Equation of state from $N_f = 2$ twisted mass lattice QCD

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for the tmfT Collaboration:

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Outline

Motivation

- Explore finite T phase transition/crossover for $N_f = 2$ QCD at vanishing chemical potential
- Order of transition in the chiral limit not known yet for $N_f = 2$
- Differences in the results for the thermodynamic equation of state from different staggered simulations
- Other discretizations of QCD worthwhile to study systematics and universality
- Useful to study onset of mass thresholds in thermodynamics

Lattice Setup

- $N_f = 2$ lattice QCD with Wilson fermions at maximal twist.

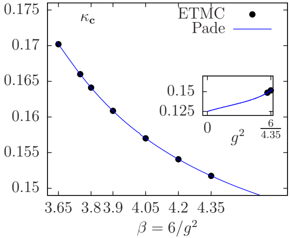
$$S_f[U,\psi,\overline{\psi}] = \sum_x \overline{\chi}(x)\left(1 - \kappa H[U] + 2i\kappa a\mu\gamma_5\tau^3\right)\chi(x)$$

- Tree level improved gauge sector:

$$S_g[U] = \beta\Big(c_0 \sum_P [1 - \frac{1}{3}\mathrm{ReTr}\,(U_P)] + c_1 \sum_{\mathrm{R}} [1 - \frac{1}{3}\mathrm{ReTr}\,(U_R)]\Big)$$

- κ tuned to critical value κ_c: Automatic $\mathcal{O}(a)$ improvement

- <u>β-scans</u> along $\kappa_c(\beta)$ with <u>fixed m_π</u>:

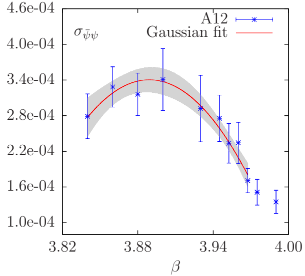


Outline

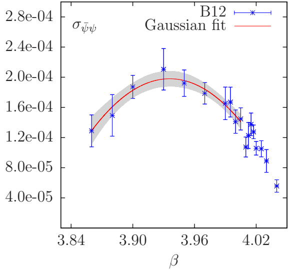
Suszeptibility of $\langle\bar{\psi}\psi\rangle$

$$\sigma_{\langle\bar{\psi}\psi\rangle} = V/T\left(\langle(\bar{\psi}\psi)^2\rangle - \langle\bar{\psi}\psi\rangle^2\right)$$

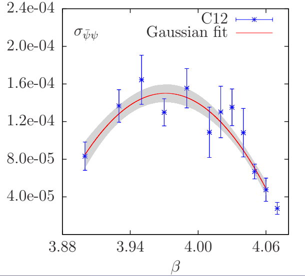
$m_\pi \approx 320$ MeV:



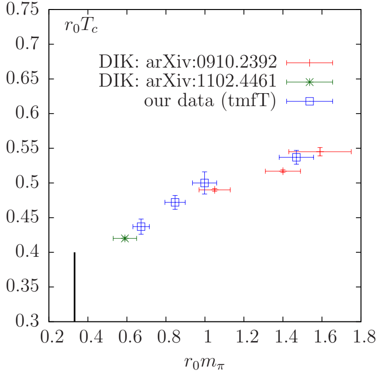
$m_\pi \approx 400$ MeV:



$m_\pi \approx 480$ MeV:

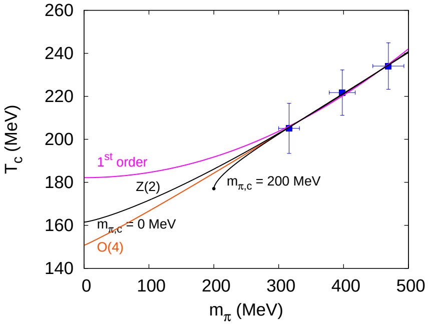


Comparison with QCDSF-DIK-Collaboration
(G. Schierholz et. al) from $\sigma_{\langle\bar{\psi}\psi\rangle}$:



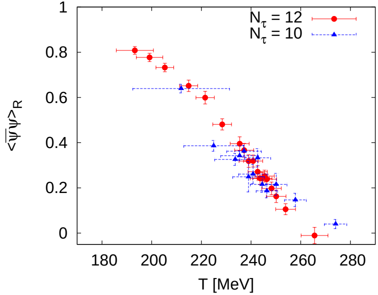
$$T_c(m_\pi) = T_c(0) + A m_\pi^{2/(\beta\delta)}$$

O(4): $T_c(0) = 152(26)$ MeV

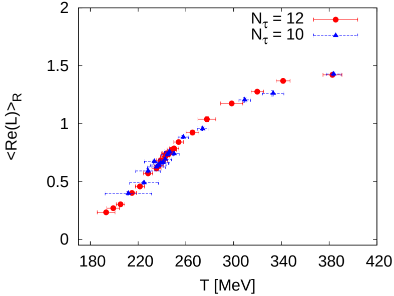


Renormalized Re(L), $\langle\bar{\psi}\psi\rangle$

$$\langle\overline{\psi}\psi\rangle_R = \frac{\langle\overline{\psi}\psi\rangle(T,\mu) - \langle\overline{\psi}\psi\rangle(0,\mu) + \langle\overline{\psi}\psi\rangle(0,0)}{\langle\overline{\psi}\psi\rangle(0,0)}$$



$$\langle \mathrm{Re}(L)\rangle_R = \mathrm{Re}(L)\exp\left(V(r_0)/2T\right)$$



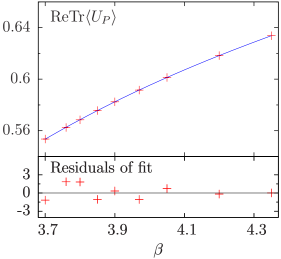
Outline

Trace Anomaly

- $$\frac{I}{T^4} = \frac{\epsilon - 3p}{T^4} = -\frac{T}{V}\left\langle \frac{d\ln Z}{d\ln a}\right\rangle_{\text{sub}}$$
$$= \left(a\frac{d\beta}{da}\right)\Bigg(c_0\left\langle \mathrm{ReTr}U_P\right\rangle_{\text{sub}} + c_1\left\langle \mathrm{ReTr}U_R\right\rangle_{\text{sub}}$$
$$+ \frac{\partial\kappa_c}{\partial\beta}\left\langle\bar{\chi}H[U]\chi\right\rangle_{\text{sub}} - \left(2a\mu\frac{\partial\kappa_c}{\partial\beta} + 2\kappa_c\frac{\partial(a\mu)}{\partial\beta}\right)\left\langle\bar{\chi}i\gamma_5\tau^3\chi\right\rangle_{\text{sub}}\Bigg)$$
- Starting point for $p(T)$ and $\epsilon(T)$ by integral method
- subtracted expectation values $\rightarrow$ interpolations for $T = 0$ data
- preliminary results for $m_\pi \approx 400$ MeV and $m_\pi \approx 700$ MeV

$T = 0$ interpolations, β-function

example: plaquette interpolation:

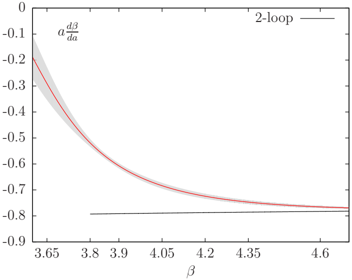


$$\left(a\frac{d\beta}{da}\right) = -\left(\frac{r_\chi}{a}\right)\left(\frac{d\left(\frac{r_\chi}{a}\right)}{d\beta}\right)^{-1}$$

$$\left(\frac{r_\chi}{a}\right)(\beta) = \frac{1+n_0 R(\beta)^2}{d_0(a_{2\mathrm{L}}(\beta)+d_1 R(\beta)^2)}$$

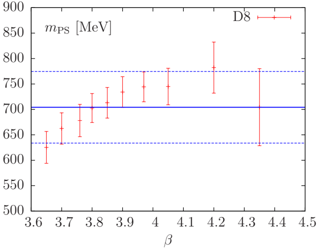
$$R(\beta) = \frac{a_{2\mathrm{L}}(\beta)}{a_{2\mathrm{L}}(3.9)}$$

[M. Cheng et al.: Phys.Rev. D77:014511, 2008]

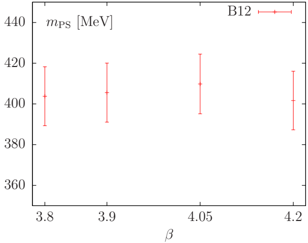


Lines of Constant Physics, constant m_π

$m_\pi \approx 700$ MeV: presently fulfilled up to 10 %



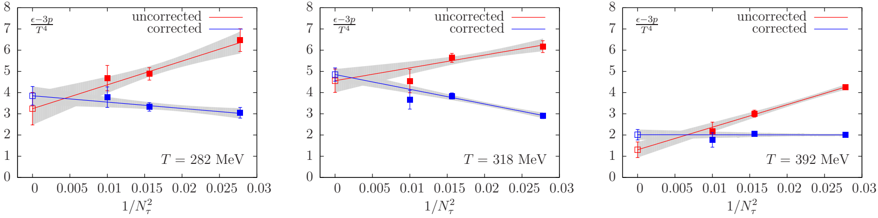
$m_\pi \approx 400$ MeV:



Trace anomaly, tree level corrections

- Observe large lattice artifacts in $\frac{I}{T^4}$
- Leading lattice corrections for $\frac{p_L}{p_{\mathrm{SB}}}$ twisted mass fermions
  [P. Hegde et al. Eur.Phys.J. C55 (2008)] [O. Philipsen, L. Zeidlewicz (2010)]
- Corrected by division by $\frac{p_L}{p_{\mathrm{SB}}}$ [S. Borsanyi: JHEP, 1011:077, 2010]

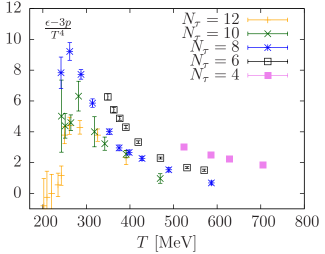
$m_\pi \approx 700$ MeV:

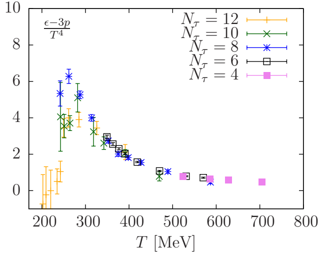


Tree level corrections II

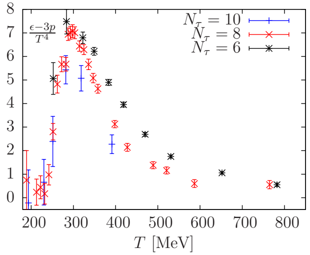
uncorrected **corrected**

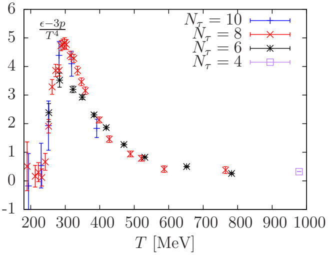
$m_\pi \approx 400$ MeV:





$m_\pi \approx 700$ MeV:





Interpolation of I/T^4, T integration (preliminary results)

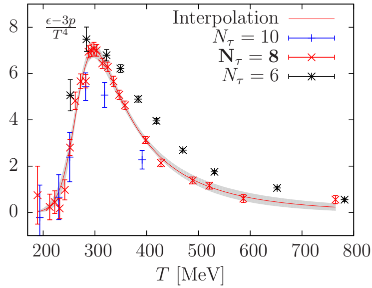
- $$\frac{p}{T^4} - \frac{p_0}{T_0^4} = \int_{T_0}^{T} d\tau \frac{\epsilon - 3p}{\tau^5}\Big|_{\mathrm{LCP}}$$

- Using interpolation for uncorrected I/T^4 :

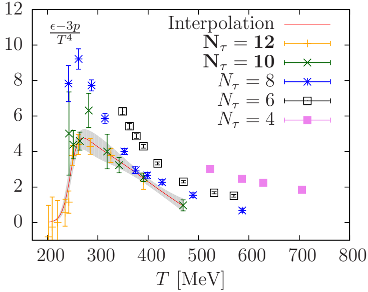
$$\frac{I}{T^4} = \exp\left(-h_1\bar{t} - h_2\bar{t}^2\right) \cdot \left(h_0 + \frac{f_0\{\tanh f_1\bar{t} + f_2\}}{1 + g_1\bar{t} + g_2\bar{t}^2}\right), \qquad \bar{t} = 300\ \mathrm{MeV}$$

[S. Borsanyi: JHEP, 1011:077, 2010]

$m_\pi \approx 700$ MeV:

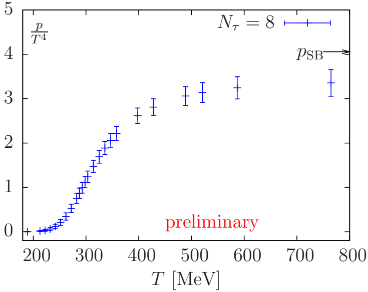


$m_\pi \approx 400$ MeV:

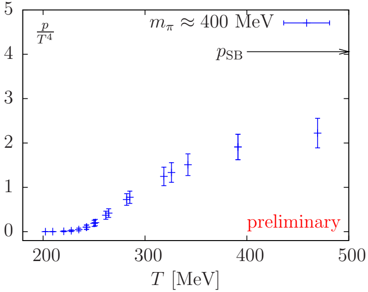


Pressure (preliminary results)

$m_\pi \approx 700$ MeV:



$m_\pi \approx 400$ MeV:



Outline

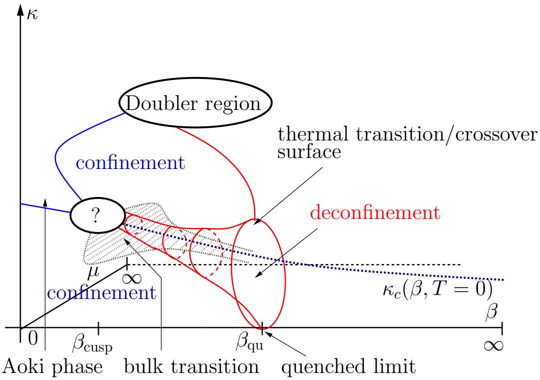
1. Introduction
2. T_c and chiral limit
3. Thermodynamic Equation of State
4. Outlook

Conclusions & Outlook

- Conclusions:
  - T_c for pion masses in the range 300 - 500 MeV
  - Thermodynamic Equation of State presented for two pion masses
  - Improvement on $T = 0$ interpolations and LCP on the way
- Outlook:
  - $N_f = 2 + 1 + 1$

Thank you

Phase Diagram:



[Phys.Rev.D80:094502, 2009]