Finite-size scaling in nucleon axial charge from 2+1-flavor DWF lattice QCD

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Talk at Lattice 2012, Cairns, QLD, June 24-29, 2012

RBC and UKQCD collaborations have been generating dynamical Domain-Wall Fermions (DWF) ensembles:

- good chiral and flavor symmetries,

that allowed us do a lot of good pion and kaon physics as well as nucleon.

We are now much closer to physical pion mass with large volume, than the previous sets ensembles:

- light, $m_\pi \sim 170$ and 250 MeV, quarks ($m_{ud}a = 0.001$ and 0.0042, and $m_{\rm res}a \sim 0.002$),
- a large, (4.6fm)^3, volume ($a^{-1} \sim 1.371(8)$ GeV),

made possible by Iwasaki + dislocation suppressing determinant ratio (DSDR) gauge action.

Here we report the current status of our nucleon calculations, by

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[§]Thanks also to XSEDE/OCI-1053575 and RICC, RIKEN, for partial supports.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu F_V(q^2) + \frac{\sigma_{\mu\lambda}q_\lambda}{2m_N}F_T(q^2)\right]u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p\left[\gamma_\mu\gamma_5 F_A(q^2) + iq_\mu\gamma_5 F_P(q^2)\right]u_n e^{iq\cdot x}.$$

$$F_V = F_1, F_T = F_2; G_E(q^2) = F_1 - \frac{q^2}{4m_N^2}F_2, G_M = F_1 + F_2.$$

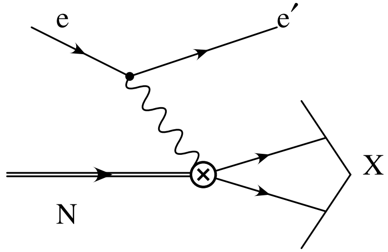
Related to mean-squared charge radius, magnetic moment, $g_V = F_V(0) = G_{\rm Fermi}\cos\theta_{\rm Cabibbo}$, $g_A = F_A(0) = 1.2694(28)g_V$, Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, ... determine much of nuclear physics.

On the lattice, with appropriate nucleon operator, for example, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, ratio of two- and three-point correlators such as $\dfrac{C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t)}{C_{\rm 2pt}(t_{\rm sink})}$ with

$$C_{\rm 2pt}(t_{\rm sink}) = \sum_{\alpha,\beta}\left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta}\langle N_\beta(t_{\rm sink})\bar{N}_\alpha(0)\rangle,$$

$$C_{\rm 3pt}^{\Gamma,O}(t_{\rm sink},t) = \sum_{\alpha,\beta}\Gamma_{\alpha\beta}\langle N_\beta(t_{\rm sink})O(t)\bar{N}_\alpha(0)\rangle,$$

give a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2$) or momentum-transfer (if any) projections.

Deep inelastic scatterings 

$: \left|\frac{\mathcal{A}}{4\pi}\right|^2 = \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu},\ W^{\mu\nu} = W^{[\mu\nu]} + W^{\{\mu\nu\}}$

- unpolarized: $W^{\{\mu\nu\}}(x,Q^2) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2} q^\mu\right)\left(P^\nu - \frac{\nu}{q^2} q^\nu\right) \frac{F_2(x,Q^2)}{\nu}$,
- polarized: $W^{[\mu\nu]}(x,Q^2) = i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right)$,

with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

Moments of the structure functions are accessible on the lattice:

$$\begin{aligned}
2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
\int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2),\\
2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^q(\mu^2/Q^2, g(\mu))\, d_n^q(\mu) - 2e_{1,n}^q(\mu^2/Q^2, g(\mu))\, \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2)
\end{aligned}$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle 1\rangle_{\delta q}(\mu) = \langle P,S|\bar\psi i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$ which may be measured by polarized Drell-Yan and RHIC Spin.

Unpolarized (F_1/F_2): on the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overset{\leftrightarrow}{D}_{\mu_2}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{trace})\right]q$$

Polarized (g_1/g_2): on the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_\sigma\overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1}-S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n}\gamma_5\gamma_{[\sigma}\overset{\leftrightarrow}{D}_{\mu_1]}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n}+\cdots-(\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^{n}\gamma_5\sigma_{\rho\nu}\overset{\leftrightarrow}{D}_{\mu_1}\cdots\overset{\leftrightarrow}{D}_{\mu_n} - (\text{traces})]q$$

Higher moment operators mix with lower dimensional ones: Only $\langle x\rangle_q$, $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

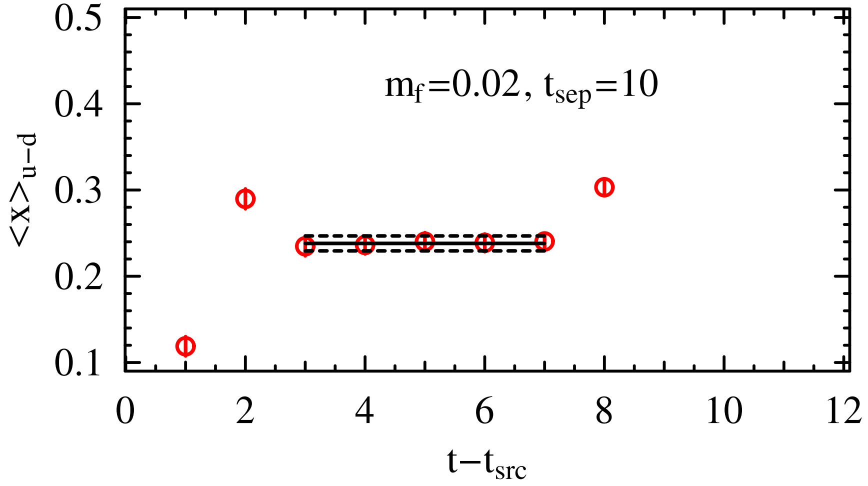
Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

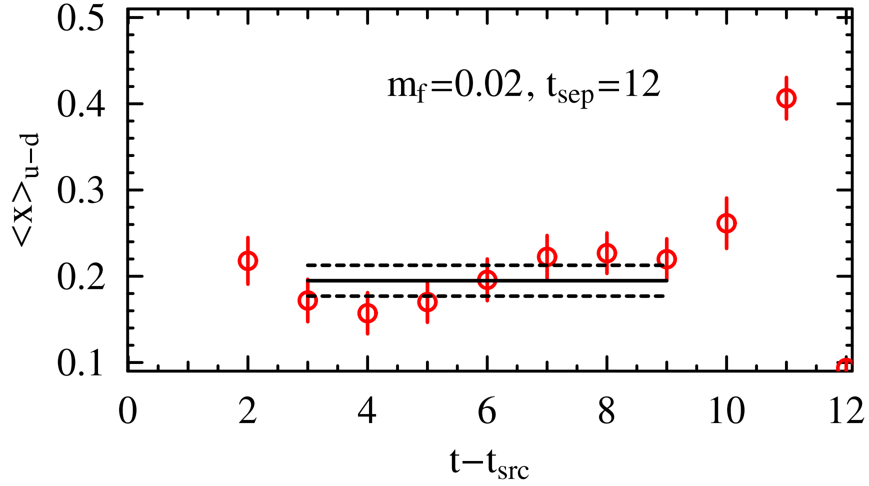
- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

Source/sink time separation:

- If too short, too much contamination from excited states, but if too long, the signal is lost.

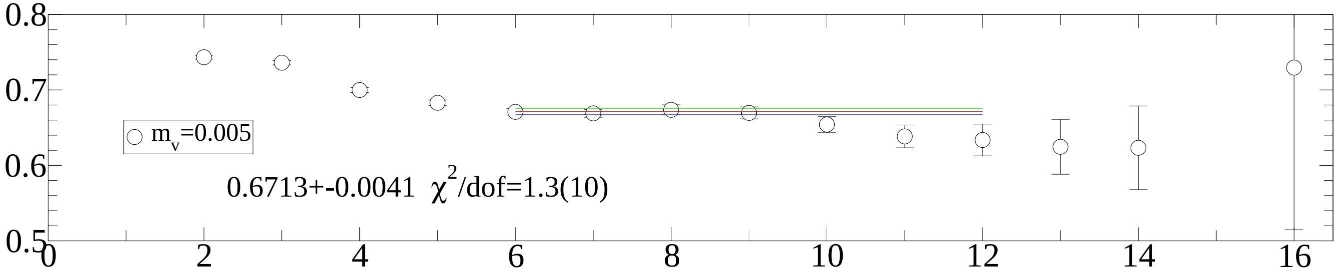




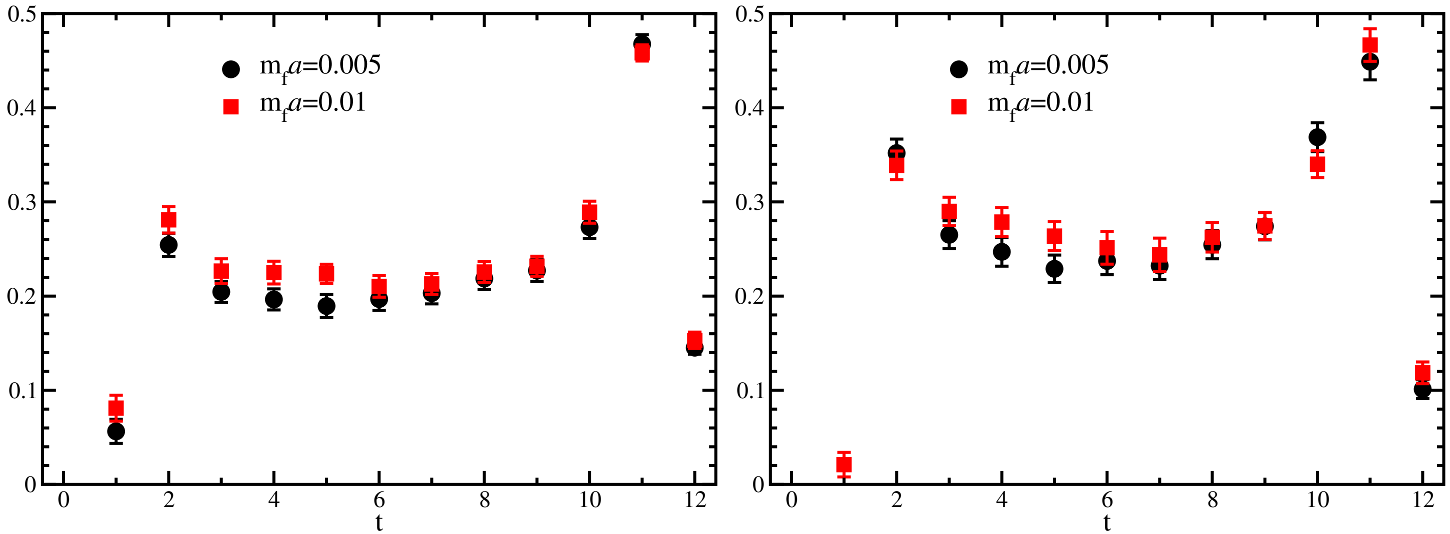
- In an earlier RBC 2-flavor DWF study at $a^{-1} \sim 1.7$ GeV, separation of 10 or 1.1 fm appeared too short.

In the RBC+UKQCD (2+1)-flavor study at 1.7 GeV we choose separation 12 : ∼1.4 fm:

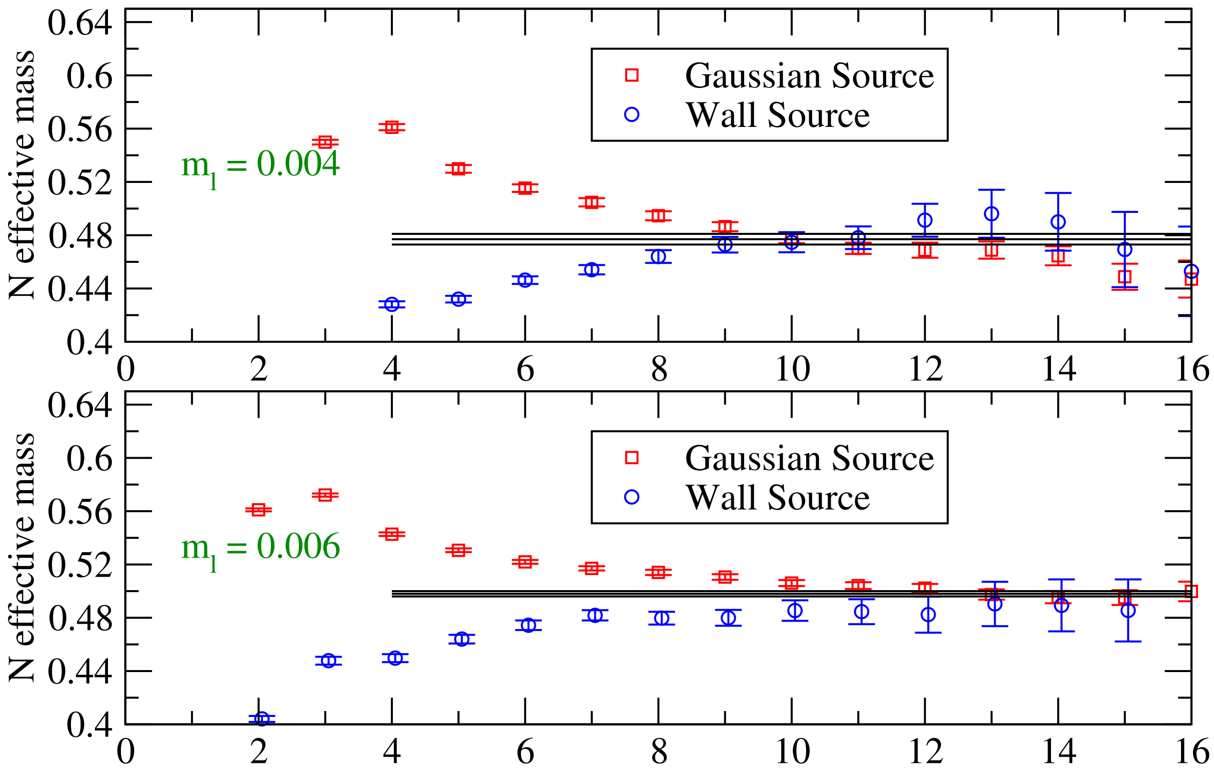
Mass signal: $m_f = 0.005$



Bare three-point functions: $\langle x \rangle_{u-d}$ (left) and $\langle x \rangle_{\Delta u - \Delta d}$ (right), for $m_f = 0.005$ (red +) and 0.01 (blue ×):



On the other hand, with RBC+UKQCD 2.3-GeV (2+1)-flavor dynamical DWF ensemble:



2-state fits suggest excited-state survives $t_{\mathrm{sink}} \geq 9$.

LHP analysis of vector form factors with $t_{\mathrm{sep}} = 12$ or 1 fm agree with RBC+UKQCD 1.7-GeV results.

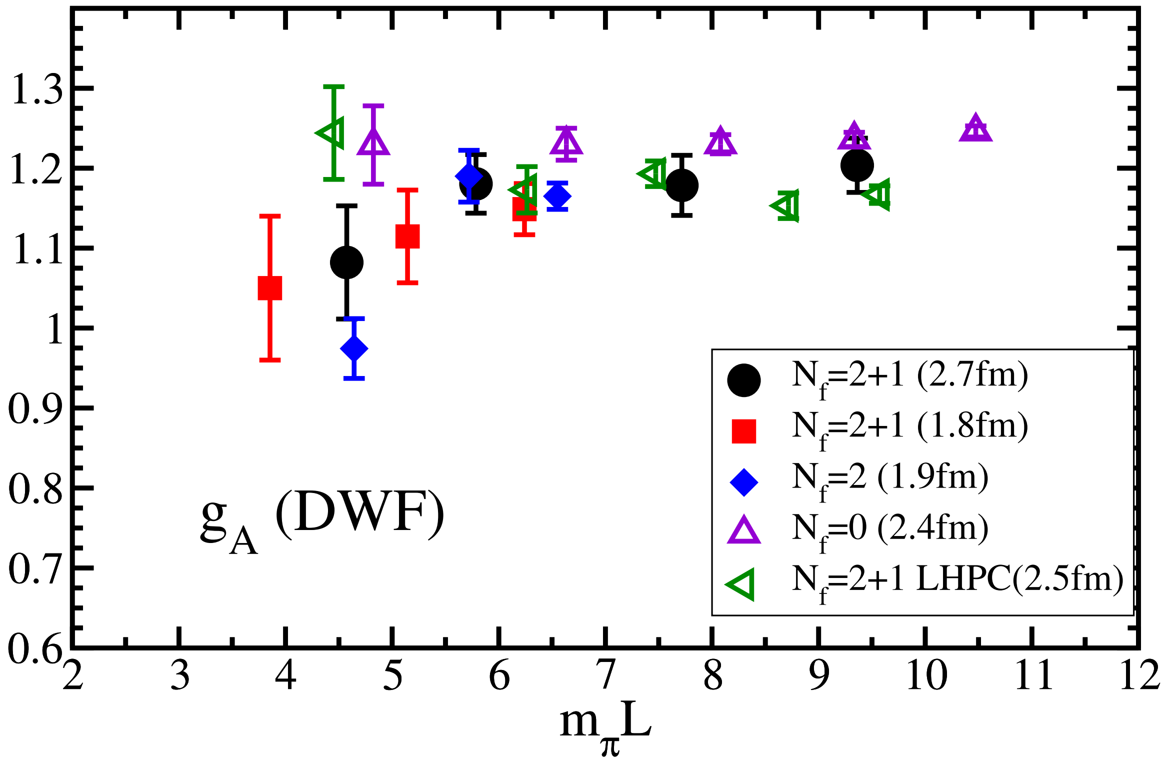
Vector current is less sensitive: conserved charge cannot tell excited-state contamination, for example.

Can we go shorter, ∼1 fm, separation, in spite of our lighter masses?

- Perhaps with better tuned source and sink smearing?
- Would be good as we have to fight growing error, $\sim \exp(-3m_\pi t)$.

Spatial volume. In Lattice 2007 Takeshi Yamazaki reported unexpectedly large finite-size effect:

- in axial charge, $g_A/g_V = 1.2694(28)$, measured in neutron β decay, decides neutron life.



Our DWF on quenched and LHPC DWF on MILC calculations are presented for comparison.

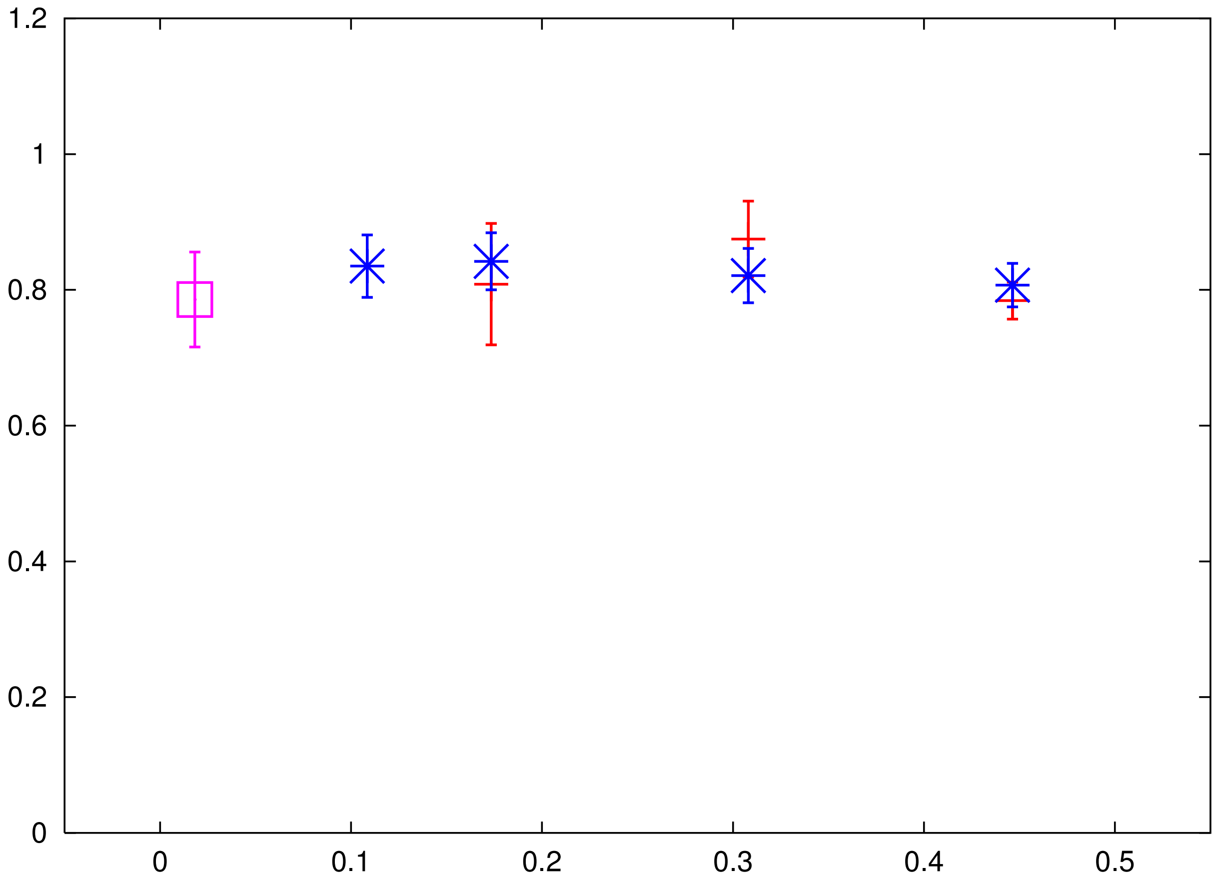
- Heavier quarks: almost consistent with experiment, no discernible quark-mass dependence.
- Lighter quarks: finite-size sets in as early as $m_\pi L \sim 5$, appear to scale in $m_\pi L$:
  - first credible evidence of pion cloud surrounding nucleons.

Structure function moments do not seem to suffer so badly, but we need large volume at least for form factors, such importnat quantities as g_A or $g_{\pi NN}$: present $(\sim 4.6\text{fm})^3$ volume is a good start.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.75(3)$ GeV, $m_{\rm res} = 0.00315(2)$, $m_{\rm strange} = 0.04$,

- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Ratio, $\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$, of momentum and helicity fractions (naturally renormalized on the lattice),



consistent with experiment, no discernible quark-mass dependence.

No finite-size effect seen, in contrast to g_A/g_V which is also naturally renormalized on the lattice.

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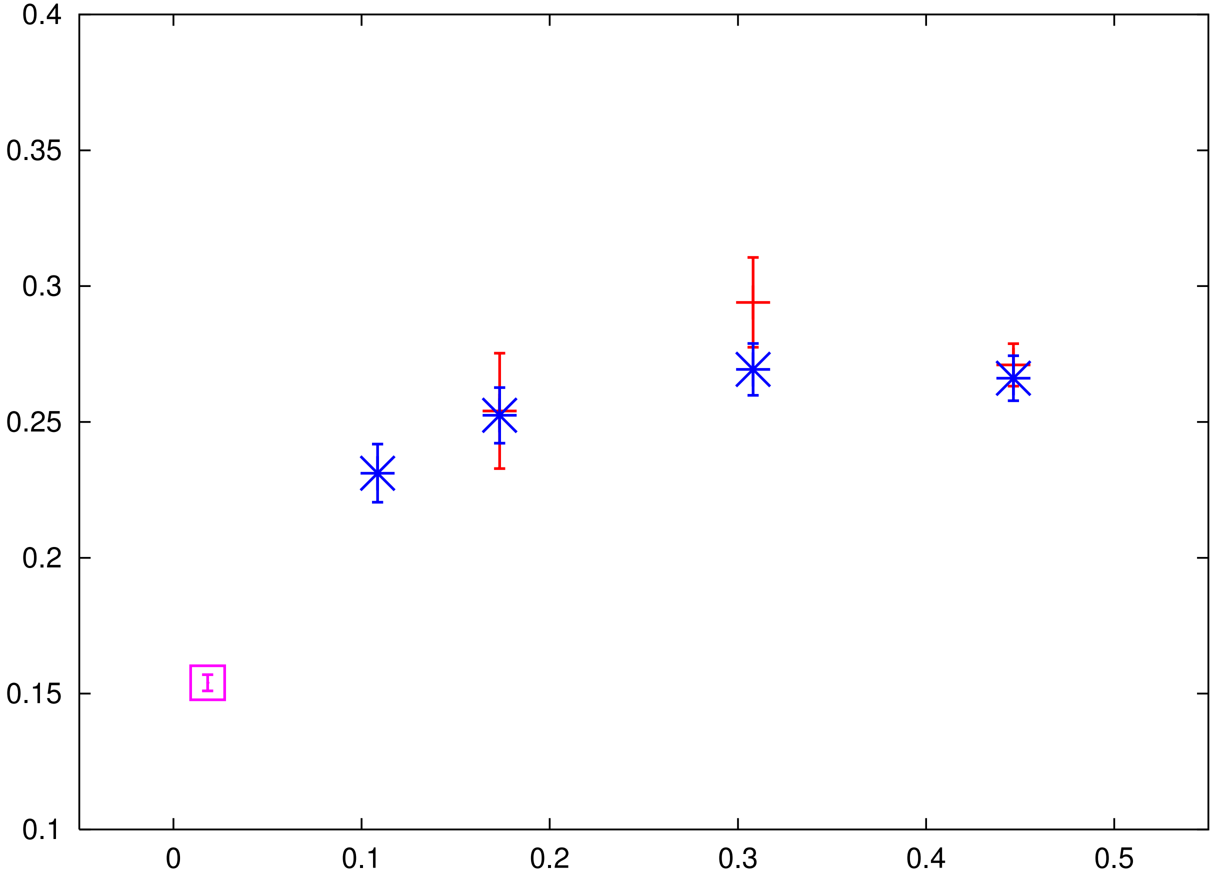
- $m_\pi = 0.67, 0.56, 0.42$ and 0.33 GeV; $m_N = 1.55, 1.39, 1.22$ and 1.15 GeV,

Momentum fraction, $\langle x \rangle_{u-d}$, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$, plotted against m_π^2,



Absolute values have improved, trending to the experimental values, with NPR, $Z^{\overline{\rm MS}}(2{\rm GeV}) = 1.15(4)$.

No finite size effect seen (16^3 (+) and 24^3 ($\times$) results agree): Likely physical light-quark effect.

A better understanding of quark mass dependence is necessary.

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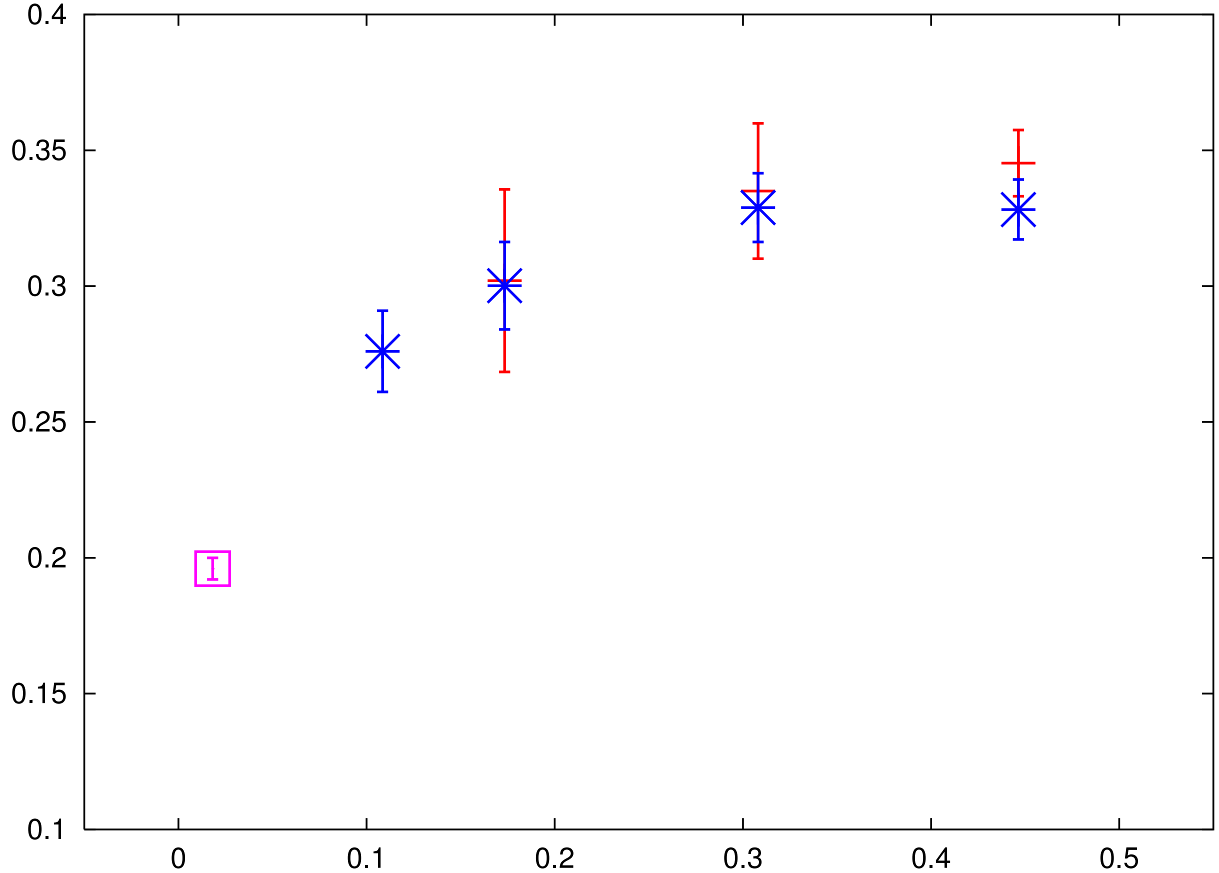
- $m_\pi = 0.67$, 0.56, 0.42 and 0.33 GeV; $m_N = 1.55$, 1.39, 1.22 and 1.15 GeV,

Helicity fraction, $\langle x \rangle_{\Delta u - \Delta d}$, with NPR, $Z^{\overline{\rm MS}(2{\rm GeV})} = 1.15(3)$, plotted against m_π^2,



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Now RBC and UKQCD collaborations are jointly generating new (2+1)-flavor DWF ensembles

- with Iwasaki and dislocation-suppressing-determinant-ratio (DSDR) gauge action, $\beta = 1.75$,
- and DWF fermion action, $L_s = 32$ and $M_5 = 1.8$, with $m_{\rm strange} = 0.045$, $m_{\rm ud} = 0.0042$ and 0.001,

using FNAL ALCF, a BG/P facility.

We have reasonable topology distribution while maintaining small residual mass, $m_{\rm res}a \sim 0.002$:

- lattice scale from Ω^-: $a^{-1} = 1.371(8)$ GeV,
- $m_\pi = 0.1816(8)$ and $0.1267(8)$, or $\sim$ 250 and 170 MeV.

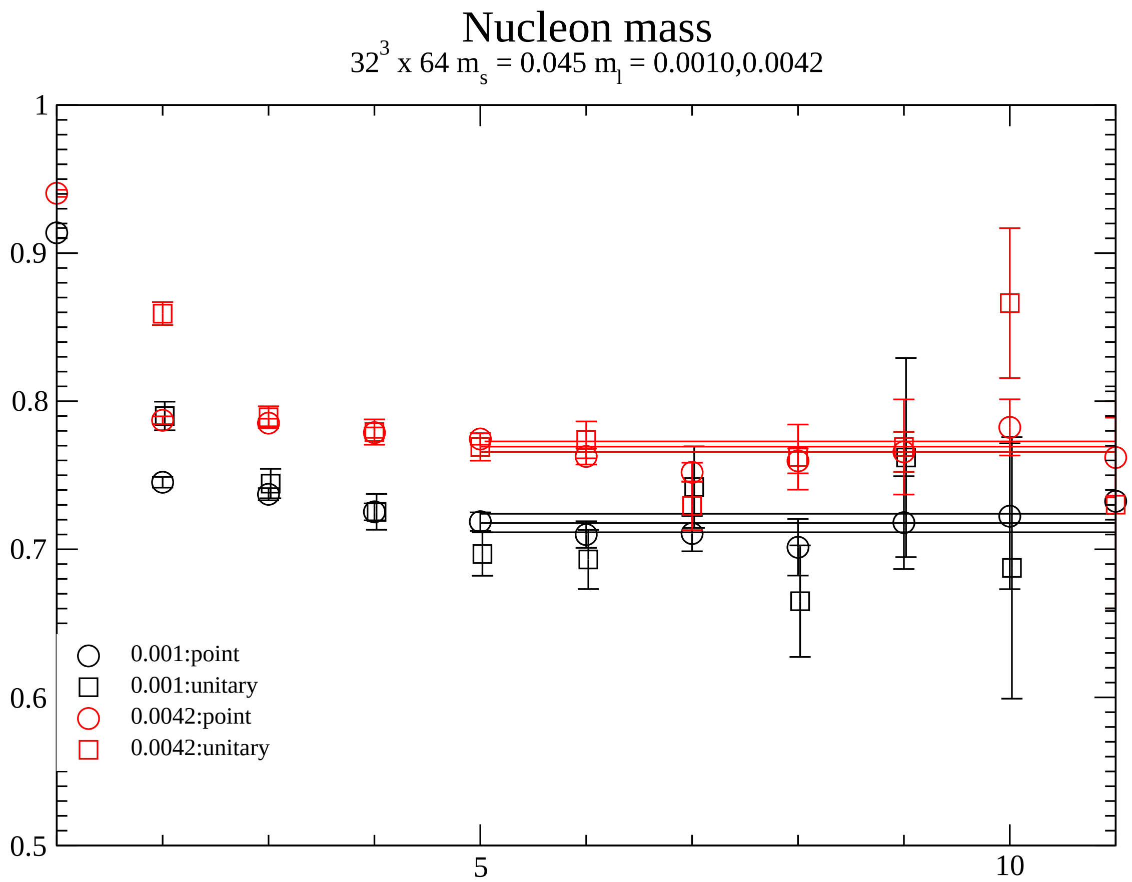
$32^3 \times 64$ volume is about 4.6 fm across in space, 9.2 fm in time.

We started nucleon structure calculations:

- finished tuing Gaussian smearing, width 6 favored over 4.
- sink separation at 9, four source positions per configuarion so far,
- 608–1920/8 for 250-MeV, 508–1412/8 for 170-MeV so far analyzed for 3pt,

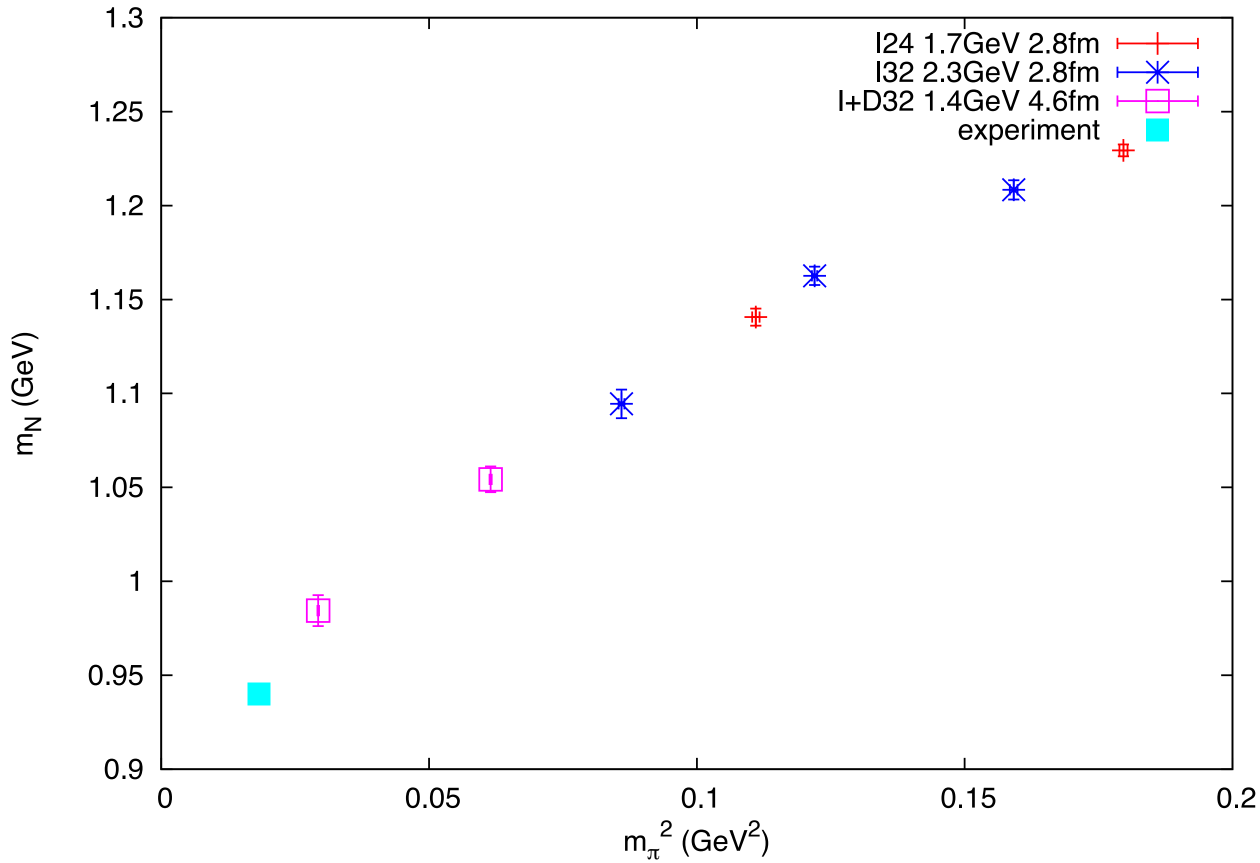
thanks to RICC/RIKEN and Teragrid/XSEDE clusters.

RBC/UKQCD (2+1)-flavor, ID+DWF dynamical, $a^{-1} = 1.371(8)$ GeV,



$m_N = 0.718(6)$ or ~ 0.98 GeV for $m_\pi \sim 170$MeV, and $m_N = 0.769(5)$ or ~ 1.05 GeV for $m_\pi \sim 250$MeV.

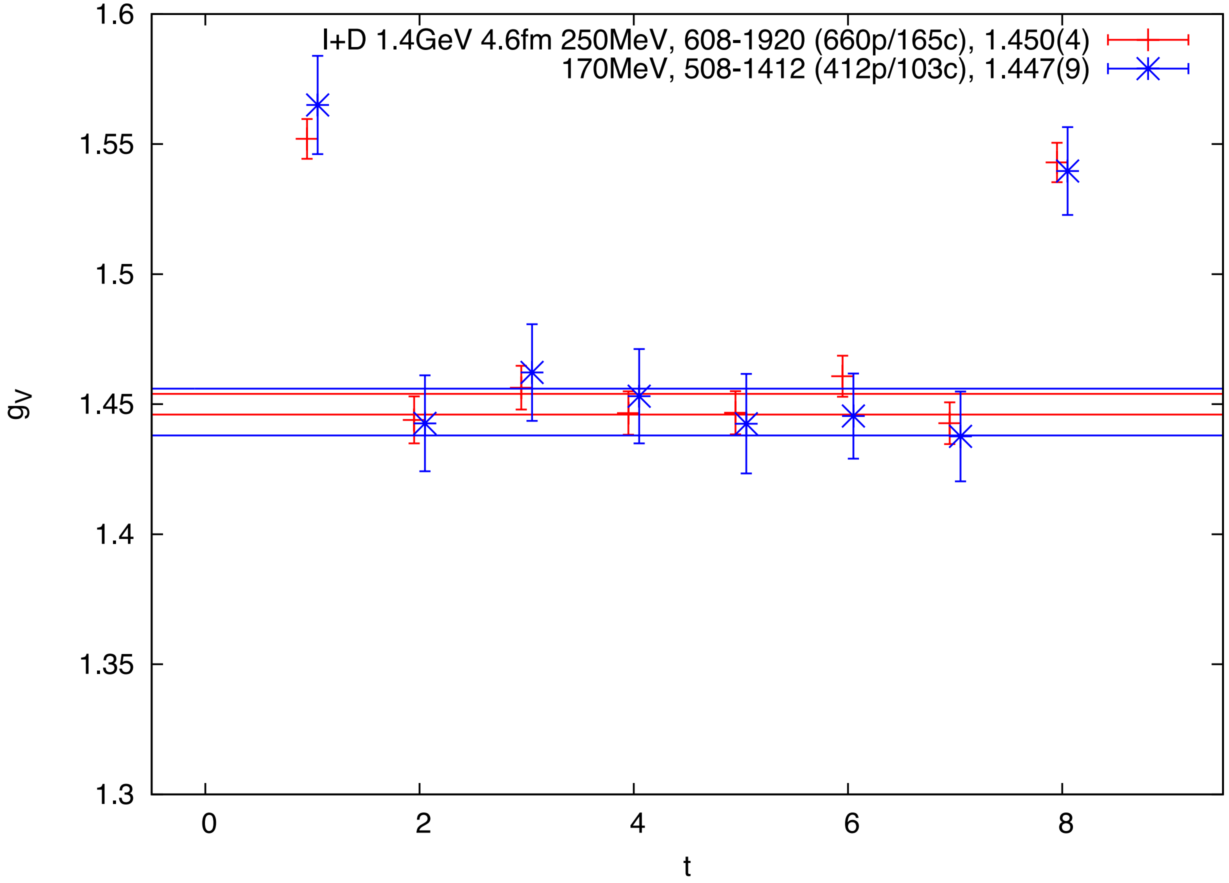
Nucleon mass: RBC/UKQCD (2+1)-flavor, ID+DWF ensembles are being analyzed for nucleon physics.



with $a^{-1} = 1.371(8)$ GeV, $(\sim 4.6\text{fm})^3$ spatial volume.
Closer to physical mass, $m_\pi = 170$ and 250 MeV, $m_N < 1.0$ GeV,

.

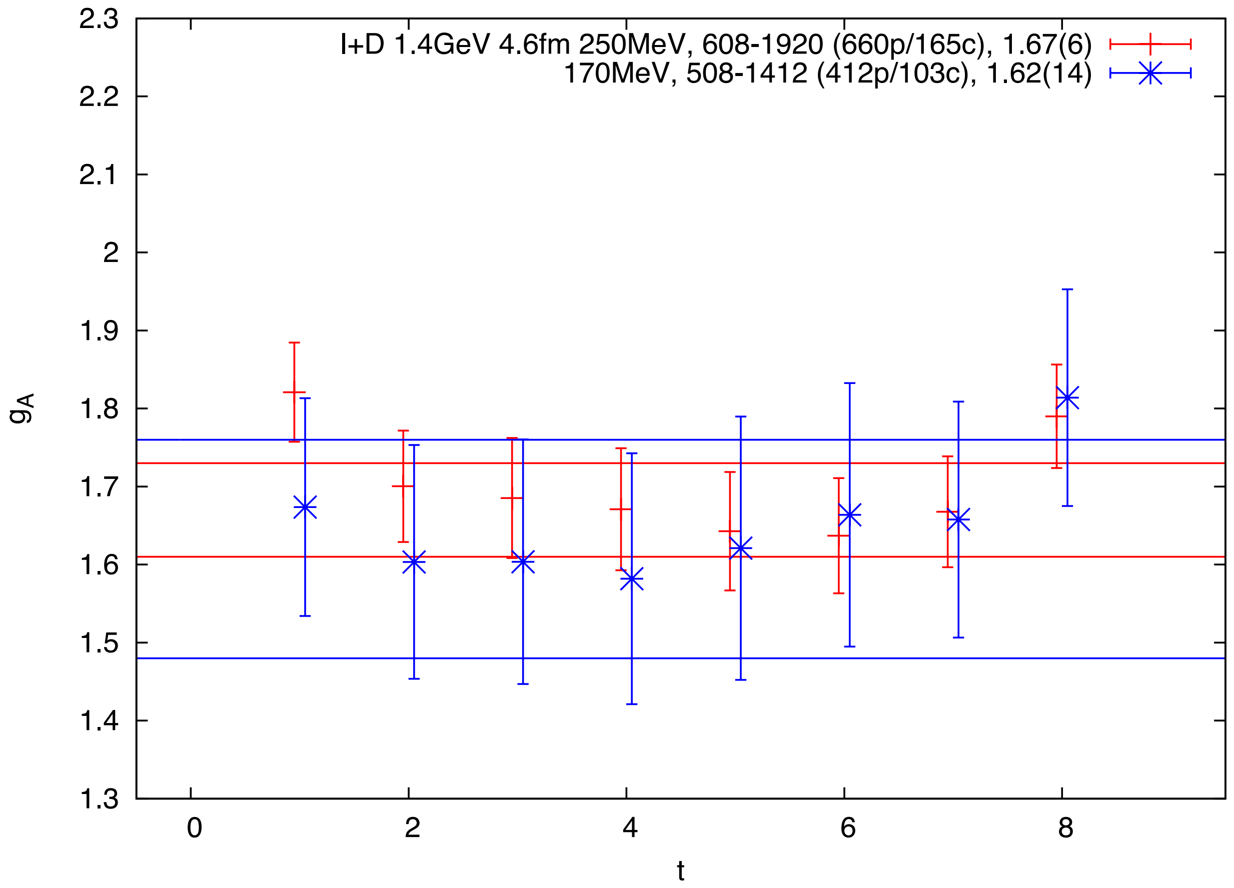
Nucleon isovector 3-pt functions are being obtained: 608-1920 for 250-MeV, 508-1412 for 170-MeV.



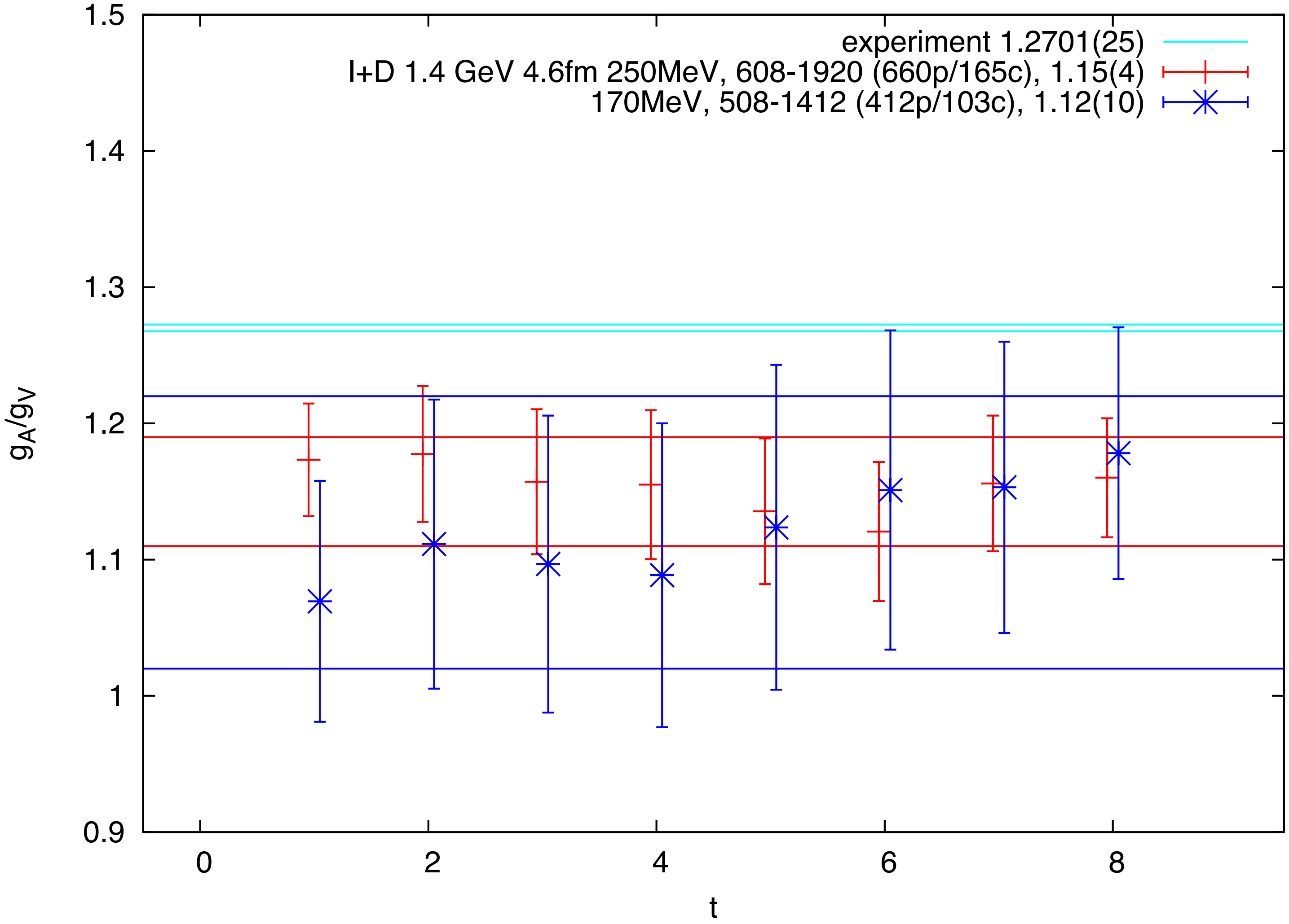
Local-current isovector vector charge, $g_V = 1.450(4)$ or $1.447(9)$, is obtained, corresponding to $Z_V = 0.686(7)$,

- in good agreement with $Z_V = 0.673(8)$ and $Z_A = 0.6878(3)$ obtained in the meson sector,
- yet again proving good chiral and flavor symmetries up to $O(a^2)$.

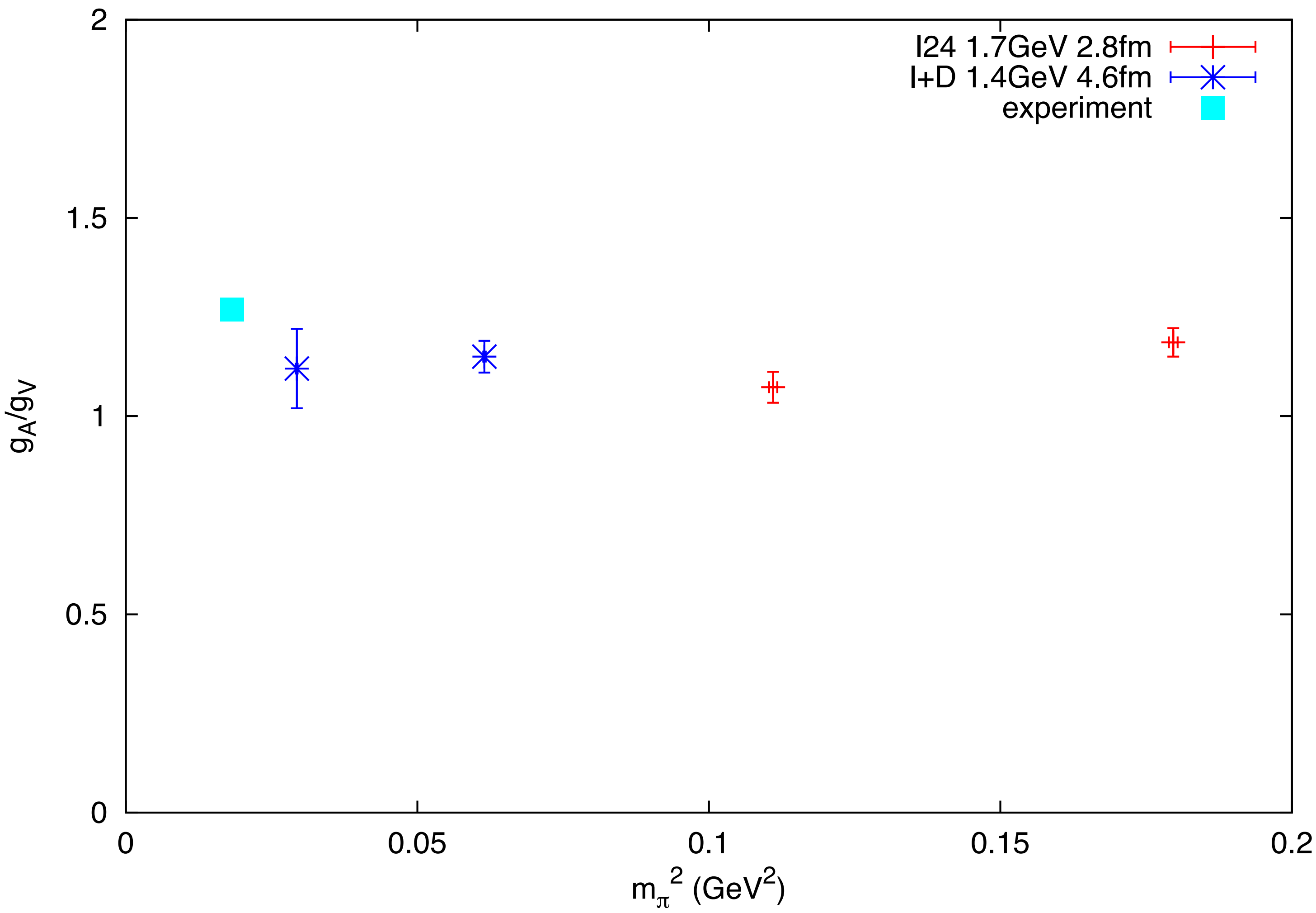
Axialvector current: Noisier than vector current, as expected,



g_A/g_V, ratio of isovector axial and vector charges, is less noisy, again as expected,

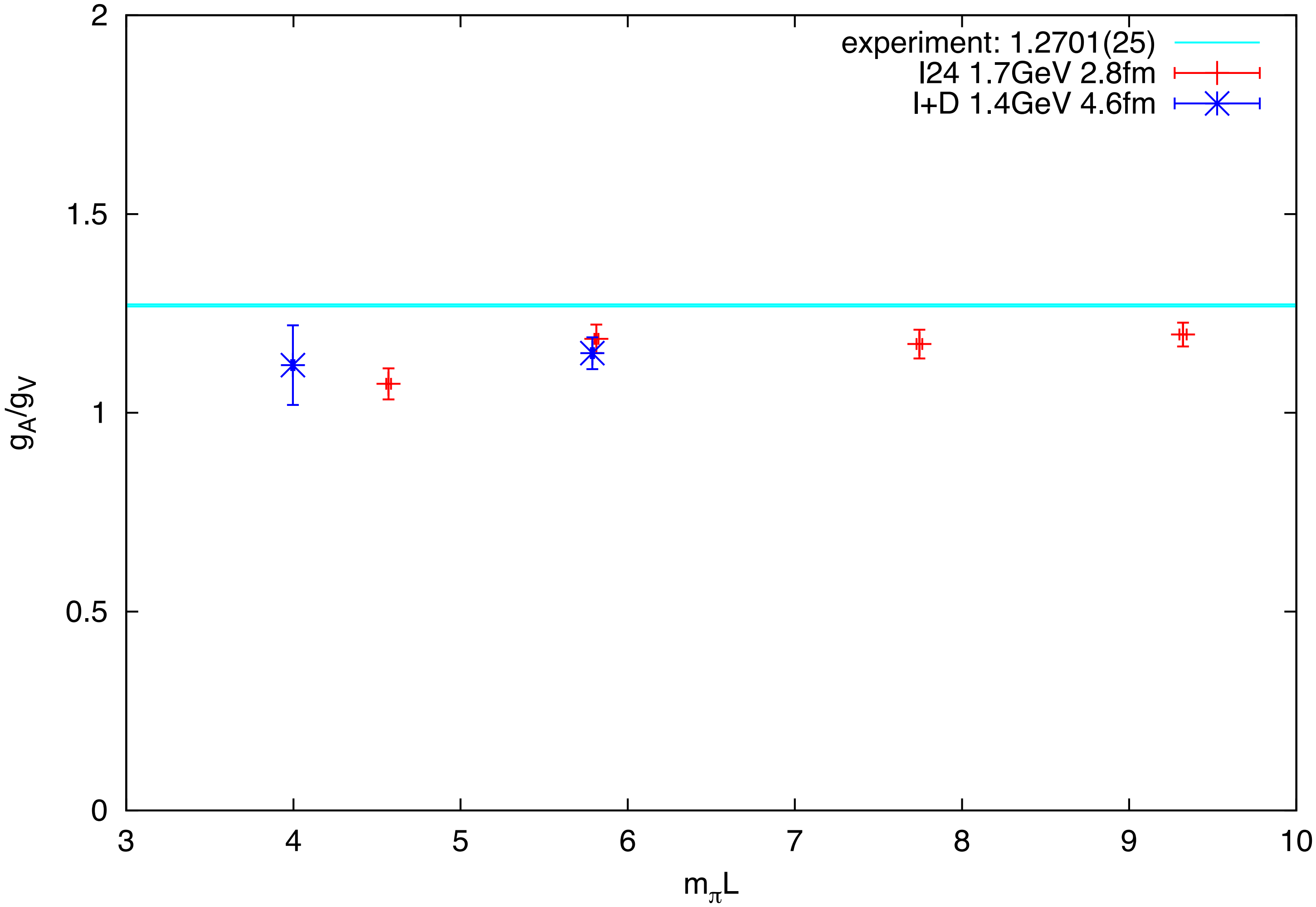


g_A/g_V: seems to stay away from the experiment as we set the pion mass lighter.



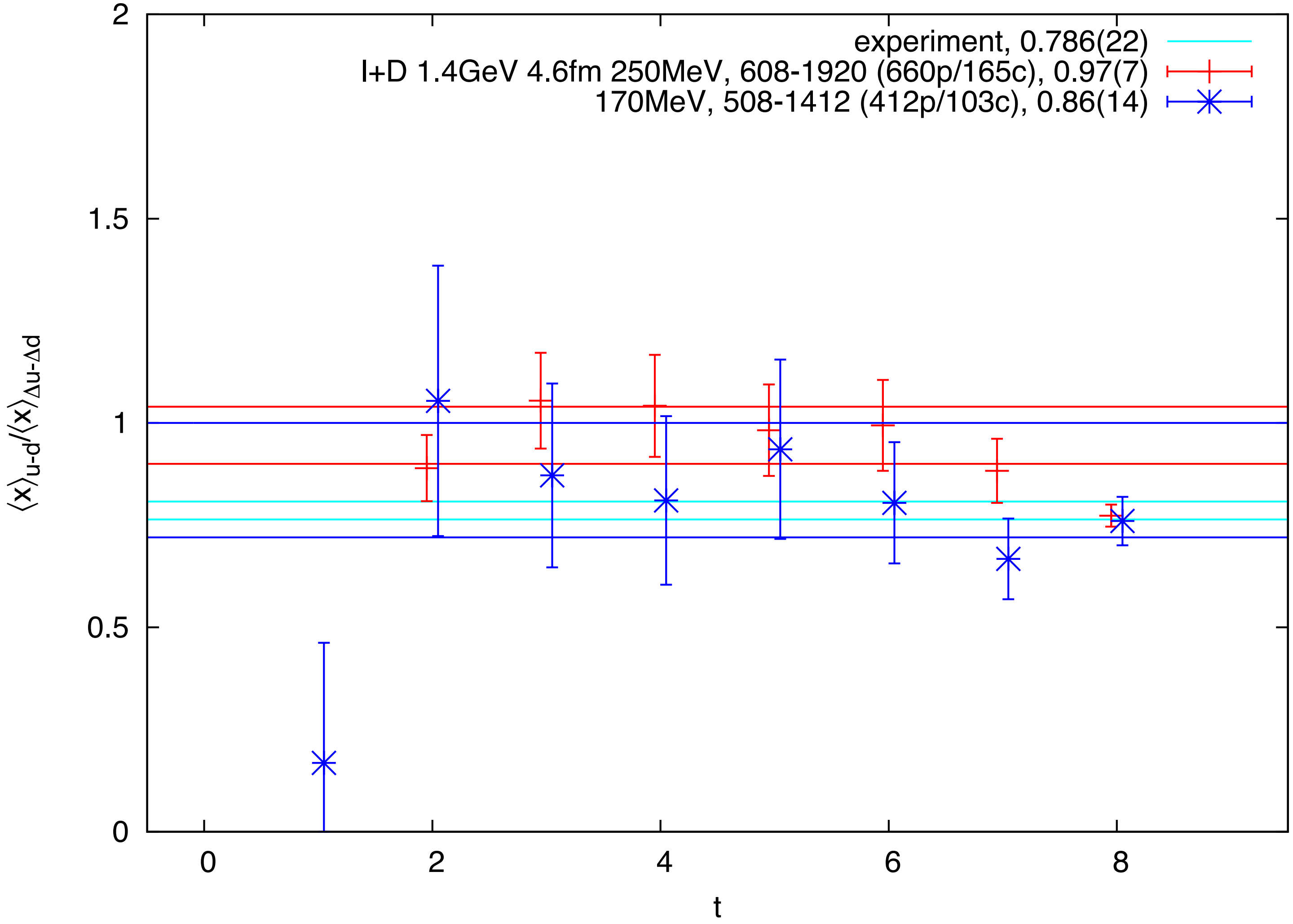
Not monotonic: appears to be a finite-size effect.

g_A/g_V: appears to show finite-size effect that is consistent with scaling in $m_\pi L$.



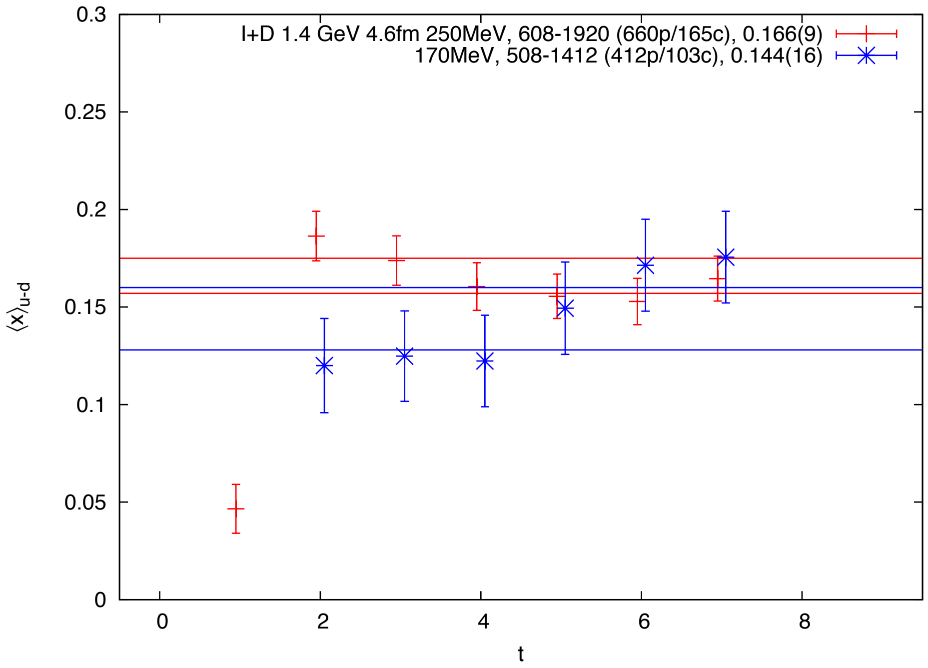
Results from two ensembles, 1.19(4) from I24 and 1.15(5) from ID, agree with each other, despite very much different m_π that significantly alter mass spectrum.

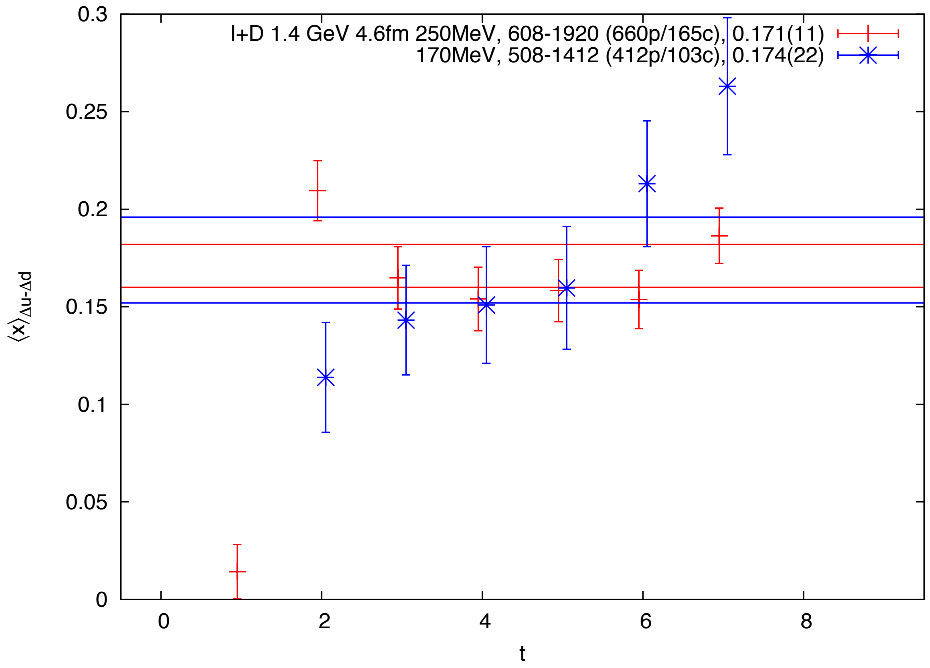
Moments of structure functions: signals are seen,



the ratio $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$ consistent with experiment, 0.786(22).

Moments of structure functions: signals are seen, though yet to be renormalized,





possibly decreasing with mass.

Conclusions: RBC+UKQCD work on nucleon structure using the 2+1f dynamical DWF ensembles,

- lattice cutoff $\sim$ 1.4 GeV, $(4.6\text{fm})^3$ spatial volume,
- good chiral and flavor symmetries up to $O(a^2)$, $m_{\text{res}}a \sim 0.00184(1)$,
- $m_\pi \sim 170$ and 250 MeV, $m_N \sim 0.99$ and 1.06 GeV.

Isovector vector charge is well-behaved: confirms good chiral and flavor symmetries with DWF.

Isovector axial charge is noisier, yet the calculation is solid:

- confirms about 10% deficit in g_A/g_V, almost 3-σ significant, and
- confirms the finite-size effect in it, scaling with $m_\pi L$;
- no sign of excited-state contamination is seen,
- suggesting the first concrete evidence for the pion cloud surrounding nucleon.

Nucleon is hardly point-like: it may grow further toward physical mass, $m_\pi \sim 140$ MeV.
It seems hard to reconcile with the conventional nuclear models.
Details of form factors are reported by Meifeng Lin.

Moments of structure functions are noisier, but calculations are well under way.

We are increasing our statistics: double at least, and possibly quadruple or further.
We are exploring calculations at smaller momentum transfer.
We seek calculations at physical pion mass, and with appropriate isospin breaking soon afterward.