A simple model with Z_N symmetry

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arXiv:hep-ph/1202.5584 (J. Phys. G),
arXiv:hep-ph/1204.0228.

Collaborators

Chiral and deconfinement transitions

Key ingredients in phase diagram	Chiral transition	Deconfinement transition
• Underlying symmetry • Order parameter	• Chiral symmetry • Chiral condensate	• Z_3 symmetry? • Polyakov loop?

Z_3 symmetry is exact only in the heavy-quark limit.
Is Z_3 symmetry a good concept for light quarks?

QCD action in Euclidean space-time

$$S_0 = \int d^4x [\sum_f \bar{q}_f(\gamma_\nu D_\nu + m_f)q_f + \frac{1}{4g^2} F^{a\ 2}_{\mu\nu}],$$

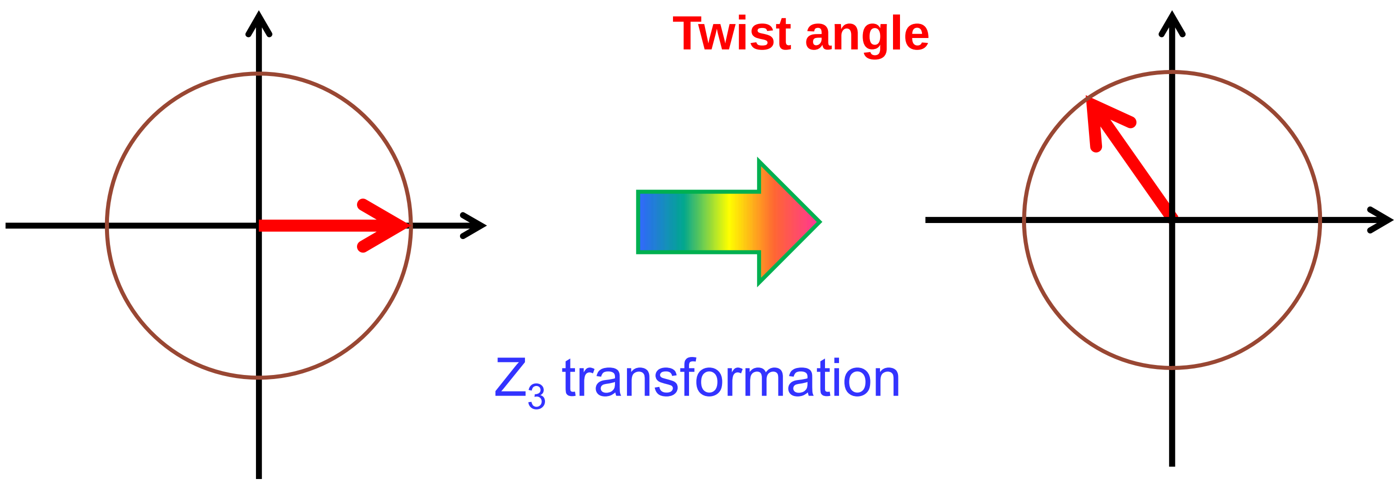
Invariant under Z_3 transformation

$$q \rightarrow Uq, \qquad A \rightarrow UAU^{-1} - \frac{i}{g}(\partial U)U^{-1},$$

$$U(x, \beta) = \exp[i 2\pi / 3] U(x, 0)$$

Fermion boundary condition

$$q_f(x, \beta = 1/T) = -\exp[i\theta] q_f(x, 0)$$



$$\theta = 0 \qquad\qquad \theta = 2\pi/3$$

Standard boundary condition

Z_3 symmetry is a proper concept for light quarks?

Purpose

To propose a simple model with Z_3 symmetry, by changing QCD slightly, and understand roles of Z_3 symmetry in phase diagram.

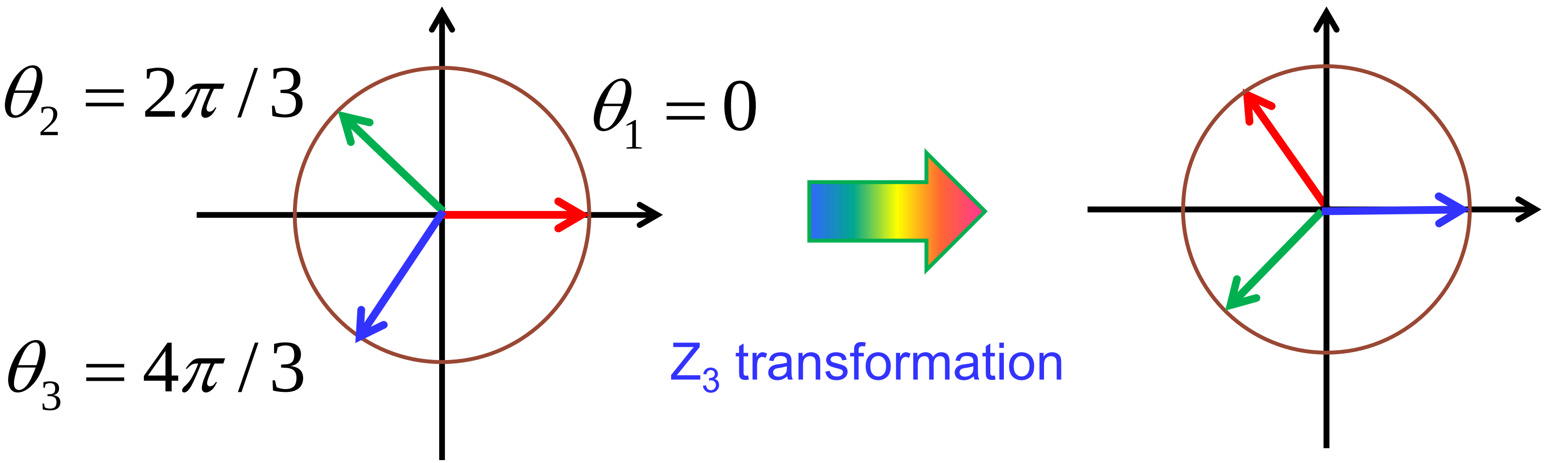
↓

Degenerate three-flavor system

$$N_c = N_f = 3, \quad m_1 = m_2 = m_3$$

Flavor-dependent twist boundary condition (TBC)

$$q_f(x, \beta = 1/T) = -\exp[i\theta_f]q_f(x, 0)$$



QCD with TBC is Z_3 invariant.

Understanding of QCD with TBC

Apply TBC to the Polyakov-loop Nambu-Jona-Lasinio (PNJL) model

TBC model=a simple model with Z_3 symmetry

The setup of TBC model

$$\boxed{D_\nu = \partial_\nu + iA_\nu = \partial_\nu + i\delta_{\nu,4}A_{4,a}\tilde{\lambda}_a/2}$$

PNJL Lagrangian

quark part (Nambu-Jona-Lasinio type)

$$\mathcal{L} = \sum_f \bar{q}_f(\gamma_\nu D_\nu - \mu_f\gamma_4 + m_f)q_f - G_\mathrm{S}\sum_f\sum_{a=0}^{8}[(\bar{q}_f\lambda_a q_f)^2 + (\bar{q}_f i\gamma_5\lambda_a q_f)^2] + G_\mathrm{D}\left[\det_{ij}\bar{q}_i(1+\gamma_5)q_j + \det_{ij}\bar{q}_i(1-\gamma_5)q_j\right] + \mathcal{U}(\Phi[A],\bar{\Phi}[A],T),$$

gluon potential

$$\boxed{\Phi = \frac{1}{3}\mathrm{Tr}_c e^{-iA_4/T}}$$

TBC

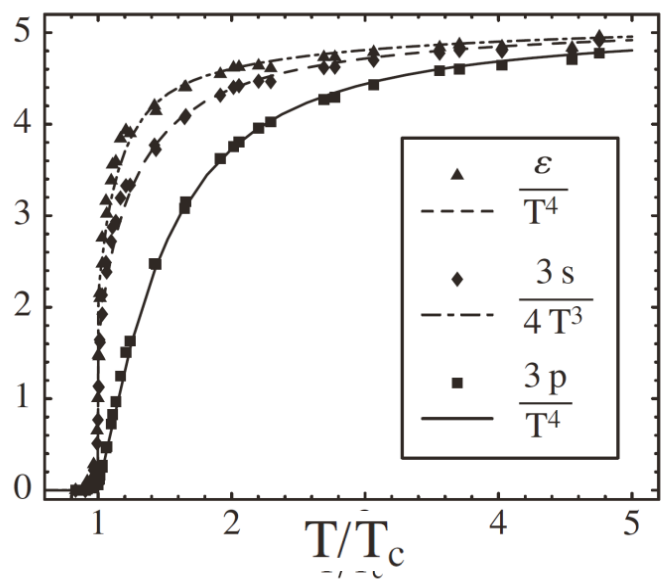
$$q_f(x,\beta=1/T) = -\exp[i\theta_f]q_f(x,0)$$

Polyakov potential

$$\mathcal{U} = T^4 \Big[-\frac{a(T)}{2} \Phi^* \Phi + b(T) \ln(1 - 6\Phi\Phi^* + 4(\Phi^3 + \Phi^{*3}) - 3(\Phi\Phi^*)^2) \Big],$$

$$a(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2, \quad b(T) = b_3 \left(\frac{T_0}{T}\right)^3.$$

Rößner, Ratti, Weise; PRD**75**



It reproduces the lattice data in the pure gauge limit.

Parameter set for 2+1 flavors

(a)	a_0	a_1	a_2	b_3	T_0(MeV)
	3.51	-2.47	15.2	-1.75	195
(b)	m_0(MeV)	Λ(MeV)	$G_s\Lambda^2$	$G_D\Lambda^5$	
	5.5	602.3	1.835	12.36	

determined from $(M_\pi, f_\pi, M_K, M_{\eta'})$

The thermodynamic potential is calculated by the mean-field approximation.

Thermodynamic potential

$$
\begin{aligned}
\Omega = -2 \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \Big[& N_c E_f \\
& + \frac{1}{\beta} \ln[1 + \boxed{C_{3,1}(\mathbf{p}) e^{i\theta_f}} \\
& + \boxed{C_{3,2}(\mathbf{p}) e^{2i\theta_f}} + C_{3,3}(\mathbf{p}) e^{3i\theta_f}] \\
& + \frac{1}{\beta} \ln[1 + \boxed{C^*_{3,1}(\mathbf{p}) e^{-i\theta_f}} e^{-\beta E_f} \\
& + \boxed{C^*_{3,2}(\mathbf{p}) e^{-2i\theta_f}} + C^*_{3,3}(\mathbf{p}) e^{-3i\theta_f}] \Big] \\
& + U(\sigma_u, \sigma_d, \sigma_s) + \mathcal{U}(\Phi, \Phi^*, T),
\end{aligned}
$$

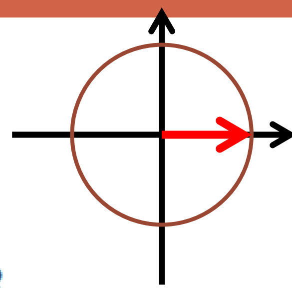
$$
E_f \equiv \sqrt{\mathbf{p}^3 + {M_f}^2}
$$

$$
\begin{aligned}
C_{3,1}(\mathbf{p}) &= 3\Phi e^{-\beta E_f}, \\
C_{3,2}(\mathbf{p}) &= 3\Phi^* e^{-2\beta E_f}, \\
C_{3,3}(\mathbf{p}) &= e^{-3\beta E_f}.
\end{aligned}
$$

No flavor dependence in the confinement phase.

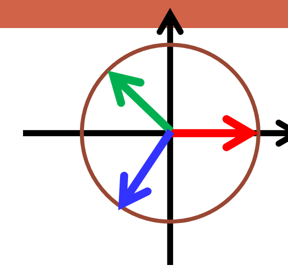
PNJL-model and TBC-model

PNJL model

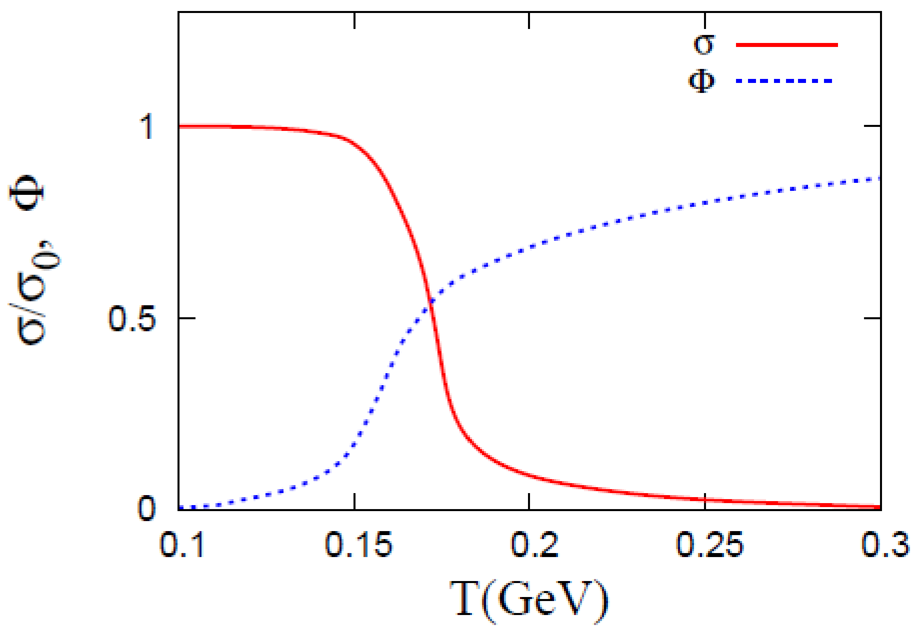


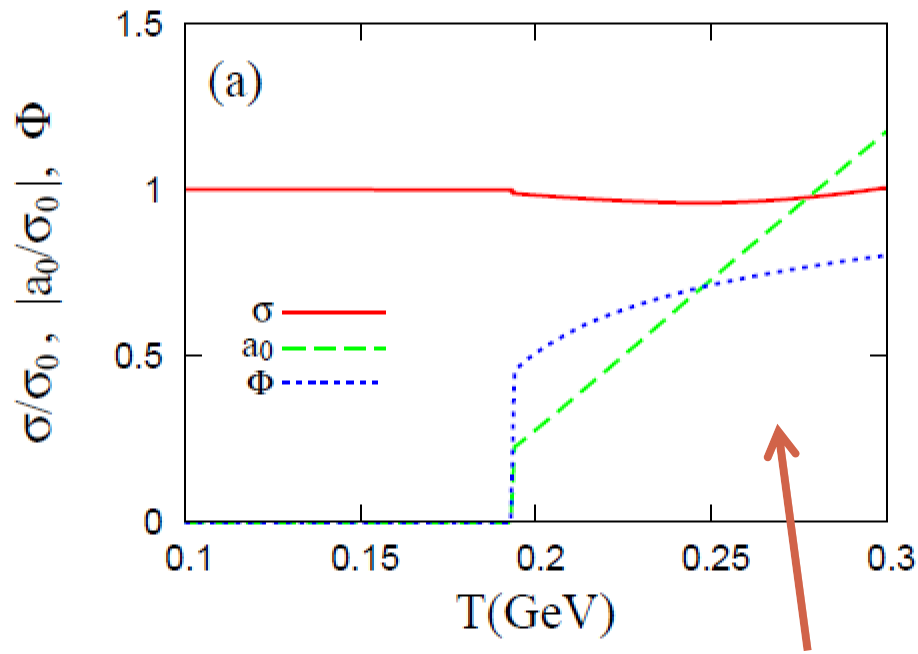
$$\sigma \equiv (\sigma_u + \sigma_d + \sigma_s)/3$$

TBC model



$$a_0 \equiv \sigma_u - \sigma_d = \sigma_u - \sigma_s$$



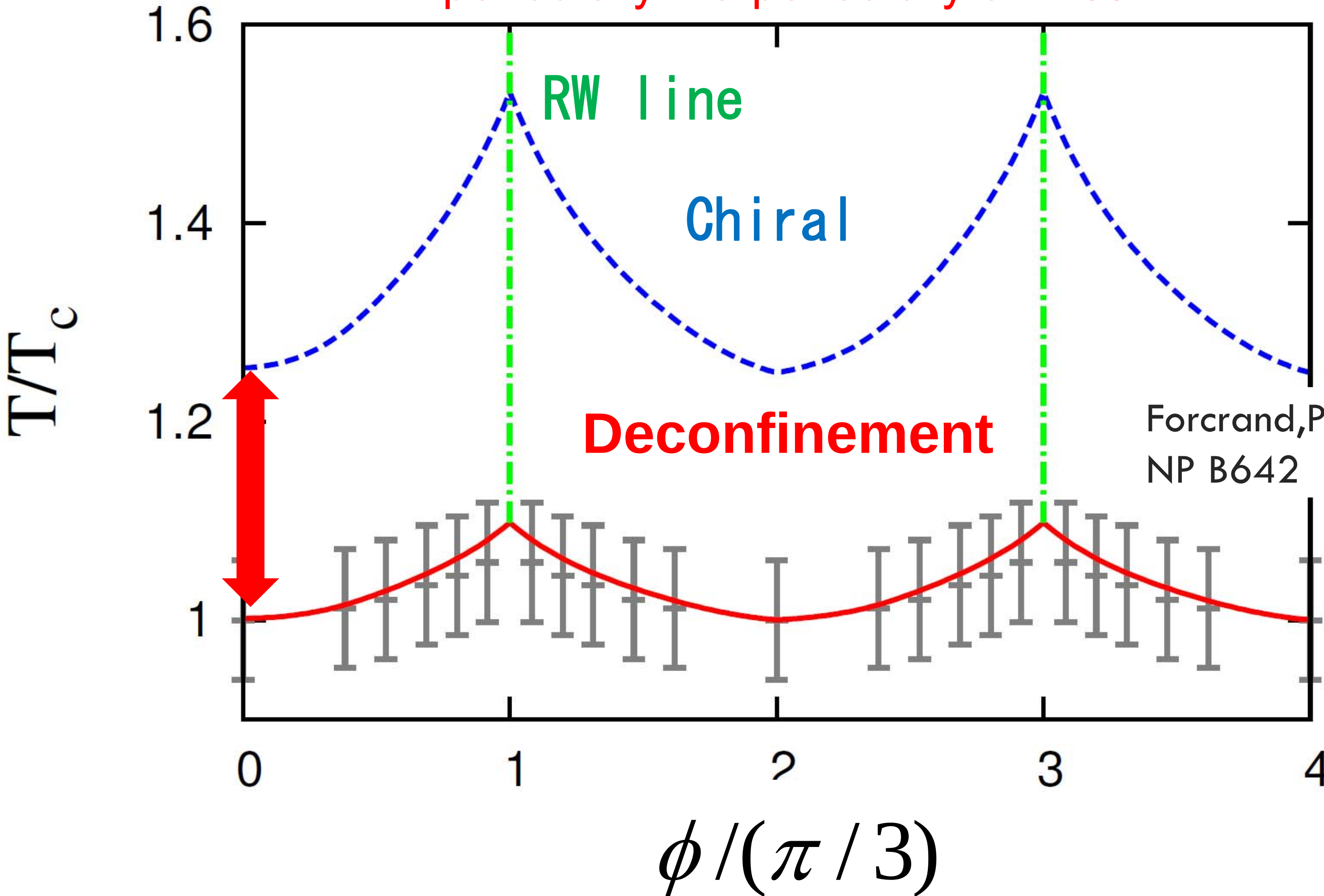


SSB of Z_3 symmetry

Phase diagram at imaginary chemical potential

$$\mu = i\phi T$$

RW periodicity: a periodicity of 2π/3



Forcrand,Philipsen,
NP B642

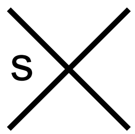
Entanglement vertex

PNJL with an effective four-quark vertex depending on Polyakov loop

Y. Sakai, T. Sasaki, H. Kouno, and M. Yahiro, Phys.Rev.D82:076003,2010.

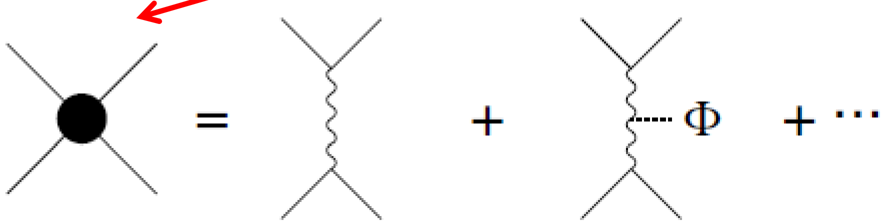
Entanglement interactions such as $G_s(\Phi)\sigma^2$

PNJL(TBC) model



G_s

$$G_s(\Phi) = G_s[1 - \alpha_1 \Phi\Phi^* - \alpha_2(\Phi^3 + \Phi^{*3})].$$



Renormalization group method:
K.-I. Kondo,
Phys. Rev. D82, 065024 (2010).

$$A_0^a \delta^{a3} = \left\langle A_0^a \right\rangle \delta^{a3} + a_0^a \delta^{a3}$$

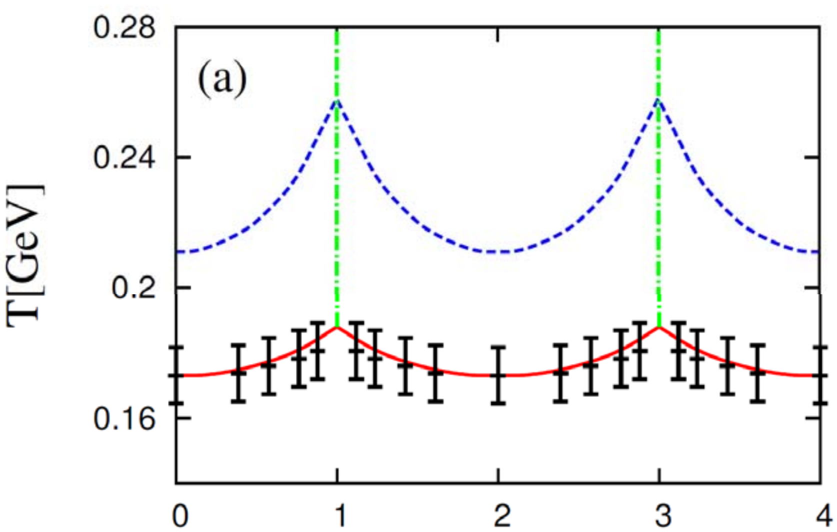
$$A_0^a \delta^{a8} = \left\langle A_0^a \right\rangle \delta^{a8} + a_0^a \delta^{a8}$$

$$\Phi = Tr\left[e^{i\beta\left\langle A_0^3 \right\rangle \lambda^3 + i\beta\left\langle A_0^8 \right\rangle \lambda^8} \right]$$

Entanglement PNJL (EPNJL)

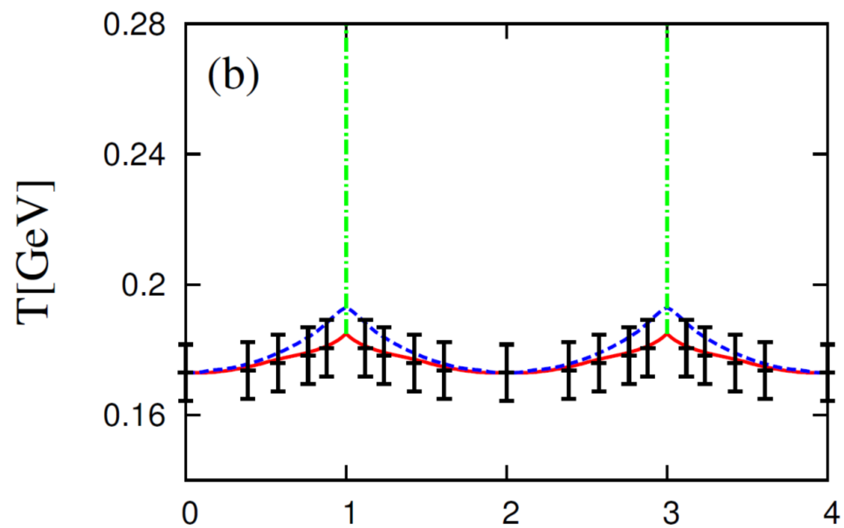
Two flavor system with imaginary chemical potential

Phase Diagram by original PNJL



$\phi/(2\pi/3)$

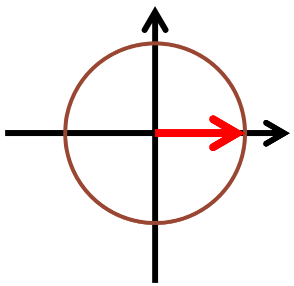
Phase Diagram by entanglement- PNJL



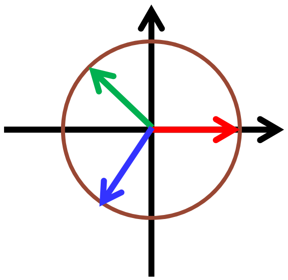
$\phi/(2\pi/3)$

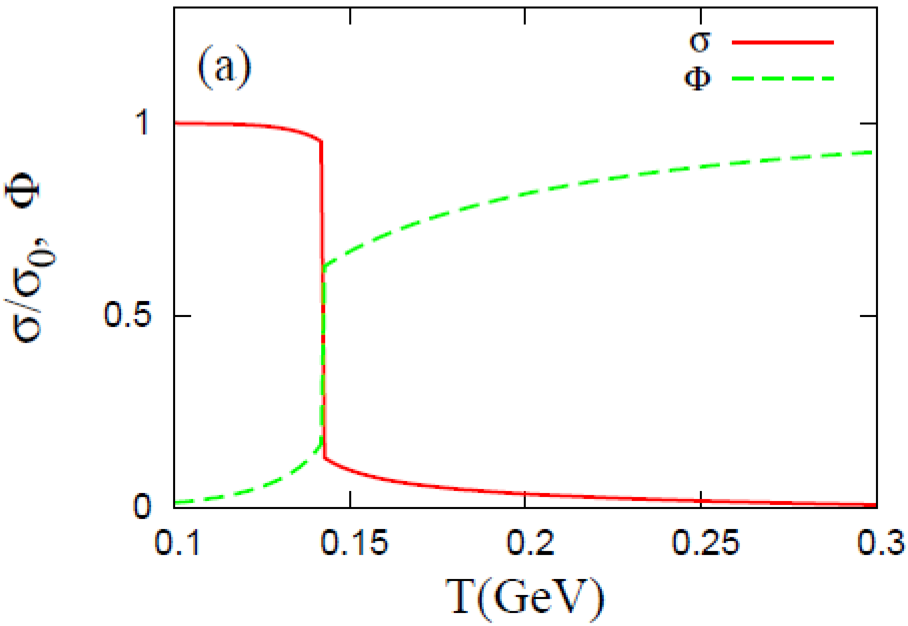
Entanglement-PNJL model for $N_f=N_c=3$

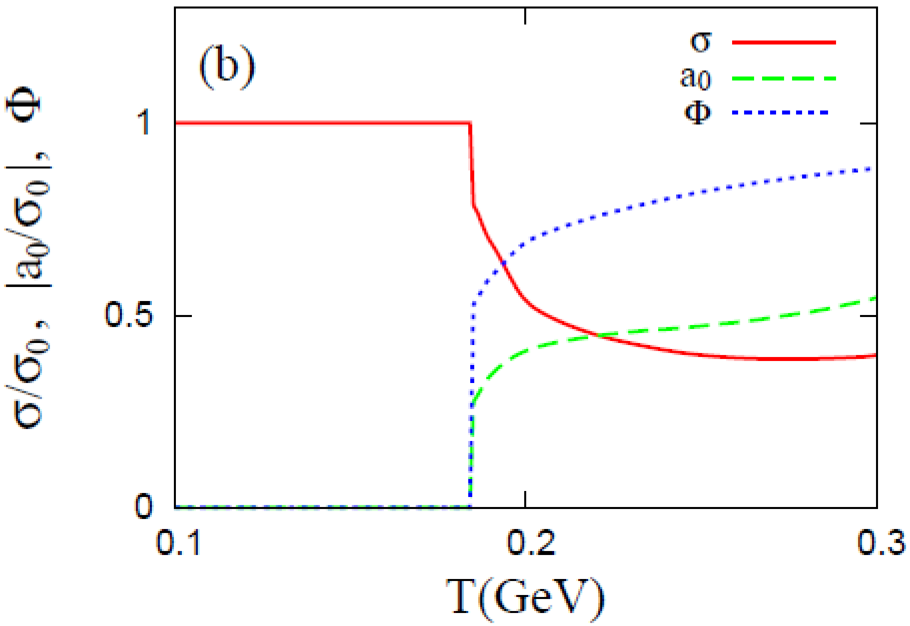
EPNJL model



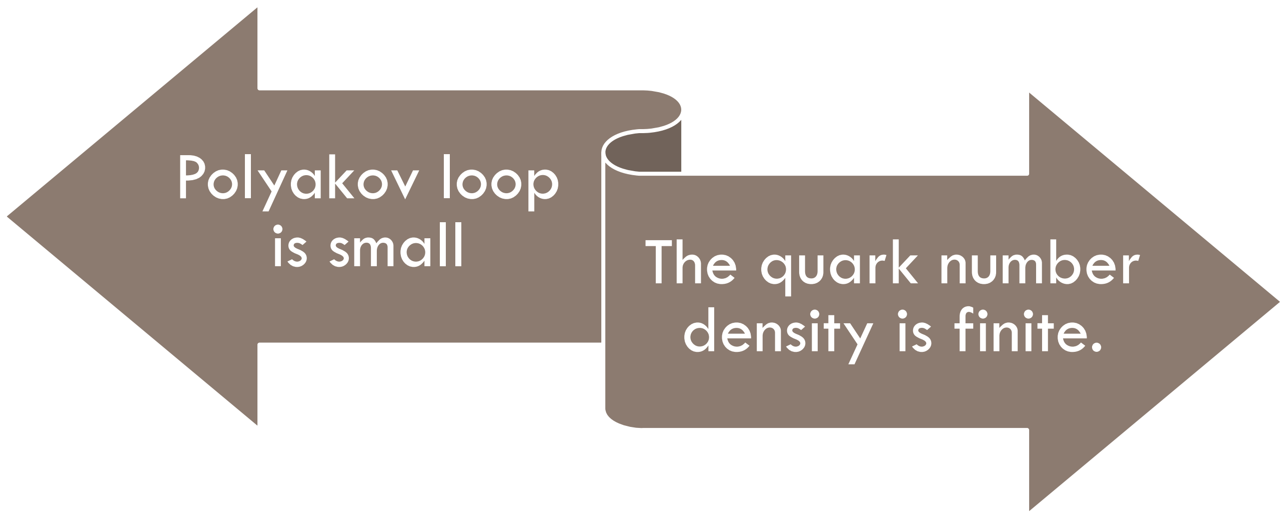
TBC model





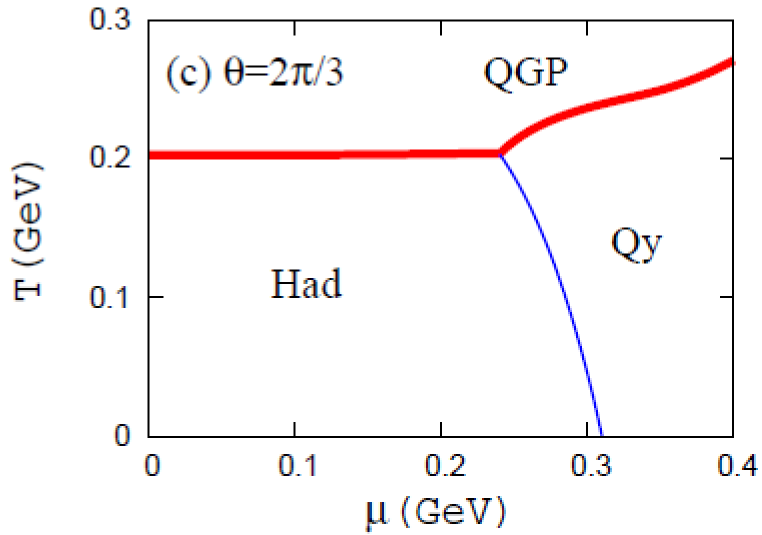


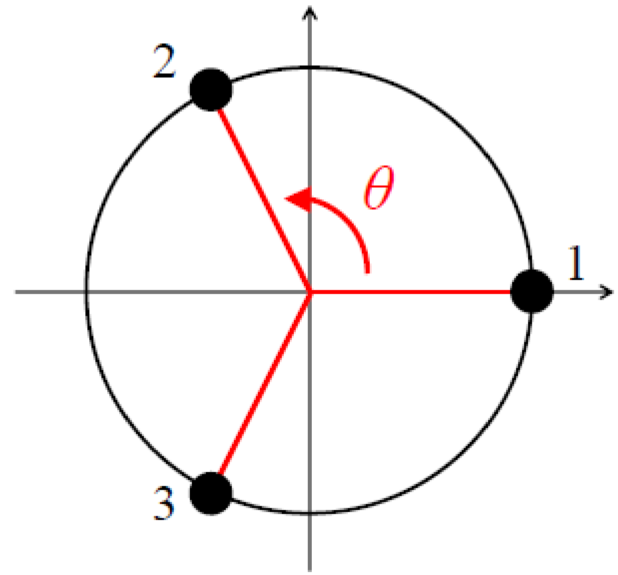
Z_3 symmetry and the quarkyonic phase



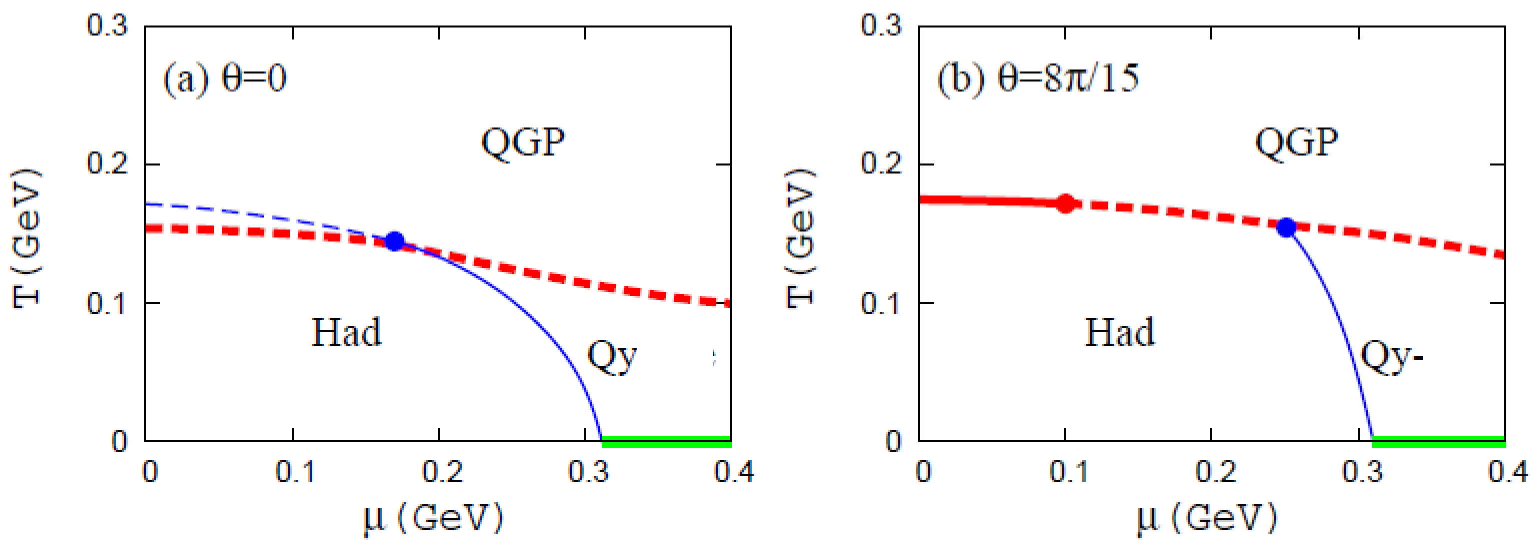
[1] L.McLerran, and R. D. Pisarski, Nucl. Phys. A796, 83 (2007).
[2] Y. Hidaka, L.McLerran, and R. D. Pisarski,Nucl. Phys. A808, 117 (2008).

TBC

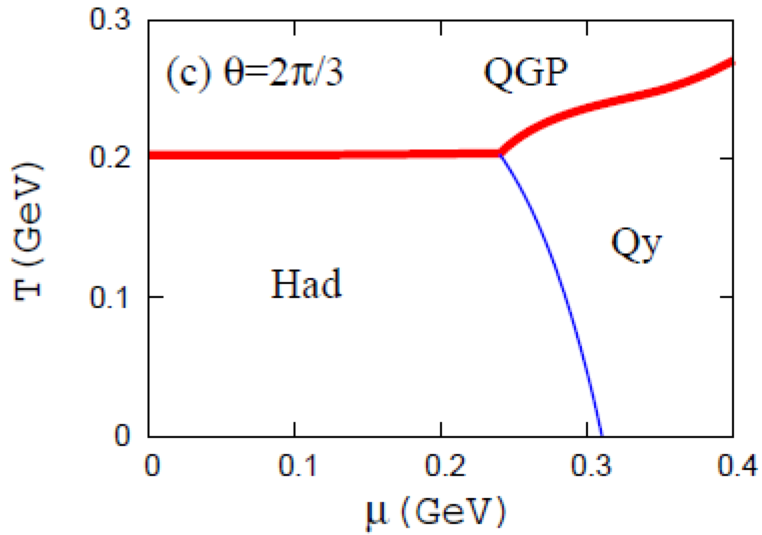

(c) θ=2π/3
QGP
Qy
Had
T (GeV)
μ (GeV)
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4

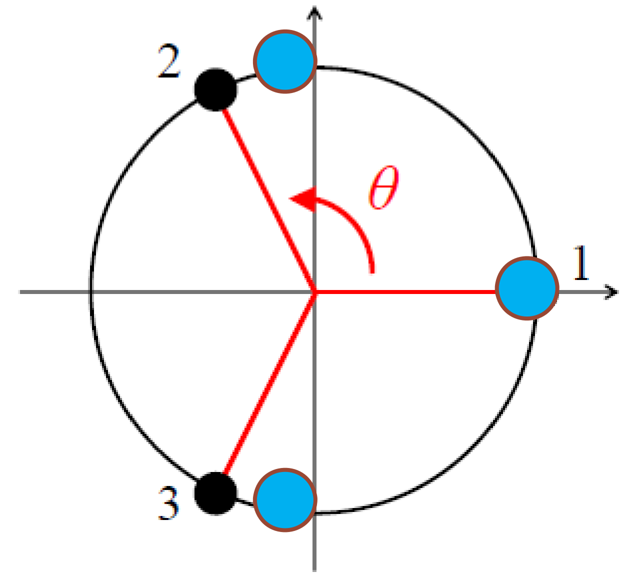

2
1
3
θ

PNJL

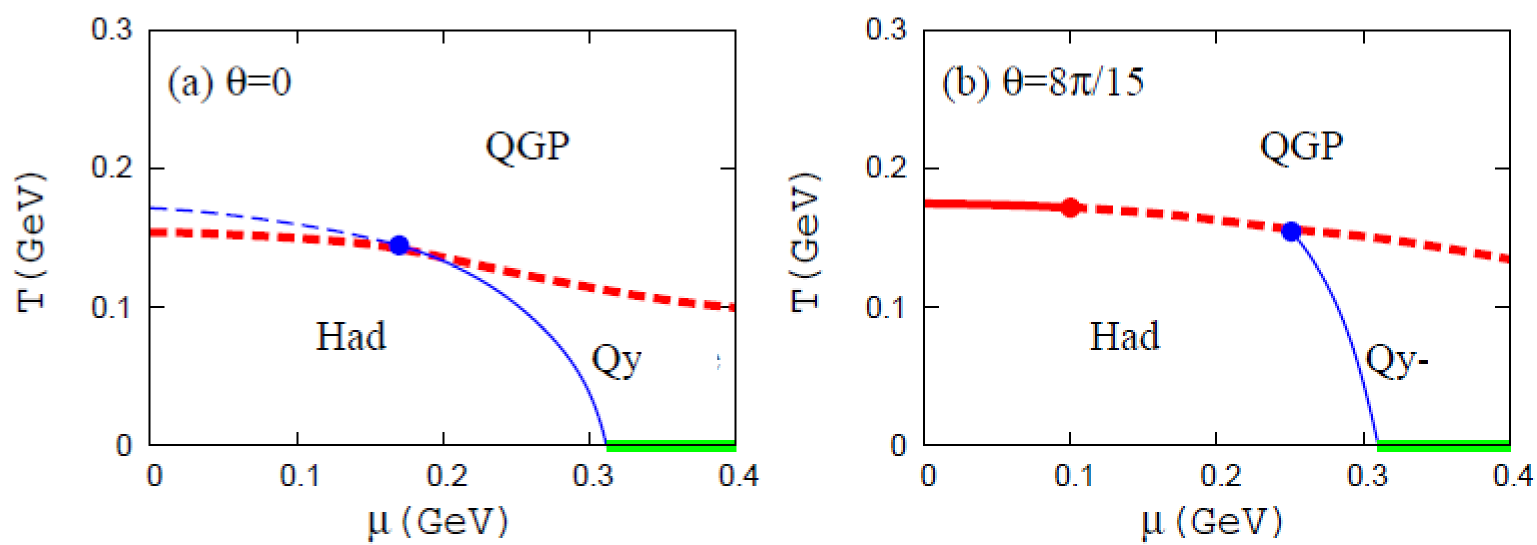

(a) θ=0
QGP
Had
Qy
T (GeV)
μ (GeV)
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
(b) θ=8π/15
QGP
Had
Qy-
T (GeV)
μ (GeV)
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4

TBC


(c) θ=2π/3
QGP
Qy
Had
0.3
0.2
0.1
0
T (GeV)
0
0.1
0.2
0.3
0.4
μ (GeV)


2
θ
1
3

PNJL


(a) θ=0
QGP
Had
Qy
0.3
0.2
0.1
0
T (GeV)
0
0.1
0.2
0.3
0.4
μ (GeV)
(b) θ=8π/15
QGP
Had
Qy-
0.3
0.2
0.1
0
T (GeV)
0
0.1
0.2
0.3
0.4
μ (GeV)

Summary

1. First, we proposed a simple model with Z_3 symmetry in order to understand roles of Z_3 symmetry in the QCD phase diagram.
2. Z_3 symmetry makes the deconfinement transition stronger and the critical temperature higher.
3. Second, we discussed the relation between Z_3 symmetry and the quarkyonic phase.
4. The quarkyonic phase is well separated from the QGP and hadron phases by the first-order deconfinement and chiral transition lines, when Z_3 symmetry is preserved exactly.
5. Throughout these analyses, QCD with TBC seems to be not far from real QCD from the qualitative point of view. This means that Z_3 symmetry may be a good concept in real QCD.