Correlation Matrix Methods for the π and ρ Form Factors in Full QCD

Benjamin Owen

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Outline

1. Introduction
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   - Extracting Form Factors
2. Calculation Details
3. Results
   - Meson mass spectrum
   - Meson Form Factors
   - Charge Radii

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$$= \sum_{\alpha=0}^{N-1} \frac{e^{-E_{M_\alpha}(\vec{p})\,t}}{2E_{M_\alpha}(\vec{p})} Z_i^\alpha(\vec{p}) Z_j^{\alpha\,\dagger}(\vec{p})$$

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- where Z_i^α, $Z_j^{\alpha\,\dagger}$ are the couplings of sink operator (χ_i) and source operator ($\chi_j^\dagger$) for the state α

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- use our basis of operators to construct these new operators

$$\left. \begin{aligned} \phi_\alpha^\dagger(\vec{p}) &= \sum_{i=1}^{N} u_i^\alpha(\vec{p}) \chi_i^\dagger(\vec{p}) \\ \phi_\alpha(\vec{p}) &= \sum_{i=1}^{N} v_i^\alpha(\vec{p}) \chi_i(\vec{p}) \end{aligned} \right\} \quad \text{optimal coupling to state } |\, M_\alpha, p \rangle$$

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- Using $v_i^\alpha(\vec{p})$, $u_j^\alpha(\vec{p})$ we are able to project out the correlation function for the state $|\,M_\alpha, p\,\rangle$

$$\mathcal{G}_\alpha(t,\vec{p}) = v_i^\alpha(\vec{p})\mathcal{G}_{ij}(t,\vec{p})u_j^\alpha(\vec{p})$$

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$$\begin{aligned} \pi: &\quad \chi_1 = \gamma_5, \quad \chi_2 = \gamma_0\gamma_5 \\ \rho: &\quad \chi_1 = \gamma_\mu, \quad \chi_2 = \gamma_0\gamma_\mu \end{aligned}$$

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- We shall use smearing of the fermion source and sink as a method of increasing our operator basis
- Our basis will comprise standard local operators with some number of sweeps of smearing

$$\{\,\chi_i\,\} \rightarrow \{\,\chi_i^n\,\},\ n \text{ number of sweeps of smearing}$$

- In this work we use of gauge-invariant Gaussian smearing at both source and sink

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$$= \sum_{\alpha=0}^{N-1} \frac{e^{-E_{M_\alpha}(\vec{p}\,')\,t_2} e^{-E_{M_\alpha}(\vec{q})\,t_1}}{2\sqrt{E_{M_\alpha}(\vec{p}) E_{M_\alpha}(\vec{p}\,')}} Z_i^{\alpha}(\vec{p}\,') Z_j^{\alpha\,\dagger}(\vec{p}) \langle\, M_\alpha, p' \,| J^{\mu}(0) |\, M_\alpha, p\,\rangle$$

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- Crucial thing to note above is that we have source and sink coupling parameters with different momenta

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- In this work we use the following ratio of three- and two-point correlation functions

$$R_\alpha^\mu(\vec{p}, \vec{p}\,') = \sqrt{\frac{\mathcal{G}_\alpha^\mu(t_2, t_1; \vec{p}\,', \vec{p})\mathcal{G}_\alpha^\mu(t_2, t_1; \vec{p}, \vec{p}\,')}{\mathcal{G}_\alpha(t_2, \vec{p}\,')\mathcal{G}_\alpha(t_2, \vec{p})}}$$

Configuration Details

- For this calculation we are working with the PACS-CS (2+1)-flavour Full QCD ensembles[1] made available through the ILDG

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- For this calculation we are working with the PACS-CS (2+1)-flavour Full QCD ensembles[1] made available through the ILDG
- These are $32^3 \times 64$ lattices with $\beta = 1.9$, corresponding to a physical lattice spacing of $0.0907(13)\ \mathrm{fm}$

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- Were using 4×4 Correlation matrix

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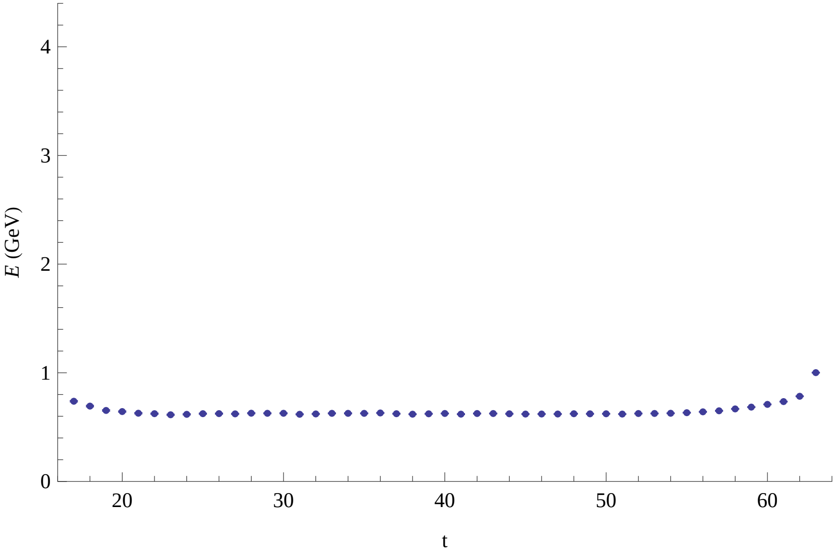

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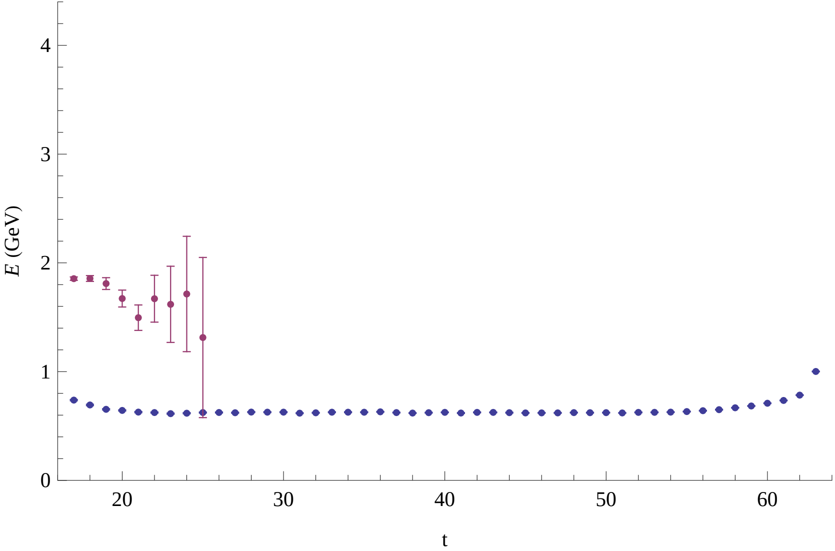

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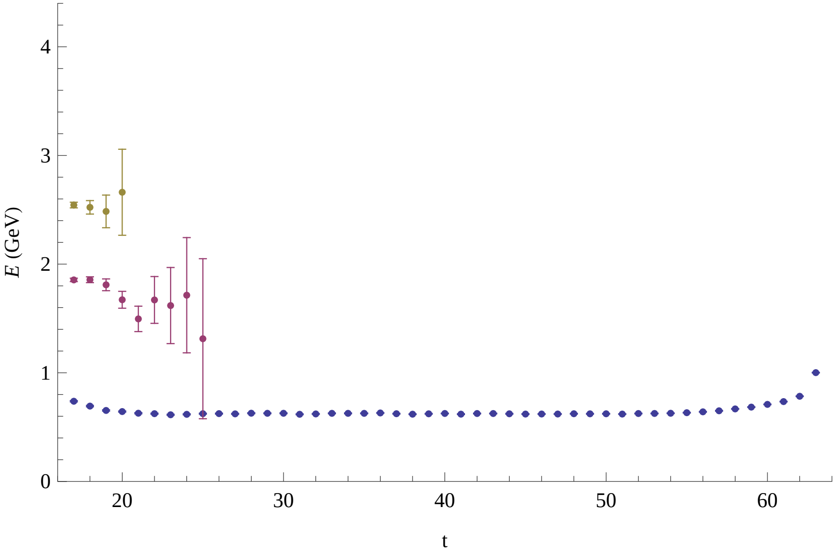

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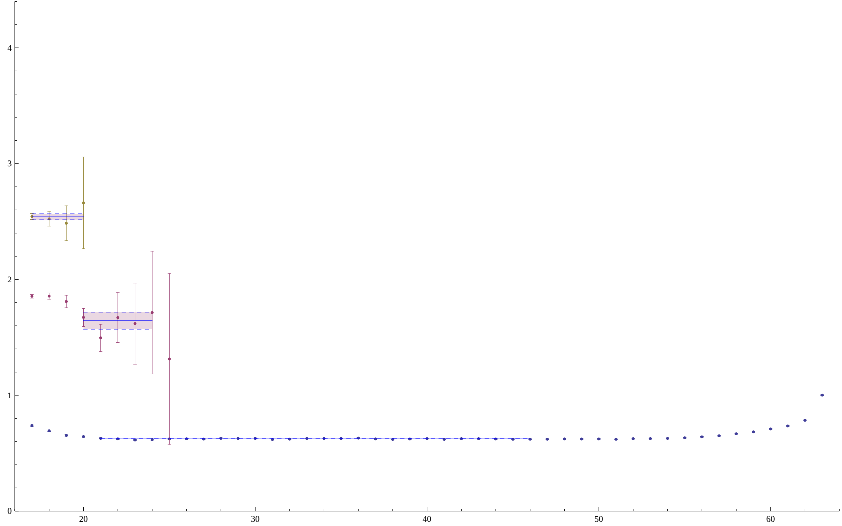

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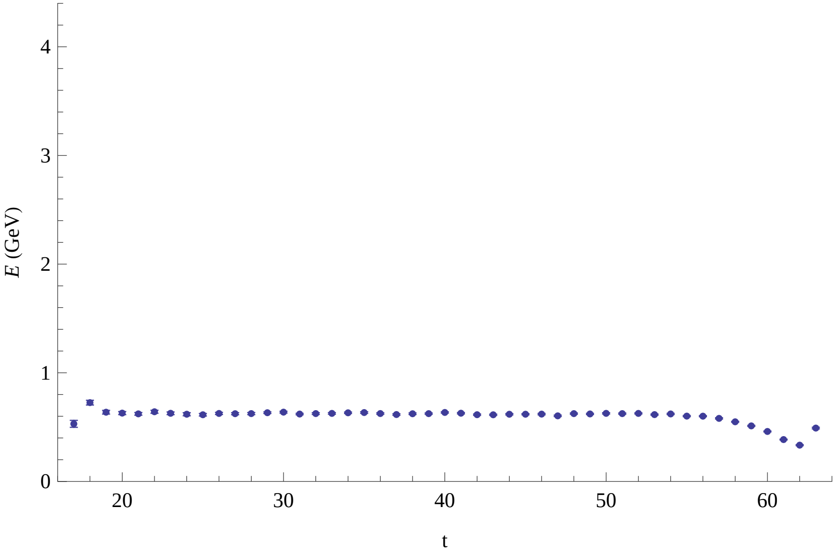

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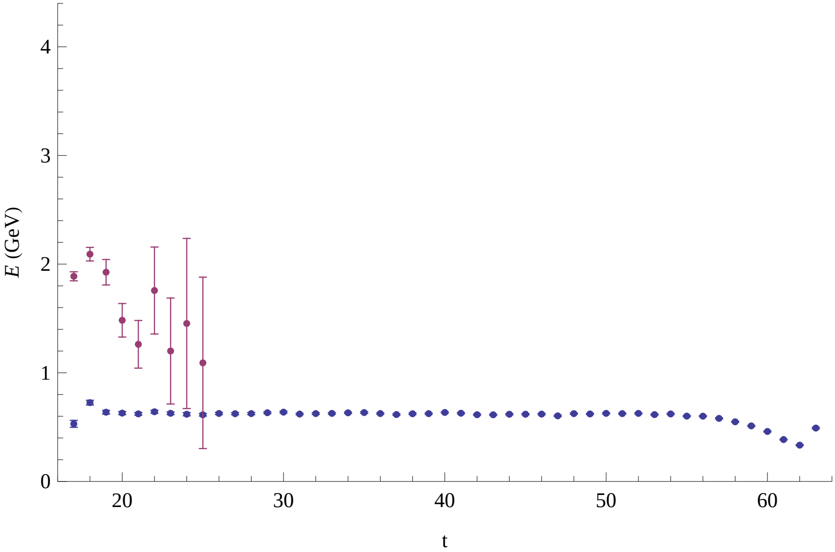

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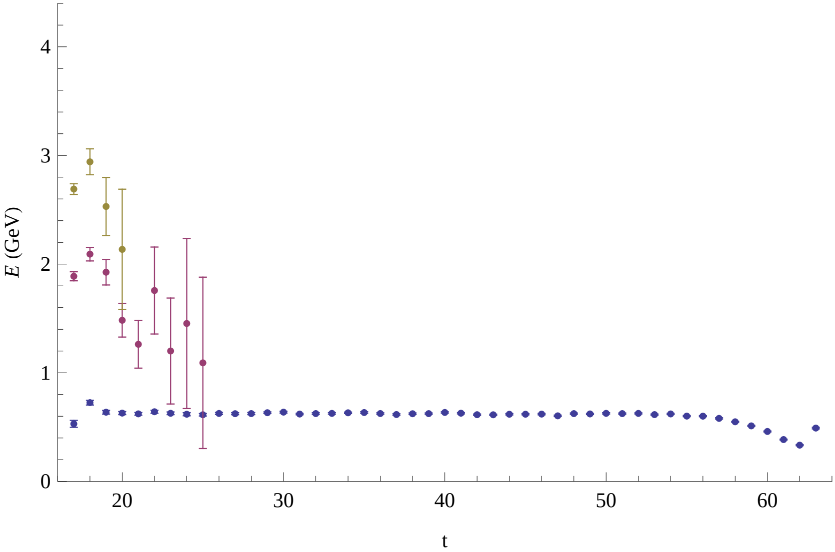

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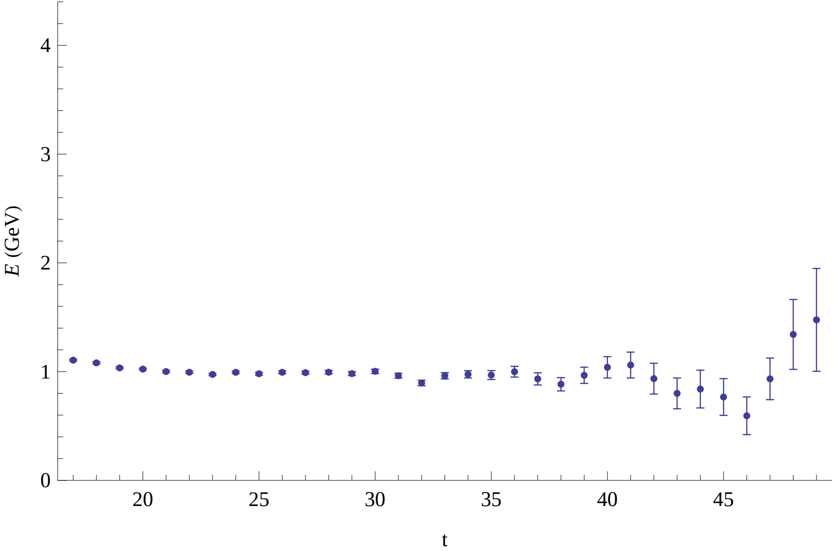

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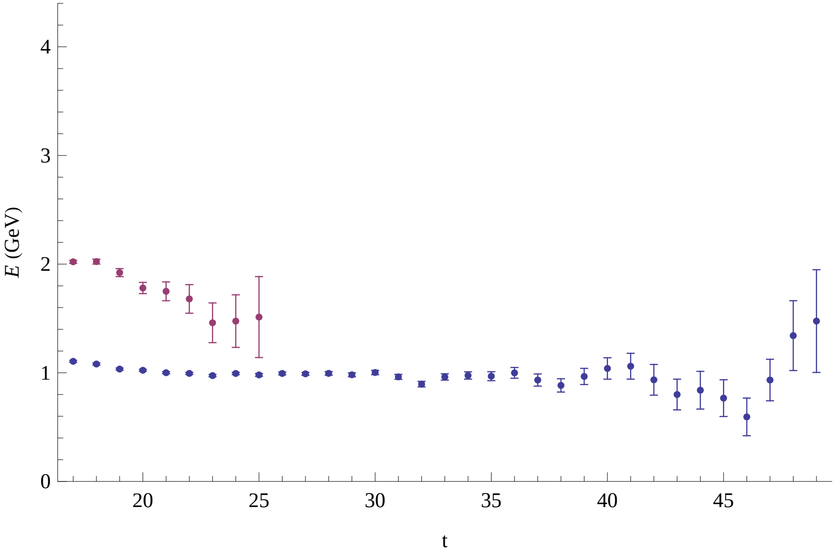

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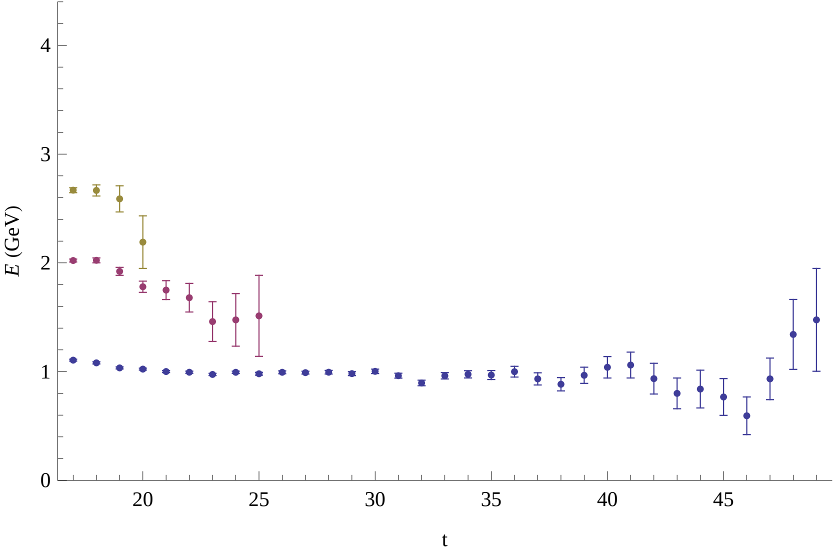

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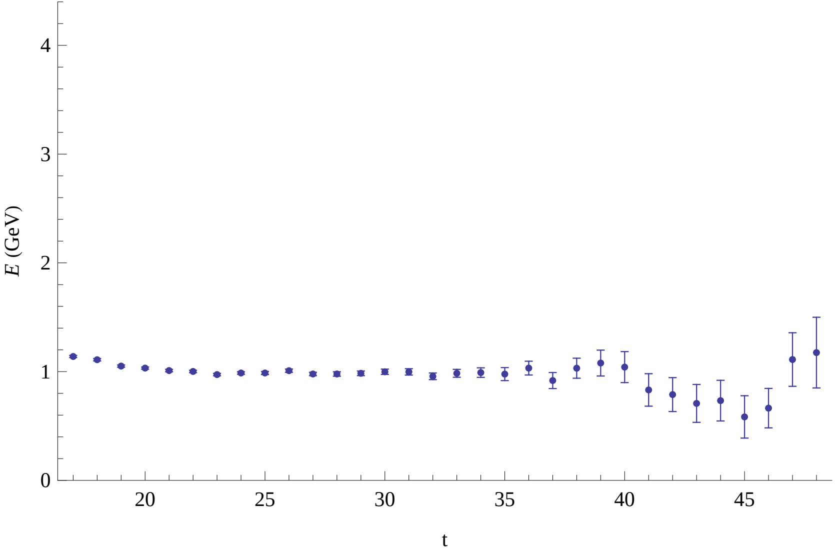

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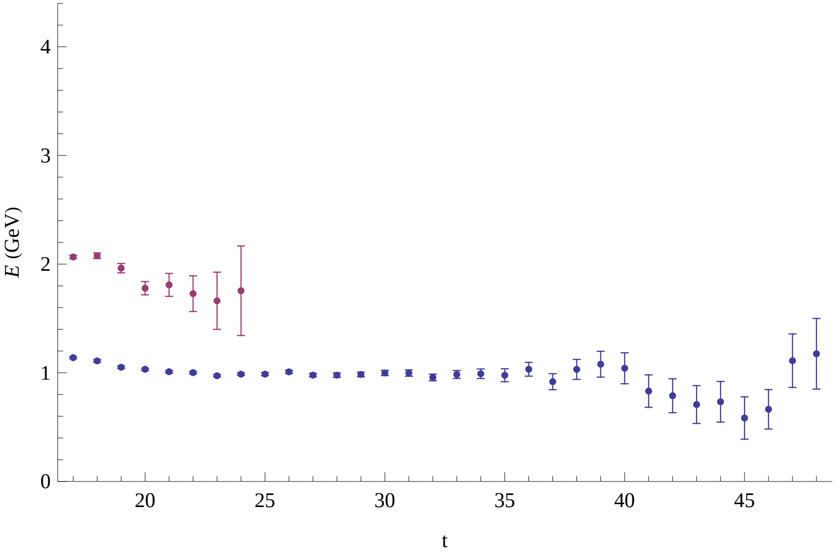

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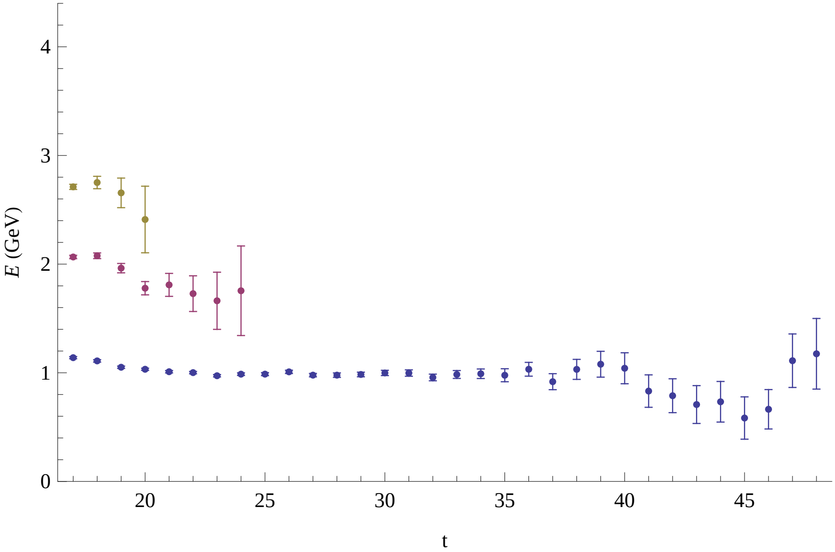

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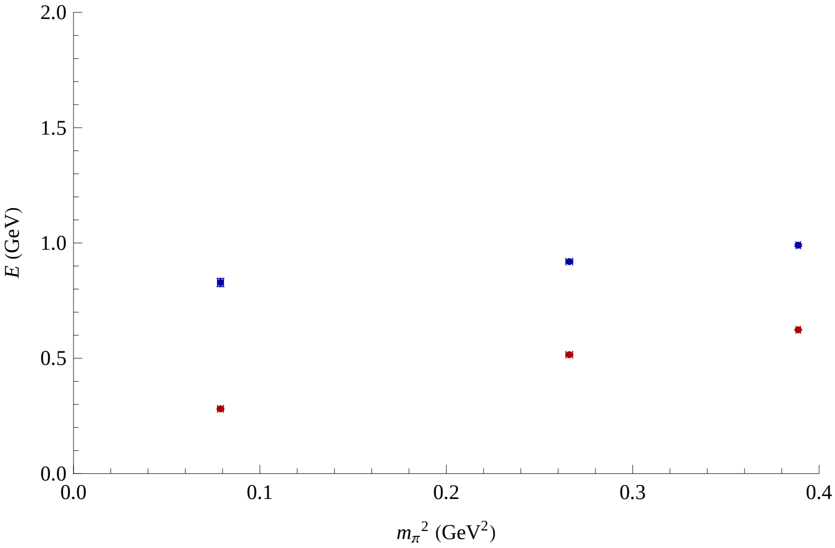

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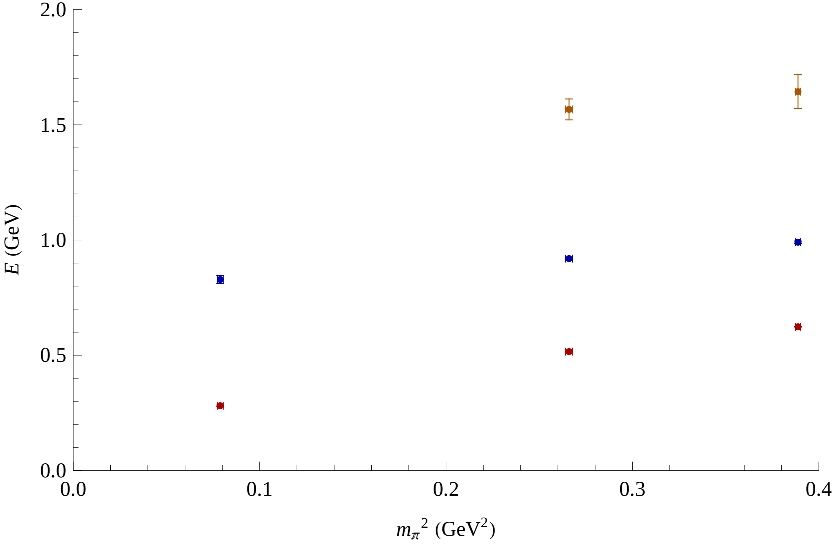

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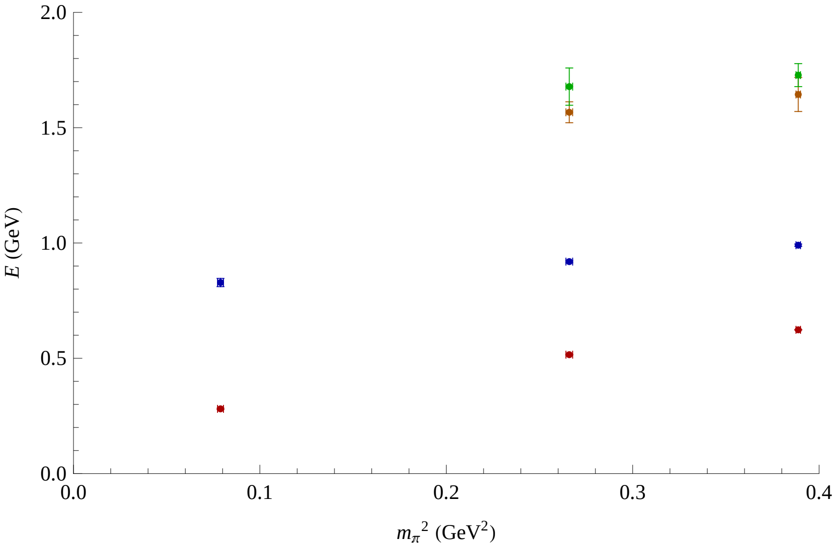

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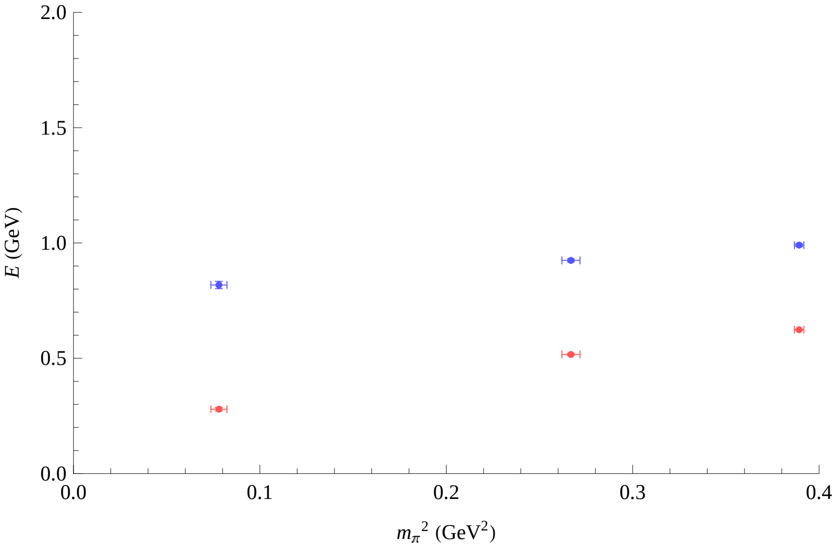

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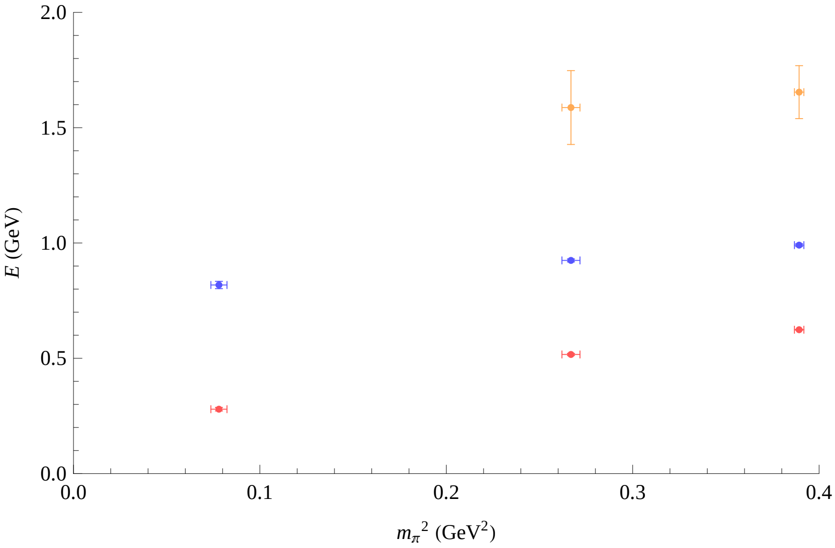

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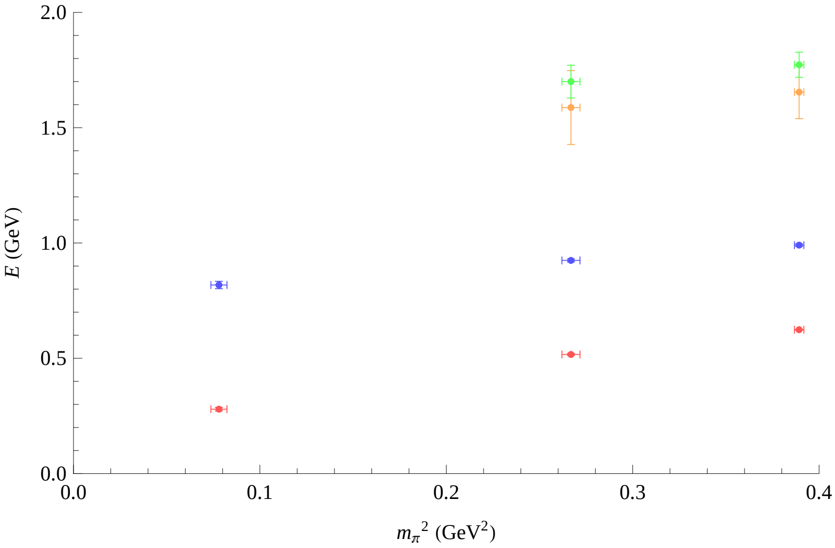

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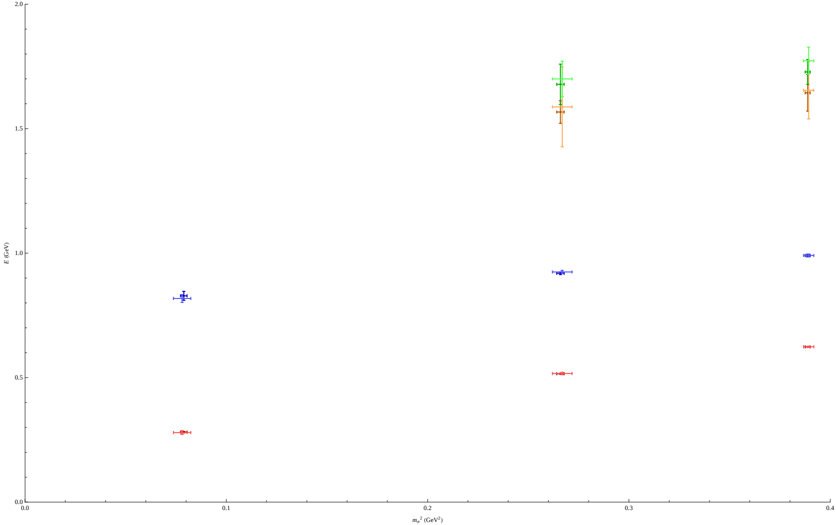

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Vertex Functions

- Pion:

$$\langle\, \pi(\vec{p}\,')\,|J^{\mu}|\,\pi(\vec{p})\,\rangle = \frac{1}{\sqrt{E_{\pi}(\vec{p}\,')E_{\pi}(\vec{p})}}[p'^{\mu} + p^{\mu}]F_{\pi}(Q^2)$$

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- Rho:

$$\langle\, \rho(\vec{p}\,', s')|J^\mu|\,\rho(\vec{p}, s)\,\rangle = \frac{1}{\sqrt{E_\rho(\vec{p}\,')E_\rho(\vec{p})}}\epsilon'^*_\sigma(p\,', s')\Gamma^{\sigma\mu\tau}(p,\, p\,')\epsilon_\tau(p,\, s)$$

where

$$\Gamma^{\sigma\mu\tau}(p,\, p\,') =$$
$$-\left\{G_1(Q^2)g^{\sigma\tau}[p'^\mu + p^\mu] + G_2(Q^2)[g^{\mu\tau}q^\sigma - g^{\mu\sigma}q^\tau] - G_3(Q^2)q^\sigma q^\tau\frac{[p'^\mu + p^\mu]}{2M_\rho^2}\right\}$$

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$$G_C(Q^2) = \frac{2}{3}\frac{\sqrt{E_\rho m_\rho}}{E_\rho + m_\rho}({R_1}^0{}_1 + {R_2}^0{}_2 + {R_3}^0{}_3)$$

$$G_M(Q^2) = \frac{\sqrt{E_\rho m_\rho}}{p_x}({R_1}^3{}_3 + {R_3}^3{}_1)$$

$$G_Q(Q^2) = m_\rho\frac{\sqrt{E_\rho m_\rho}}{p_x^2}(2{R_1}^0{}_1 - {R_2}^0{}_2 - {R_3}^0{}_3)$$

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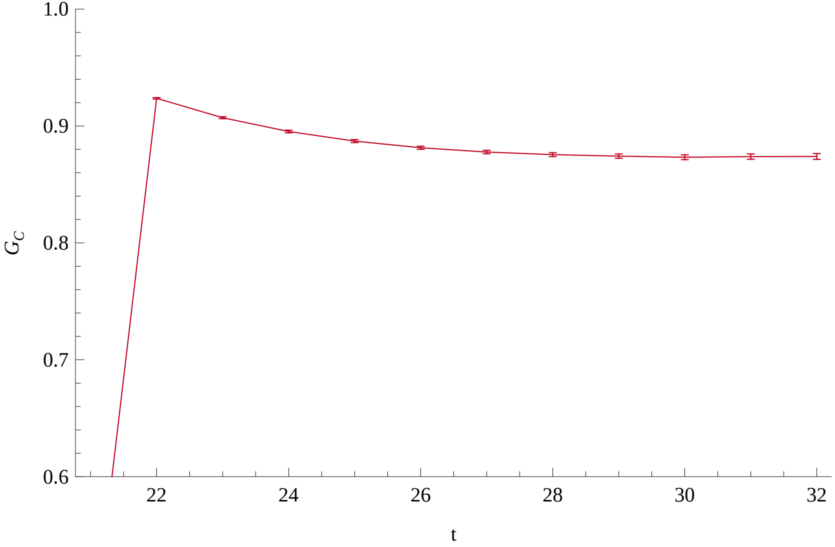

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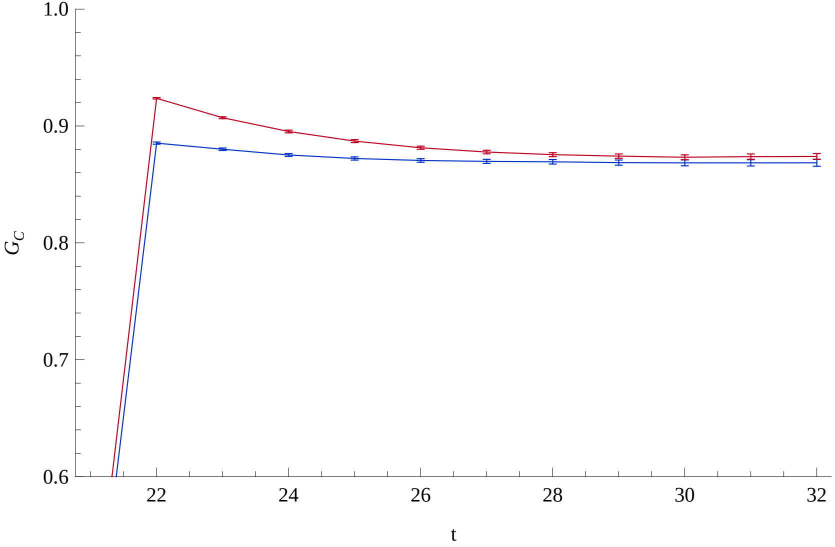

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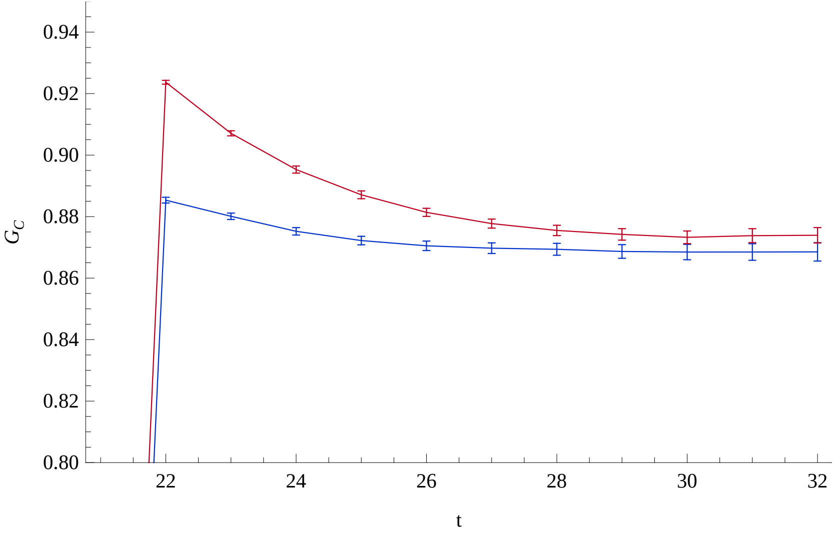

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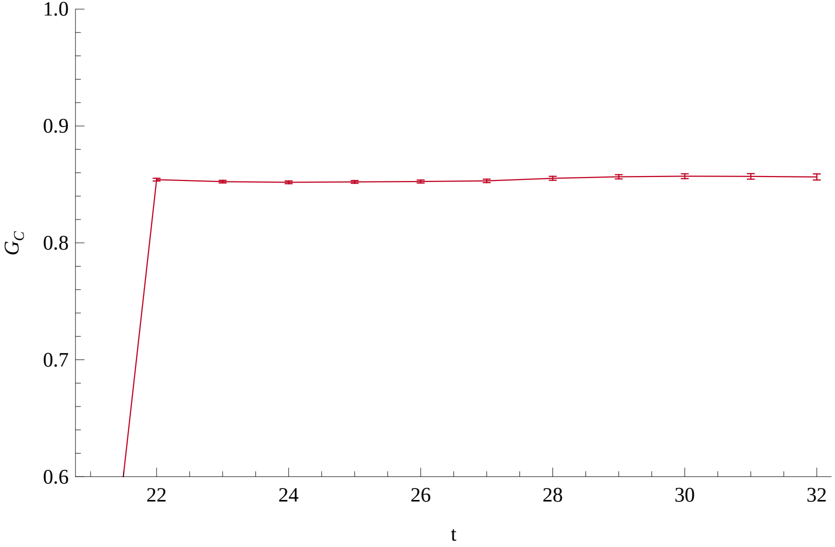

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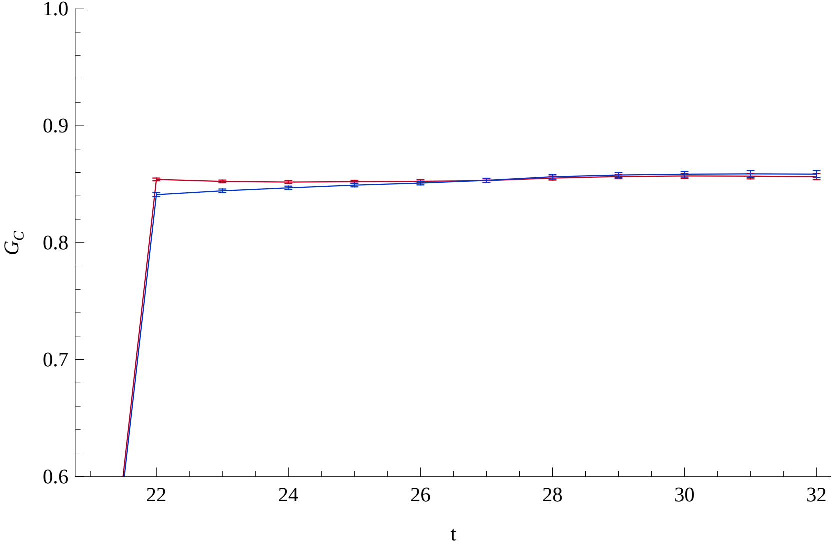

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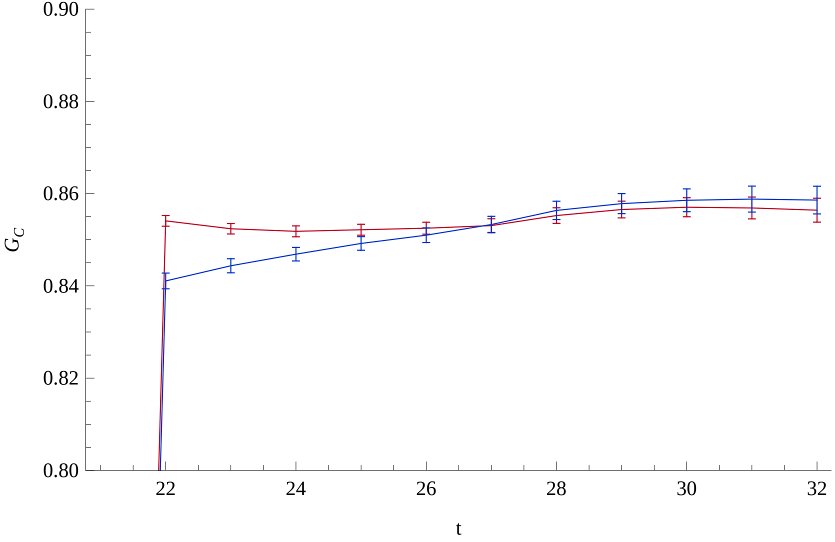

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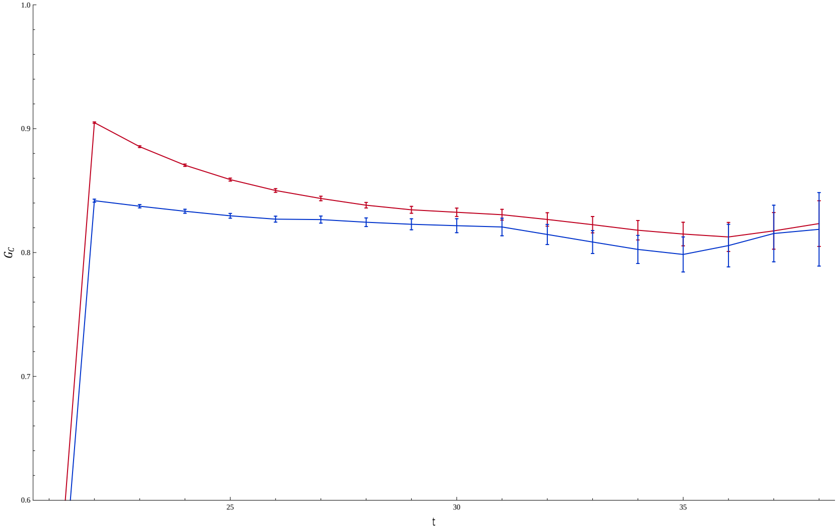

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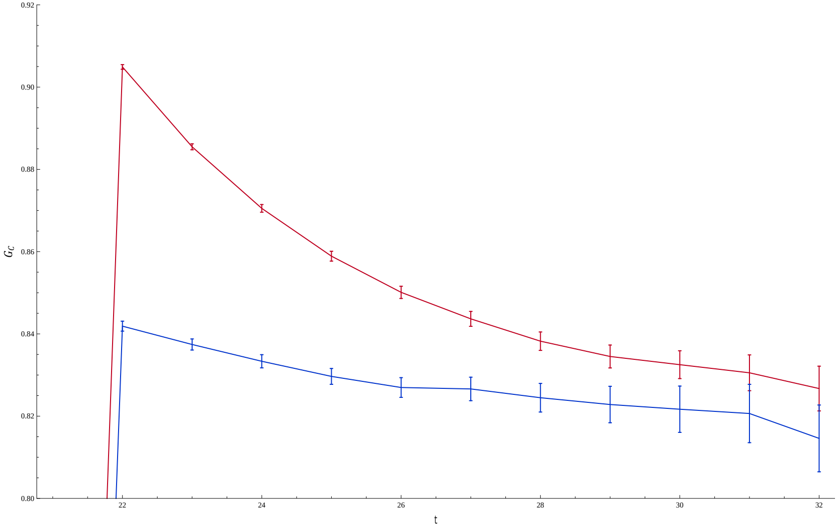

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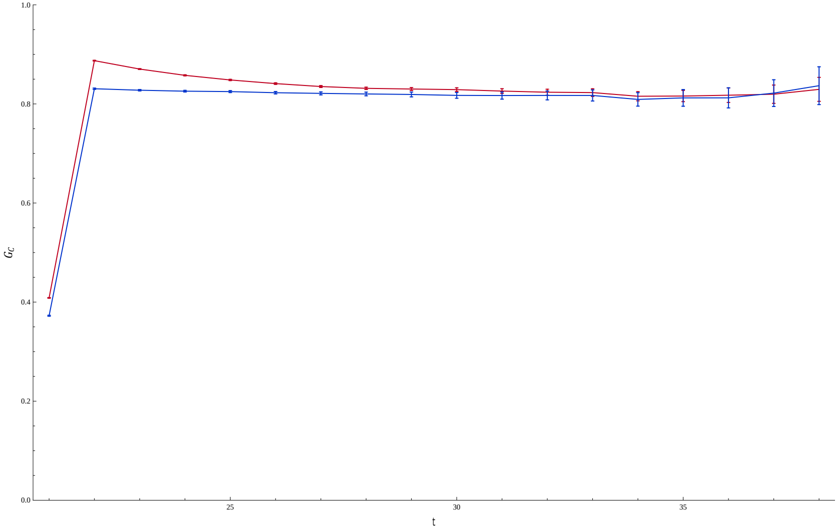

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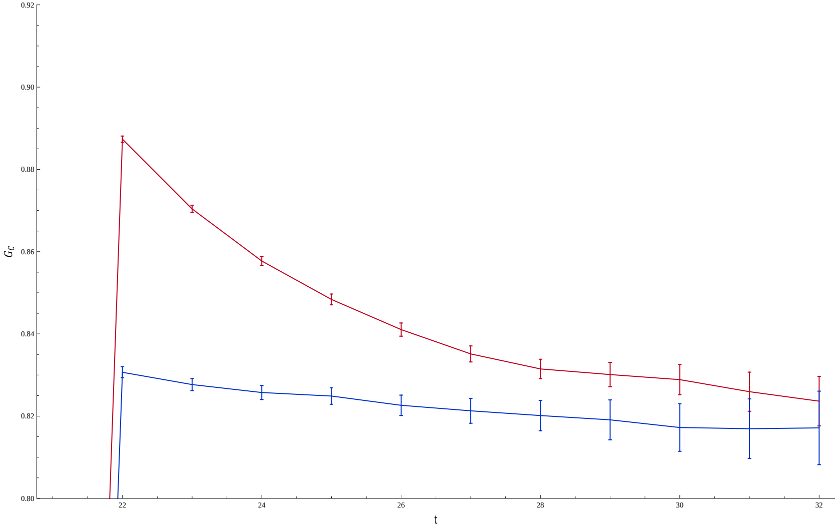
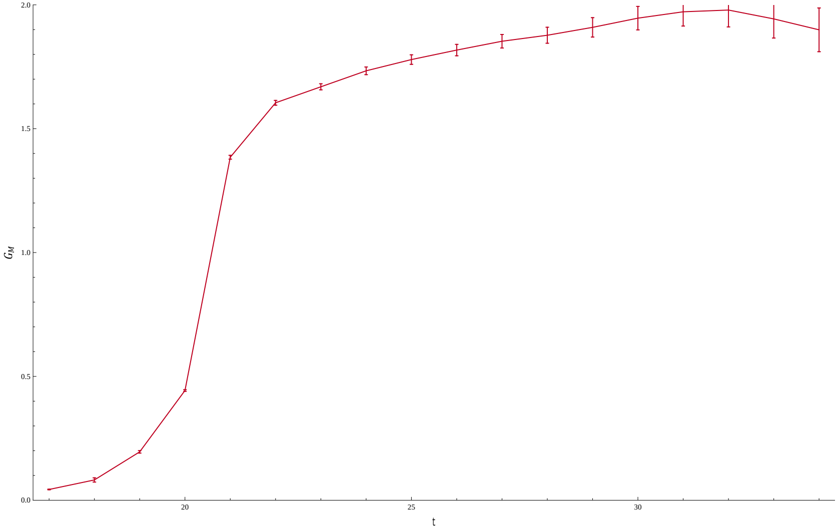


rho G_M Form Factor - $\{\chi_1^{16}, \chi_1^{35}, \chi_1^{100}, \chi_1^{200}\}$



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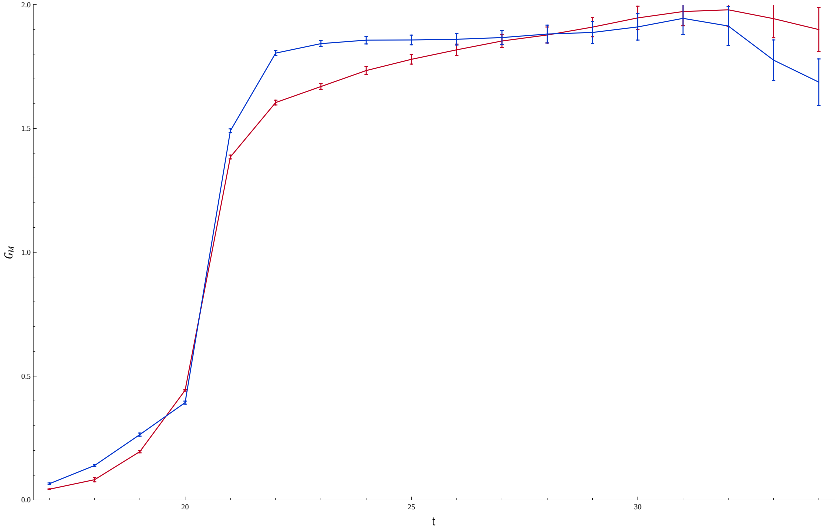

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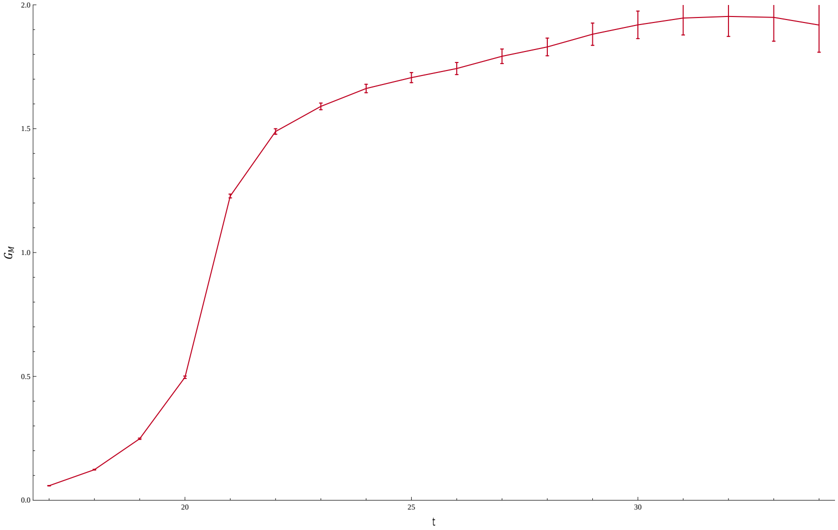

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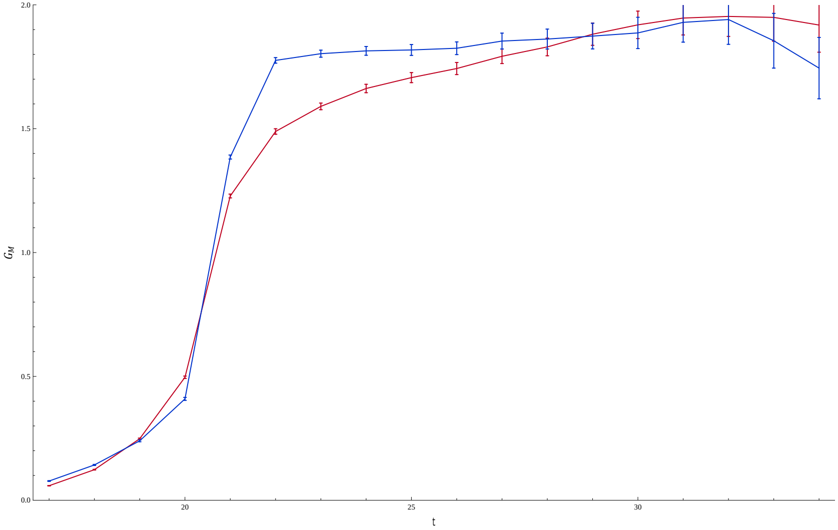

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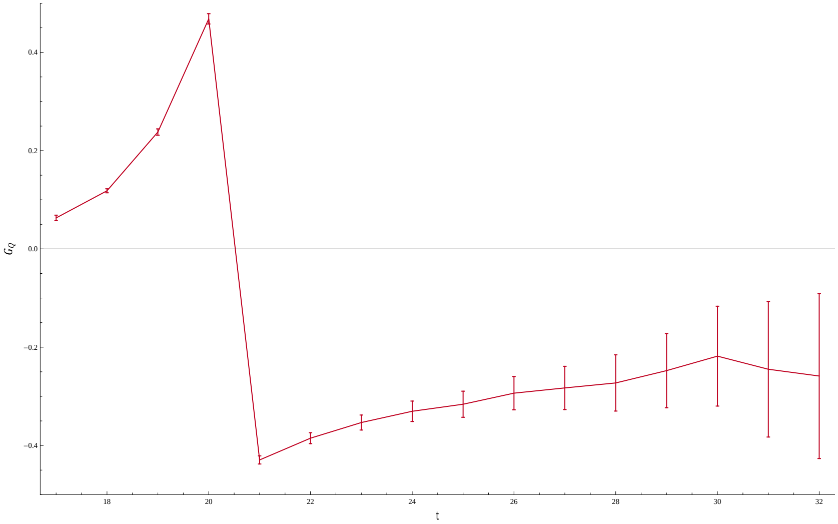

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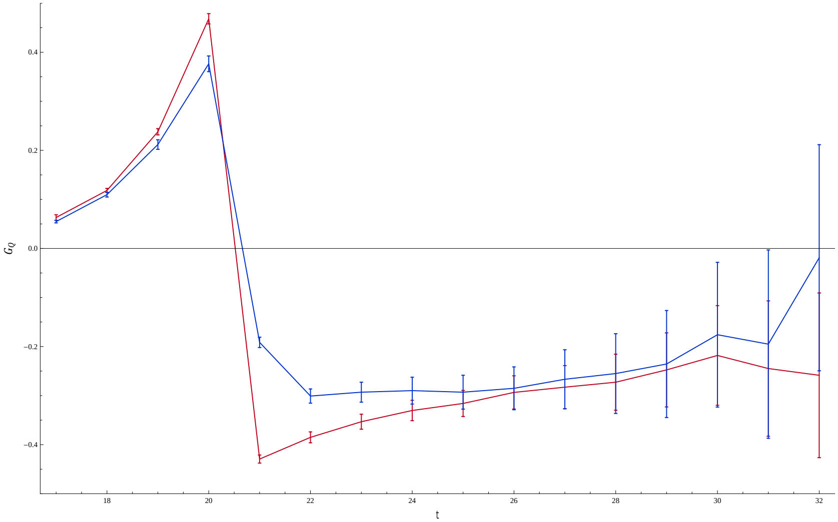

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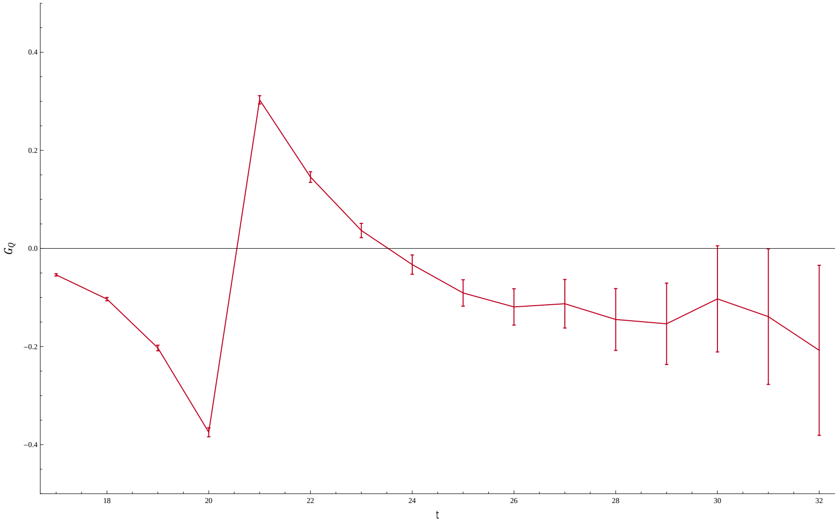

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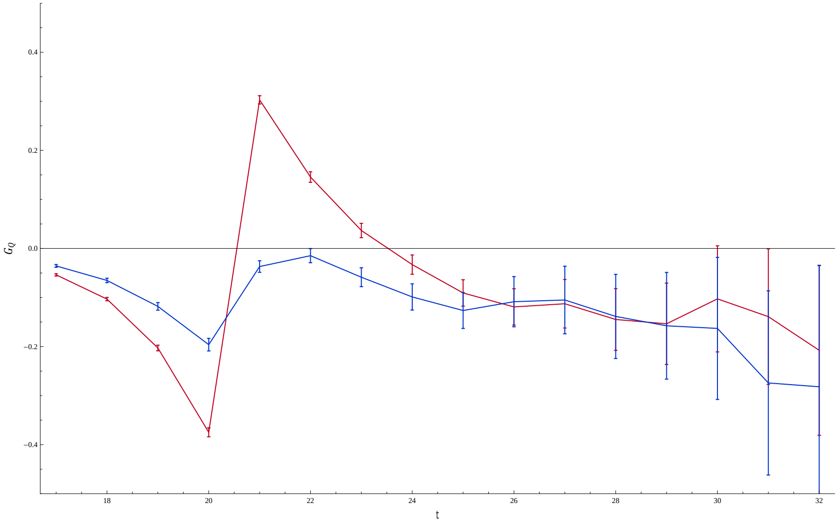

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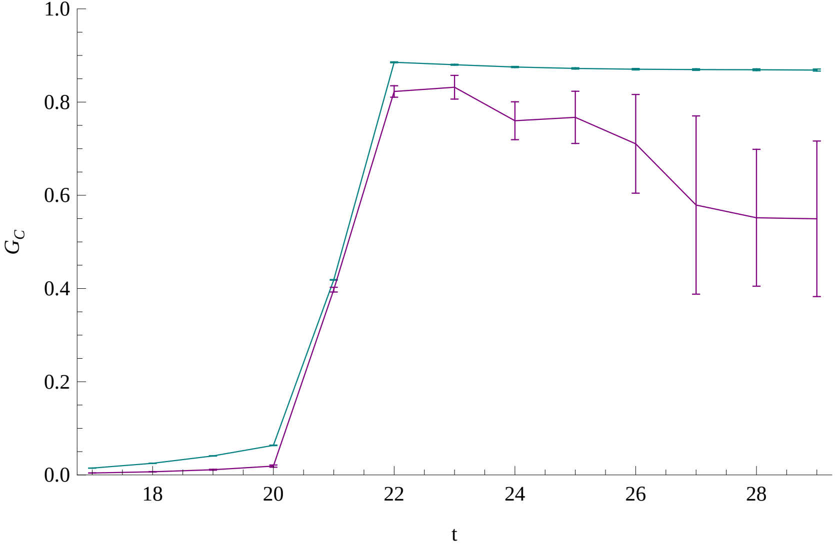

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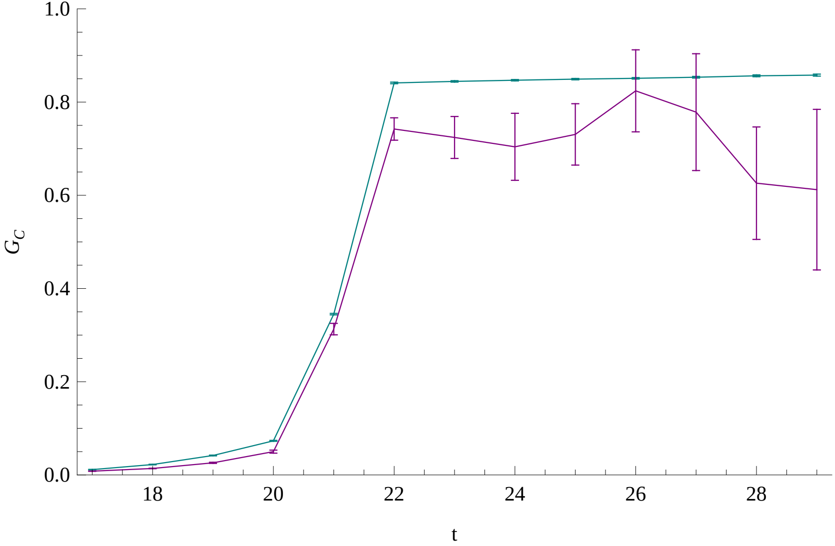

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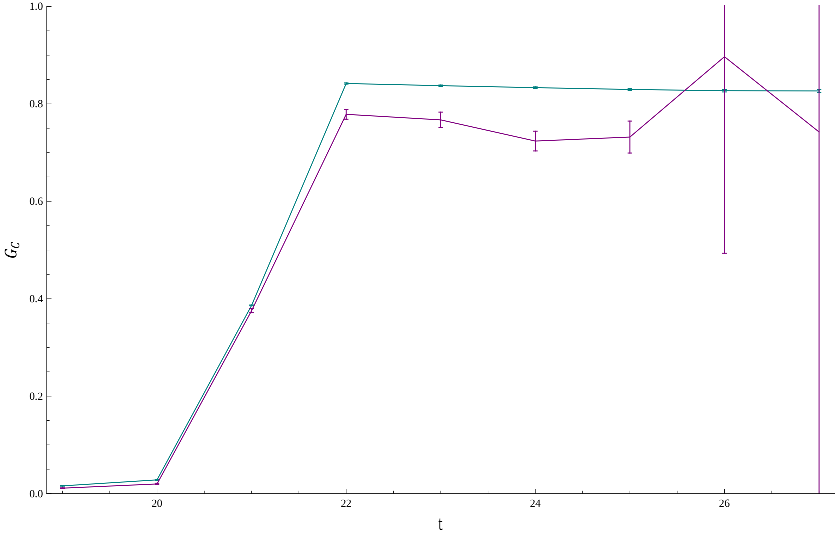

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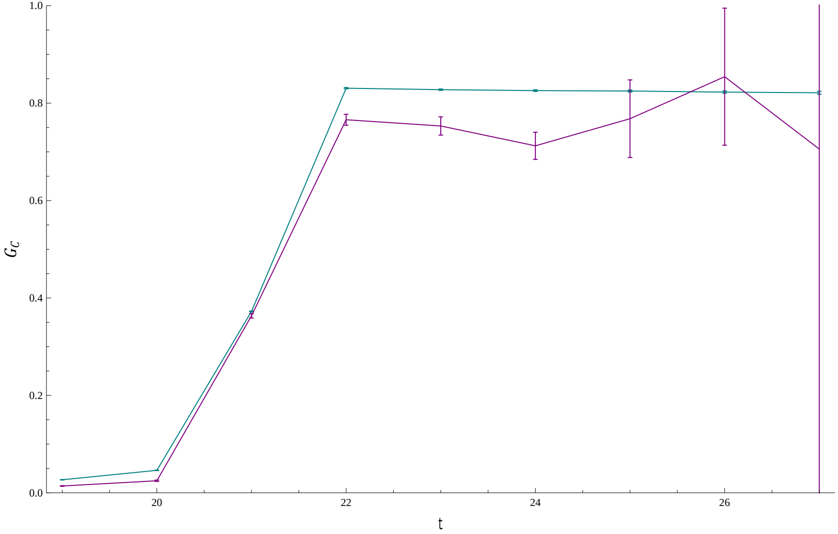

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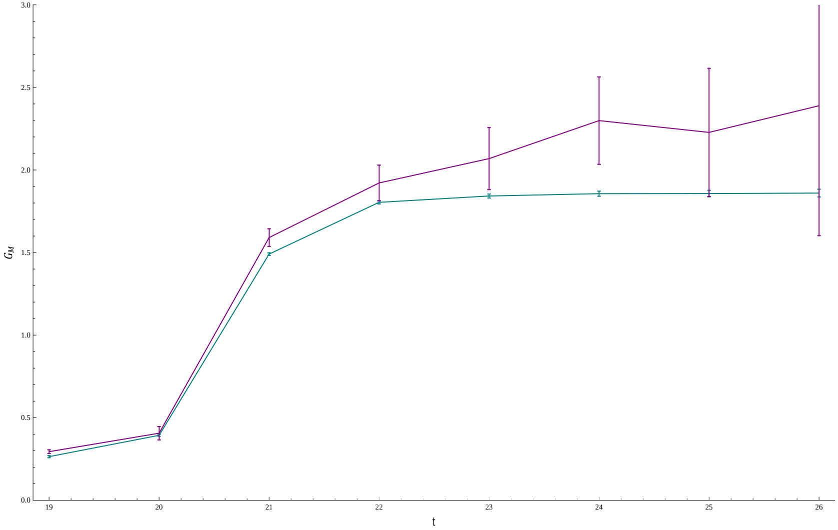

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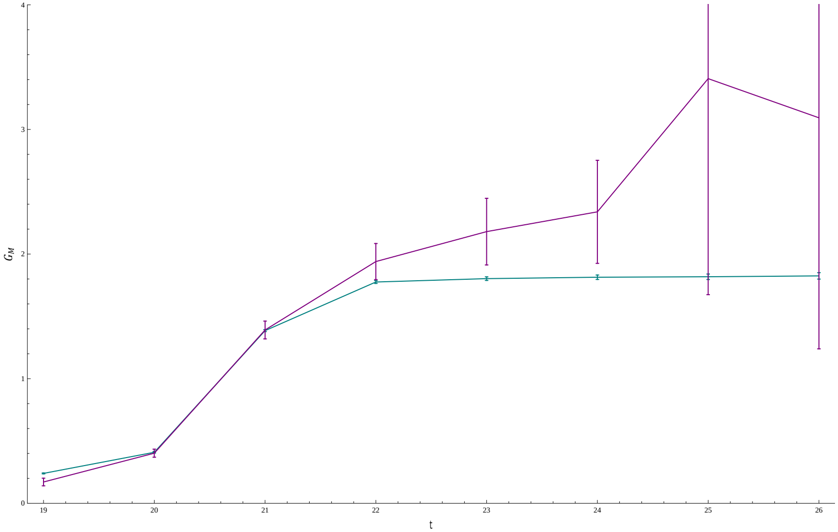

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$$\langle r^2 \rangle = \frac{6}{Q^2} \left(\frac{1}{G_C(Q^2)} - 1 \right)$$

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$$\pi : \langle r_{rms} \rangle = 0.507 \pm 0.008 \,\text{fm}$$

$$\rho : \langle r_{rms} \rangle = 0.586 \pm 0.010 \,\text{fm}$$

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- Extracted G_C for the first excited state of the π and ρ mesons
- Extracted $\langle r^2 \rangle$ for these excited states allowing for comparison with the ground states