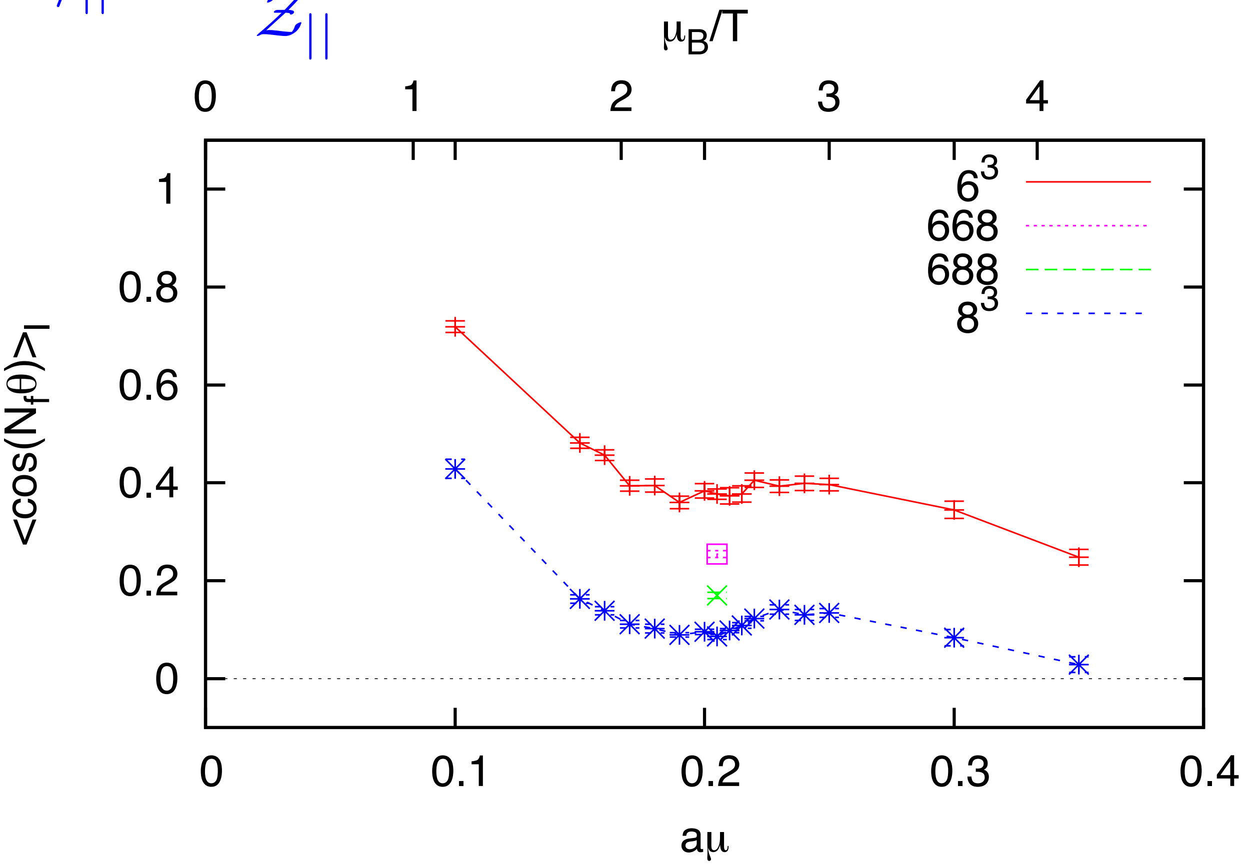
Kentucky group PRD82.054502(2010)

$$\beta = 1.6 \quad \kappa = 0.1371$$
$$N_{\mathrm{T}} = 4 \quad c_{\mathrm{sw}} = 1.9655$$



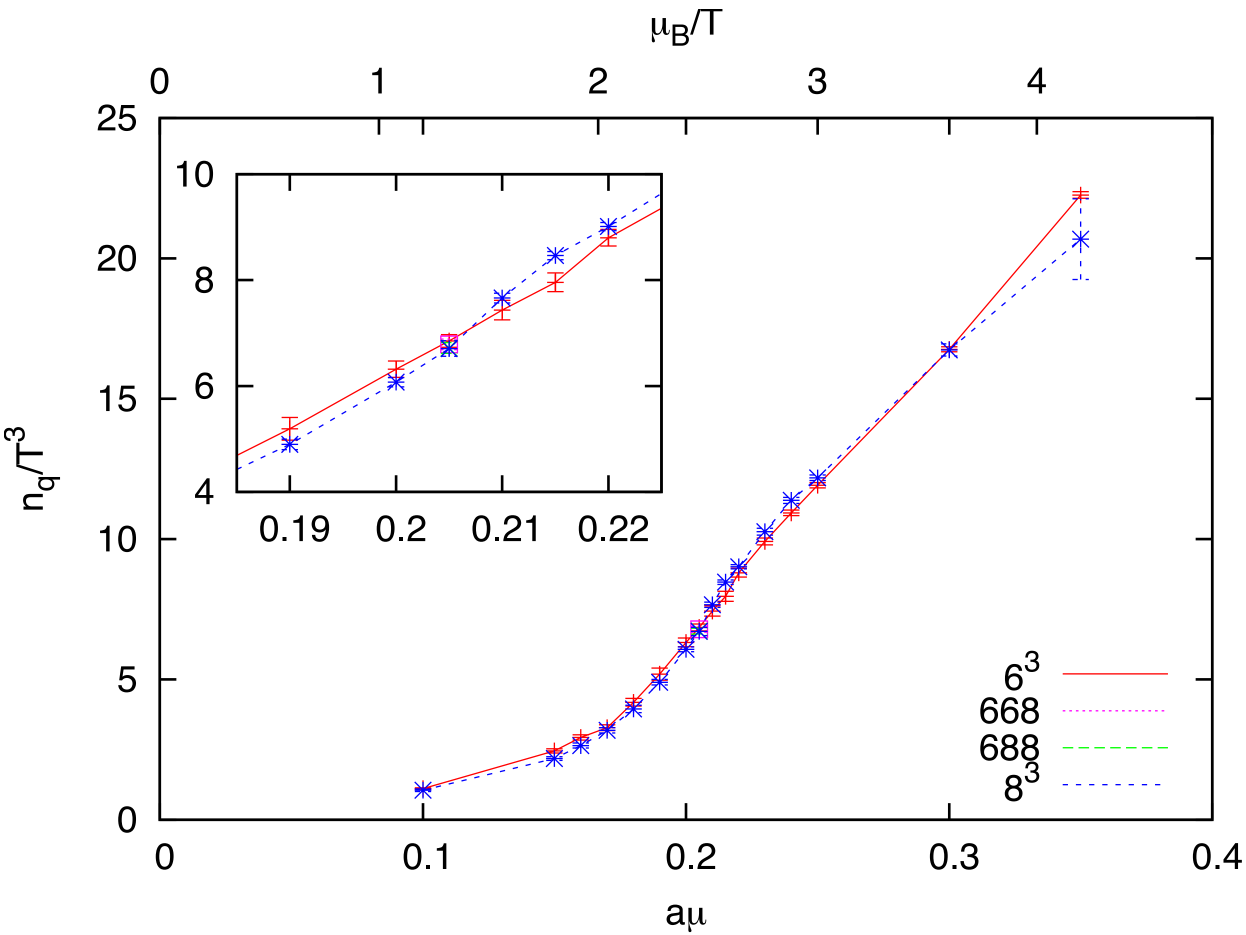
Phase-reweighting factor

$$\langle e^{i4\theta} \rangle_{||} = \frac{\mathcal{Z}_{\rm QCD}}{\mathcal{Z}_{||}}$$

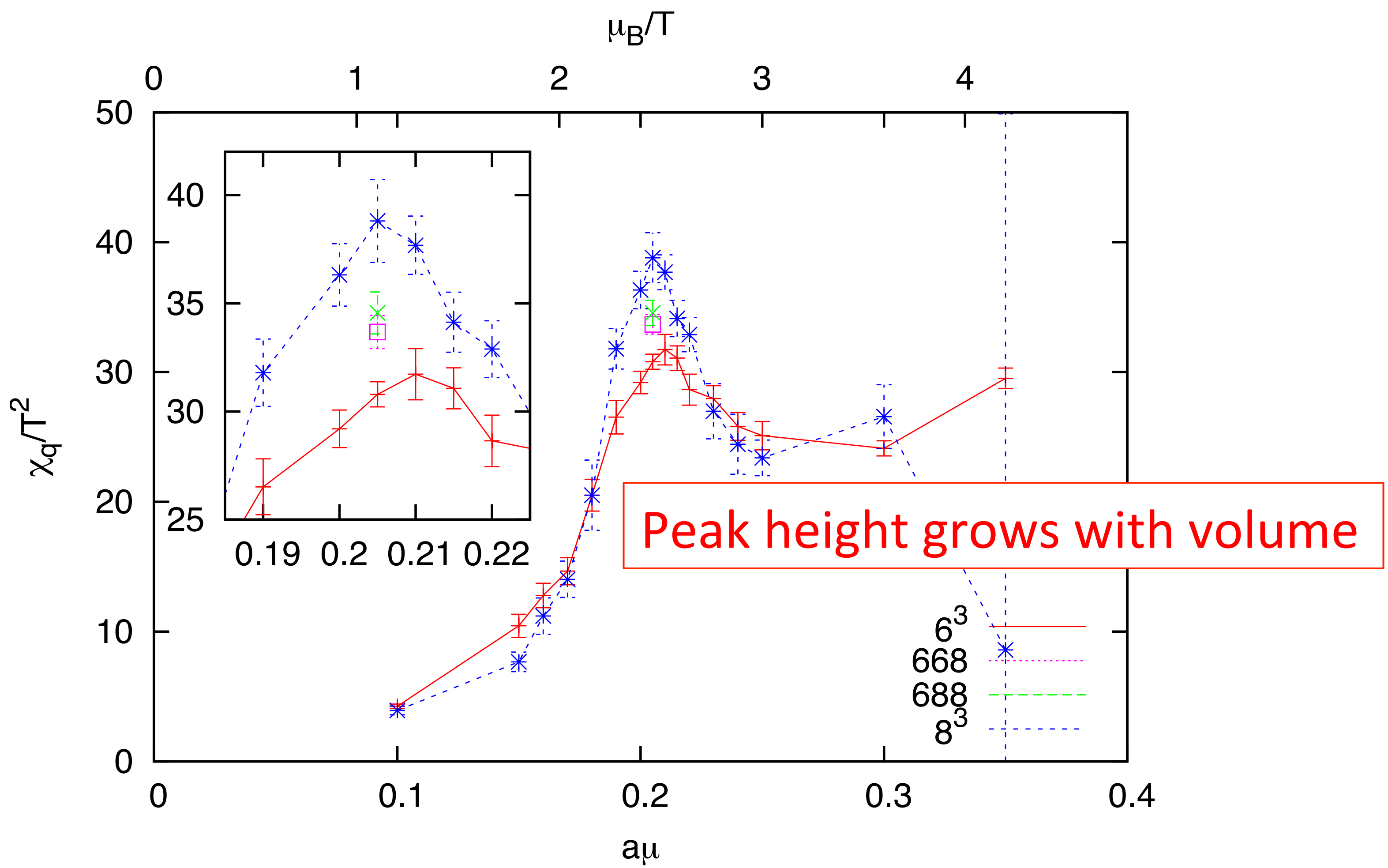


$$a\mu_c = am_\pi/2 \sim 0.7$$

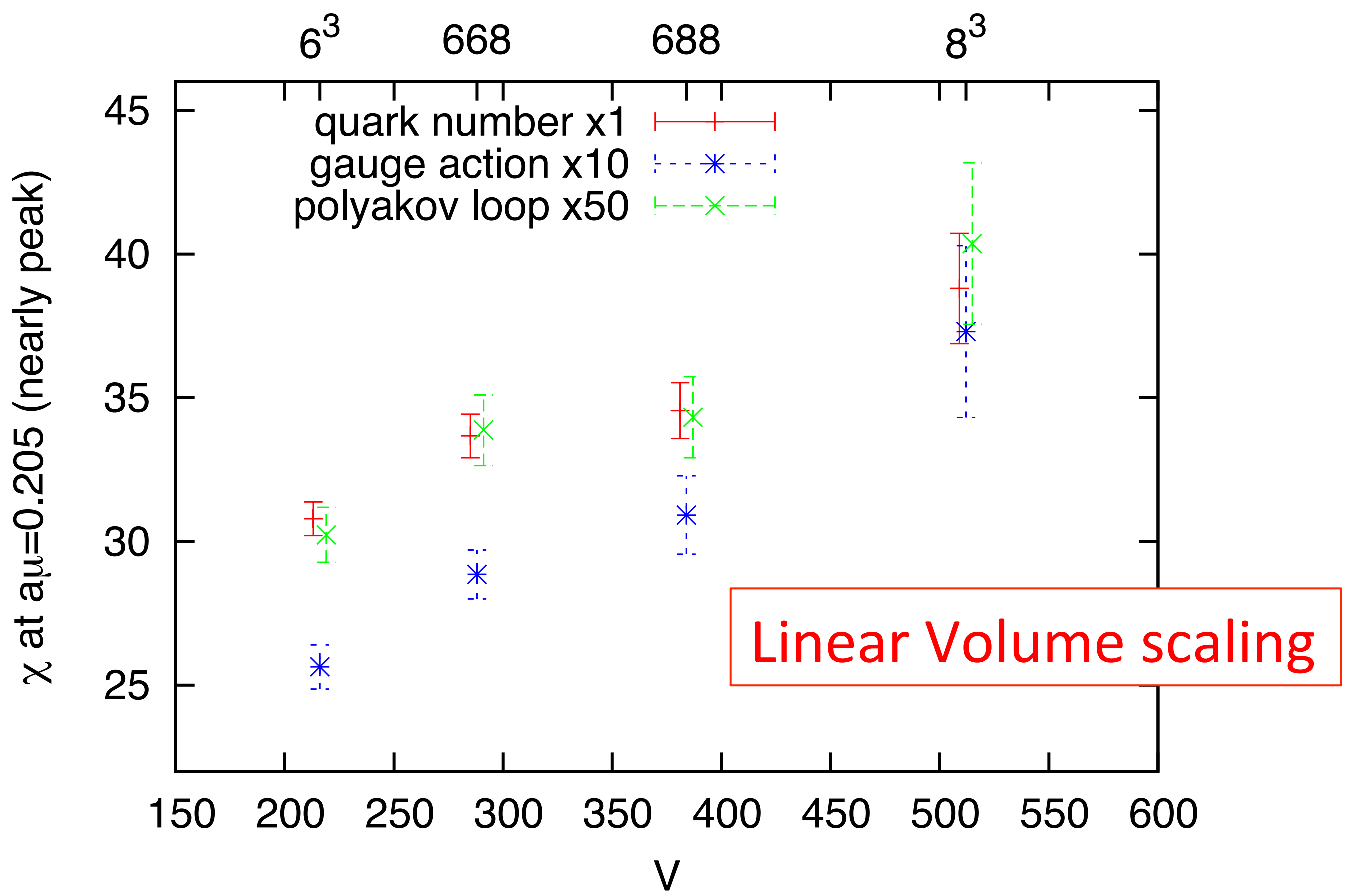
Quark Number density



Susceptibility of quark number

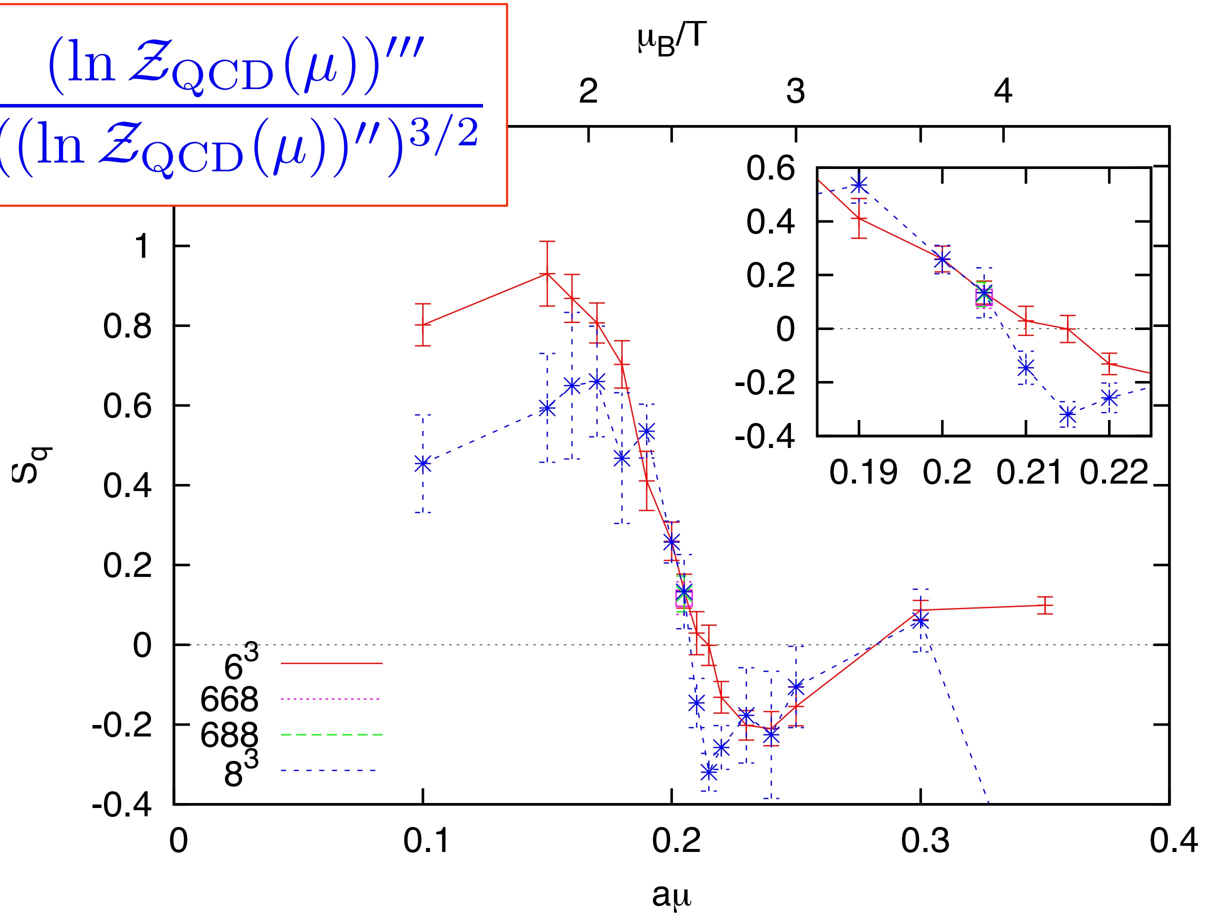


Volume scaling for peak of susceptibility



Skewness for quark number

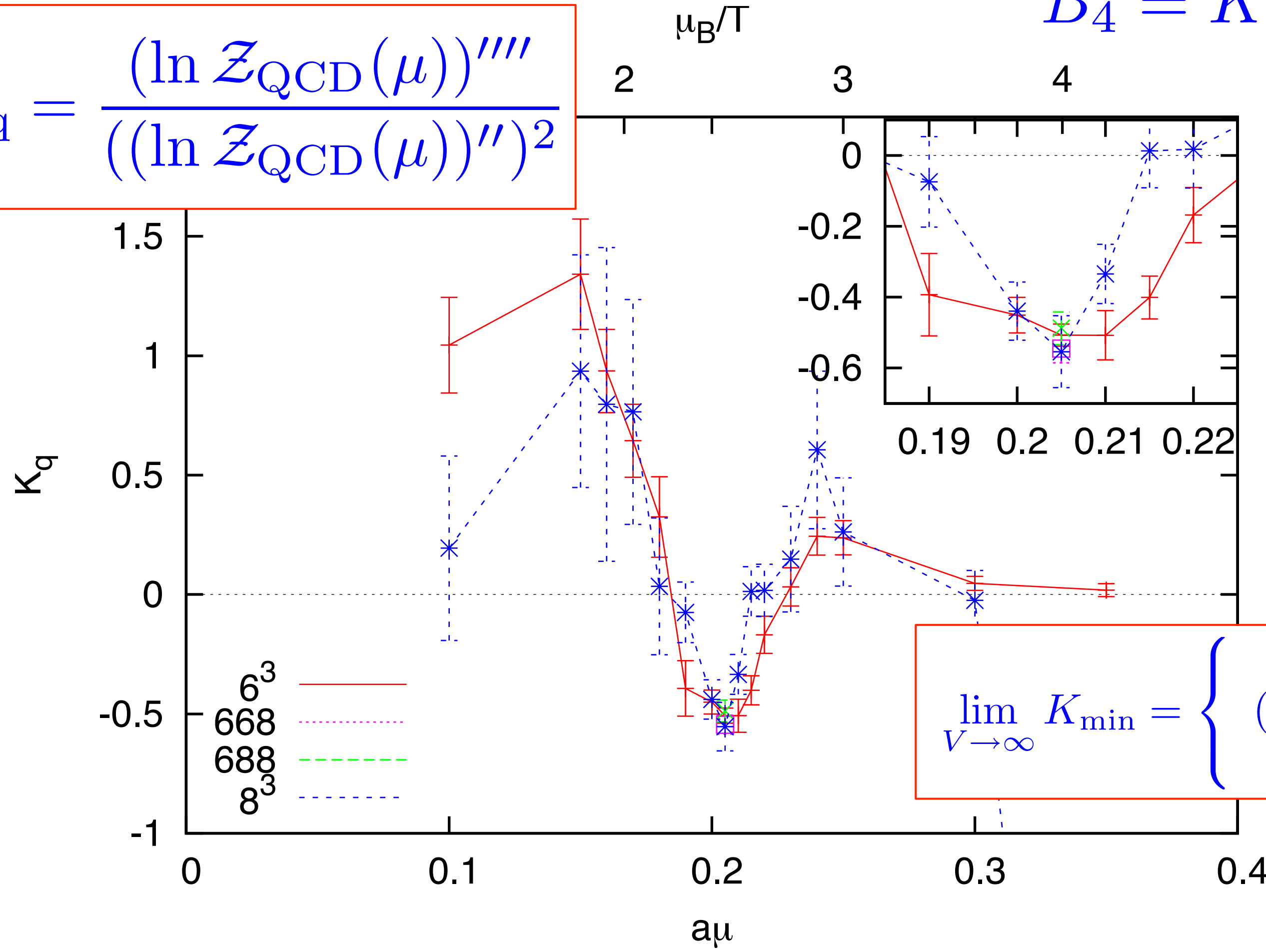
$$S_q = \frac{(\ln \mathcal{Z}_{\rm QCD}(\mu))'''}{((\ln \mathcal{Z}_{\rm QCD}(\mu))'')^{3/2}}$$



Kurtosis for quark number

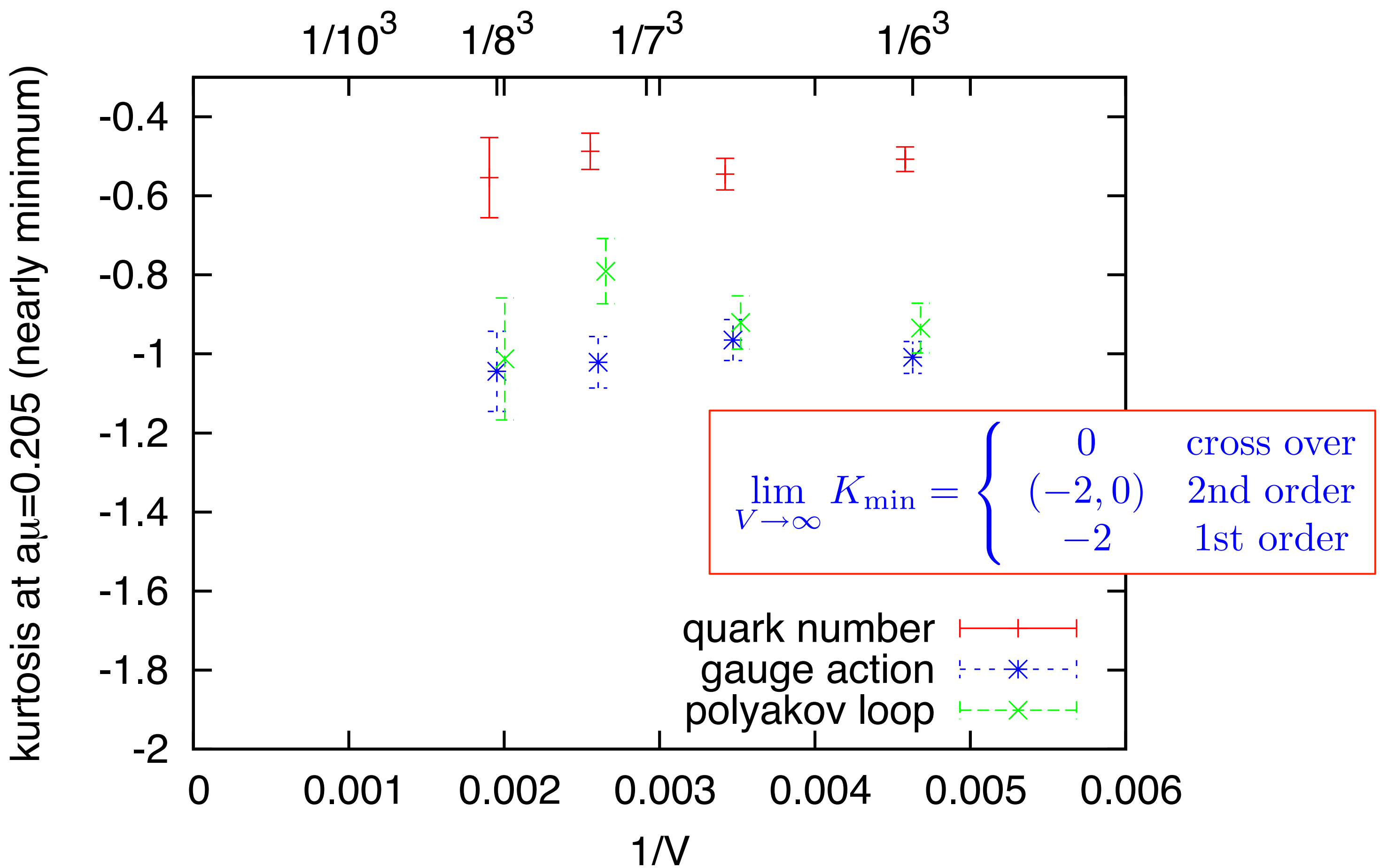
$$K_q = \frac{(\ln Z_{\rm QCD}(\mu))''''}{((\ln Z_{\rm QCD}(\mu))'')^2}$$

$$B_4 = K + 3$$



$$\lim_{V\to\infty} K_{\rm min} = \begin{cases} 0 & \text{cross over} \\ (-2, 0) & \text{2nd order} \\ -2 & \text{1st order} \end{cases}$$

Scaling for the minimum of kurtosis

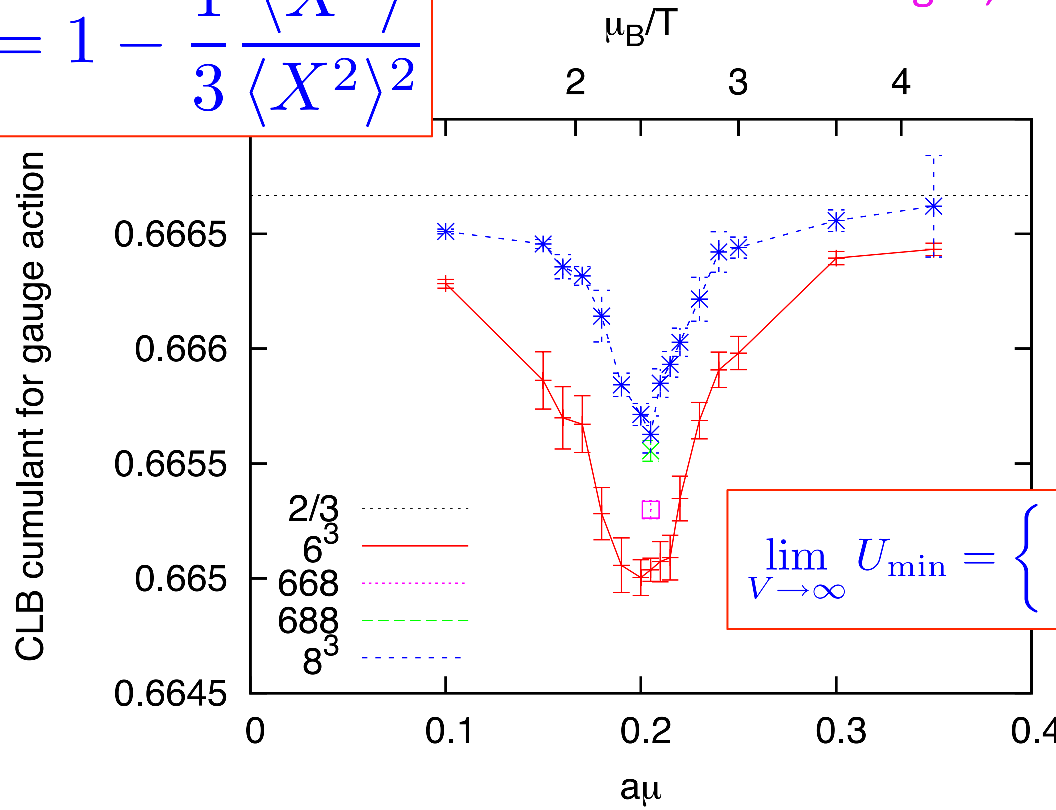


Challa Landau Binder cumulant

Challa, Landau & Binder 86

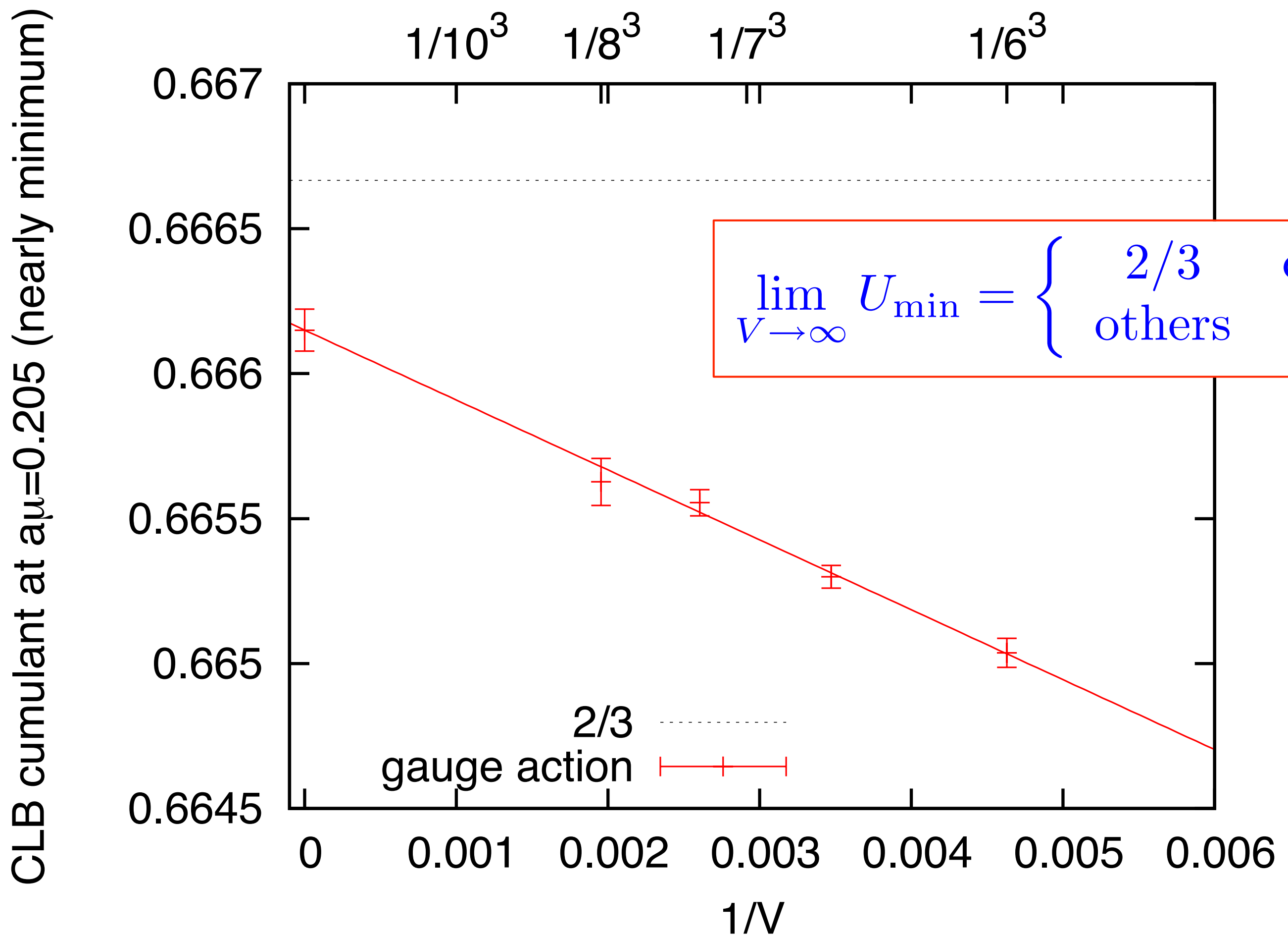
Fukugita, Okawa & Ukawa 89

$$U_X = 1 - \frac{1}{3}\frac{\langle X^4 \rangle}{\langle X^2 \rangle^2}$$



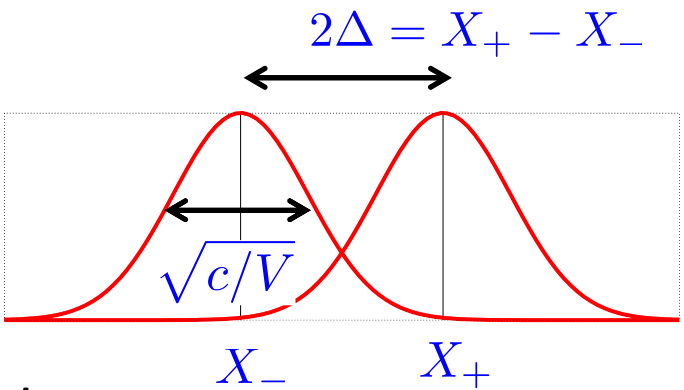
$$\lim_{V\to\infty} U_{\min} = \begin{cases} 2/3 & \text{cross over} \\ \text{others} & \text{1st order} \end{cases}$$

Scaling for the minimum of CLB cumulant



$$\lim_{V\to\infty} U_{\rm min} = \begin{cases} 2/3 & \text{cross over} \\ \text{others} & \text{1st order} \end{cases}$$

Distribution argument



- For a double peak distribution

$$P(x) \propto e^{-\frac{(x-X_-)^2}{2c/V}} + e^{-\frac{(x+X_+)^2}{2c/V}}$$

- Susceptibility, Kurtosis & CLB are given by

$$\chi = c + \Delta^2 V$$

$$X = \frac{1}{2}(X_+ + X_-)$$

$$K = -2\left[1 - \frac{2c}{\Delta^2}\frac{1}{V} + O\left(\left(\frac{c}{\Delta^2}\frac{1}{V}\right)^2\right)\right]$$

$$U = \frac{2}{3}\frac{X^4 + \Delta^4}{(X^2+\Delta^2)^2}\left[1 - \frac{2c}{X^2+\Delta^2}\frac{1}{V} + O\left(\left(\frac{c}{X^2+\Delta^2}\frac{1}{V}\right)^2\right)\right]$$

In our case here $\quad \frac{c}{\Delta^2} \sim V \qquad X \gg \Delta$

Summary

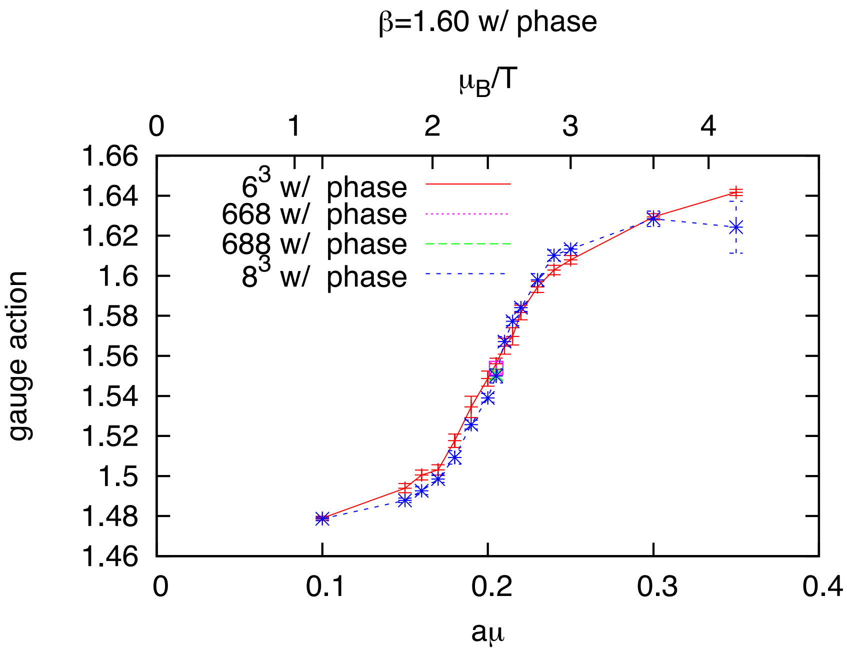
- Susceptibility : linear volume scaling is observed
- Kurtosis : volume is too small to see 1/V scaling
- CLB cumulant : 1/V scaling is clearly observed
- At this parameter set, the transition is 1^{st} order
- Further statistics and refined analysis to consolidate the above results/conclusion
- For Lee-Yang zero analysis see

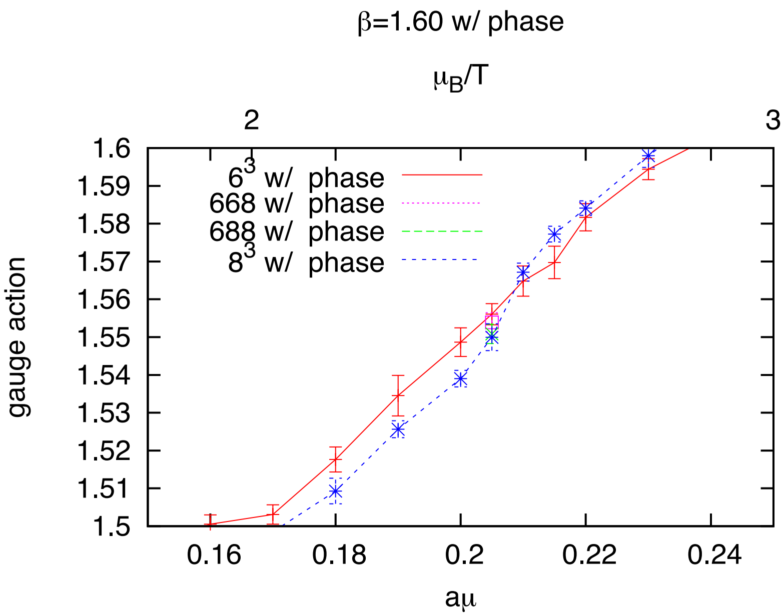
  talk by X-Y. Jin on 6/27 11:40 at room 8
- For 3-flavor study see

  talk by Y. Nakamura on 6/27 8:50 at room 8

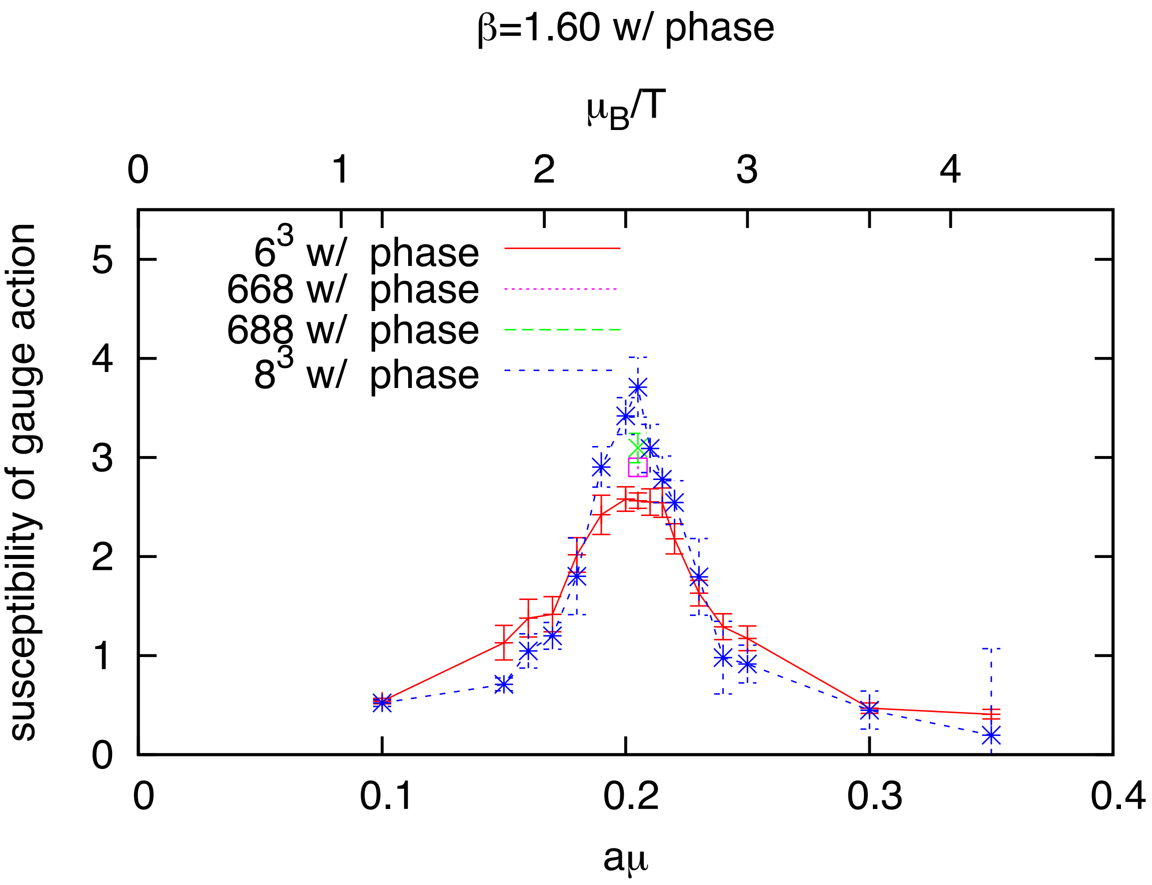
BACK UP SLIDES

Gauge action

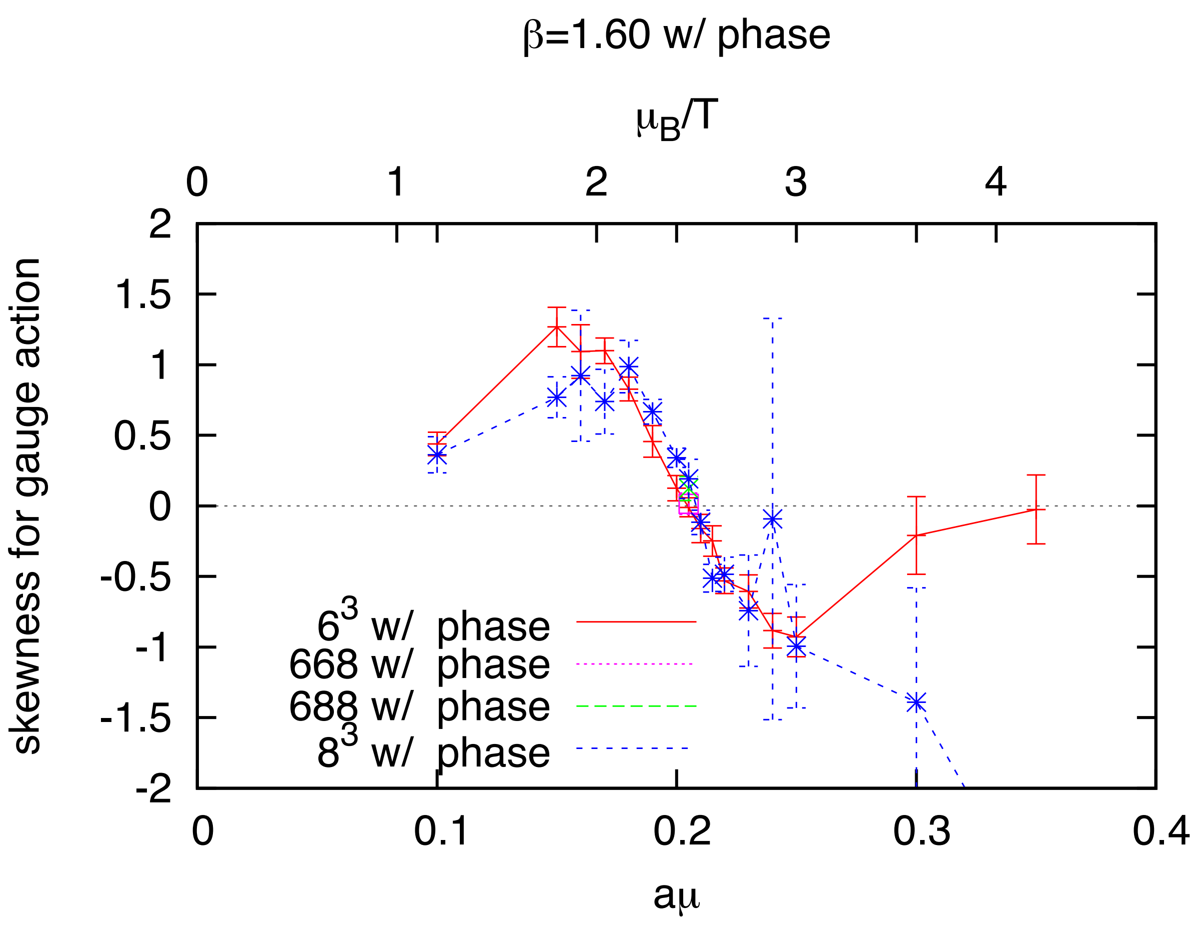




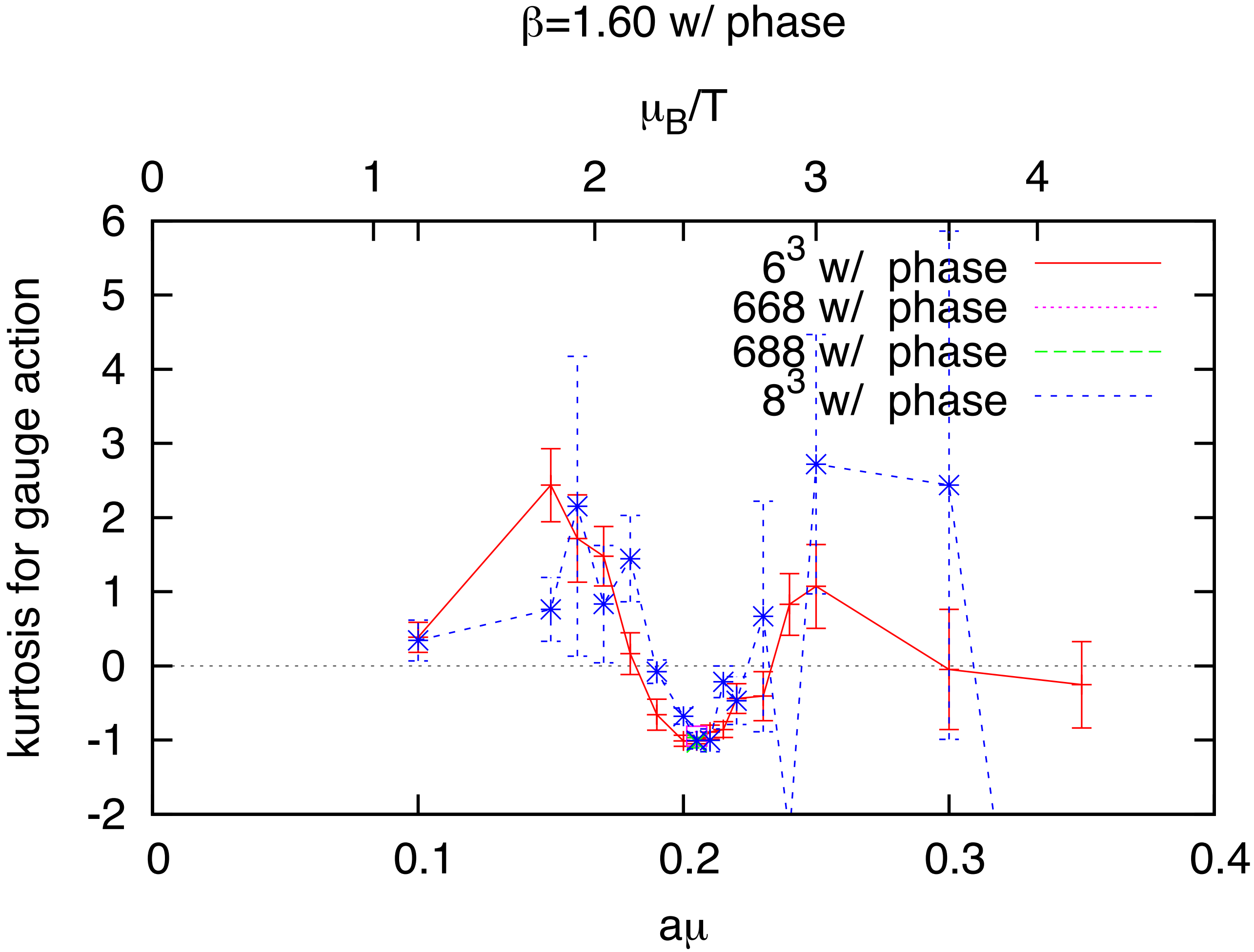
Susceptibility of gauge action



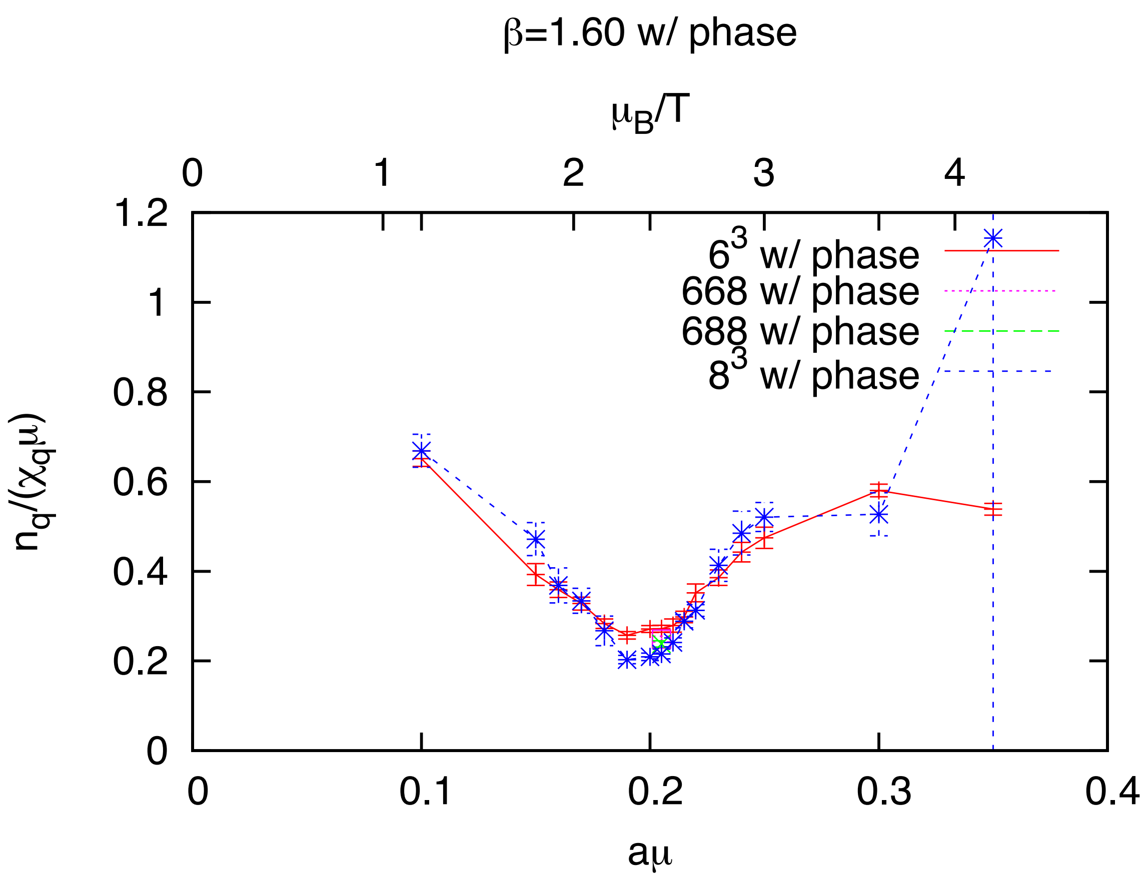
Skewness of gauge action



Kurtosis of gauge action

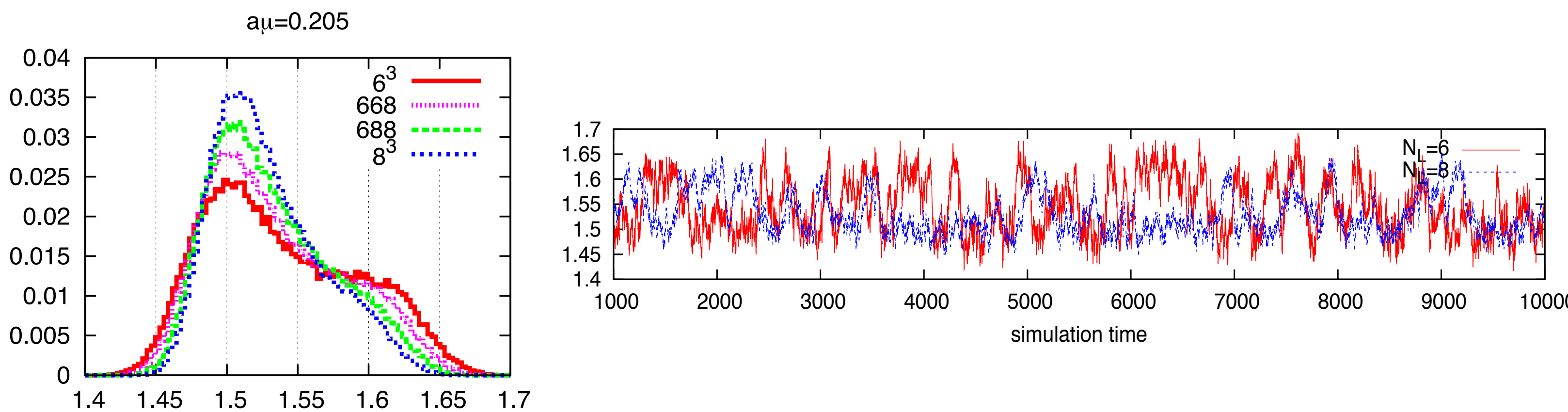


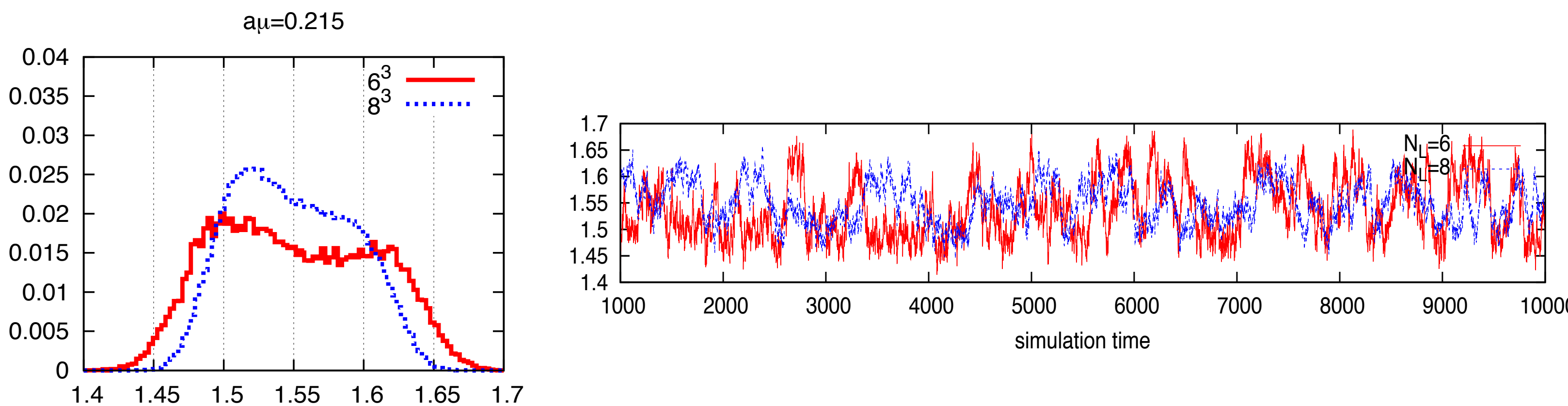
Ratio



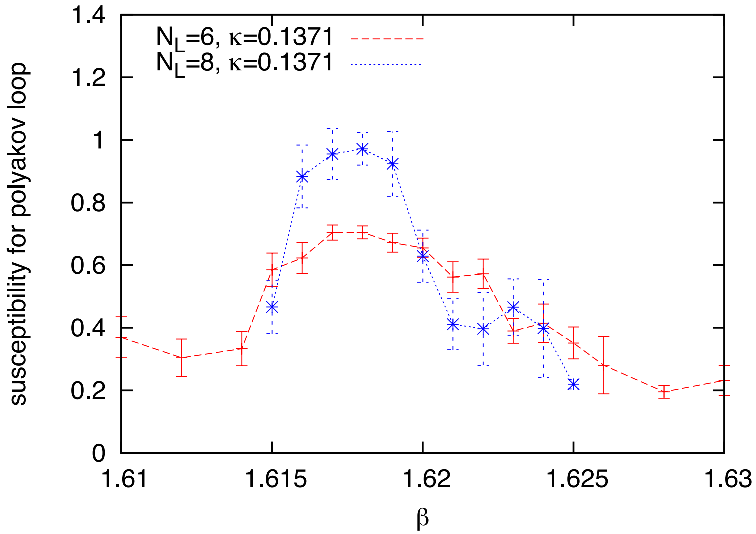
β=1.60 w/ phase
μB/T
0
1
2
3
4
1.2
1
0.8
0.6
0.4
0.2
0
6³ w/ phase
668 w/ phase
688 w/ phase
8³ w/ phase
nq/(χqμ)
0
0.1
0.2
0.3
0.4
aμ

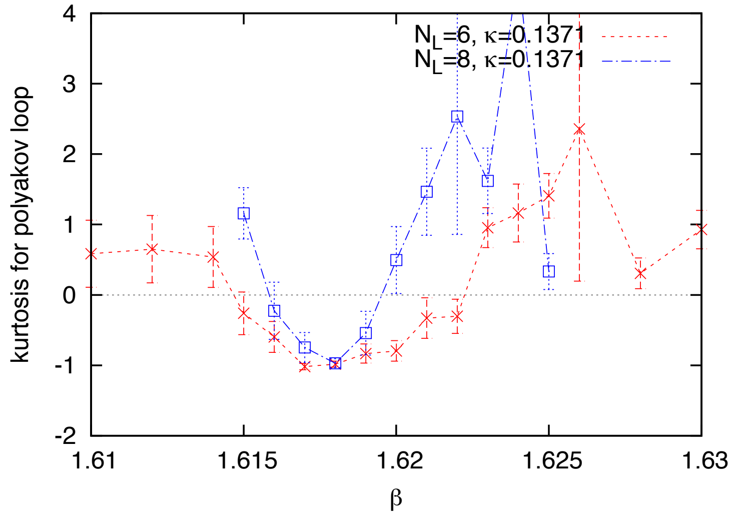
History & histogram of gauge action on the phase quenched configuration





Zero density finite temperature transition





Constant physics with rescaling cut off

Constant physics

$$
\begin{aligned}
T &= 1/aN_{\mathrm{T}} \\
\mu & \\
V &= L^3 = (aN_{\mathrm{L}})^3 \\
m &
\end{aligned}
$$

Rescaling by factor b

$$
\begin{aligned}
a &\longrightarrow a/b \\
a\mu &\longrightarrow a\mu/b \\
N_{\mathrm{T}} &\longrightarrow bN_{\mathrm{T}} \\
N_{\mathrm{L}} &\longrightarrow bN_{\mathrm{L}} \\
\kappa &\longrightarrow \kappa' \approx \kappa(am \longrightarrow am/b)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\text{Bound of } |\theta|_{\text{after}}}{\text{Bound of } |\theta|_{\text{before}}} &= \frac{12(bN_{\mathrm{L}})^3(2\kappa')^{bN_{\mathrm{T}}}\sinh(\mu/T)}{12(N_{\mathrm{L}})^3(2\kappa)^{N_{\mathrm{T}}}\sinh(\mu/T)} \\
&= b^3(2\kappa)^{N_{\mathrm{T}}(b-1)}
\end{aligned}
$$

Constant physics?