Monte Carlo simulation of abelian Gauge-Higgs lattice models using dual representations

Alexander Schmidt and Christof Gattringer

University of Graz
Institute of Physics

June 25, 2012

What? Why?

What are we doing?

1. We rewrite the action of an abelian gauge theory to an alternative representation in terms of **dual variables** i.e., loops and surfaces.
2. We do lattice Monte-Carlo simulations for non-vanishing chemical potential using the dual variables.

Why are we doing this?

1. The *dual approach* solves the complex action-problem.
2. This is the first dual simulation for a system with gauge- and matter fields, i.e., surfaces and loops.
3. Improve techniques for simulations with dual variables.

Partition sum Z of the conventional representation

Partition sum Z, Gauge-action S_G and Higgs-action S_H

$$
\begin{aligned}
Z &= \sum_{\{\phi,U\}} e^{-S_G-S_H} \quad , \\
S_G &= -\frac{\beta}{2} \sum_x \sum_{\sigma<\tau} \left[U_{x,\sigma\tau} + U^\star_{x,\sigma\tau} \right] \quad , \\
S_H &= -\kappa \sum_{x,\nu} \left[e^{\mu\delta_{\nu,4}} \phi^\star_x U_{x,\nu} \phi_{x+\widehat{\nu}} + e^{-\mu\delta_{\nu,4}} \phi^\star_x U^\star_{x-\widehat{\nu},\nu} \phi_{x-\widehat{\nu}} \right] \quad .
\end{aligned}
$$

In this example: **Gauge fields $U_{x,\nu}$ and Higgs fields $\phi_x \in Z_3$**

$$
\phi_x = e^{is_x 2\pi/3} , \quad U_{x,\sigma} = e^{ia_{x,\sigma} 2\pi/3} , \quad s_x, a_{x,\sigma} \in \{-1,0,1\} \quad .
$$

Other abelian groups can be treated in a similiar manner.

We will make use of the identities I_1 and I_2

Identity I_1

$$\exp\left(\kappa e^{i\frac{2\pi}{3}s} + \kappa e^{-i\frac{2\pi}{3}s}\right) = C_\kappa \sum_{k=-1}^{1} B_\kappa^{|k|}\, e^{i\frac{2\pi}{3}sk} \quad , \quad s = -1, 0, 1 \,,$$

$$C_\kappa = \frac{e^{2\kappa} + 2e^{-\kappa}}{3} \quad , \qquad B_\kappa = \frac{e^{2\kappa} - e^{-\kappa}}{e^{2\kappa} + 2e^{-\kappa}} \,.$$

Identity I_2

$$\exp\left(\kappa e^{\mu} e^{i\frac{2\pi}{3}s} + \kappa e^{-\mu} e^{-i\frac{2\pi}{3}s}\right) = \sum_{k=-1}^{1} M_k\, e^{i\frac{2\pi}{3}sk} \quad , \quad s = -1, 0, 1 \,,$$

$$M_k = \frac{1}{3}\left[e^{2\kappa\cosh(\mu)} + 2e^{-\kappa\cosh(\mu)} \cos\left(\sqrt{3}\kappa\sinh(\mu) - k\frac{2\pi}{3}\right)\right] \,.$$

(character expansion)

To give an idea how we are using I_1 and I_2

The Boltzmann factor e^{-S} with

$$S_H \;=\; -\kappa \sum_{x,\nu} \left[e^{\mu\delta_{\nu,4}} \phi_x^\star U_{x,\nu} \phi_{x+\widehat{\nu}} + e^{-\mu\delta_{\nu,4}} \phi_x^\star U_{x-\widehat{\nu},\nu}^\star \phi_{x-\widehat{\nu}} \right] \quad .$$

Is written as a product of local terms such as

$$\prod_x \exp\left(\kappa\, e^{\mu} e^{i\frac{2\pi}{3}[s_{x+\widehat{4}} - s_x + a_{x,4}]} + \kappa\, e^{-\mu} e^{-i\frac{2\pi}{3}[s_{x+\widehat{4}} - s_x + a_{x,4}]} \right)$$

$$= \prod_x M_{k_{x,4}}\, e^{i\frac{2\pi}{3}\left[s_{x+\widehat{4}} - s_x + a_{x,4}\right] k_{x,4}} \quad ,$$

which we expand using I_1 and I_2.

A similiar expansion is used for the gauge part of the action.

The full dual partition sum

$$Z = \sum_{\{p\}} \sum_{\{k\}} \mathcal{C}_P[p,k] \, \mathcal{C}_F[k] \, \mathcal{W}_P[p] \, \mathcal{W}_F[k] \, .$$

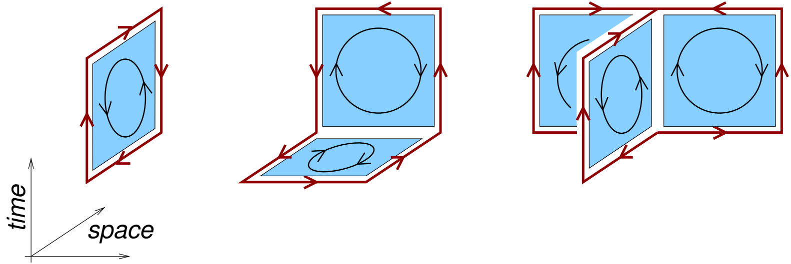
$\mathcal{C}$ ... constraints $\mathcal{W}$ ... weights (non-negative)

New dual degrees of freedom

- Plaquette occupation numbers $p_{x,\sigma\tau} \in \{-1, 0, 1\}$
- Flux occupation numbers $k_{x,\nu} \in \{-1, 0, 1\}$

Structure of the constraints.

Some admissible low-lying configurations:



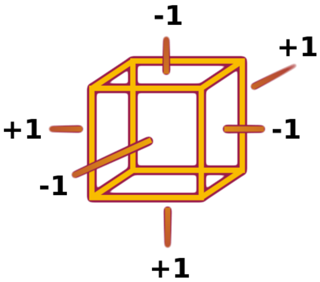
The two constraints:

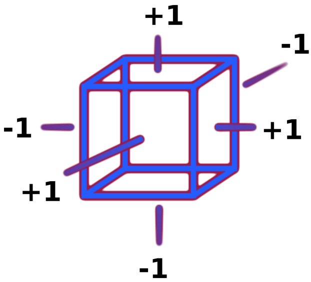
$\mathcal{C}_F[k]$: At each site the total flux from the k-variables has to be a multiple of 3.

$\mathcal{C}_P[p,k]$: For each link the combined flux of k-variables and plaquette occupation numbers p also has to be a multiple of 3.

Cube update + surface update (Gauge-field)

Two possible cube updates respecting the dual constraints (4 embeddings)



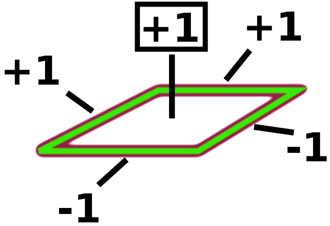


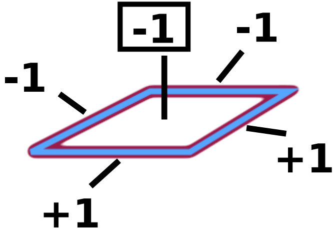
+ Surface update

Change plaquettes on coordinate planes by ± 1. This however updates only a finite volume correction.

Plaquette update (Gauge-field + Higgs-field)

Here we combine **link and plaquette updates** respecting the dual constraints. (6 embeddings)





Observables we look at

For the gauge sector we study the plaquette expecation value $\langle U \rangle$ and the corresponding susceptibility χ_U

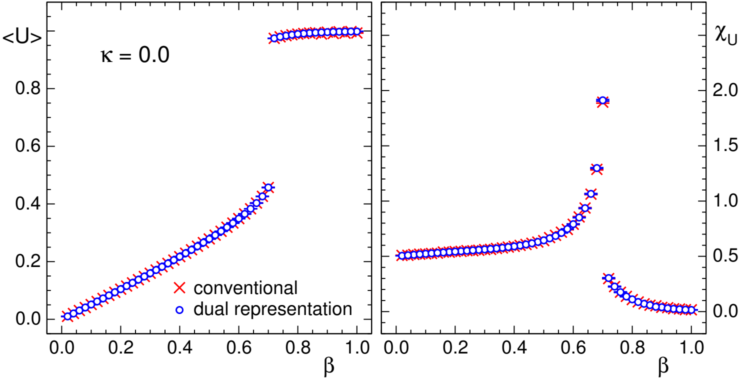
$$\langle U \rangle = \frac{1}{6 N_s^3 N_t} \frac{\partial}{\partial \beta} \ln Z \quad , \quad \chi_U = \frac{1}{6 N_s^3 N_t} \frac{\partial^2}{\partial \beta^2} \ln Z \,,$$

while in the Higgs-sector we additionally look at the particle number density $\langle n \rangle$ and its susceptibility χ_n

$$n = \frac{1}{N_s^3 N_t} \frac{\partial}{\partial \mu} \ln Z \quad , \quad \chi_n = \frac{1}{N_s^3 N_t} \frac{\partial^2}{\partial \mu^2} \ln Z \,.$$

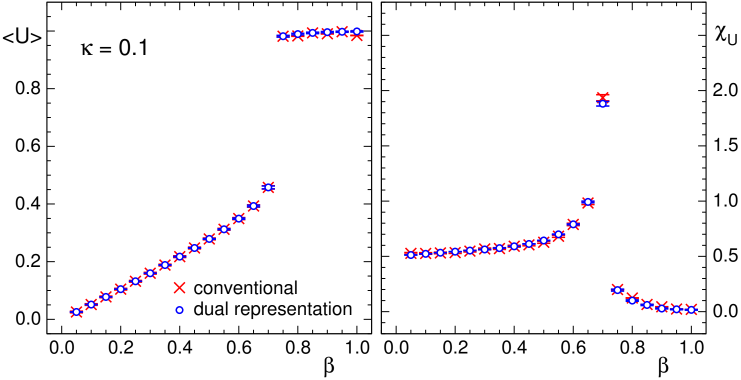
Perform the derivatives in the dual representation $\Rightarrow$ Observables are related to first and second moments of the dual variables.

Comparison for pure gauge-theory at $\kappa = 0$



- System undergoes a first order phase transition.
- Conventional simulation and flux-simulation results perfectly agree.

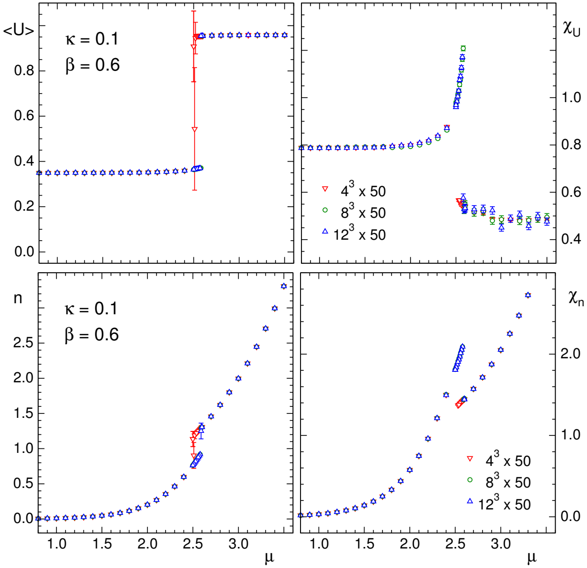
Comparison of the full simulation at $\kappa \neq 0$



- 1st order transition persists.
- Conventional simulation and flux-simulation results again perfectly agree.

Density driven transition at strong coupling

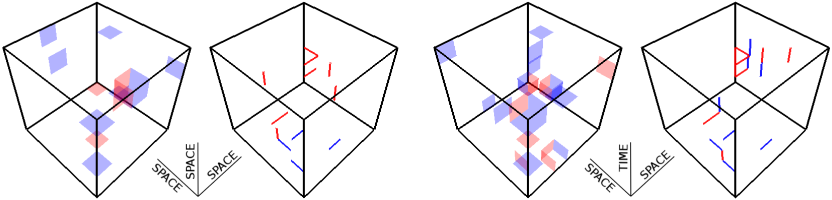
Narrow
1st order transition
separating a dilute
and a condensed phase.



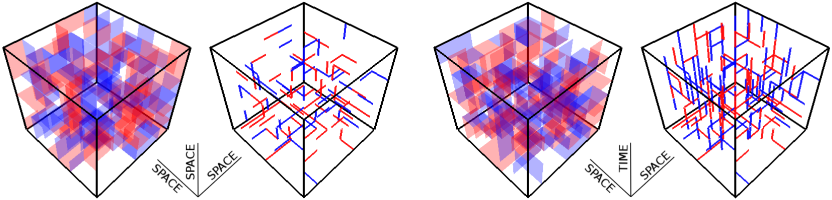
3d-illustration of plaquettes and fluxes

$\mu = 2.4$

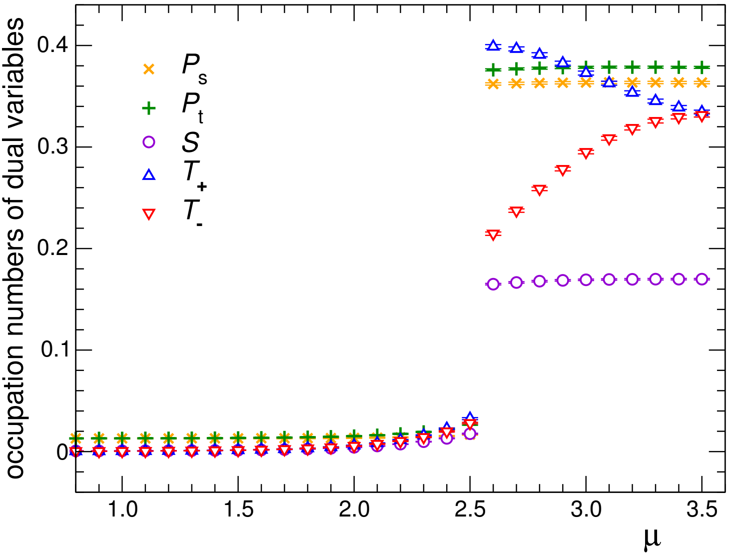
+1 **-1**



$\mu = 2.7$



A closer look at occupation numbers



Summary and Outlook

Summary

- We can deal with the sign-problem of the Z3-Gauge-Higgs model making use of the dual representation.
- It is possible to perform Monte-Carlo simulations directly in terms of the flux and surface variables.
- Interesting condensation transitions (also at weak coupling).
- Analysis of plaquette and flux occupation numbers reveals interesting interplay of gauge and matter fields in the condensation transitions.

Outlook

- Further studies of the Z3-Gauge-Higgs model.
- Try to develop similiar techniques for non-abelian gauge theories.