The leading-order hadronic contributions to a_μ and $\Delta\alpha_{\mathrm{QED}}(Q^2)$ from $N_f = 2 + 1 + 1$ twisted mass fermions

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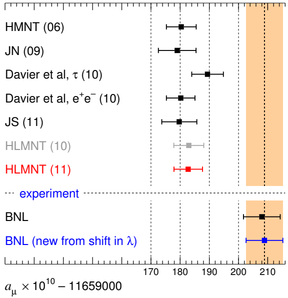
Why the muon's anomalous magnetic moment?

- The anomalous magnetic moment of the muon, a_μ, can be measured very precisely: [B. Lee Roberts, Chinese Phys. C 34, 2010]

$$a_\mu^{\text{exp}} = 116592089(63) \times 10^{-11}$$

$$a_\mu^{\text{SM}} = 116591828(49) \times 10^{-11}$$

[Hagiwara et al., arXiv:1105.3149 [hep-ph], 2011]



There is a $\approx 3\sigma$ discrepancy between a_μ^{exp} and a_μ^{SM}:

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(80) \times 10^{-11}$$

Charm quark necessary to reach required precision

Current discrepancy [Hagiwara et al., arXiv:1105.3149 [hep-ph], 2011]

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 261(80) \times 10^{-11}$$

- charm quark contribution from weighting experimental scattering data by flavours [Jegerlehner, Szafron, Eur. Phys. J C71, 2011]

$$a_\mu^{\text{hvp,c}} \gtrsim 100 \times 10^{-11}$$

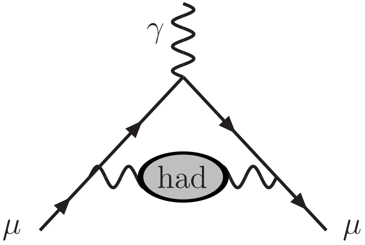
- comparable to hadronic light-by-light scattering contribution [Prades et al., arXiv:0901.0306 [hep-ph], 2009]

$$a_\mu^{\text{hlbl}} = 105(26) \times 10^{-11}$$

- and also to electroweak contribution [Jegerlehner, Nyffeler, Phys. Rept. 477 , 2009]

$$a_\mu^{\text{EW}} = 153(2) \times 10^{-11}$$

Leading hadronic contributions $a_\mu^{\rm hvp}$ and $\Delta\alpha_{\rm QED}^{\rm hvp}(Q^2)$



- can be computed directly in Euclidean space [T. Blum, PRL 91, 2003]

$$a_\mu^{\rm hvp} = \alpha_0^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{m_\mu^2}\right) \Pi_{\rm R}(Q^2)$$

where $\Pi_{\rm R}(Q^2) = \Pi(Q^2) - \Pi(0)$

- main ingredient: hadronic vacuum polarisation tensor

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot(x-y)} \langle J_\mu^{\rm em}(x) J_\nu^{\rm em}(y)\rangle = (Q_\mu Q_\nu - Q^2 g_{\mu\nu})\Pi(Q^2)$$

- running of $\alpha_{\rm QED}$ given by

$$\alpha_{\rm QED}(Q^2) = \frac{\alpha_0}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- leading hadronic contribution

$$\Delta\alpha_{\rm QED}^{\rm hvp}(Q^2) = 4\pi\alpha_0 \Pi_{\rm R}\left(Q^2\right)$$

Mixed-action set-up

- configurations generated by ETMC [Baron et al., JHEP 1006, 2010]
- light quarks: twisted mass action for mass-degenerate fermion doublet [Frezzotti, Rossi, JHEP 007, 2004]

$$S_F[\chi, \overline{\chi}, U] = \sum_x \overline{\chi}(x) \left[D_W + m_0 + i\mu_q \gamma_5 \tau^3 \right] \chi(x)$$

- heavy sea quarks: twisted mass action for non-degenerate fermion doublet [Frezzotti, Rossi, Nucl. Phys. Proc. Suppl. 128, 2004]

$$S_F[\chi_h, \overline{\chi}_h, U] = \sum_x \overline{\chi}_h(x) \left[D_W + m_0 + i\mu_\sigma \gamma_5 \tau^1 + \mu_\delta \tau^3 \right] \chi_h(x)$$

- heavy valence quarks: Osterwalder-Seiler action

  [Frezzotti, Rossi, JHEP 070, 2004]

$$S_F[\chi_h, \overline{\chi}_h, U] = \sum_x \overline{\chi}_h(x) \left[D_W + m_0 + i \begin{pmatrix} \mu_c & 0 \\ 0 & -\mu_s \end{pmatrix} \gamma_5 \right] \chi_h(x)$$

- tune bare mass parameters $\mu_{c/s}$ such that physical kaon and D-meson masses are reproduced

How the observables are determined

- use conserved (point-split) vector current

$$J^C_\mu(x) = \frac{1}{2}\Big(\overline{\chi}(x+\hat{\mu})(\mathbb{1}+\gamma_\mu)U^\dagger_\mu(x)Q_{\rm el}\chi(x) - \overline{\chi}(x)(\mathbb{1}-\gamma_\mu)U_\mu(x)Q_{\rm el}\chi(x+\hat{\mu})\Big)$$

- use redefinition [Feng, Jansen, Petschlies, Renner, PRL 107, 2011]

$$a^{\rm hvp}_{\overline{\mu}} = \alpha_0^2 \int_0^\infty \frac{dQ^2}{Q^2} w\left(\frac{Q^2}{H^2}\frac{H^2_{\rm phys}}{m^2_\mu}\right)\Pi_{\rm R}(Q^2)$$

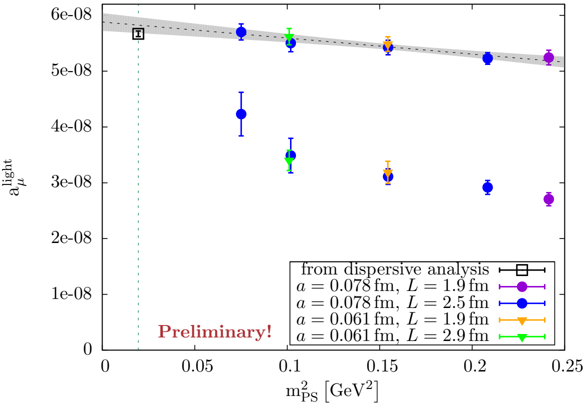
which goes to $a^{\rm hvp}_\mu$ for $m_{PS} \to m_\pi$, i.e. when $H \to H_{\rm phys}$

- and analogously for $\Delta\alpha^{\rm hvp}_{\rm QED}$ [Feng, Jansen, Petschlies, Renner, arXiv:1206.3113[hep-lat], 2012]

$$\Delta\overline{\alpha}^{\rm hvp}_{\rm QED}(Q^2) = 4\pi\alpha_0\Pi_{\rm R}\left(Q^2\frac{H^2}{H^2_{\rm phys}}\right)$$

Preliminary results for a_μ^{hvp}

The light quark contribution from four dynamical quark flavours



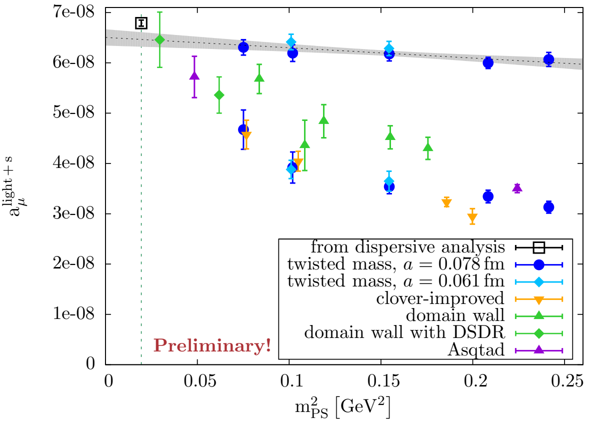
$H = m_V$

$H = 1$

- preliminary result: $a_\mu^{\mathrm{hvp}} = 5.83(13) \cdot 10^{-8}$
- consistent with $N_f = 2$ result: $a_\mu^{\mathrm{hvp}} = 5.72(16) \cdot 10^{-8}$

Preliminary results for $a_\mu^{\rm hvp}$

$a_\mu^{\rm hvp}$ for $N_f = 2+1$ compared to [Boyle, Del Debbio, Kerrane, Zanotti, Phys. Rev. D85, 2012], [Della Morte, Jäger, Jüttner, Wittig, JHEP 1203, 2012], and [Aubin, Blum, Golterman, Peris, arXiv:1205.3695, 2012]

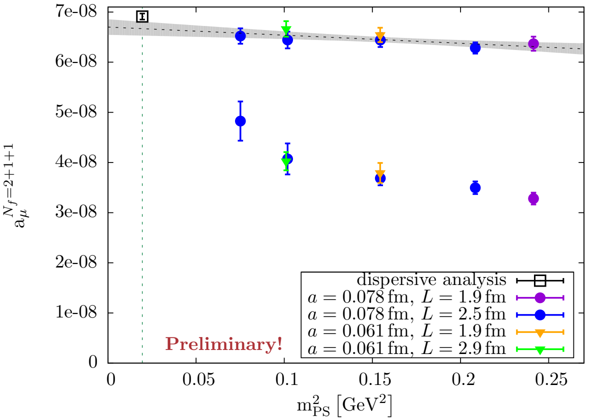


$H = m_V$

$H = 1$

- preliminary result: $a_\mu^{\rm hvp} = 6.47(13) \cdot 10^{-8}$

The four flavour contribution



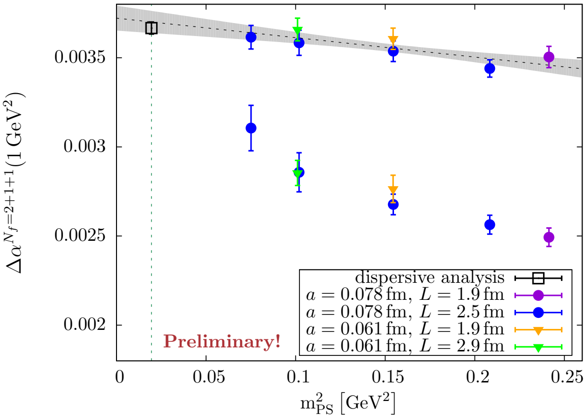
$H = m_V$

$H = 1$

- preliminary result: $a_\mu^{\mathrm{hvp}} = 6.67(14) \cdot 10^{-8}$
- Jegerlehner's result: [Jegerlehner, Szafron, Eur. Phys. J C71, 2011]
  $a_\mu^{\mathrm{hvp}} = 6.91(05) \cdot 10^{-8}$

Preliminary results for $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}$

The leading hadronic contribution to $\Delta\alpha_{\mathrm{QED}}$ at $Q^2 = 1\,\mathrm{GeV}^2$



$H = m_V$

$H = 1$

- preliminary result: $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}(1\,\mathrm{GeV}^2) = 3.70(06) \cdot 10^{-3}$
- Jegerlehner's result: [Jegerlehner, Nuovo Cim. 034C, 2011]
  $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}(1\,\mathrm{GeV}^2) = 3.67(03) \cdot 10^{-3}$

Summary & Outlook

Summary

- First $N_f = 2 + 1 + 1$ lattice calculation of a_μ^{hvp}.
- Vacuum polarisation data also used for $\Delta\alpha_{\mathrm{QED}}^{\mathrm{hvp}}(Q^2)$.
- Modified method [Feng, Jansen, Petschlies, Renner, PRL 107, 2011] works for $N_f = 2 + 1 + 1$ computation.

Outlook

- improvement of the results
  - systematic uncertainty
  - disconnected contributions
  - twisted boundary conditions
  - include isospin breaking and electromagnetism
- different observables
  - compute Adler function, weak mixing angle, S-parameter
  - light-by-light scattering