# Axial couplings of heavy hadrons

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work in collaboration with David Lin & Stefan Meinel
[PRL 108 172003, PRD 85 114508, and work in progress]

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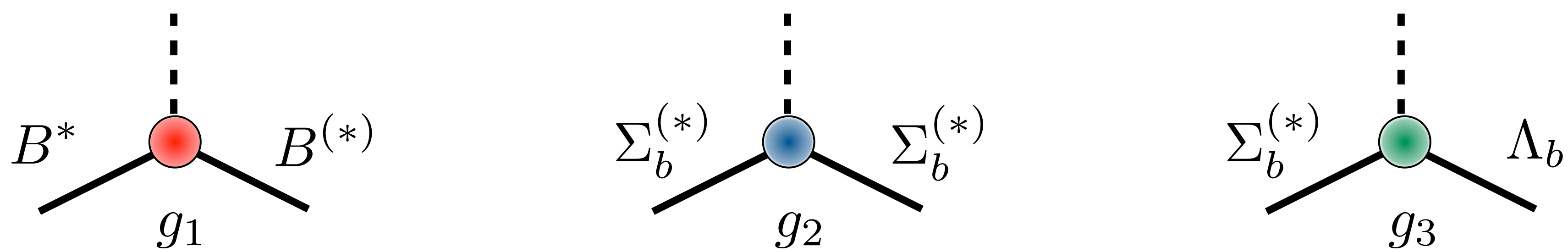
# Chiral dynamics of heavy hadrons

- Axial couplings *defined* in static limit




$\sim \hat{g} \sim g_\pi \sim g_{B^* B\pi}$

$$\langle P^{*d}(0,s)|\mathsf{A}_\mu^{-(\chi\mathrm{PT})}(0)|P^u(0)\rangle|_{\mathrm{LO}} = -2\, g_1\, \varepsilon_\mu^*(s).$$

$B^*$ → $P^{*d}$; $B$ → $P^u$

$$\langle S^{dd}(0,s)|\mathsf{A}^{\mu-(\chi\mathrm{PT})}(0)|S^{du}(0,s')\rangle|_{\mathrm{LO}} = -\frac{i}{\sqrt{2}}\, g_2\, v_\lambda\, \epsilon^{\lambda\mu\nu\rho}\, \overline{U}_\nu(s) U_\rho(s').$$

$$\langle S^{dd}(0,s)|\mathsf{A}^{\mu-(\chi\mathrm{PT})}(0)|T^{du}(0,s')\rangle|_{\mathrm{LO}} = -g_3\, \overline{U}^\mu(s)\, \mathcal{U}(s').$$

$$\begin{pmatrix} \Sigma_b^+ & \frac{1}{\sqrt{2}}\Sigma_b^0 \\ \frac{1}{\sqrt{2}}\Sigma_b^0 & \Sigma_b^- \end{pmatrix}^{(*)} \qquad \begin{pmatrix} 0 & \Lambda_b \\ -\Lambda_b & 0 \end{pmatrix}$$

- Govern leading interactions in heavy-light meson/ baryon chiral Lagrangian [Wise; Burdman & Donoghue; Cheng et al.]

# Previous knowledge of $g_1$

- Experimental extraction of $g_1$ from $D^* \to D\pi,\ D^* \to D\gamma$
  - $g_1$=0.5(?) [Arnesen et al.]
- Lattice calculations for $g_1$

| Reference | $n_f$, action | $[m_\pi^{(\mathrm{vv})}]^2$ (GeV$^2$) | $g_1$ |
|---|---|---|---|
| De Divitiis *et al.*, 1998 [14] | 0, clover | 0.58 - 0.81 | $0.42 \pm 0.04 \pm 0.08$ |
| Abada *et al.*, 2004 [15] | 0, clover | 0.30 - 0.71 | $0.48 \pm 0.03 \pm 0.11$ |
| Negishi *et al.*, 2007 [16] | 0, clover | 0.43 - 0.72 | $0.517 \pm 0.016$ |
| Ohki *et al.*, 2008 [17] | 2, clover | 0.24 - 1.2 | $0.516 \pm 0.005 \pm 0.033 \pm 0.028 \pm 0.028$ |
| Bećirević *et al.*, 2009 [18] | 2, clover | 0.16 - 1.2 | $0.44 \pm 0.03^{+0.07}_{-0.00}$ |
| Bulava *et al.*, 2010 [19] | 2, clover | 0.063 - 0.49 | $0.51 \pm 0.02$ |

  - Need fully quantified uncertainties

# Current knowledge of $g_{1,2,3}$

- Model estimates for $g_{1,2,3}$ [Cho normalisation]

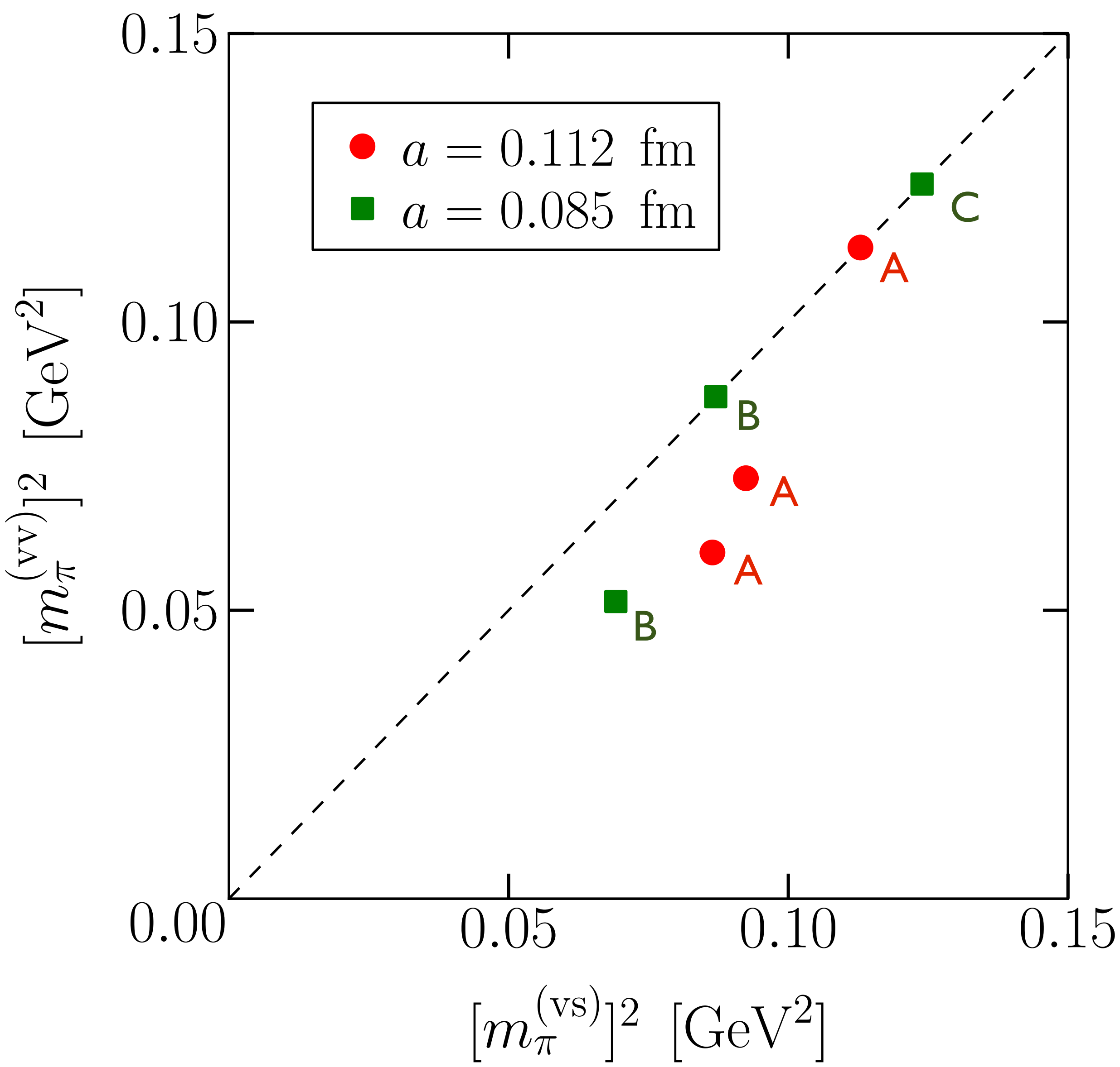
| Reference | Method | $g_1$ | $g_2$ | $g_3$ |
|---|---|---|---|---|
| Yan *et al.*, 1992 [5] | Nonrelativistic quark model | 1 | 2 | $\sqrt{2}$ |
| Colangelo *et al.*, 1994 [45] | Relativistic quark model | 1/3 | ... | ... |
| Bećirević, 1999 [46] | Quark model with Dirac eq. | $0.6 \pm 0.1$ | ... | ... |
| Guralnik *et al.*, 1992 [47] | Skyrme model | ... | 1.6 | 1.3 |
| Colangelo *et al.*, 1994 [48] | Sum rules | 0.15 - 0.55 | ... | ... |
| Belyaev *et al.*, 1994 [49] | Sum rules | $0.32 \pm 0.02$ | ... | ... |
| Dosch and Narison, 1995 [50] | Sum rules | $0.15 \pm 0.03$ | ... | ... |
| Colangelo and Fazio, 1997 [51] | Sum rules | 0.09 - 0.44 | ... | ... |
| Pirjol and Yan, 1997 [52] | Sum rules | ... | $< \sqrt{6 - g_3^2}$ | $< \sqrt{2}$ |
| Zhu and Dai, 1998 [53] | Sum rules | ... | $1.56 \pm 0.30 \pm 0.30$ | $0.94 \pm 0.06 \pm 0.20$ |
| Cho and Georgi, 1992 [54] | $\mathcal{B}[D^* \to D\,\pi]$, $\mathcal{B}[D^* \to D\,\gamma]$ | $0.34 \pm 0.48$ | ... | ... |
| Arnesen *et al.*, 2005 [57] | $\mathcal{B}[D^*_{(s)} \to D_{(s)}\pi]$, $\mathcal{B}[D^*_{(s)} \to D_{(s)}\gamma]$, $\Gamma[D^*]$ | 0.51 | ... | ... |
| Li *et al.*, 2010 [58] | $\mathrm{d}\Gamma[B \to \pi \ell \nu]$ | $< 0.87$ | ... | ... |

- All over the place!

- Reliable calculation needed

# Actions and ensembles

- Domain-wall light quarks [RBC/UKQCD]
  - Lattice chiral symmetry
- Static heavy quarks with $n_{HYP}$=0,1,2,3,5,10 levels of HYP smearing
- Two lattice spacings a = 0.085, 0.112 fm
- Six <u>valence</u> quark masses $m_\pi$ = 0.23–0.35 GeV
- Single (2.5 fm)$^3$ volume



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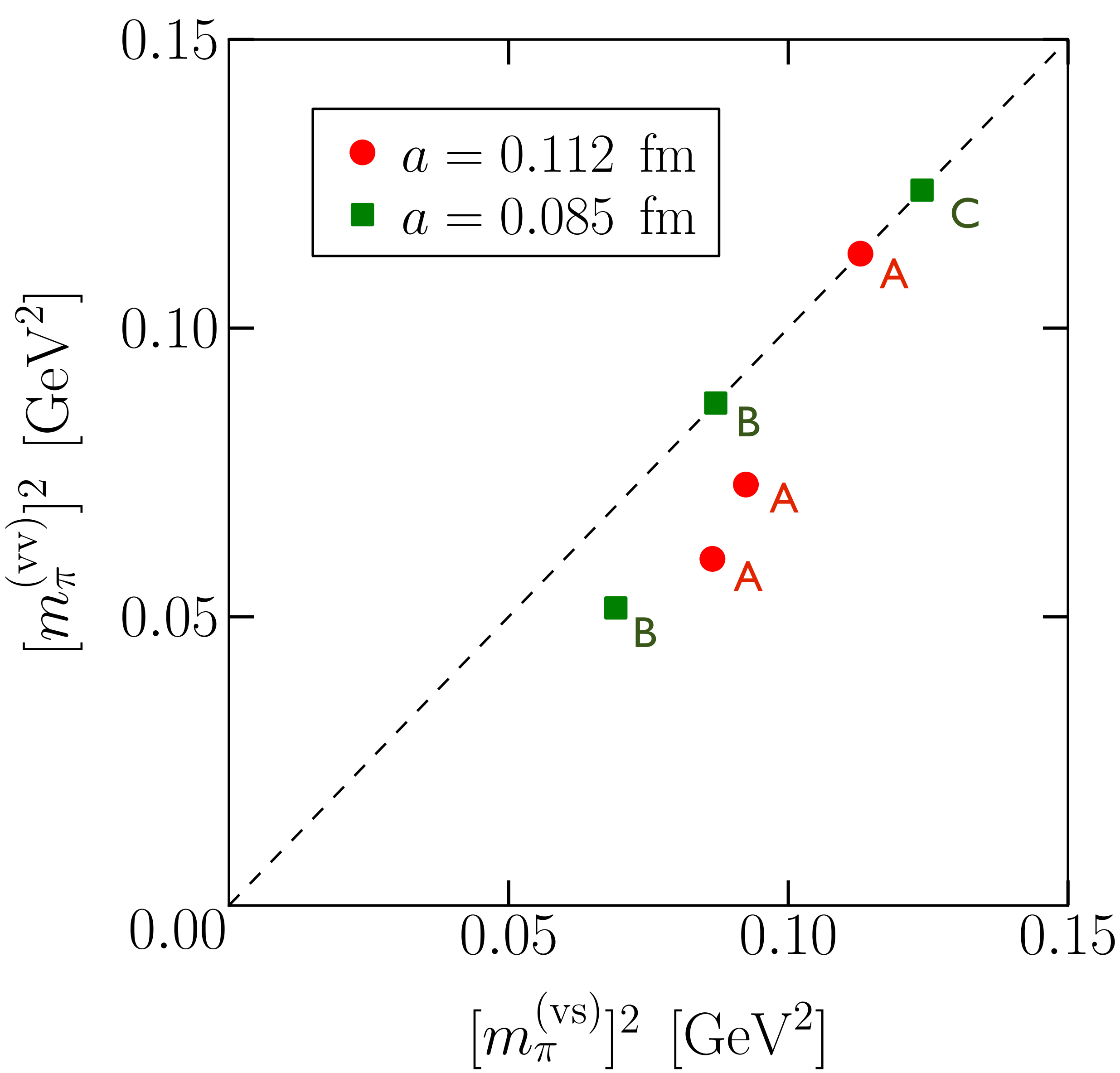



- O(a) improved* axial current:

$$Z_A = \begin{cases} 0.7019(26) & \text{for } a = 0.112 \text{ fm}, \\ 0.7396(17) & \text{for } a = 0.085 \text{ fm}. \end{cases}$$ [RBC]

# Correlation functions

- Interpolating operators in static limit

$$P^i = \overline{Q}_{a\alpha}\,(\gamma_5)_{\alpha\beta}\,\tilde{q}^i_{a\beta}, \qquad S^{ij}_{\mu\,\alpha} = \epsilon_{abc}\,(C\gamma_\mu)_{\beta\gamma}\,\tilde{q}^i_{a\beta}\,\tilde{q}^j_{b\gamma}\,Q_{c\alpha},$$

$$P^{*i}_\mu = \overline{Q}_{a\alpha}\,(\gamma_\mu)_{\alpha\beta}\,\tilde{q}^i_{a\beta}, \qquad T^{ij}_\alpha = \epsilon_{abc}\,(C\gamma_5)_{\beta\gamma}\,\tilde{q}^i_{a\beta}\,\tilde{q}^j_{b\gamma}\,Q_{c\alpha}.$$

- Two point and three point correlation functions

$$C[P^u\,P^\dagger_u](t) = \sum_{\mathbf{x}} \langle\, P^u(\mathbf{x},t)\,P^\dagger_u(0)\,\rangle,$$

$$C[P^{*d}\,P^{*\dagger}_d]^{\mu\nu}(t) = \sum_{\mathbf{x}} \langle\, P^{*d\,\mu}(\mathbf{x},t)\,P^{*\,\nu\dagger}_d(0)\,\rangle,$$

$$C[S^{dd}\,\overline{S}_{dd}]^{\mu\nu}_{\alpha\beta}(t) = \sum_{\mathbf{x}} \langle\, S^{dd\,\mu}_\alpha(\mathbf{x},t)\,\overline{S}^\nu_{dd\,\beta}(0)\,\rangle,$$

$$C[S^{du}\,\overline{S}_{du}]^{\mu\nu}_{\alpha\beta}(t) = \sum_{\mathbf{x}} \langle\, S^{du\,\nu}_\alpha(\mathbf{x},t)\,\overline{S}^\nu_{du\,\beta}(0)\,\rangle,$$

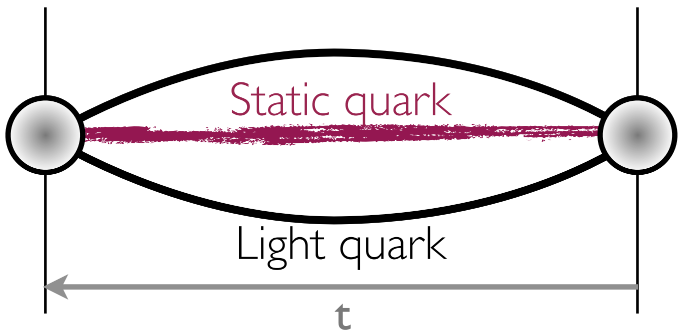
$$C[T^{du}\,\overline{T}_{du}]_{\alpha\beta}(t) = \sum_{\mathbf{x}} \langle\, T^{du}_\alpha(\mathbf{x},t)\,\overline{T}_{du\,\beta}(0)\,\rangle.$$

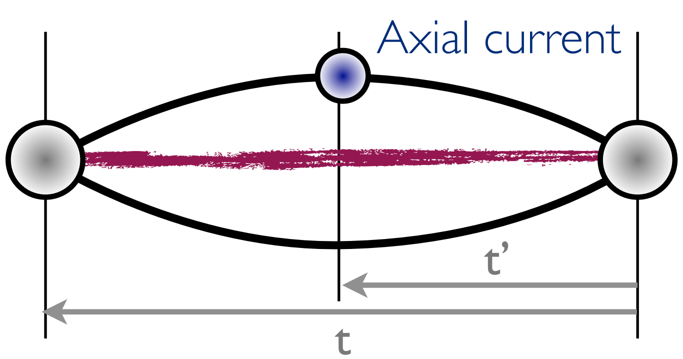
$$C[P^{*d}\,A\,P^\dagger_u]^{\mu\nu}(t,\,t') = \sum_{\mathbf{x}}\sum_{\mathbf{x}'} \langle\, P^{*d\,\mu}(\mathbf{x},t)\,A^\nu(\mathbf{x}',t')\,P^\dagger_u(0)\,\rangle,$$

$$C[S^{dd}A\,\overline{S}_{du}]^{\mu\nu\rho}_{\alpha\beta}(t,\,t') = \sum_{\mathbf{x}}\sum_{\mathbf{x}'} \langle\, S^{dd\,\mu}_\alpha(\mathbf{x},t)\,A^\nu(\mathbf{x}',t')\,\overline{S}^\rho_{du\,\beta}(0)\,\rangle,$$

$$C[S^{dd}A\,\overline{T}_{du}]^{\mu\nu}_{\alpha\beta}(t,\,t') = \sum_{\mathbf{x}}\sum_{\mathbf{x}'} \langle\, S^{dd\,\mu}_\alpha(\mathbf{x},t)\,A^\nu(\mathbf{x}',t')\,\overline{T}_{du\,\beta}(0)\,\rangle,$$

$$C[T^{du}A^\dagger\,\overline{S}_{dd}]^{\mu\nu}_{\alpha\beta}(t,\,t') = \sum_{\mathbf{x}}\sum_{\mathbf{x}'} \langle\, T^{du}_\alpha(\mathbf{x},t)\,A^{\mu\dagger}(\mathbf{x}',t')\,\overline{S}^\nu_{dd\,\beta}(0)\,\rangle.$$



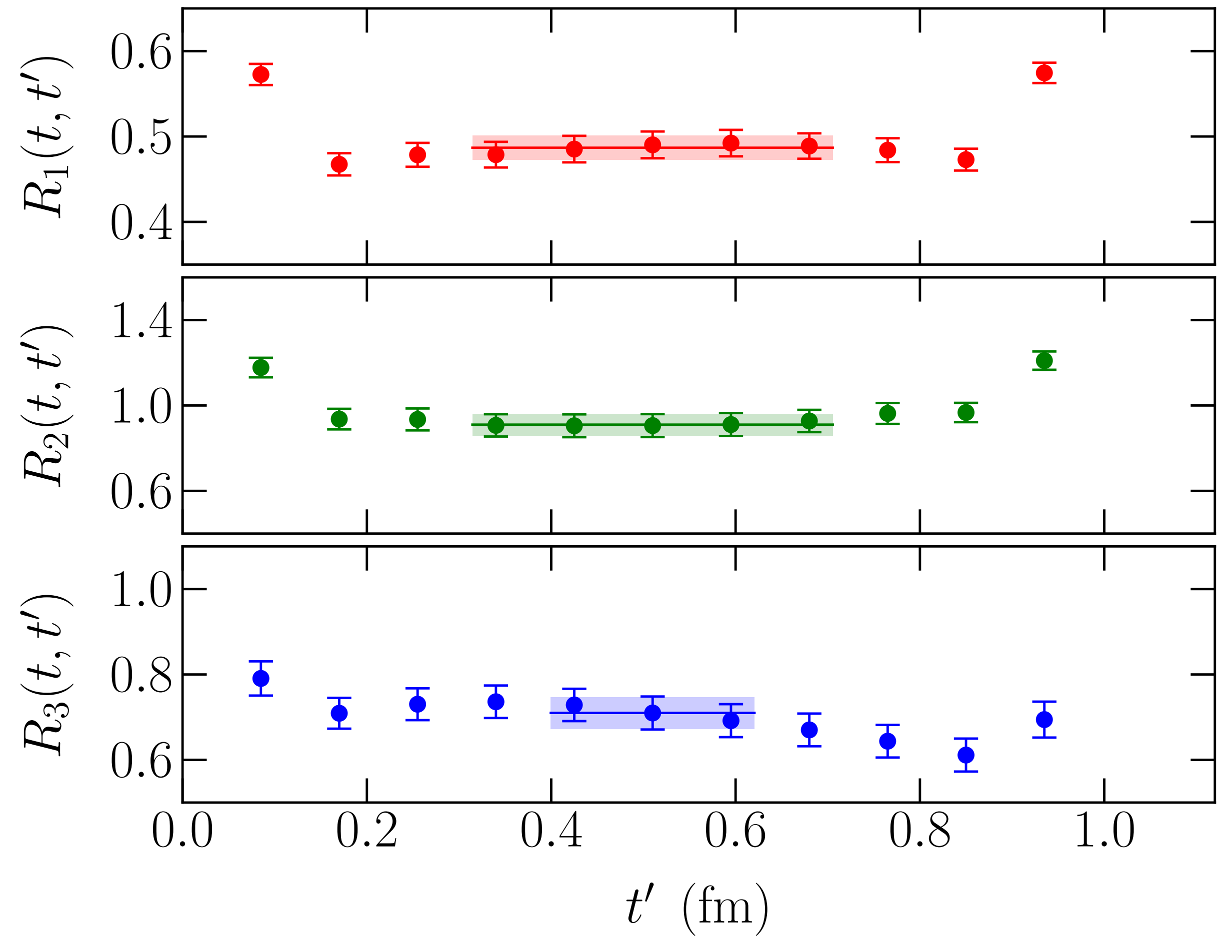

- Calculate with forward propagators from 2 sources

# Correlator ratios

- Ratios of 3pt to 2pt correlation functions give effective couplings

$$R_1(t,\, t') = -\frac{\frac{1}{3}\sum_{\mu=1}^{3} C[P^{*d}\, A\, P_u^\dagger]^{\mu\mu}(t,\, t')}{C[P^u\, P_u^\dagger](t)}$$

$$\xrightarrow{t,t'\to\infty} (g_1)_{\text{eff}}$$




- Ratios for varying operator insertion time (t')

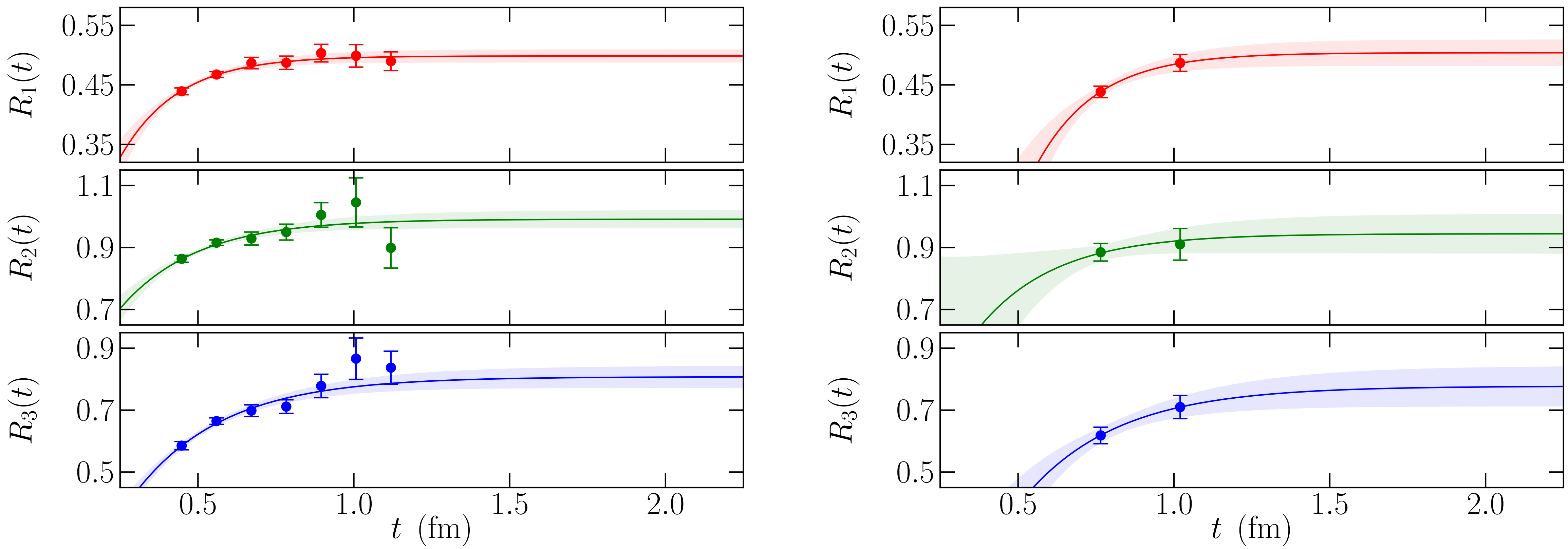
- Negligible t' dependence away from source/sink
  - No evidence for large transition matrix elements

# Source-sink separation

- Extract effective axial couplings $(g_i)_{\mathrm{eff}}$ from t extrapolation

$$R_i(t, a, m_\pi, n_{\mathrm{HYP}}) = (g_i)_{\mathrm{eff}}(a, m_\pi, n_{\mathrm{HYP}}) - A_i(a, m_\pi, n_{\mathrm{HYP}}) e^{-\delta_i(a, m_\pi, n_{\mathrm{HYP}})t}$$

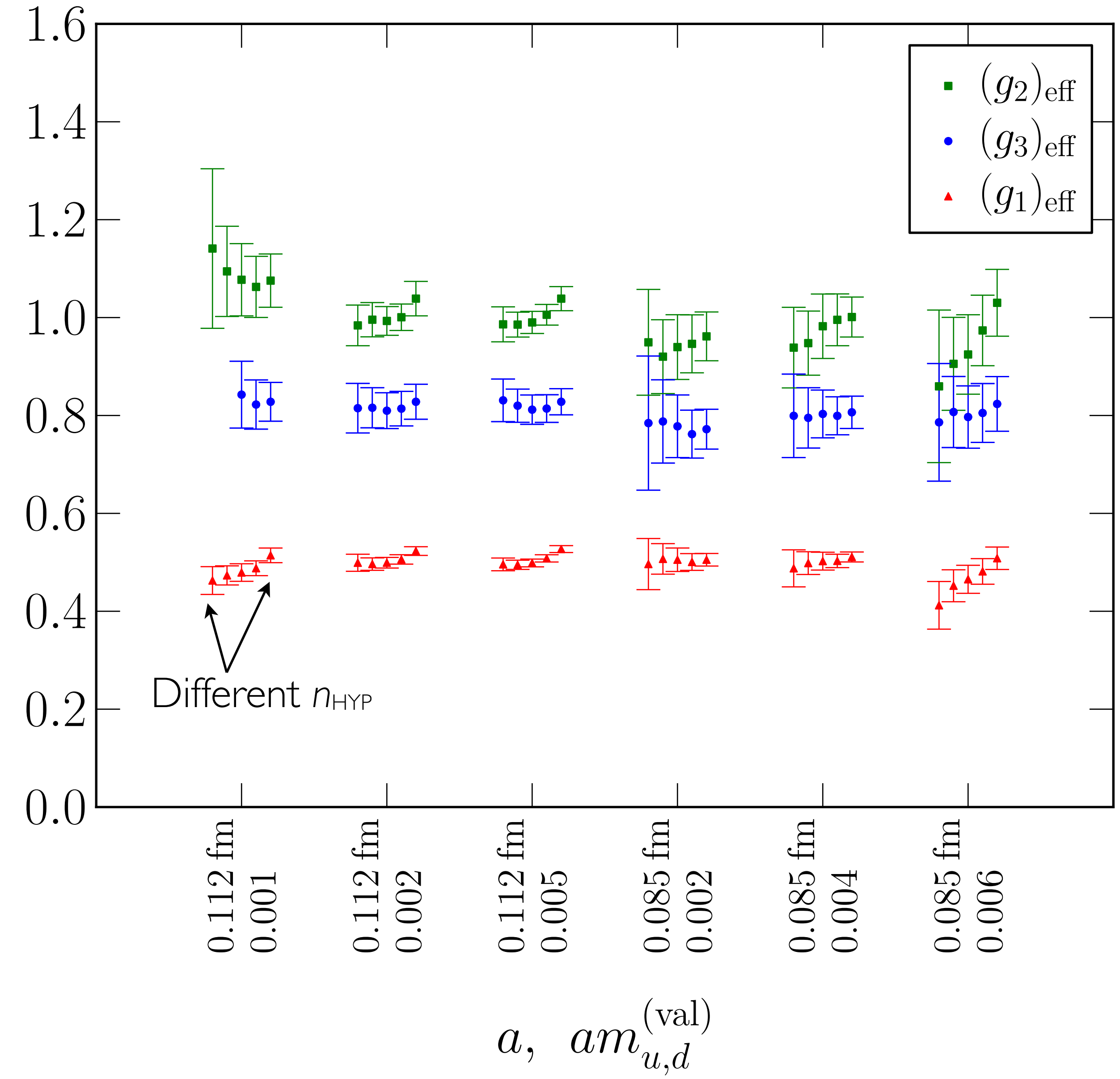
- Constrain $\boldsymbol{\delta_i}$ for a=0.086 fm from $\boldsymbol{\delta_i}$ at a=0.112 fm




- Fitted gaps: $\delta_i \sim 0.7$—$1.0$ GeV

# Source-sink separation

- Extracted effective couplings $(g_i)_{\text{eff}}(a, m_\pi, n_{\text{HYP}})$
- Estimate systematic uncertainty in extrapolation
  - Remove 1 or 2 points
  - Add second exp with Gaussian priors
  - 2, 3, 5% for $g_{1,2,3}$



# Chiral and continuum extrapolation

- Use NLO partially quenched SU(4|2) HHχPT at finite volume and include polynomial discretisation effects

$$(g_1)_{\rm eff}(a,m,n_{\rm HYP}) = g_1\Big[1-\frac{2}{f^2}\,\mathcal{I}(m_\pi^{(\rm vs)})+\frac{g_1^2}{f^2}\Big\{4\,\mathcal{H}(m_\pi^{(\rm vs)},\,0)-4\,\delta_{VS}^2\mathcal{H}_{\eta'}(m_\pi^{(\rm vv)},\,0)\Big\}$$
$$+\,c_1^{(\rm vv)}\,[m_\pi^{(\rm vv)}]^2+c_1^{(\rm vs)}\,[m_\pi^{(\rm vs)}]^2+d_{1,\,n_{\rm HYP}}\,a^2\Big].$$

$$(g_2)_{\rm eff}(a,m,n_{\rm HYP}) = g_2\Big[1-\frac{2}{f^2}\,\mathcal{I}(m_\pi^{(\rm vs)})+\frac{g_2^2}{f^2}\Big\{\frac{3}{2}\,\mathcal{H}(m_\pi^{(\rm vs)},\,0)-\delta_{VS}^2\mathcal{H}_{\eta'}(m_\pi^{(\rm vv)},\,0)\Big\}$$
$$+\frac{g_3^2}{f^2}\Big\{2\,\mathcal{H}(m_\pi^{(\rm vs)},\,-\Delta)-\mathcal{H}(m_\pi^{(\rm vv)},\,-\Delta)-2\,\mathcal{K}(m_\pi^{(\rm vs)},\,-\Delta,\,0)\Big\}$$
$$+\,c_2^{(\rm vv)}\,[m_\pi^{(\rm vv)}]^2+c_2^{(\rm vs)}\,[m_\pi^{(\rm vs)}]^2+d_{2,\,n_{\rm HYP}}\,a^2\Big],$$

$$(g_3)_{\rm eff}(a,m,n_{\rm HYP}) = g_3\Big[1-\frac{2}{f^2}\,\mathcal{I}(m_\pi^{(\rm vs)})+\frac{g_3^2}{f^2}\Big\{\mathcal{H}(m_\pi^{(\rm vs)},\,-\Delta)-\frac{1}{2}\mathcal{H}(m_\pi^{(\rm vv)},\,-\Delta)$$
$$+\frac{3}{2}\mathcal{H}(m_\pi^{(\rm vv)},\,\Delta)+3\,\mathcal{H}(m_\pi^{(\rm vs)},\,\Delta)-\mathcal{K}(m_\pi^{(\rm vs)},\,\Delta,\,0)\Big\}$$
$$+\frac{g_2^2}{f^2}\Big\{-\mathcal{H}(m_\pi^{(\rm vs)},\,\Delta)-\mathcal{H}(m_\pi^{(\rm vv)},\,\Delta)+\mathcal{H}(m_\pi^{(\rm vs)},\,0)-\delta_{VS}^2\mathcal{H}_{\eta'}(m_\pi^{(\rm vv)},\,0)\Big\}$$
$$+\,c_3^{(\rm vv)}\,[m_\pi^{(\rm vv)}]^2+c_3^{(\rm vs)}\,[m_\pi^{(\rm vs)}]^2+d_{3,\,n_{\rm HYP}}\,a^2\Big].$$

Partial quenching

Loop functions

Lattice spacing effects depend on $n_{\rm HYP}$

$g_{2,3}$ extrapolation is coupled

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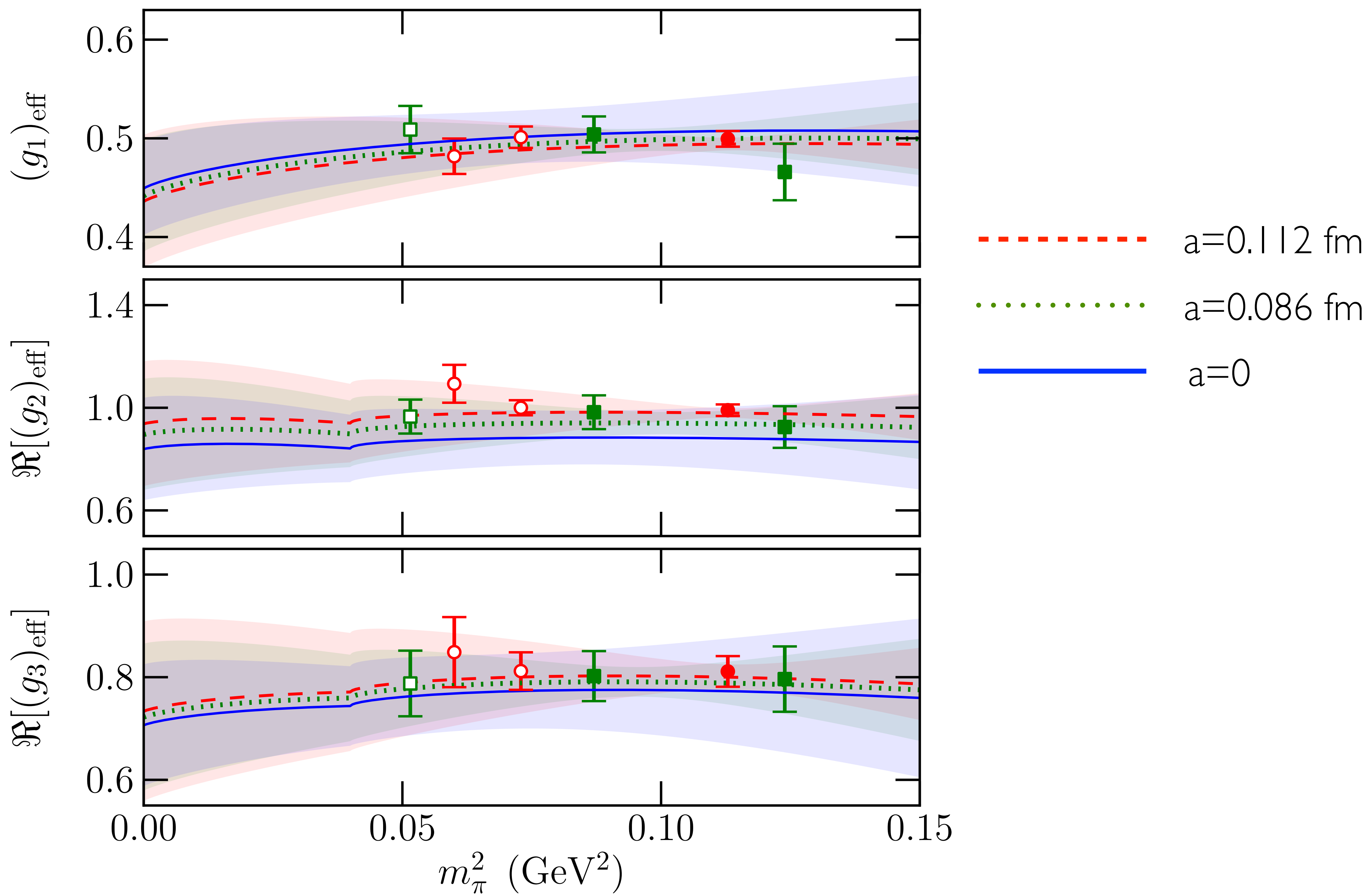
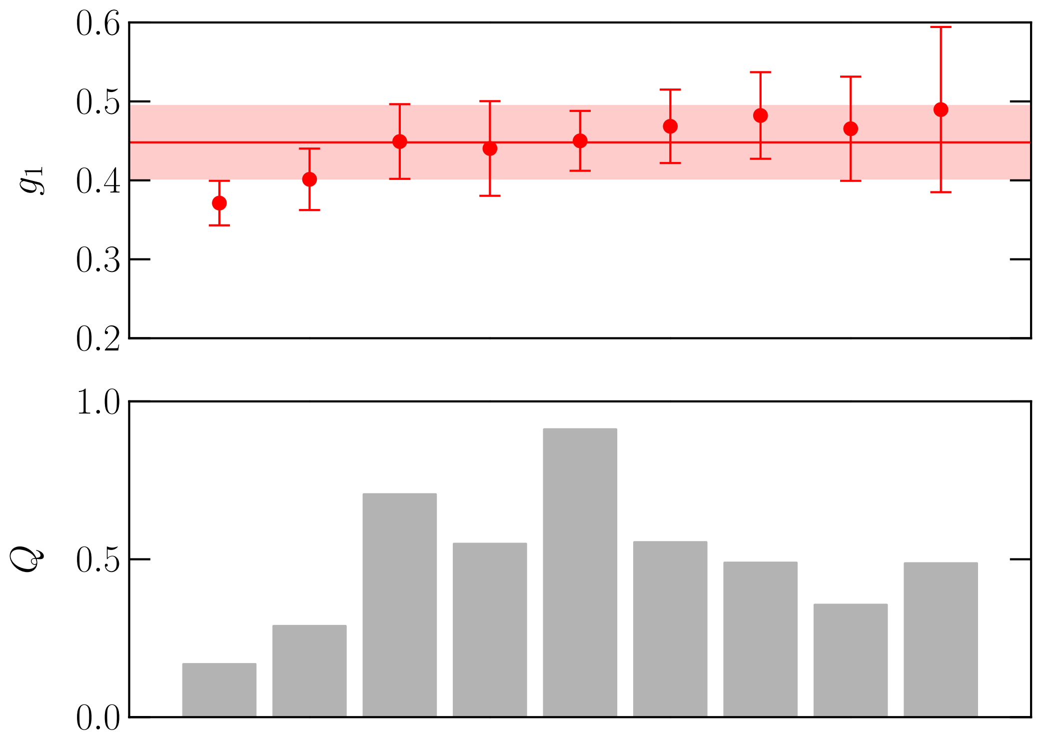
# Chiral and continuum extrapolation



# Systematic uncertainties

- Excited states in fits
- Heavy quark discretisation: various levels of HYP smearing
- Higher order terms in chiral extrapolation (quark mass and lattice spacing)
- Finite volume
- Unphysical strange quark mass



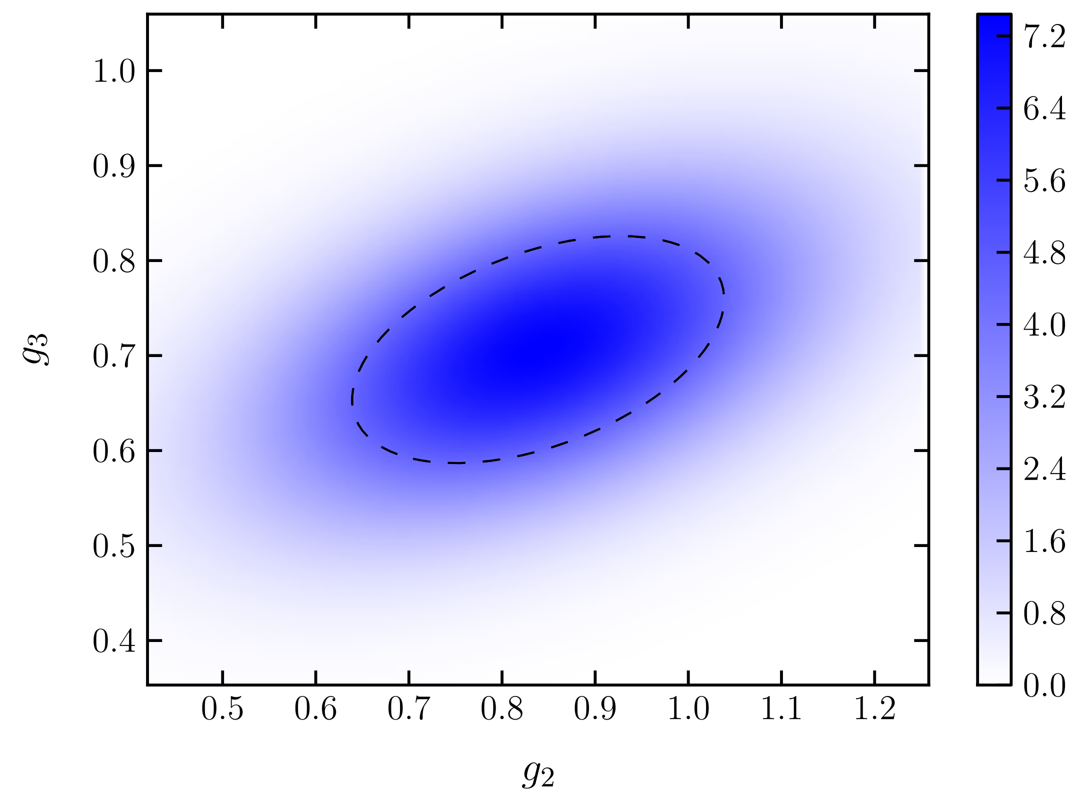
# Axial couplings

- Final extracted values

$$
\begin{aligned}
g_1 &= 0.449 \pm 0.047_{\,\text{stat}} \pm 0.019_{\,\text{syst}} \\
g_2 &= 0.84 \pm 0.20_{\,\text{stat}} \pm 0.04_{\,\text{syst}} \\
g_3 &= 0.71 \pm 0.12_{\,\text{stat}} \pm 0.04_{\,\text{syst}}
\end{aligned}
$$

- Sources of systematic errors

| Source | $g_1$ | $g_2$ | $g_3$ |
|---|---|---|---|
| NNLO terms in fits of $m_\pi$- and $a$-dependence | 3.6% | 2.8% | 3.7% |
| Higher excited states in fits to $R_i(t)$ | 1.7% | 2.8% | 4.9% |
| Unphysical value of $m_s^{(\text{sea})}$ | 1.5% | 1.5% | 1.5% |
| Total | 4.2% | 4.3% | 6.3% |




- Dominated by statistical errors

# Decay widths

- Strong decays allowed for heavy baryons

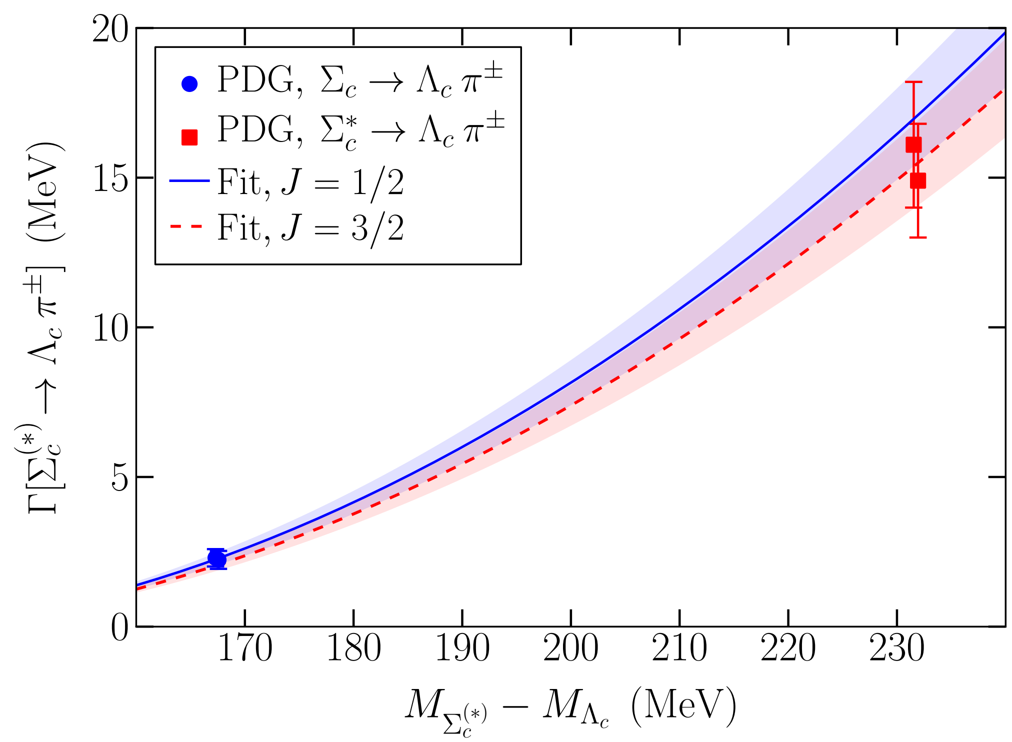
$$\Gamma[S \to T\,\pi] = c_{\mathrm{f}}^2\,\frac{1}{6\pi f_\pi^2}\left(g_3 + \frac{\kappa_J}{m_Q}\right)^2 \frac{M_T}{M_S}\,|\mathbf{p}_\pi|^3$$

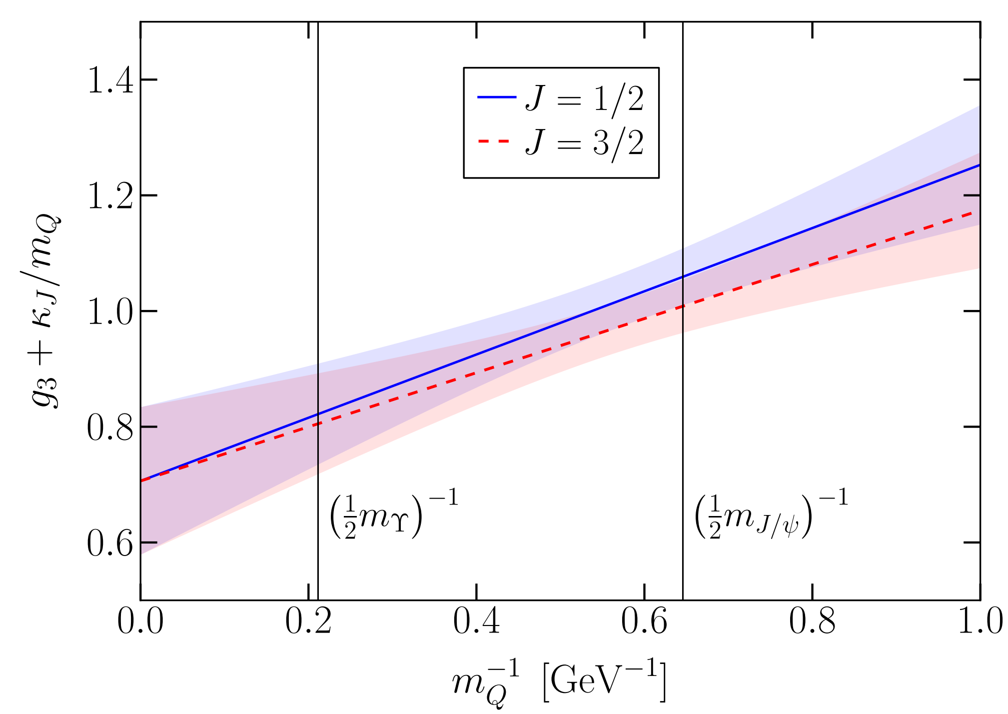
$$c_{\mathrm{f}} = \begin{cases} 1 & \text{for} \quad \Sigma_Q^{(*)} \to \Lambda_Q\,\pi^\pm, \\ 1 & \text{for} \quad \Sigma_Q^{(*)} \to \Lambda_Q\,\pi^0, \\ 1/\sqrt{2} & \text{for} \quad \Xi_Q^{\prime(*)} \to \Xi_Q\,\pi^\pm, \\ 1/2 & \text{for} \quad \Xi_Q^{\prime(*)} \to \Xi_Q\,\pi^0. \end{cases}$$

- 1/$m_Q$ corrections important: determine from charm sector

- Effective coupling vs 1/$m_Q$

- Valid only at LO in HHχPT

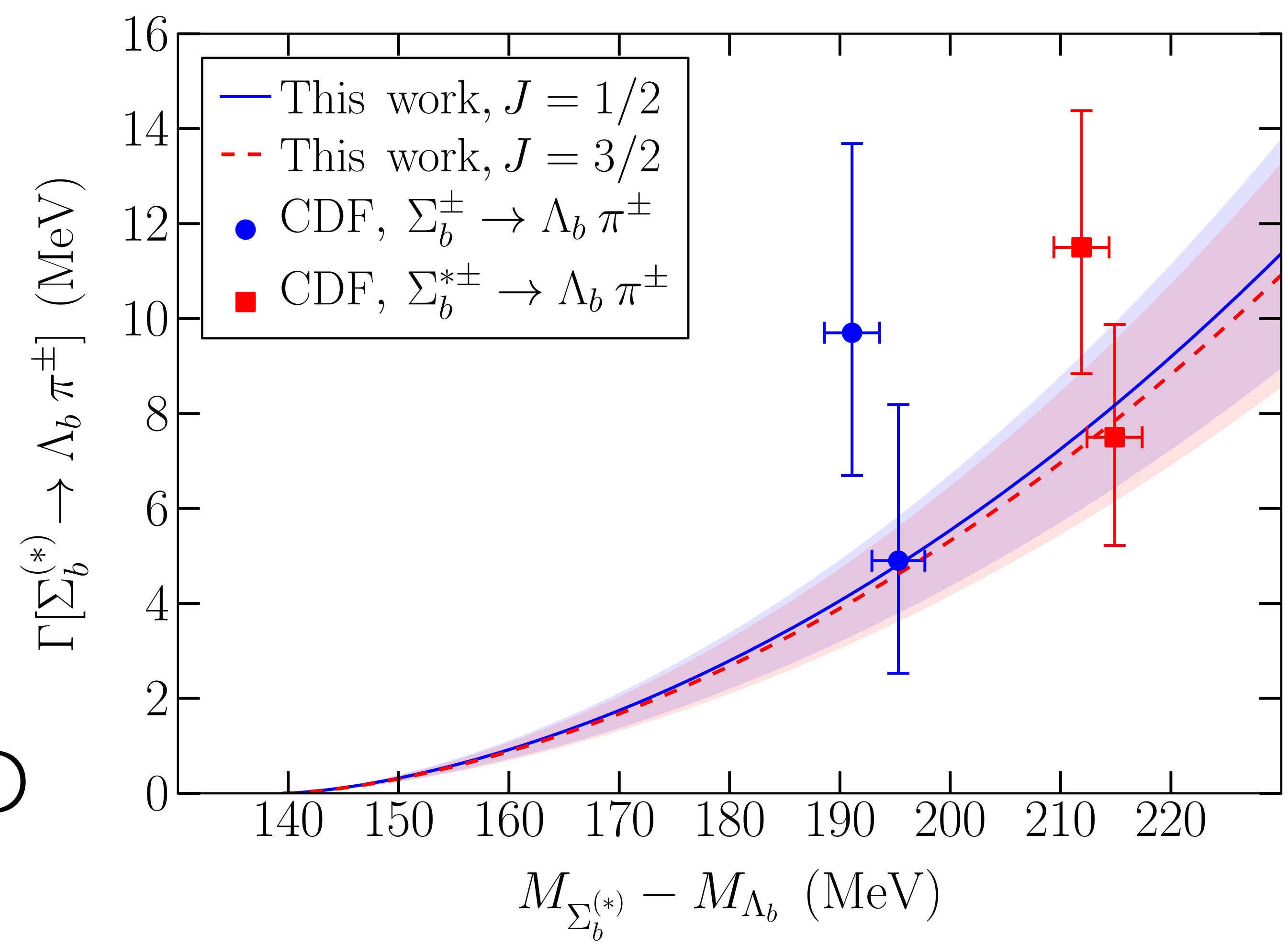


# Decay widths

- Calculate (and predict) bottom and charm baryon decay widths

| Hadron | This work | Experiment |
|---|---|---|
| $\Sigma_b^+$ | 4.2(1.0) | $9.7^{+3.8+1.2}_{-2.8-1.1}$ [13] |
| $\Sigma_b^-$ | 4.8(1.1) | $4.9^{+3.1}_{-2.1} \pm 1.1$ [13] |
| $\Sigma_b^{*+}$ | 7.3(1.6) | $11.5^{+2.7+1.0}_{-2.2-1.5}$ [13] |
| $\Sigma_b^{*-}$ | 7.8(1.8) | $7.5^{+2.2+0.9}_{-1.8-1.4}$ [13] |
| $\Xi_b'$ | 1.1 (CL=90%) | ... |
| $\Xi_b^*$ | 2.8 (CL=90%) | ... |
| $\Xi_c^{*+}$ | 2.44(26) | $< 3.1$ (CL=90%) [70] |
| $\Xi_c^{*0}$ | 2.78(29) | $< 5.5$ (CL=90%) [71] |




- Uses determinations of $\Xi_b'$, $\Xi_b^*$ masses from LQCD [Lewis & Woloshyn 09]

- Update: CDF observation of $\Xi_b^*$ leads to

$$\Gamma[\Xi_b^{*0} \to \Xi_b^- \pi^+,\ \Xi_b^0 \pi^0] = 0.51 \pm 0.16 \ \text{MeV}.$$

# Heavy hadron axial couplings

- First complete calculation of heavy hadron axial couplings controlling all systematics

$$
\begin{aligned}
g_1 &= 0.449 \pm 0.047_{\,\mathrm{stat}} \pm 0.019_{\,\mathrm{syst}} \\
g_2 &= 0.84 \;\; \pm 0.20_{\,\mathrm{stat}} \;\; \pm 0.04_{\,\mathrm{syst}} \\
g_3 &= 0.71 \;\; \pm 0.12_{\,\mathrm{stat}} \;\; \pm 0.04_{\,\mathrm{syst}}
\end{aligned}
$$

  - Considerably smaller than quark model estimates
  - Pleasant consequences for convergence of HHχPT
- Allows pre- (and post-) dictions of strong decay widths (also $\Gamma[\Xi_c^* \to \Xi_c \gamma]$ )
- Currently extending to look at SU(3) breaking effects

[fin]

# Actions and ensembles

- Further details

| $L^3 \times T$ | $am_s^{(\mathrm{sea})}$ | $am_{u,d}^{(\mathrm{sea})}$ | $am_{u,d}^{(\mathrm{val})}$ | $a$ (fm) | $m_\pi^{(\mathrm{ss})}$ (MeV) | $m_\pi^{(\mathrm{vs})}$ (MeV) | $m_\pi^{(\mathrm{vv})}$ (MeV) |
|---|---|---|---|---|---|---|---|
| $24^3 \times 64$ | 0.04 | 0.005 | 0.001 | 0.1119(17) | 336(5) | 294(5) | 245(4) |
| $24^3 \times 64$ | 0.04 | 0.005 | 0.002 | 0.1119(17) | 336(5) | 304(5) | 270(4) |
| $24^3 \times 64$ | 0.04 | 0.005 | 0.005 | 0.1119(17) | 336(5) | 336(5) | 336(5) |
| $32^3 \times 64$ | 0.03 | 0.004 | 0.002 | 0.0849(12) | 295(4) | 263(4) | 227(3) |
| $32^3 \times 64$ | 0.03 | 0.004 | 0.004 | 0.0849(12) | 295(4) | 295(4) | 295(4) |
| $32^3 \times 64$ | 0.03 | 0.006 | 0.006 | 0.0848(17) | 352(7) | 352(7) | 352(7) |

- Numbers of measurements

| $L^3 \times T$ | $am_{u,d}^{(\mathrm{val})}$ | $t/a$ | $N_{\mathrm{meas}}$ (approx.) |
|---|---|---|---|
| $24^3 \times 64$ | 0.001 | 10 | 550 |
| $24^3 \times 64$ | 0.001 | 9, 8, 7, 6 | 240 |
| $24^3 \times 64$ | 0.001 | 5 | 460 |
| $24^3 \times 64$ | 0.001 | 4 | 120 |
| $24^3 \times 64$ | 0.002 | 10 | 880 |
| $24^3 \times 64$ | 0.002 | 9, 8, 7, 6, 4 | 240 |
| $24^3 \times 64$ | 0.002 | 5 | 480 |
| $24^3 \times 64$ | 0.005 | 10 | 960 |
| $24^3 \times 64$ | 0.005 | 9, 8, 7, 6, 4 | 240 |
| $24^3 \times 64$ | 0.005 | 5 | 480 |
| $32^3 \times 64$ | 0.002 | 12 | 1200 |
| $32^3 \times 64$ | 0.002 | 9, 6 | 480 |
| $32^3 \times 64$ | 0.004 | 12 | 1200 |
| $32^3 \times 64$ | 0.004 | 9, 6 | 480 |
| $32^3 \times 64$ | 0.006 | 13 | 700 |

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- Two lattice spacings
  a = 0.085, 0.112 fm
- Six valence quark masses
  $m_\pi$ = 0.23–0.35 GeV
- Single (2.5 fm)$^3$ volume

| Ensemble | $a$ (fm) | $L^3 \times T$ | $am_{u,d}^{(\mathrm{sea})}$ | $m_\pi^{(\mathrm{ss})}$ (MeV) |
|---|---|---|---|---|
| A | 0.1119(17) | $24^3 \times 64$ | 0.005 | 336(5) |
| B | 0.0849(12) | $32^3 \times 64$ | 0.004 | 295(4) |
| C | 0.0848(17) | $32^3 \times 64$ | 0.006 | 352(7) |

| Ensemble | $am_{u,d}^{(\mathrm{val})}$ | $m_\pi^{(\mathrm{vs})}$ (MeV) | $m_\pi^{(\mathrm{vv})}$ (MeV) | $t/a$ |
|---|---|---|---|---|
| A | 0.001 | 294(5) | 245(4) | 4, 5, ..., 10 |
| A | 0.002 | 304(5) | 270(4) | 4, 5, ..., 10 |
| A | 0.005 | 336(5) | 336(5) | 4, 5, ..., 10 |
| B | 0.002 | 263(4) | 227(3) | 6, 9, 12 |
| B | 0.004 | 295(4) | 295(4) | 6, 9, 12 |
| C | 0.006 | 352(7) | 352(7) | 13 |

# H-L hadrons in lattice QCD

- Light quark mass dependence of H-L(L) observables controlled by pion loops, coupled through $g_{1,2,3}$
- Important for control of current lattice QCD calculations at unphysical quark masses

# Correlator ratios

- Ratios of 3pt to 2pt correlation functions give effective couplings

$$R_1(t,\,t') = -\frac{\frac{1}{3}\sum_{\mu=1}^{3} C[P^{*d}\,A\,P_u^{\dagger}]^{\mu\mu}(t,\,t')}{C[P^u\,P_u^{\dagger}](t)} \quad \xrightarrow{t,t'\to\infty} \quad (g_1)_{\rm eff}$$

$$R_2(t,\,t') = 2\,\frac{\frac{i}{6}\sum_{\mu,\nu,\rho=1}^{3} \epsilon_{0\mu\nu\rho}\, C[S^{dd}A\;\overline{S}_{du}]^{\mu\nu\rho}(t,\,t')}{\frac{1}{3}\sum_{\mu=1}^{3} C[S^{dd}\,\overline{S}_{dd}]^{\mu\mu}(t)} \quad \xrightarrow{t,t'\to\infty} \quad (g_2)_{\rm eff}$$

- For transition coupling, double ratio

$$R_3(t,\,t') = \sqrt{\frac{\left[\frac{1}{3}\sum_{\mu=1}^{3} C[S^{dd}A\;\overline{T}_{du}]^{\mu\mu}(t,\,t')\right]\left[\frac{1}{3}\sum_{\mu=1}^{3} C[T^{du}A^{\dagger}\;\overline{S}_{dd}]^{\mu\mu}(t,\,t')\right]}{\left[\frac{1}{3}\sum_{\mu=1}^{3} C[S^{dd}\,\overline{S}_{dd}]^{\mu\mu}(t)\right]\;\left[C[T^{du}\,\overline{T}_{du}](t)\right]}} \quad \xrightarrow{t,t'\to\infty} \quad (g_3)_{\rm eff}$$

- Excited state contributions important for t,t'<∞

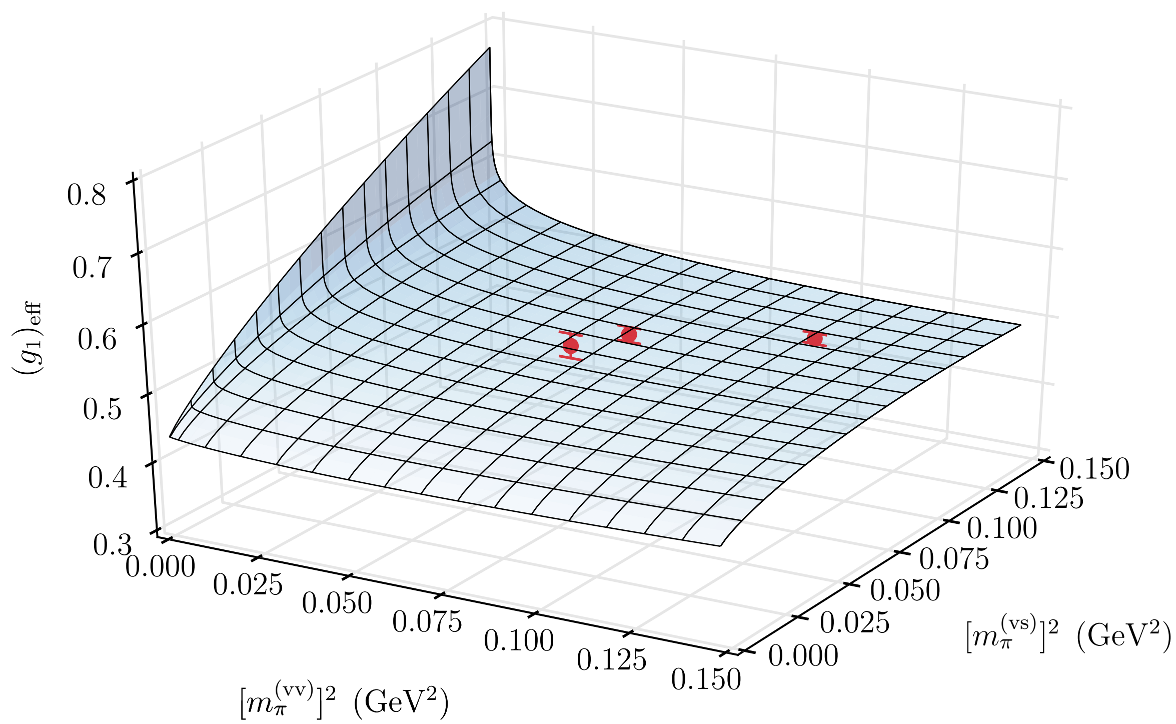
  *E.g.*

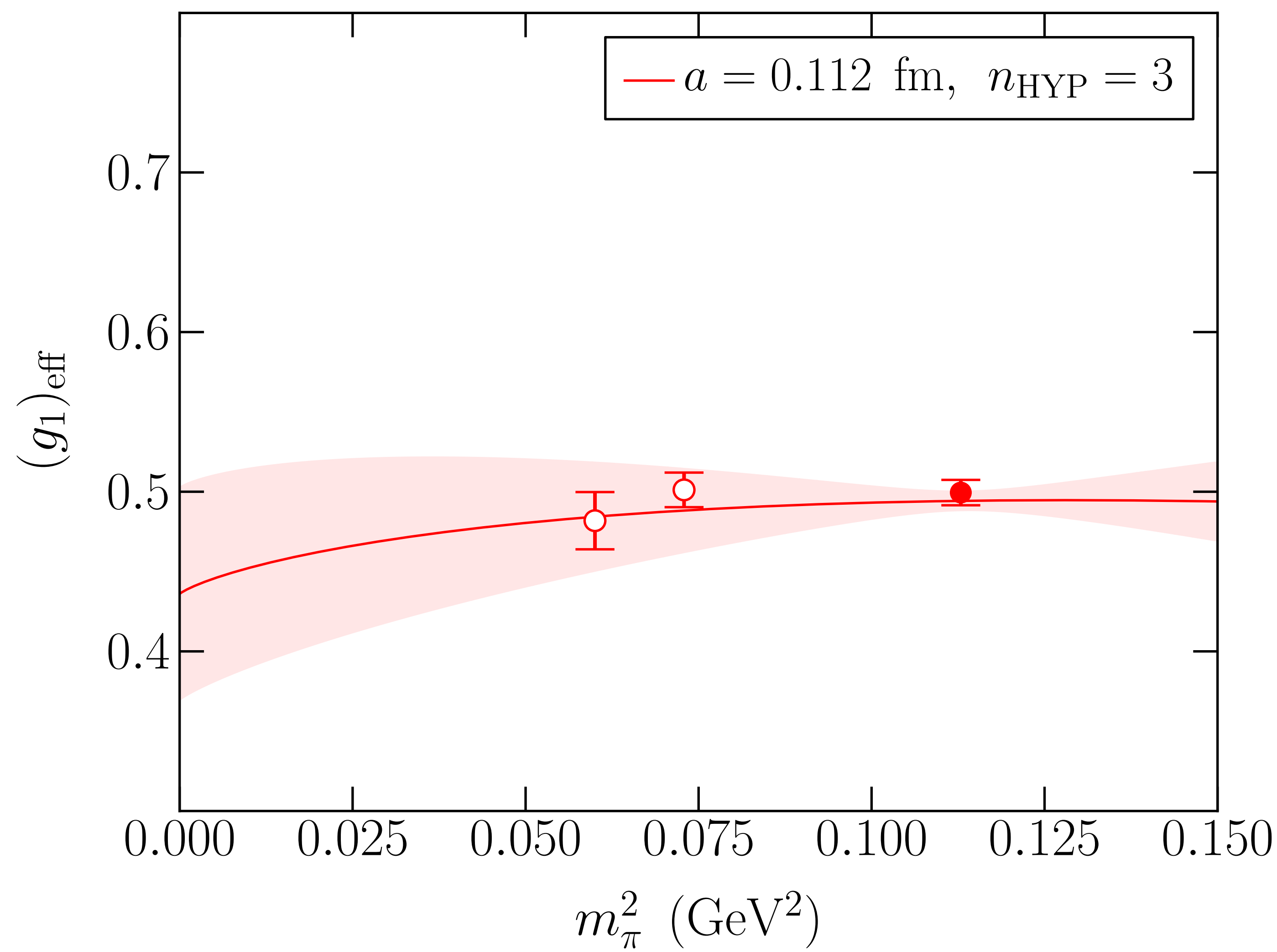
$$R_2(t,\,t/2) = A_{11}^{(SS)} + \left|\frac{Z_{S,2}}{Z_{S,1}}\right|^2 (A_{22}^{(SS)} - A_{11}^{(SS)})\,e^{-\delta_S\,t} + 2\,\Re\left[\frac{Z_{S,1}Z_{S,2}^*}{|Z_{S,1}|^2}\,A_{12}^{(SS)}\right]\,e^{-\frac{1}{2}\delta_S\,t} + \ldots$$

with energy gap $\delta_S = E_{S,2} - E_{S,1}$

# Chiral and continuum extrapolation

g1

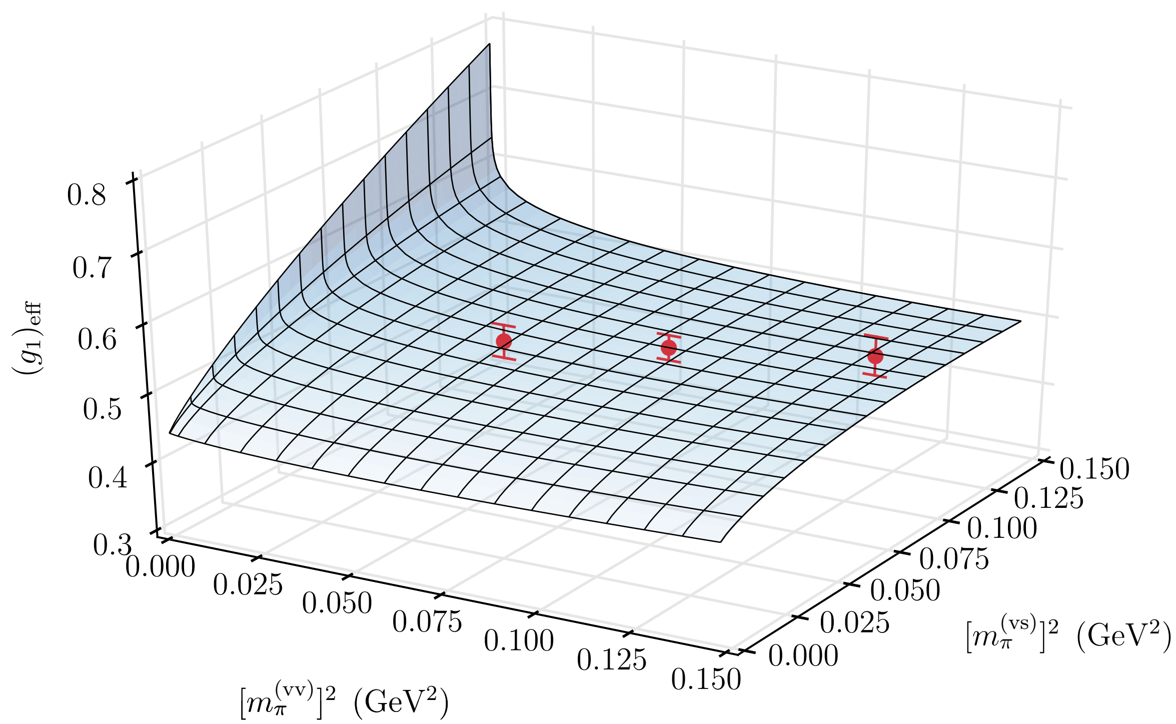


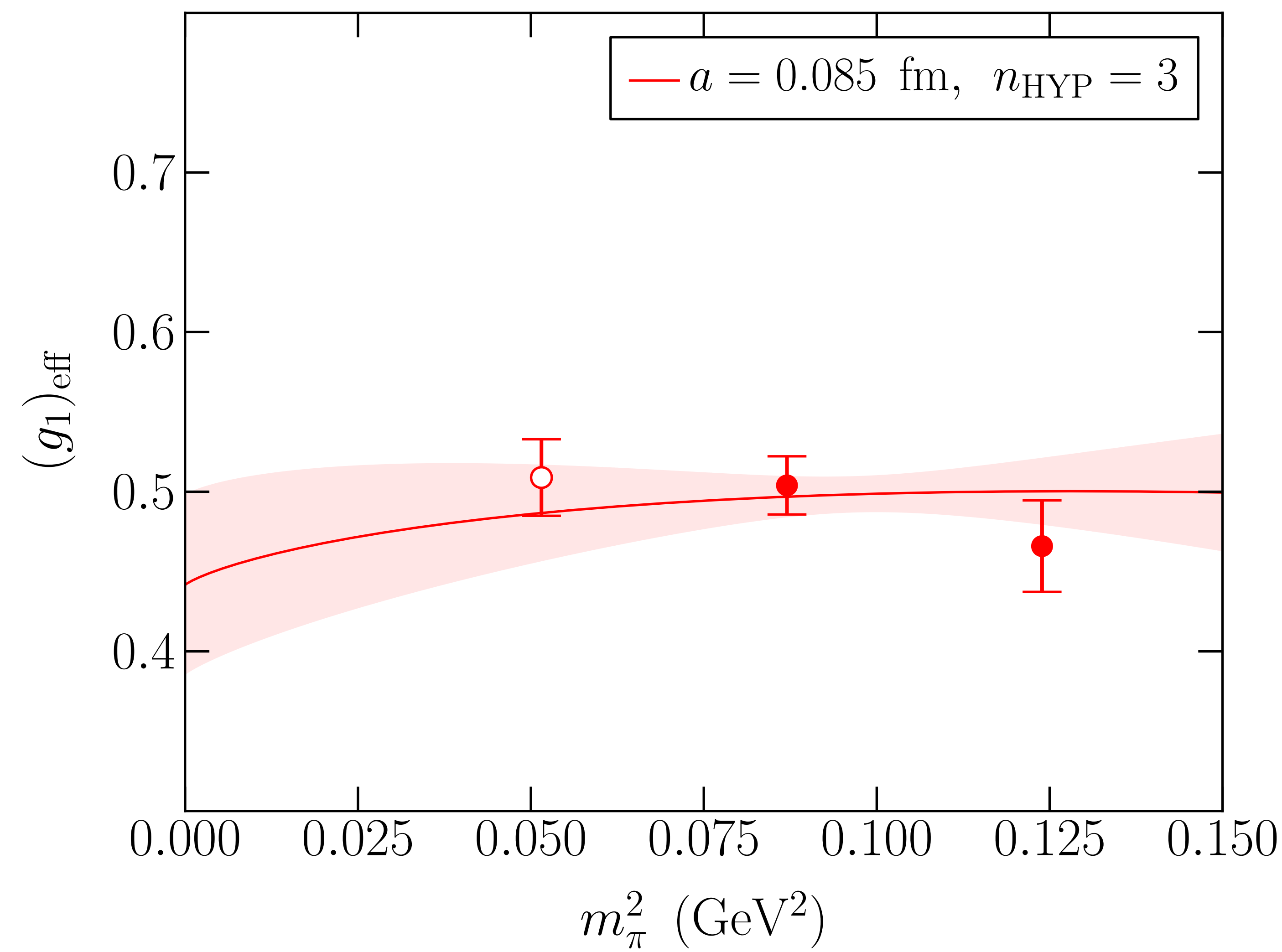




projection on unitary line

# Chiral and continuum extrapolation

## $g_1$

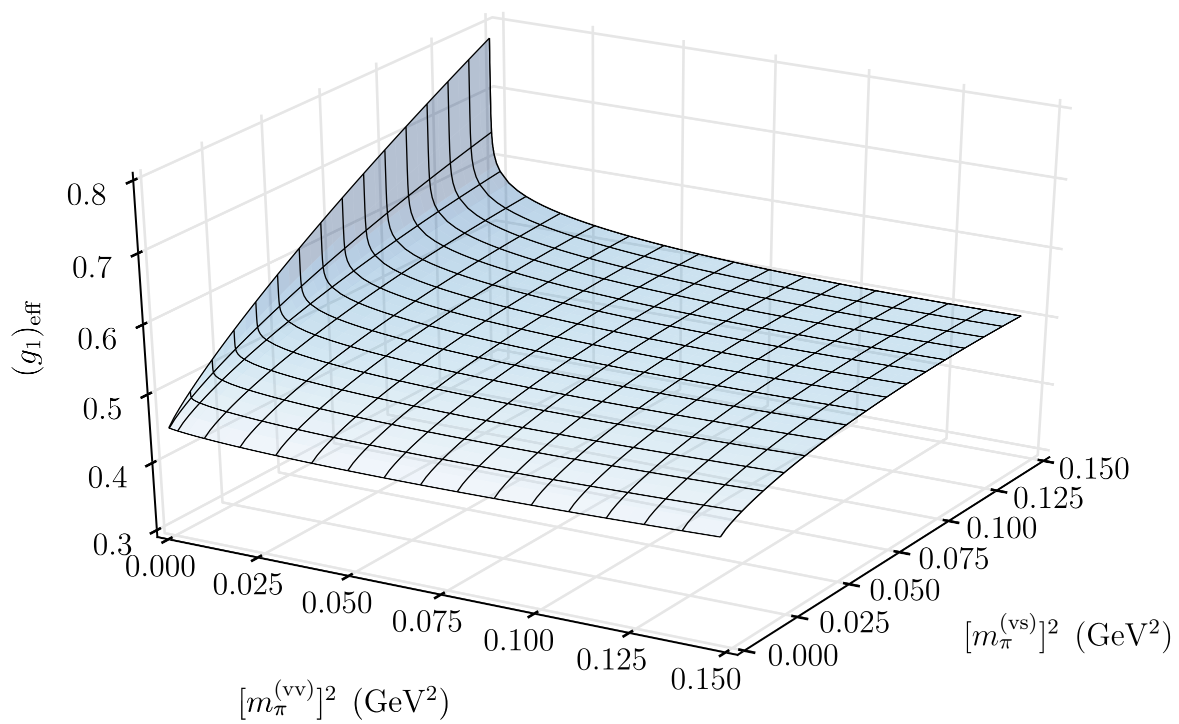


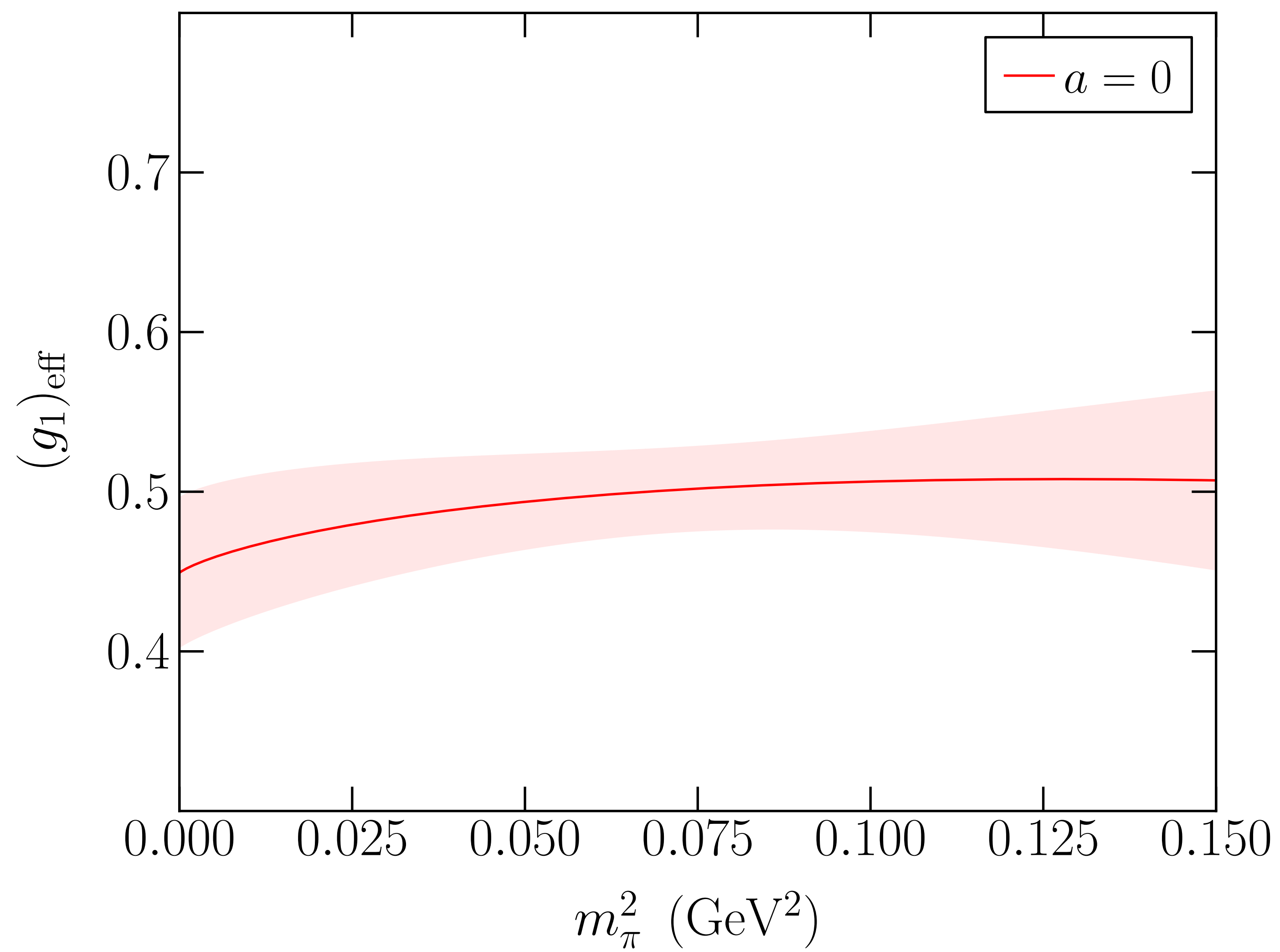




projection on unitary line

# Chiral and continuum extrapolation

$g_1$

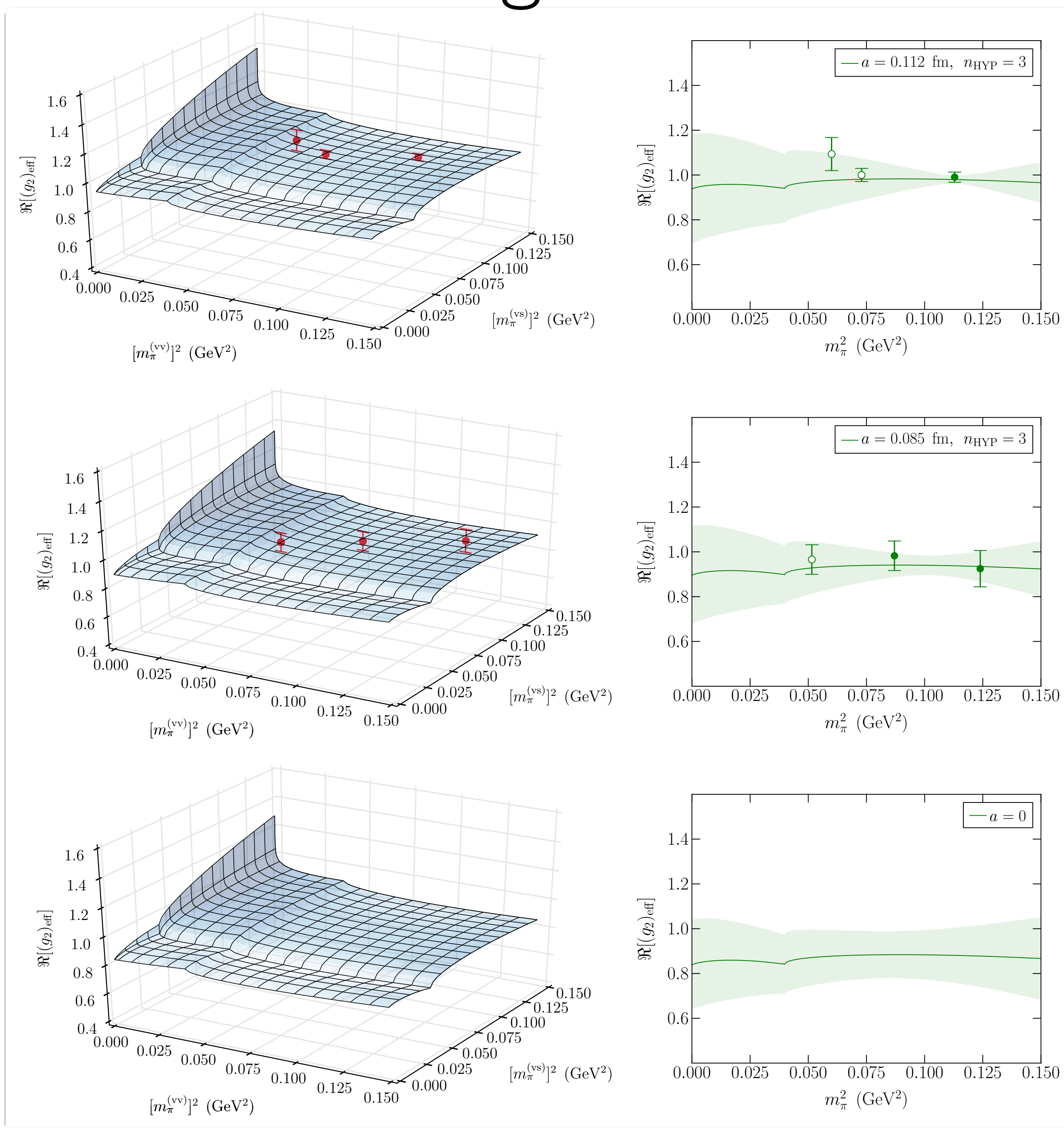


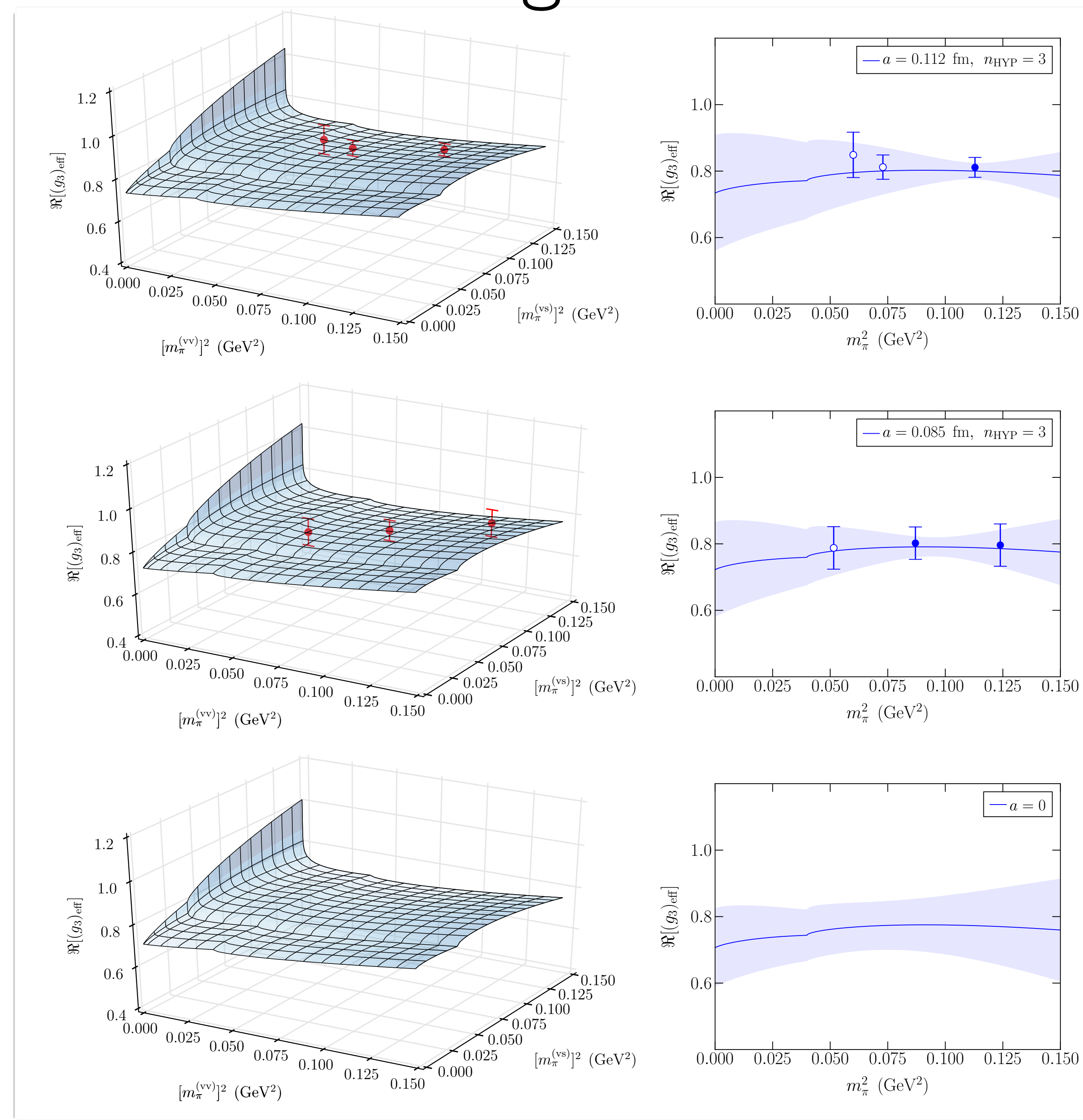

projection on unitary line

# Chiral and continuum extrapolation

g2



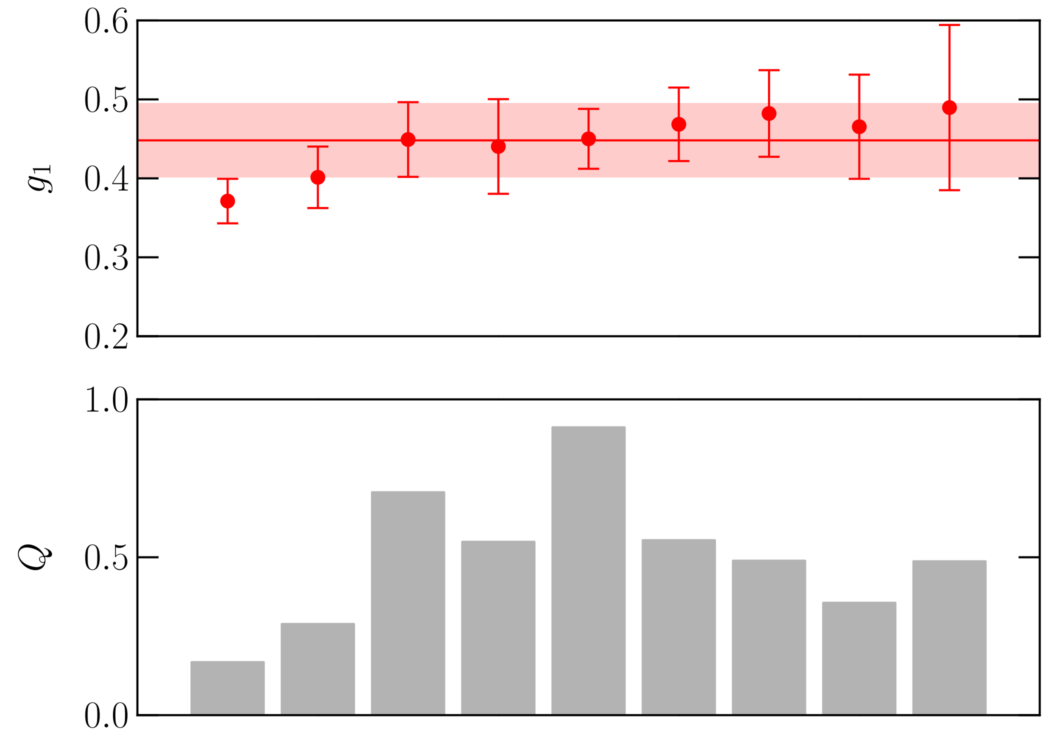

g3



# Chiral and continuum extrapolation

- Various choices of HQ actions to use in fits
- Heavy meson coupling $g_1$

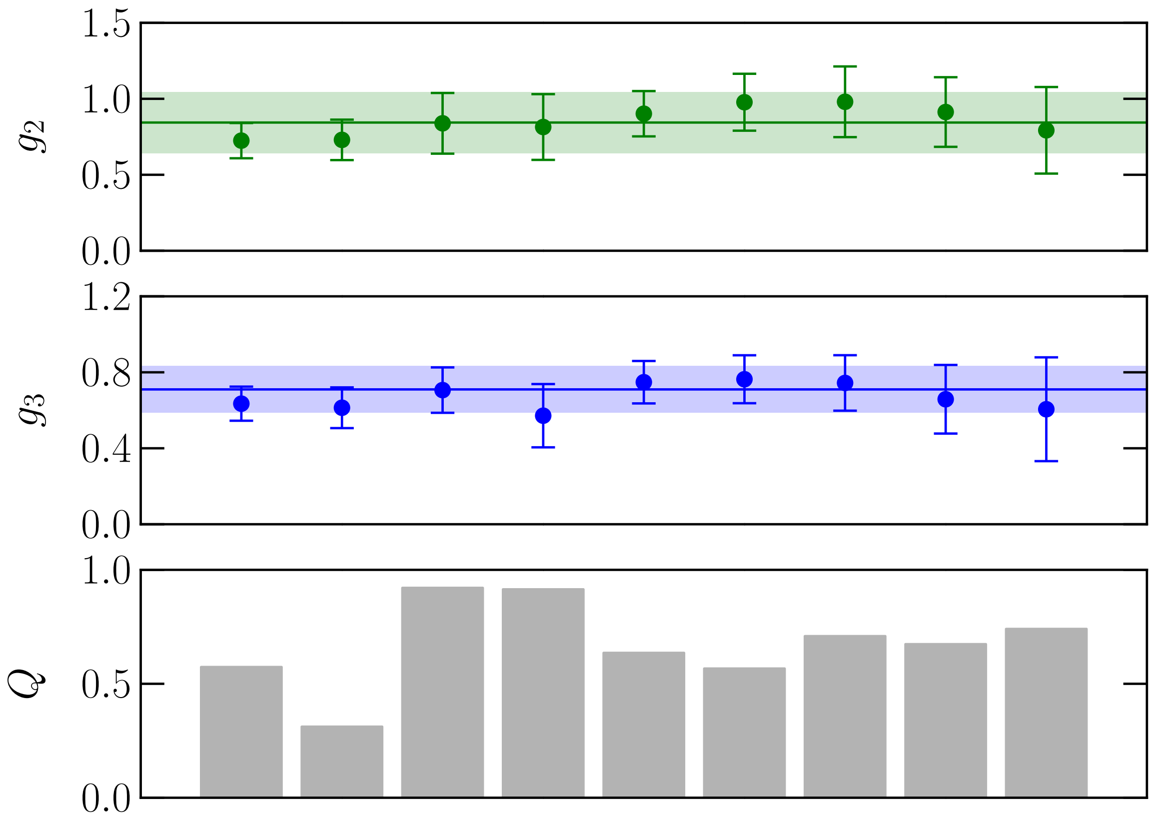
| $n_{\mathrm{HYP}}$ | $g_1$ | d.o.f. | $\chi^2$/d.o.f. | $Q$ |
|---|---|---|---|---|
| 1, 2, 3, 5, 10 | 0.371(28) | 30 − 8 | 1.3 | 0.17 |
| 1, 2, 3, 5 | 0.401(39) | 24 − 7 | 1.2 | 0.29 |
| 1, 2, 3 | 0.449(47) | 18 − 6 | 0.75 | 0.70 |
| 1, 2 | 0.440(60) | 12 − 5 | 0.85 | 0.54 |
| 10 | 0.450(38) | 6 − 4 | 0.09 | 0.91 |
| 5 | 0.468(47) | 6 − 4 | 0.61 | 0.55 |
| 3 | 0.482(55) | 6 − 4 | 0.73 | 0.49 |
| 2 | 0.465(66) | 6 − 4 | 1.0 | 0.36 |
| 1 | 0.49(10) | 6 − 4 | 0.72 | 0.49 |



# Chiral and continuum extrapolation

- Various choices of HQ actions to use in fits
- Heavy baryon couplings $g_{2,3}$

| $n_{\text{HYP}}$ | $g_2$ | $g_3$ | d.o.f. | $\chi^2$/d.o.f. | $Q$ |
|---|---|---|---|---|---|
| 1, 2, 3, 5, 10 | 0.72(12) | 0.635(90) | 58 − 16 | 0.94 | 0.57 |
| 1, 2, 3, 5 | 0.73(13) | 0.61(11) | 46 − 14 | 1.1 | 0.31 |
| 1, 2, 3 | 0.84(20) | 0.71(12) | 34 − 12 | 0.61 | 0.92 |
| 1, 2 | 0.81(22) | 0.57(17) | 22 − 10 | 0.50 | 0.91 |
| 10 | 0.90(15) | 0.75(11) | 12 − 8 | 0.64 | 0.64 |
| 5 | 0.98(19) | 0.76(13) | 12 − 8 | 0.74 | 0.57 |
| 3 | 0.98(23) | 0.74(15) | 12 − 8 | 0.54 | 0.71 |
| 2 | 0.91(23) | 0.66(18) | 12 − 8 | 0.51 | 0.67 |
| 1 | 0.79(29) | 0.61(27) | 12 − 8 | 0.42 | 0.74 |



# Higher order terms

- Add higher order analytic terms in quark masses and lattice spacings

$$
\begin{aligned}
(g_i)_{\rm eff}^{(\rm NLO+HO)}(a,m,n_{\rm HYP}) = (g_i)_{\rm eff}^{(\rm NLO)}(a,m,n_{\rm HYP}) \\
+g_i \Big[ c_i^{(\rm vv,vv)}\,[m_\pi^{(\rm vv)}]^4 + c_i^{(\rm vs,vs)}\,[m_\pi^{(\rm vs)}]^4 + c_i^{(\rm vv,vs)}\,[m_\pi^{(\rm vv)}]^2[m_\pi^{(\rm vs)}]^2 \\
+ d_{i,\,n_{\rm HYP}}^{(\rm vv)}\,a^2\,[m_\pi^{(\rm vv)}]^2 + d_{i,\,n_{\rm HYP}}^{(\rm vs)}\,a^2\,[m_\pi^{(\rm vs)}]^2 + h_{i,\,n_{\rm HYP}}\,a^4 \Big].
\end{aligned}
$$

- Refit with priors on $c_i$, $d_i$ and $h_i$

$$
\begin{aligned}
c_i^{(\rm vv,vv)} &= 0 \pm w/\Lambda_\chi^4, \\
c_i^{(\rm vs,vs)} &= 0 \pm w/\Lambda_\chi^4, \\
c_i^{(\rm vv,vs)} &= 0 \pm w/\Lambda_\chi^4, \\
d_{i,\,n_{\rm HYP}}^{(\rm vv)} &= 0 \pm w\,\Lambda_{\rm QCD}^2/\Lambda_\chi^2, \\
d_{i,\,n_{\rm HYP}}^{(\rm vs)} &= 0 \pm w\,\Lambda_{\rm QCD}^2/\Lambda_\chi^2, \\
h_{i,\,n_{\rm HYP}} &= 0 \pm w\,\Lambda_{\rm QCD}^4.
\end{aligned}
$$

| $w$ | $g_1$ | $\delta\sigma(g_1)$ | $g_2$ | $\delta\sigma(g_2)$ | $g_3$ | $\delta\sigma(g_3)$ |
|---|---|---|---|---|---|---|
| 0 | 0.449(47) | 0 | 0.84(20) | 0 | 0.71(12) | 0 |
| 1 | 0.449(47) | 0.0020 | 0.84(20) | 0.0023 | 0.71(12) | 0.0045 |
| 5 | 0.452(48) | 0.0089 | 0.84(20) | 0.014 | 0.70(12) | 0.017 |
| 10 | 0.455(50) | 0.016 | 0.84(20) | 0.024 | 0.70(12) | 0.026 |
| 50 | 0.464(72) | 0.054 | 0.82(22) | 0.099 | 0.68(15) | 0.094 |
| 100 | 0.452(94) | 0.082 | 0.78(26) | 0.17 | 0.63(21) | 0.17 |

$$\delta\sigma(g_i) = \sqrt{\sigma^2(g_i)^{(\rm NLO+HO)} - \sigma^2(g_i)^{(\rm NLO)}},$$

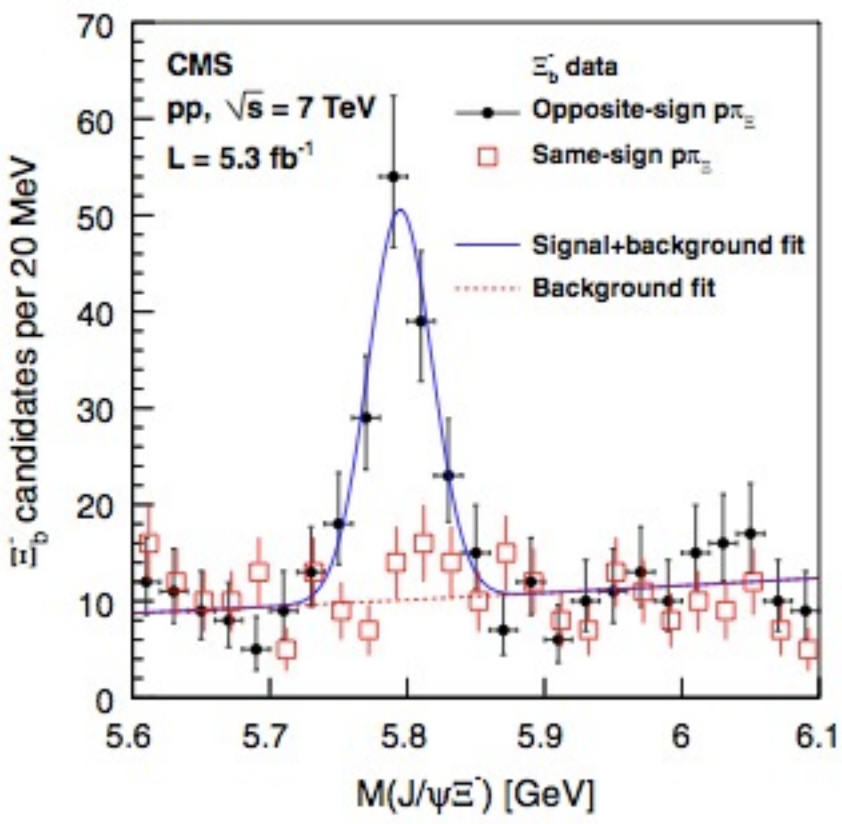
- w =10 gives systematic uncertainty (w=1 is NDA)

# Finite volume effects

- Finite volume effects computed in HHχPT

| $m_\pi^{(\mathrm{vs})}$ (MeV) | $m_\pi^{(\mathrm{vv})}$ (MeV) | $\frac{(g_1)_{\mathrm{eff}}^{(\infty)}-(g_1)_{\mathrm{eff}}^{(L)}}{(g_1)_{\mathrm{eff}}^{(\infty)}}$ | $\frac{(g_2)_{\mathrm{eff}}^{(\infty)}-(g_2)_{\mathrm{eff}}^{(L)}}{(g_2)_{\mathrm{eff}}^{(\infty)}}$ | $\frac{(g_3)_{\mathrm{eff}}^{(\infty)}-(g_3)_{\mathrm{eff}}^{(L)}}{(g_3)_{\mathrm{eff}}^{(\infty)}}$ |
|---|---|---|---|---|
| 294 | 245 | 0.0057 | 0.015 | 0.0074 |
| 304 | 270 | 0.0040 | 0.0070 | 0.0027 |
| 336 | 336 | 0.0016 | 0.00037 | $-0.00079$ |
| 263 | 227 | 0.0072 | 0.028 | 0.013 |
| 295 | 295 | 0.0031 | 0.00027 | $-0.0012$ |
| 352 | 352 | 0.0013 | 0.00033 | $-0.00071$ |

- Very small, higher order FV negligible



CMS
pp, $\sqrt{s}$ = 7 TeV
L = 5.3 fb$^{-1}$
$\Xi_b^-$ data
Opposite-sign $p\pi_\Xi$
Same-sign $p\pi_\Xi$
Signal+background fit
Background fit
$\Xi_b^-$ candidates per 20 MeV
M(J/ψΞ$^-$) [GeV]
0
10
20
30
40
50
60
70
5.6
5.7
5.8
5.9
6
6.1