

Transverse momentum-dependent parton distribution functions from lattice QCD

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“What is the probability of finding a quark with a given momentum k in a nucleon?”

Light cone coordinates:

$$w^\pm = \frac{1}{\sqrt{2}}(w^0 \pm w^3)$$

Nucleon with large momentum along 3-axis: P^+ large, $P_T = 0$

Quark momentum components: $k^+ \sim P^+/m_N$, $k_T \sim 1$, $k^- \sim m_N/P^+$

Ask for distribution of quarks

$$f(x, k_T)$$

- longitudinal momentum fraction $x = k^+/P^+$
- transverse momentum k_T

Definition of TMDs

Heuristically,

$$\Phi^{[\Gamma]}(x, k_T, P, S, \dots) \equiv \text{“} \int dk^- \langle P, S | \bar{q}(k) \Gamma q(k) | P, S \rangle \Big|_{k^+ \equiv x P^+} \text{”}$$

Decompose in terms of TMDs, for example

$$\Phi^{[\gamma^+]}(x, k_T, P, S, \dots) = f_1(x, k_T^2, \dots) - \frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^\perp(x, k_T^2, \dots)$$

Definition of TMDs

More precisely, in terms of local operators,

$$\Phi^{[\Gamma]}(x, k_T, P, S, \dots) \equiv \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} \exp(i x (b \cdot P) - i b_T \cdot k_T) \frac{\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots)}{\bar{\mathcal{S}}(b^2, \dots)} \Big|_{b^+=0}$$

$$\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$$

- “Soft factor” $\bar{\mathcal{S}}$ required to subtract divergences of Wilson line $\mathcal{U}$
- $\bar{\mathcal{S}}$ is typically a combination of vacuum expectation values of Wilson line structures
- Here, will consider only ratios in which soft factors cancel

Definition of TMDs

All leading twist structures:

$$\Phi[\gamma^+] = f_1 - \left[\frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^\perp \right]_{\text{odd}}$$

$$\Phi[\gamma^+ \gamma^5] = \Lambda g_1 + \frac{k_T \cdot S_T}{m_N} g_{1T}$$

$$\Phi[i\sigma^{i+} \gamma^5] = S_i h_1 + \frac{(2k_i k_j - k_T^2 \delta_{ij}) S_j}{2m_N^2} h_{1T}^\perp + \frac{\Lambda k_i}{m_N} h_{1L}^\perp + \left[\frac{\epsilon_{ij} k_j}{m_N} h_1^\perp \right]_{\text{odd}}$$

Definition of TMDs

All leading twist structures:

$\begin{matrix} & q \rightarrow \\ \text{N} & \\ \downarrow & \end{matrix}$	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_1 \quad h_{1T}^\perp$

← Boer-Mulders
(T-odd)

↑
Sivers (T-odd)

Fourier-transformed TMDs

$$\tilde{f}(x, b_T^2, \dots) \equiv \int d^2 k_T \exp(i b_T \cdot k_T) f(x, k_T^2, \dots)$$

$$\tilde{f}^{(n)}(x, b_T^2, \dots) \equiv n! \left(-\frac{2}{m_N^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2, \dots)$$

In limit $|b_T| \rightarrow 0$, recover k_T -moments:

$$\tilde{f}^{(n)}(x, 0, \dots) \equiv \int d^2 k_T \left(\frac{k_T^2}{2m_N^2} \right)^n f(x, k_T^2, \dots) \equiv f^{(n)}(x)$$

ill-defined for large k_T , so will not attempt to extrapolate to $b_T = 0$, but give results at finite $|b_T|$.

Also, we can only access limited range of $b \cdot P$, so cannot Fourier-transform to obtain x -dependence. Therefore, consider only first x -moments (accessible at $b \cdot P = 0$):

$$f^{[1]}(k_T^2, \dots) \equiv \int_{-1}^1 dx f(x, k_T^2, \dots)$$

Relation to physical processes

Context: All this is largely academic if we can't connect it to a physical measurement.

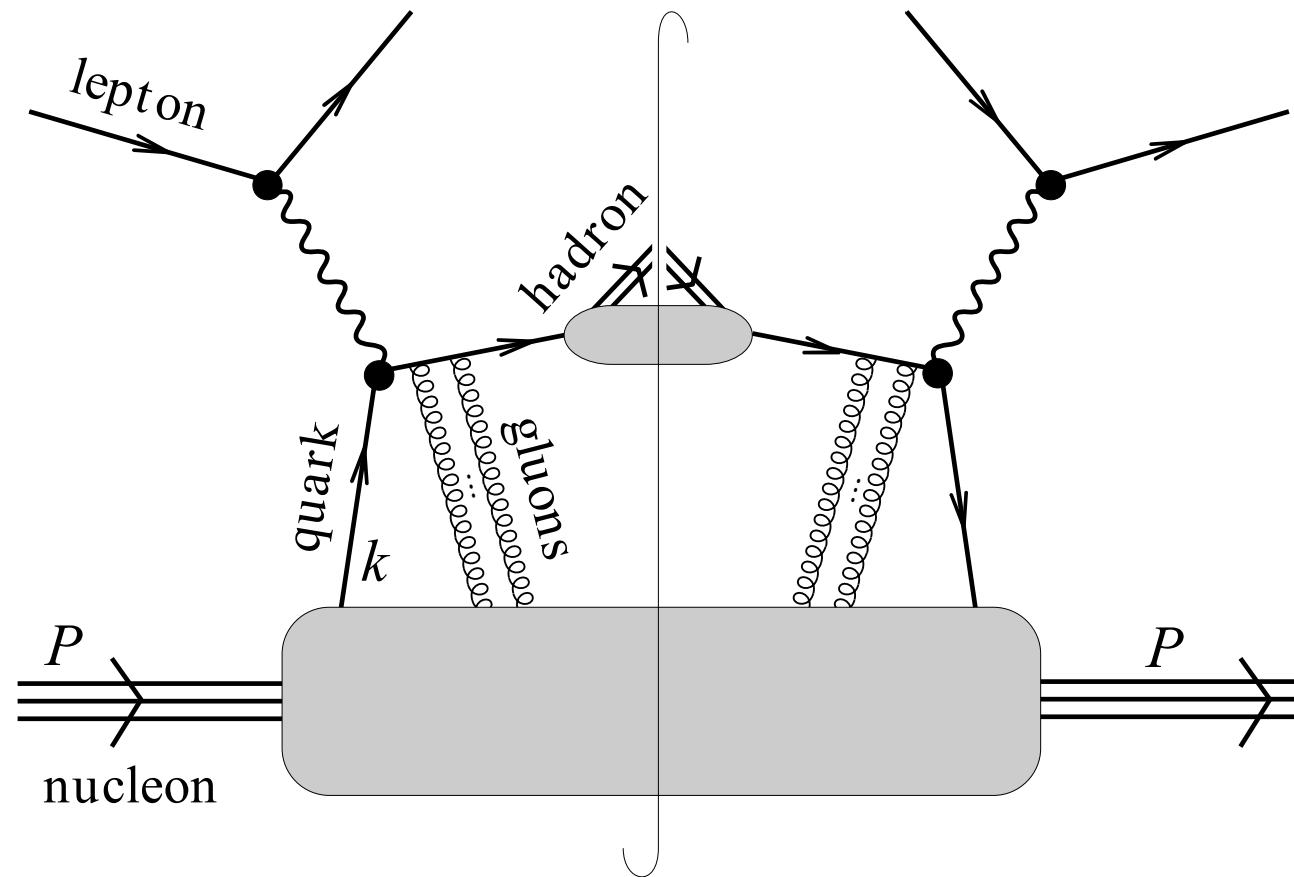
Factorization framework which allows one to separate cross section into hard amplitude, fragmentation function, TMD ?

In general, no! (e.g., processes with multiple hadrons in both initial and final state)

Factorization arguments have been given only for selected processes (SIDIS, Drell-Yan).

Physical process should also inform appropriate choice of gauge link $\mathcal{U}[0, \dots, b]$

SIDIS (Semi-inclusive deep inelastic scattering)

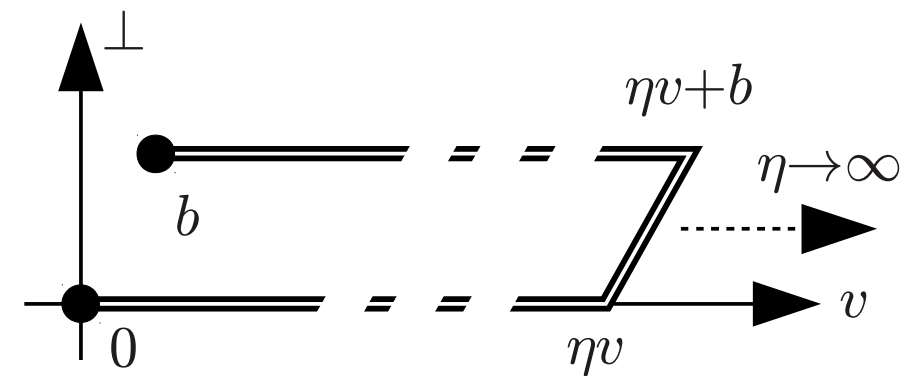


$$l + N(P) \longrightarrow l' + h(P_h) + X$$

Gauge link structure:

In matrix element $\bar{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \dots, b] q(b) | P, S \rangle$

Staple-shaped gauge link $\mathcal{U}[0, \eta v, \eta v + b, b]$



incorporates SIDIS final state effects

Relation to physical processes

Staple-shaped links incorporate SIDIS final state effects:

- Gauge link roughly follows direction of ejected quark, (close to) light cone
- Effective resummed description of gluon exchanges between ejected quark and remainder of nucleon in evolving final state
- Beyond tree level: Rapidity divergences force taking staple direction slightly off the light cone. Approach of Aybat, Collins, Qiu, Rogers makes v space-like. Parametrize in terms of Collins-Soper parameter

$$\hat{\zeta} \equiv \frac{P \cdot v}{|P||v|}$$

Light-like staple for $\hat{\zeta} \rightarrow \infty$. Perturbative evolution equations for large $\hat{\zeta}$.

- In this approach, have “modified universality”, $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$ (initial state interactions in DY case). SIDIS: $\eta v \cdot P \rightarrow \infty$, DY: $\eta v \cdot P \rightarrow -\infty$.

Without initial/final state effects, T-odd Sivers and Boer-Mulders functions would vanish!

Invariant amplitudes

Return to correlator

$$\widetilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu) \equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$

Decompose in terms of invariant amplitudes; at leading twist,

$$\frac{1}{2P^+} \widetilde{\Phi}_{\text{unsubtr.}}^{[\gamma^+]} = \widetilde{A}_{2B} + im_N \epsilon_{ij} b_i S_j \widetilde{A}_{12B}$$

$$\frac{1}{2P^+} \widetilde{\Phi}_{\text{unsubtr.}}^{[\gamma^+ \gamma^5]} = -\Lambda \widetilde{A}_{6B} + i[(b \cdot P)\Lambda - m_N(b_T \cdot S_T)] \widetilde{A}_{7B}$$

$$\begin{aligned} \frac{1}{2P^+} \widetilde{\Phi}_{\text{unsubtr.}}^{[i\sigma^{i+} \gamma^5]} &= im_N \epsilon_{ij} b_j \widetilde{A}_{4B} - S_i \widetilde{A}_{9B} \\ &\quad - im_N \Lambda b_i \widetilde{A}_{10B} + m_N[(b \cdot P)\Lambda - m_N(b_T \cdot S_T)] b_i \widetilde{A}_{11B} \end{aligned}$$

Invariant amplitudes and TMDs

Conversely, invariant amplitudes directly give selected x -integrated TMDs in Fourier (b_T) space (showing just the ones relevant for Sivers, Boer-Mulders shifts):

$$\tilde{f}_1^{[1](0)}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\tilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\tilde{S}(b^2, \dots)$$

$$\tilde{f}_{1T}^{\perp1}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = -2\tilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\tilde{S}(b^2, \dots)$$

$$\tilde{h}_1^{\perp1}(b_T^2, \hat{\zeta}, \dots, \eta v \cdot P) = 2\tilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)/\tilde{S}(b^2, \dots)$$

Generalized shifts

Form ratios in which soft factors, (Γ -independent) multiplicative renormalization factors cancel

Sivers shift:

$$\langle k_y \rangle_{TU} \equiv m_N \frac{f_{1T}^{\perp1}}{f_1^{[1](0)}} = \frac{\int dx \int d^2 k_T k_y \Phi^{[\gamma^+]}(x, k_T, S_T = (1, 0))}{\int dx \int d^2 k_T \Phi^{[\gamma^+]}(x, k_T, S_T = (1, 0))}$$

Average transverse momentum of unpolarized (“ U ”) quarks orthogonal to the transverse (“ T ”) spin of nucleon; normalized to the number of valence quarks. “Dipole moment” in $b_T^2 = 0$ limit, “shift”.

Issue: k_T -moments in this ratio singular; generalize to ratio of Fourier-transformed TMDs at *nonzero* b_T^2 ,

$$\langle k_y \rangle_{TU}(b_T^2, \dots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp1}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)}$$

(remember singular $b_T \rightarrow 0$ limit corresponds to taking k_T -moment). “Generalized shift”.

Generalized shifts from amplitudes

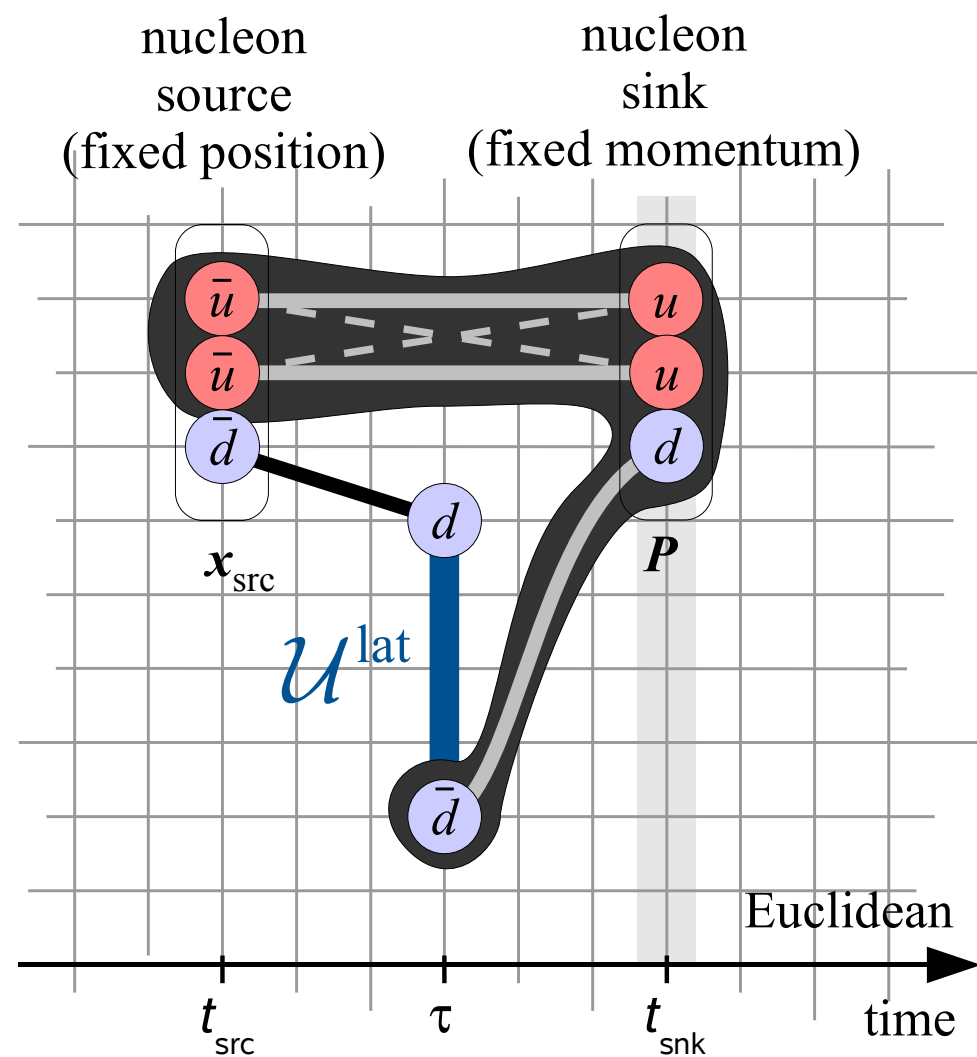
Now, can also express this in terms of invariant amplitudes:

$$\langle k_y \rangle_{TU}(b_T^2, \dots) \equiv m_N \frac{\tilde{f}_{1T}^{\perp1}(b_T^2, \dots)}{\tilde{f}_1^{[1](0)}(b_T^2, \dots)} = -m_N \frac{\tilde{A}_{12B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\tilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Analogously, Boer-Mulders shift:

$$\langle k_y \rangle_{UT}(b_T^2, \dots) = m_N \frac{\tilde{A}_{4B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}{\tilde{A}_{2B}(-b_T^2, 0, \hat{\zeta}, \eta v \cdot P)}$$

Lattice setup



- Evaluate directly $\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \hat{\zeta}, \mu)$
 $\equiv \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$
- Euclidean time: Place entire operator at one time slice, i.e., $b, \eta v$ purely spatial
- Since generic b, v space-like, no obstacle to boosting system to such a frame!
- Parametrization of correlator in terms of $\tilde{A}_i$ invariants permits direct translation of results back to original frame
- Form desired ratios of $\tilde{A}_i$ invariants
- Extrapolate $\eta \rightarrow \infty, \hat{\zeta} \rightarrow \infty$ numerically

Lattice setup

Use three MILC 2+1-flavor gauge ensembles with $a \approx 0.12$ fm:

$m_\pi = 369$ MeV ; $28^3 \times 64$; 2184 samples

$m_\pi = 369$ MeV ; $20^3 \times 64$; 5264 samples

$m_\pi = 518$ MeV ; $20^3 \times 64$; 3888 samples

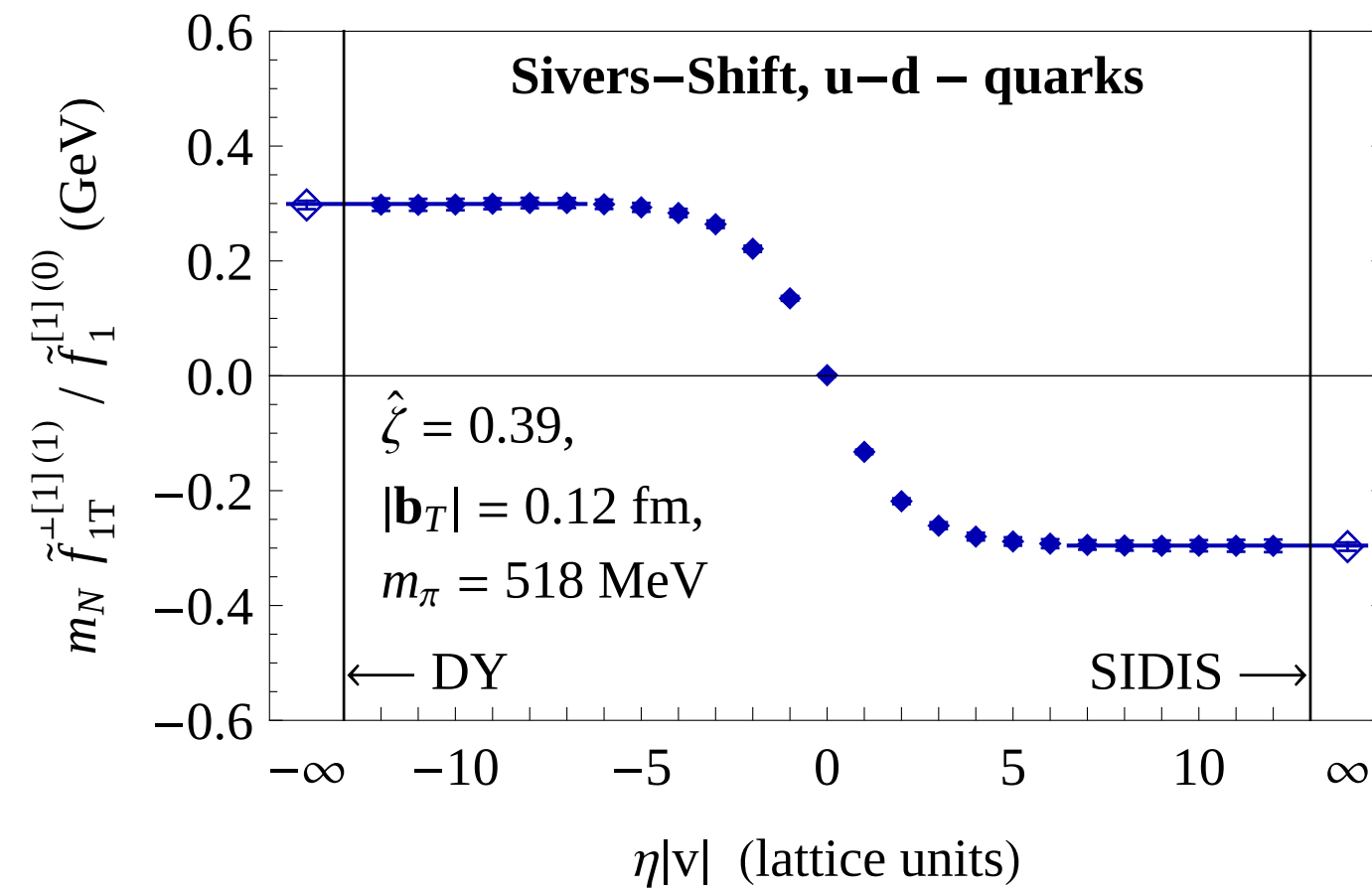
Sink momenta P : $(0, 0, 0)$, $(-1, 0, 0)$, $(-2, 0, 0)$, $(1, -1, 0)$

Variety of b , ηv ; note $b \perp P$, $b \perp v$ (lowest x -moment, kinematical choices/constraints)

Largest $\hat{\zeta} = 0.78$

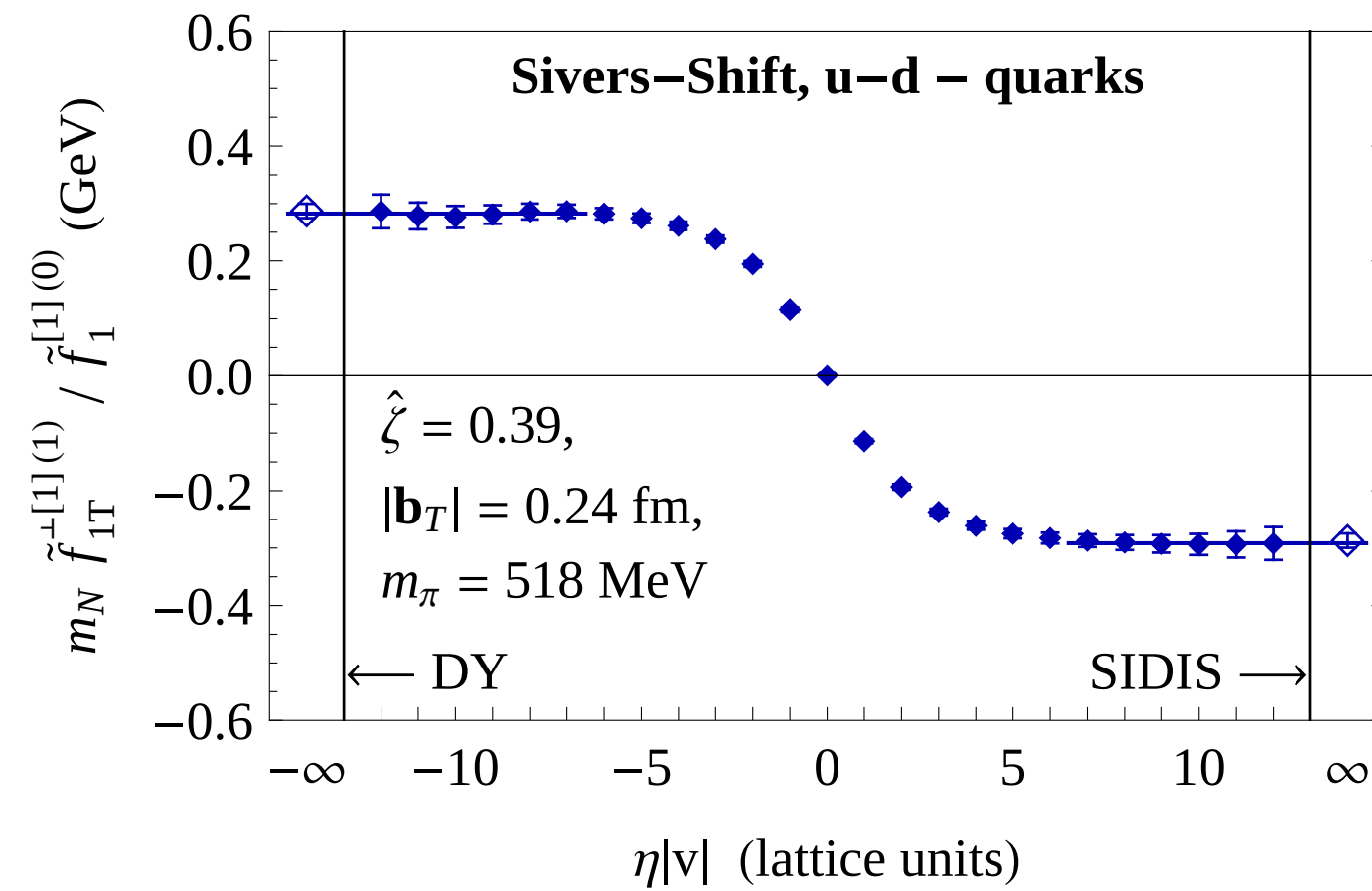
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



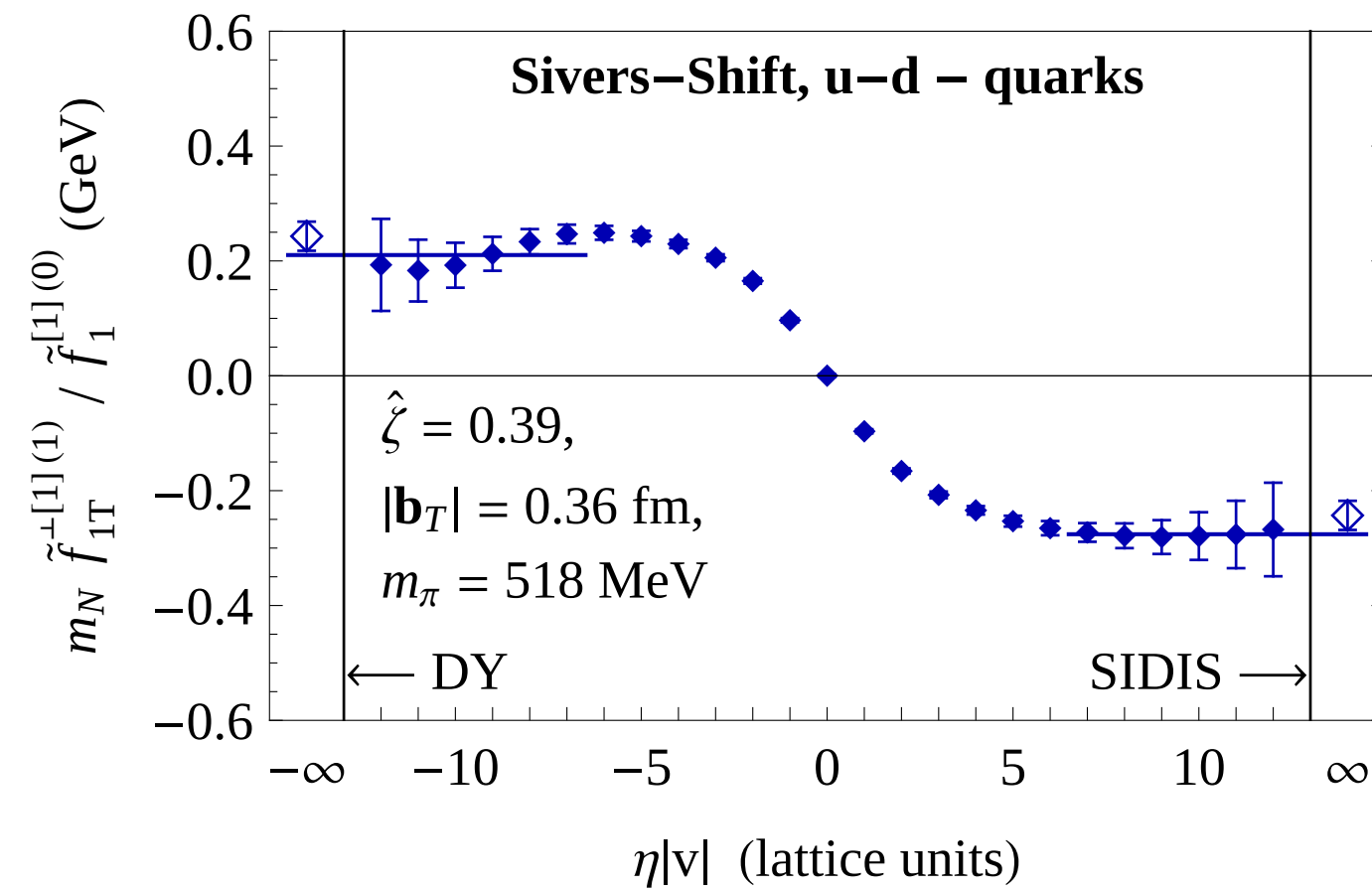
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



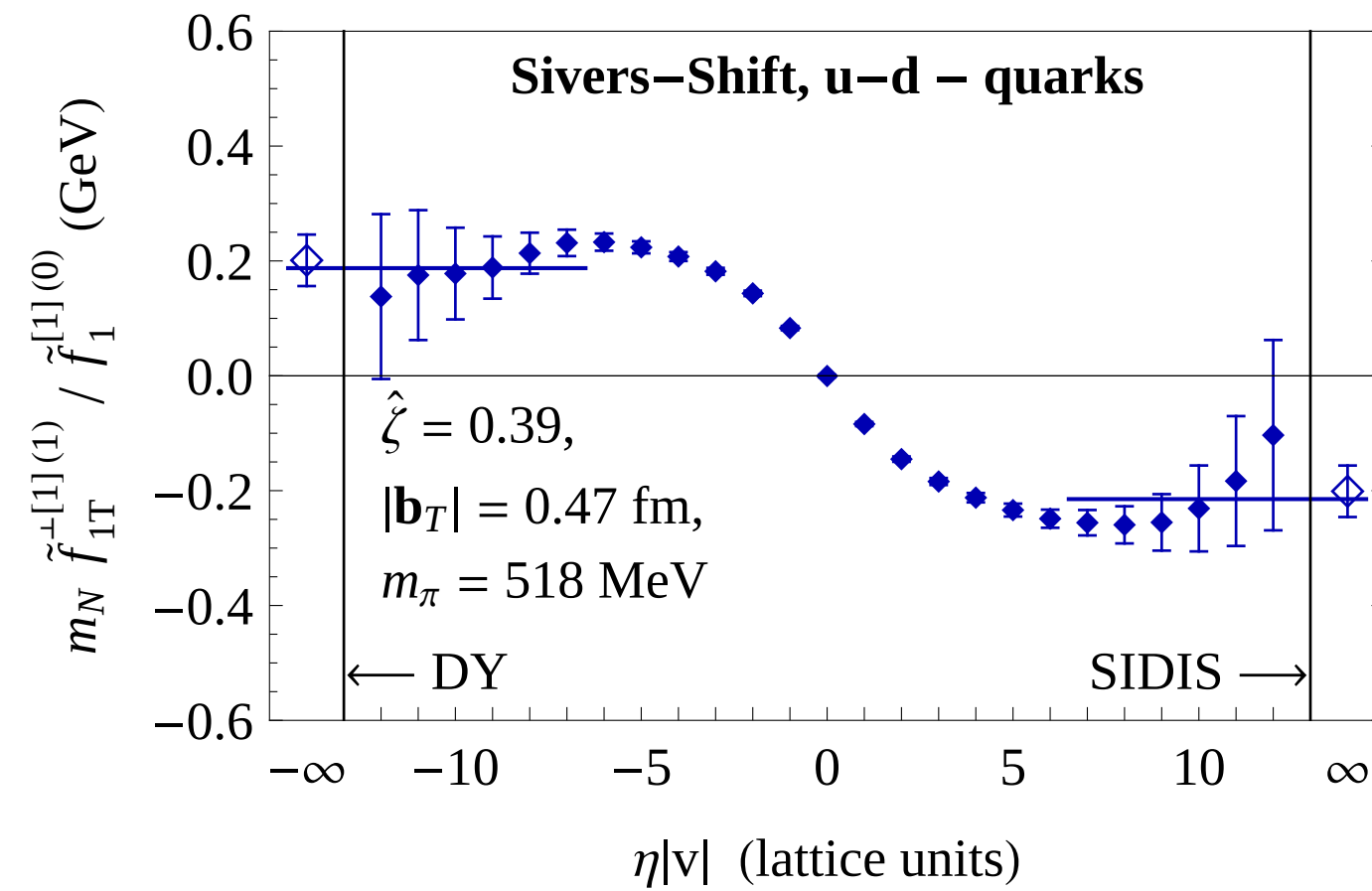
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



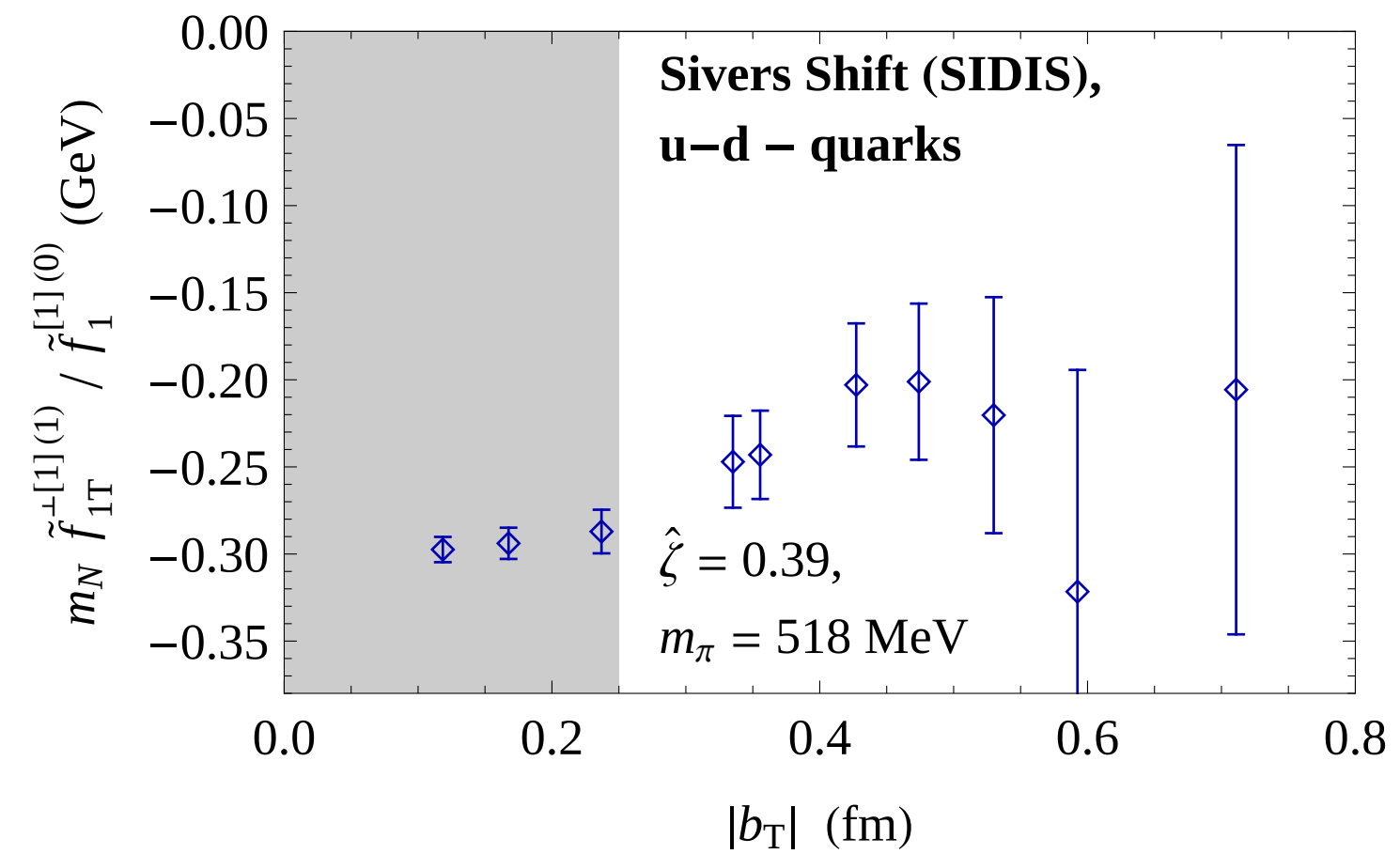
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $|b_T|$



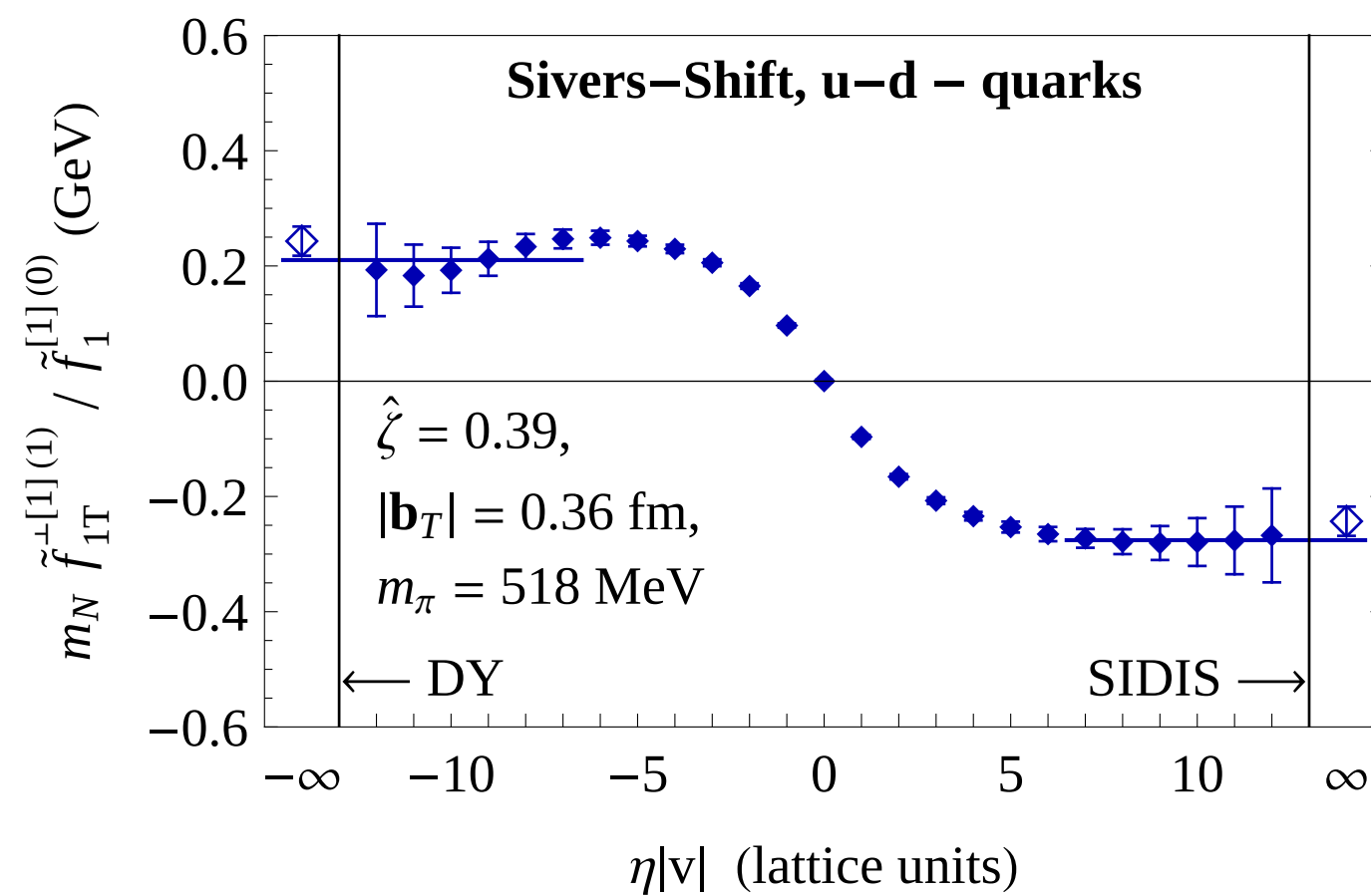
Results: Sivers shift

Dependence of SIDIS limit on $|b_T|$



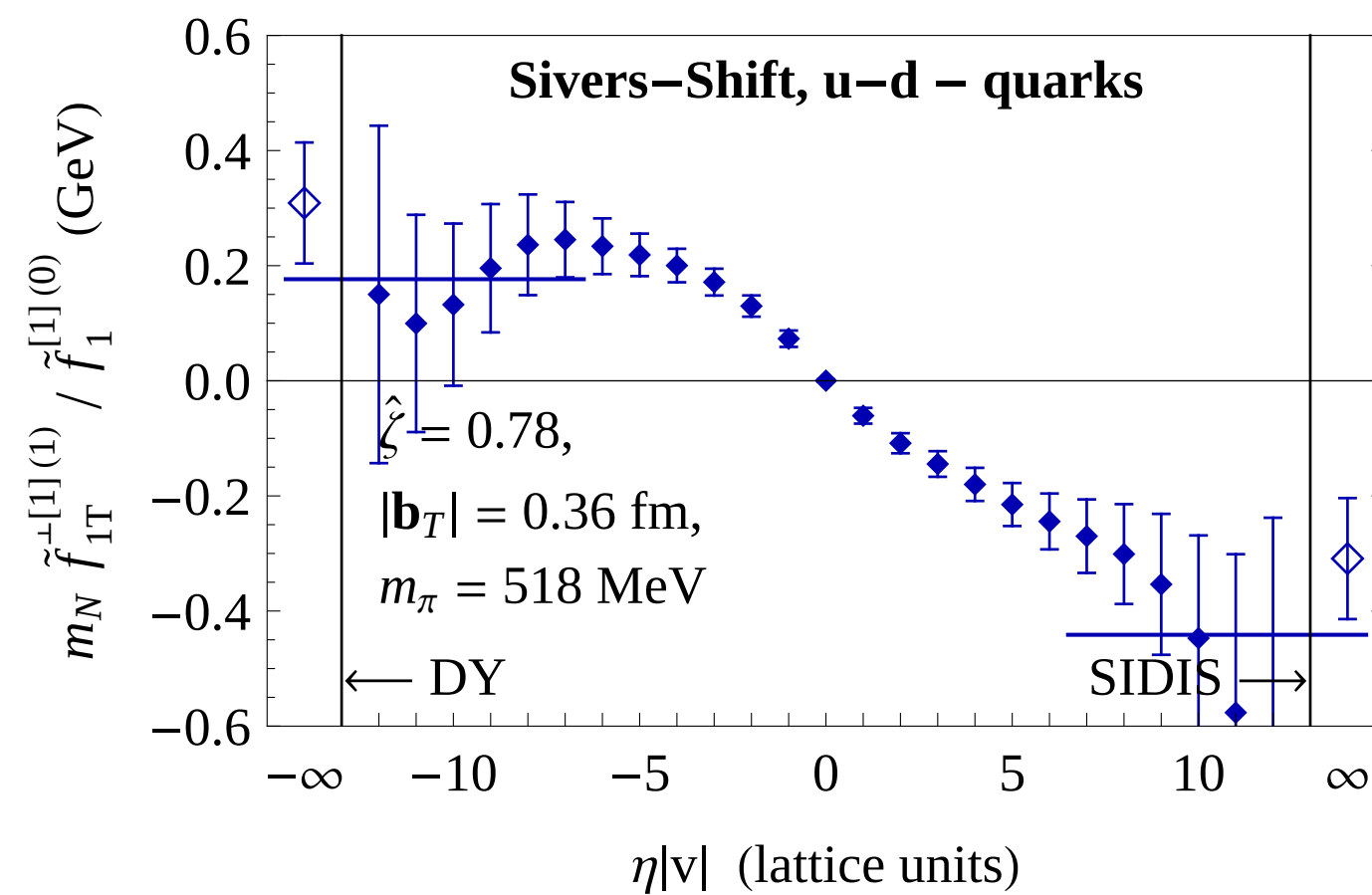
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $\hat{\zeta}$



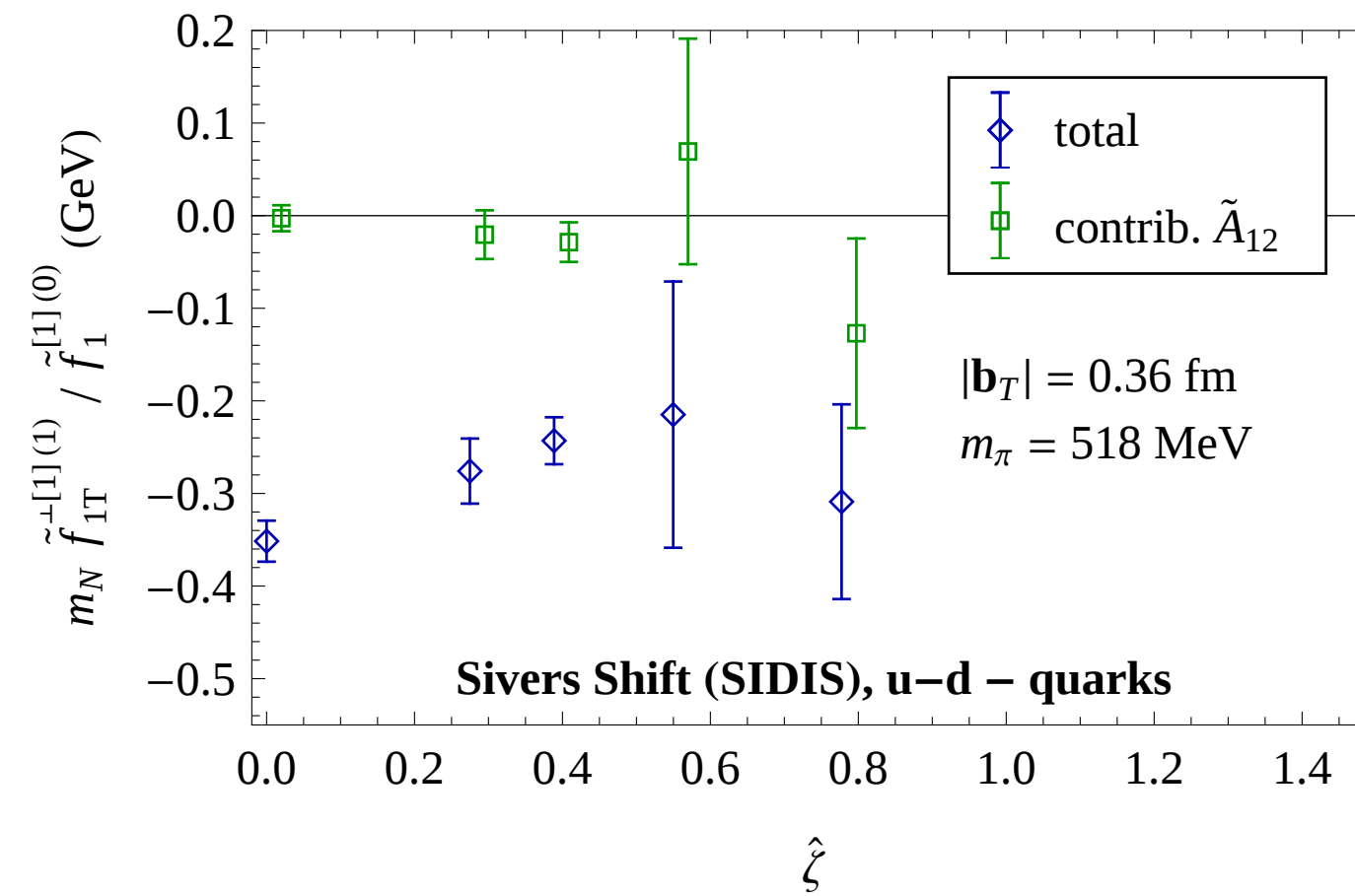
Results: Sivers shift

Dependence on staple extent; sequence of panels at different $\hat{\zeta}$



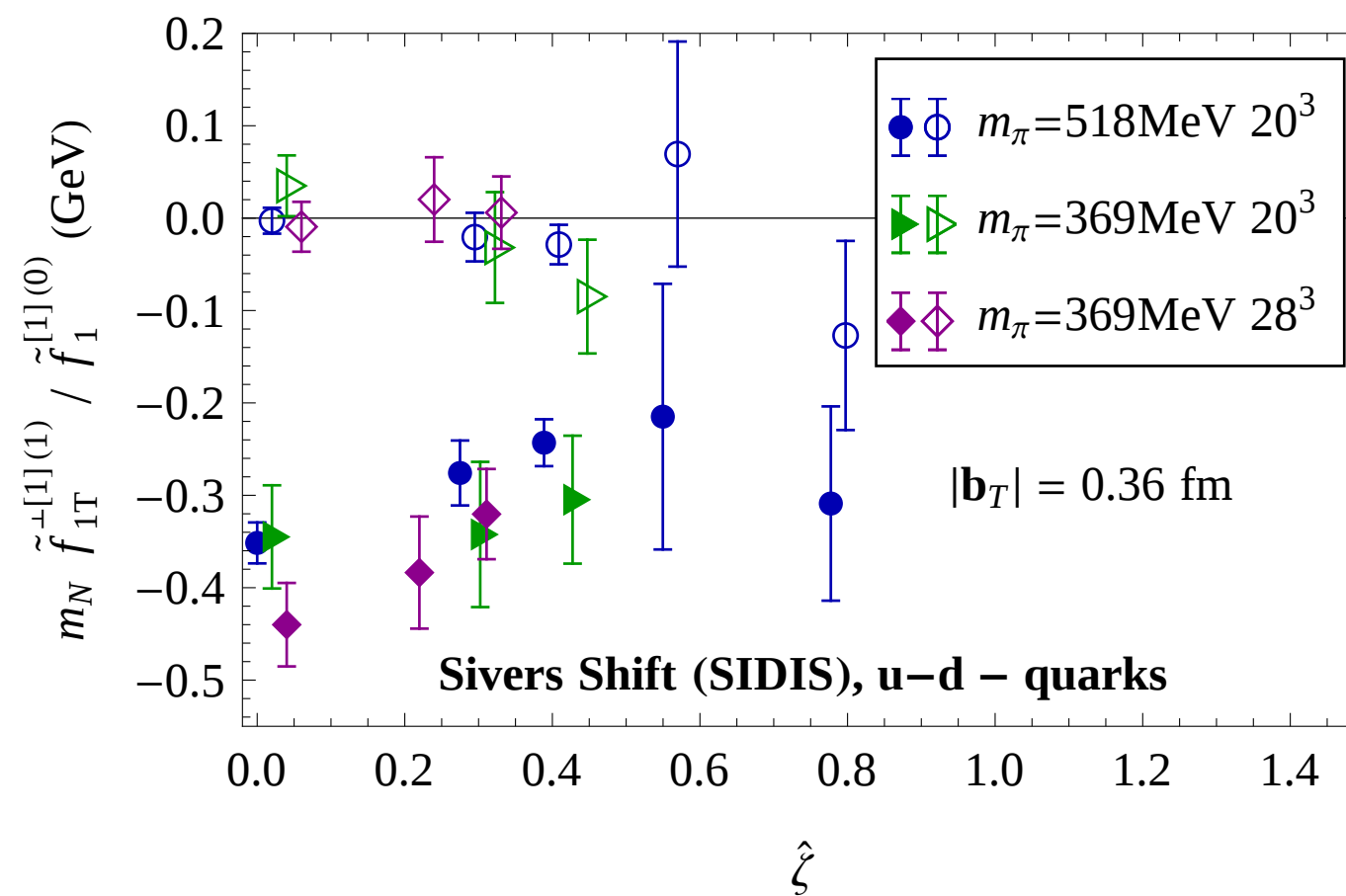
Results: Sivers shift

Dependence of SIDIS limit on $\hat{\zeta}$



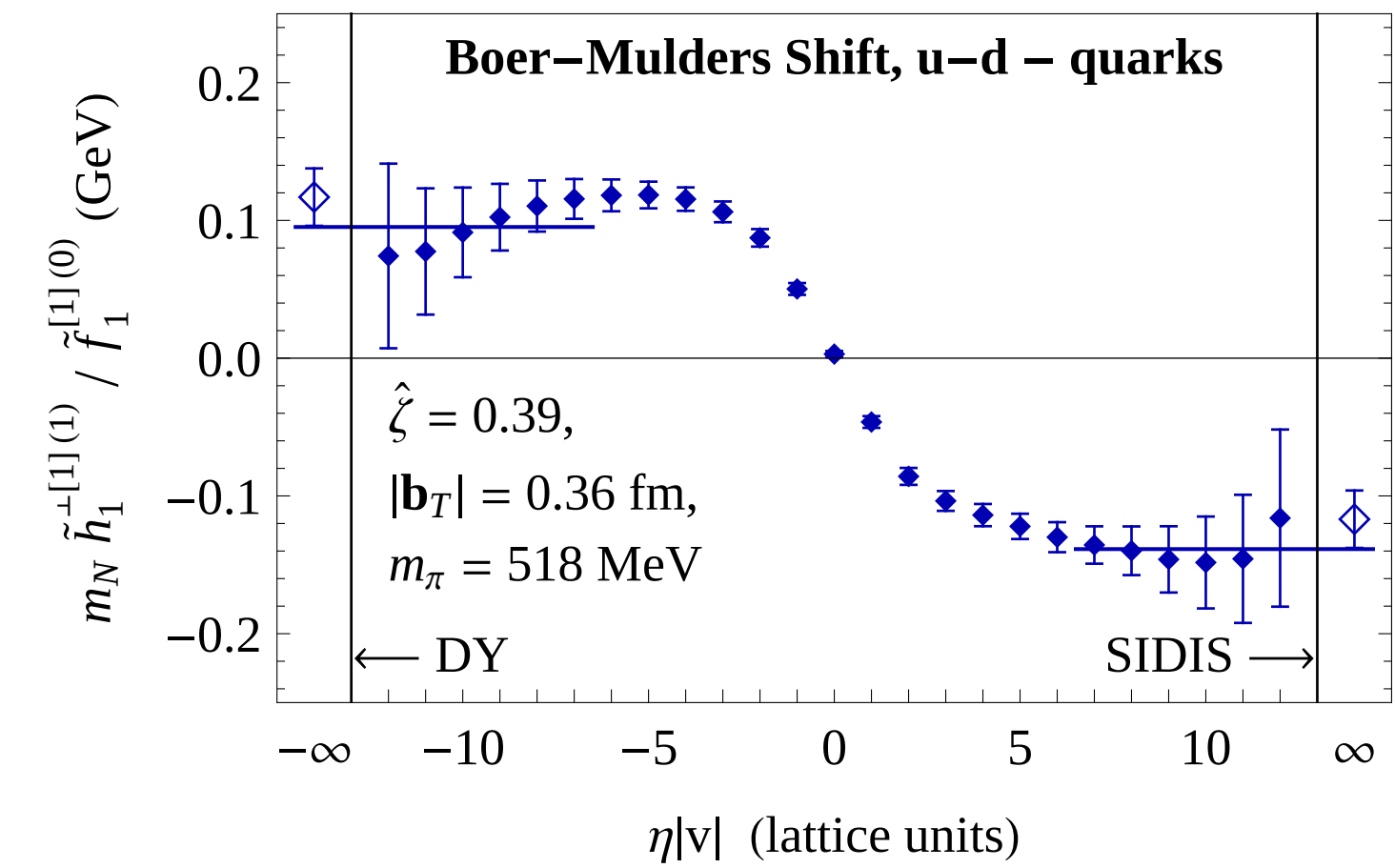
Results: Sivers shift

Dependence of SIDIS limit on $\hat{\zeta}$, all three ensembles



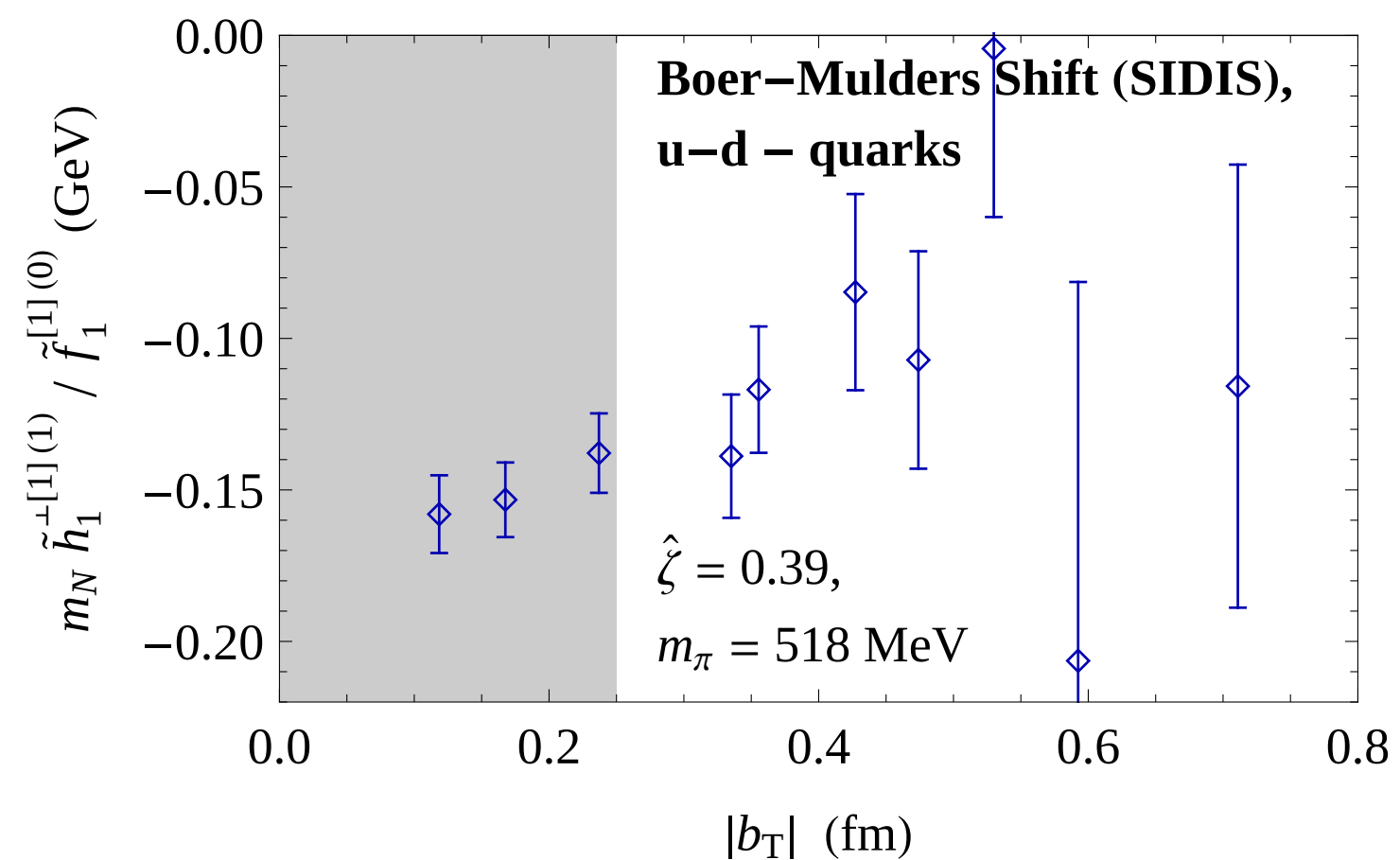
Results: Boer-Mulders shift

Dependence on staple extent



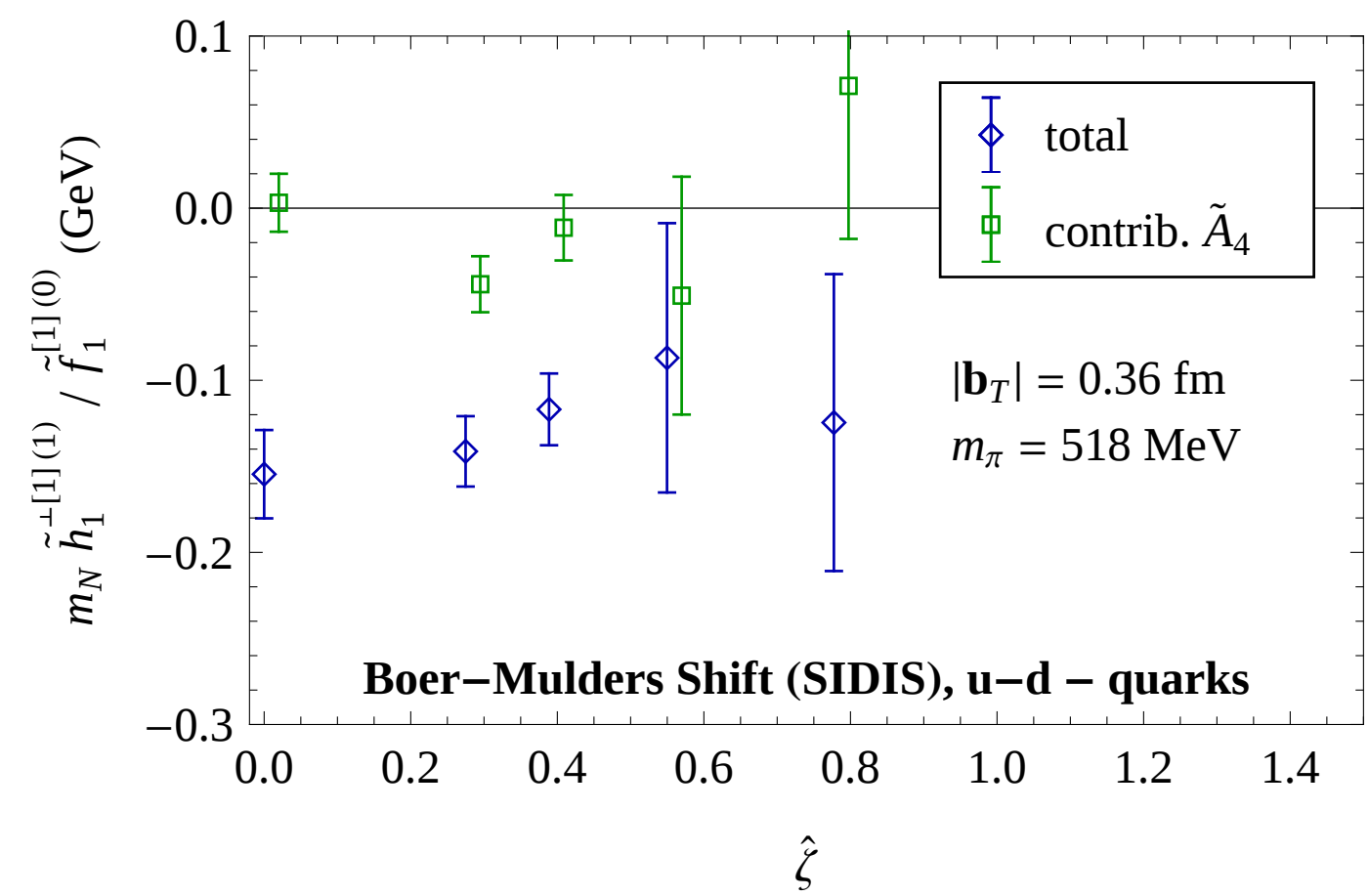
Results: Boer-Mulders shift

Dependence of SIDIS limit on $|b_T|$



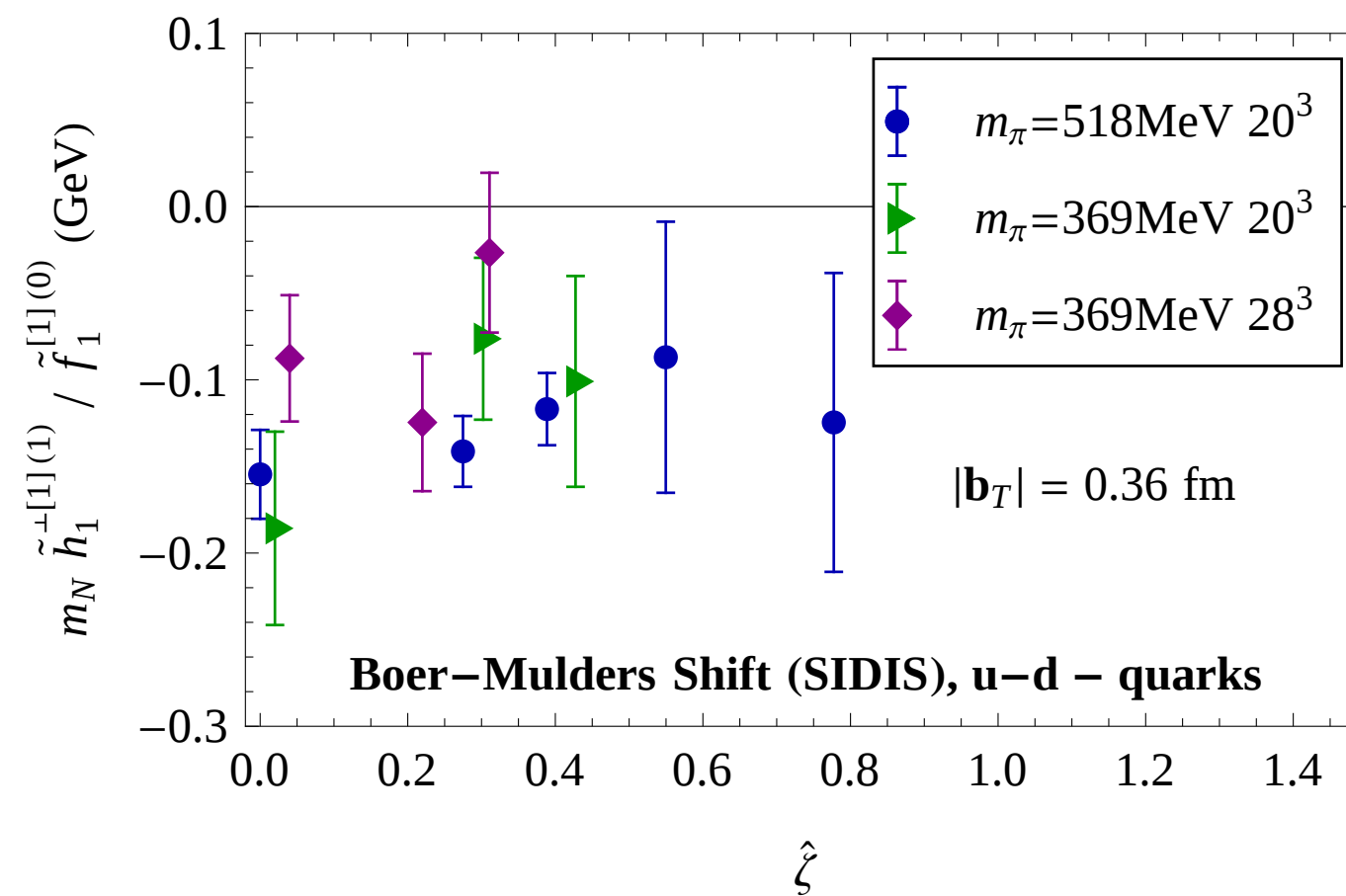
Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$



Results: Boer-Mulders shift

Dependence of SIDIS limit on $\hat{\zeta}$, all three ensembles



Conclusions

- Exploratory study of TMDs using staple-shaped gauge link structures
- Accessed T-odd Sivers, Boer-Mulders observables; SIDIS, DY limits distinguished by sign of $v \cdot P$. For u-d quark combination, SIDIS Sivers and Boer-Mulders TMDs both sizeable and negative.
- To avoid soft factors, multiplicative renormalization constants, constructed appropriate ratios of Fourier-transformed TMDs (“shifts”).
- v taken off light cone: Dependence on Collins-Soper parameter $\hat{\zeta}$. In addition to $\eta v \rightarrow \infty$, need to also consider $\hat{\zeta} \rightarrow \infty$.
- $\eta v \rightarrow \infty$ seems under good control; plateaux reached at moderate values.
- $\hat{\zeta} \rightarrow \infty$ remains a challenge. No clear trends seen in the data sets available. Need much larger $\hat{\zeta}$. Presently investigating pion with this in mind.
- No significant volume dependence, pion mass dependence detected within the limited set of (three) cases considered