Status of Nucleon Structure Calculations with 2+1 Flavors of Domain Wall Fermions

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Eigo Shintani  BNL [LMA/AMA, poster]
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The numerical calculations were performed on

- BG/P at ANL and U. Edinburgh [gauge configurations]
- TeraGrid/XSEDE supported by National Science Foundation grant number OCI-1053575 [propagators]
- RIKEN Cluster of Clusters at RIKEN [propagators]
# Outline

- **Introduction**
- **Calculation Details**
- **Preliminary Results**
  - Isovector Dirac and Pauli Form Factors
- **Error Reduction Techniques**
- **Conclusions**
Introduction

- Nucleon structure calculations suffer from various sources of systematic errors, among which
  - chiral extrapolation
  - finite volume effects
  - excited-state contaminations

are the most actively researched topics in the past few years.

- Ideally we’d like to do the calculations at physical pion mass with infinitely large volume. Realistically, our goal is to
  - push the pion mass closer to the physical point.
  - simulate at a large box.
  - keep the excited-state contaminations under control: sufficiently large source-sink separation or extrapolation from multiple separations.

- Such calculations are very challenging:
  high numerical cost per propagator at small $m_\pi$, nucleon signal decreases exponentially with $m_\pi$. 
Lattice Setup

- **Gauge Ensembles**: 2+1-flavor Domain Wall Fermion gauge ensembles generated by the RBC and UKQCD Collaborations.
  - Iwasaki gauge action, with Dislocation-Suppressing-Determinant Ratio (ID)
  - $\beta = 1.75 \rightarrow a^{-1} \approx 1.37$ GeV.

<table>
<thead>
<tr>
<th>$am_l$</th>
<th>$am_s$</th>
<th>$L^3 \times T$</th>
<th>$L_s$</th>
<th>$m_\pi$ [MeV]</th>
<th>$m_\pi L$</th>
<th>$a$ [fm]</th>
<th>$am_{res}$</th>
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<tbody>
<tr>
<td>0.001</td>
<td>0.042</td>
<td>$32^3 \times 64$</td>
<td>32</td>
<td>170</td>
<td>4.0</td>
<td>0.146</td>
<td>0.0018</td>
</tr>
<tr>
<td>0.0042</td>
<td>0.042</td>
<td>$32^3 \times 64$</td>
<td>32</td>
<td>250</td>
<td>5.8</td>
<td>0.146</td>
<td>0.0018</td>
</tr>
</tbody>
</table>

- **Quark Propagators**:
  - Gaussian-smeared source with APE-smeared gauge links
  - $(t_{snk} - t_{src})/a = 9 \Rightarrow t_{snk} - t_{src} \approx 1.3$ fm
  - 4 sources per configuration at $t/a = 0, 16, 32, 48$
  - Number of configurations analyzed:
    - $am_l = 0.001$ : 103 $\Rightarrow$ 412 correlation functions
    - $am_l = 0.0042$ : 165 $\Rightarrow$ 660 correlation functions
We use the standard proton interpolating operator, with smearing $S = \text{Gaussian (G) or Local (L)}$

$$\chi_S(x) = \epsilon_{abc} \left( [u_a^S(x)]^T C \gamma_5 d_b^S(x) \right) u_c^S(x)$$

Nucleon two-point functions:

$$C_S(t - t_{src}, p) = \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \text{Tr} \left[ \mathcal{P}_4 \langle 0 \mid \chi_S(\vec{x}, t) \chi_G(\vec{0}, t_{src}) \mid 0 \rangle \right]$$

Nucleon three-point functions:

$$C_J^{P\alpha}_{\mu} = \sum_{\vec{x}, \vec{z}} e^{i\vec{q} \cdot \vec{z}} \text{Tr} \left[ \mathcal{P}_\alpha \langle 0 \mid \chi_G(\vec{x}, t_{snk}) J_\mu(\vec{z}, t) \chi_G(\vec{0}, t_{src}) \mid 0 \rangle \right]$$

with the projection operators:

$$\mathcal{P}_4 = (1 + \gamma_4)/2$$
$$\mathcal{P}_{53} = (1 + \gamma_4)\gamma_5\gamma_3/2$$
Connected vs. Disconnected

- Two types of contractions contribute to the three-point functions:

- We do not yet include disconnected digrams in our calculations.
- In the isovector case ($p - n$), only connected diagrams contribute.
  [focus of the talk]
Determination of Form Factors

Nucleon vector form factors:

\[ \langle p | V^+_{\mu}(x) | n \rangle = \bar{u}_p \left[ F_1(q^2) + \frac{\sigma_{\mu\lambda} q_{\lambda}}{2M_N} F_2(q^2) \right] u_n e^{iq \cdot x} \]

\( F_1(q^2), F_2(q^2) \): Dirac and Pauli form factors.

Nucleon Sachs form factors:

\[ G_E(q^2) = F_1(q^2) - \frac{q^2}{4M_N^2} F_2(q^2) \]
\[ G_M(q^2) = F_1(q^2) + F_2(q^2) \]

We define the following ratio

\[ R^{P,\alpha}_{J^{\mu}}(q, t) = K \cdot \frac{C_{J^{P,\alpha}}(\vec{q}, t)}{C_G(t_{\text{snk}} - t_{\text{src}}, 0)} \left[ \frac{C_L(t_{\text{snk}} - t, q) C_G(t - t_{\text{src}}, 0) C_L(t_{\text{snk}} - t_{\text{src}}, 0)}{C_L(t_{\text{snk}} - t, 0) C_G(t - t_{\text{src}}, q) C_L(t_{\text{snk}} - t_{\text{src}}, q)} \right]^{1/2} \]

with

\[ K = M_N \sqrt{2E(q)(M_N + E(q))} \]
Determination of Form Factors

- The ratios conveniently defined to be directly related to the Sachs Form Factors:

\[ G_E(q,t) = \frac{R_{V4}^P(q,t)}{M_N(M_N + E(q))}, \]
\[ G_M(q,t) = \frac{1}{2} \left( \frac{R_{V1}^P(q,t)}{q_2M_N} - \frac{R_{V2}^P(q,t)}{q_1M_N} \right), \]

- And the Dirac and Pauli form factors can be obtained by:

\[ F_1(q^2) = \frac{G_E(q) + \tau G_M(q)}{1 + \tau}, \text{ for all } q \]
\[ F_2(q^2) = \frac{G_M(q) - G_E(q)}{1 + \tau}, \text{ for } q \neq 0 \]

where \( \tau = q^2/(4M_N^2) \).
Isovector Dirac and Pauli Form Factors

$$F^u_d(q^2) \text{ Plateaus}$$

$$E(q) = \sqrt{n^2 \left( \frac{2\pi}{L} \right)^2 + M_N^2}$$

- Good plateaus for all values of $n^2$. No signs of excited-state contaminations.
- Choose fit range $t = [2, 7]$. 

Nucleon Structure with 2+1-Flavor Domain Wall Fermions

Yale/RBRC
**$F^{u-d}_2(q^2)$ Plateaus**

- Good plateaus for all values of $n^2$ at $am_l = 0.0042$.
- Signs of excited-state contaminations at $am_l = 0.001$?
  Statistical noise is still dominating.
Isovector Dirac and Pauli Form Factors

\[ F_{1}^{u-d}(q^2) \]

Nucleon Structure with 2+1-Flavor Domain Wall Fermions

- Large volume → small \( q^2 \)
- Results for two masses almost indistinguishable.
Comparison with Previous DWF Calculations

- Mild pion mass dependence
- Translates into mild mass dependence for the radii.
Similarly for $F_{2}^{u-d}(q^{2})$
Mean-squared radii are determined from dipole fits to the form factors:

\[ F_i(q^2) = \frac{F_i(0)}{(1 + q^2/M_i^2)^2} \]

\[ \langle r_i^2 \rangle = \frac{12}{M_i^2} \]

Dirac and Pauli Radii
\[
\langle r_1^2 \rangle^{1/2} \text{ undershoots the experiment by 25\%.}
\]
\[
\langle r_2^2 \rangle^{1/2} \text{ is approaching the experiment.}
\]
\[
\text{For } m_\pi = 170 \text{ MeV, we may need to worry about finite volume effects.}
\]
\[
\text{Statistical errors are substantial for the ID32 data points.}
\]
\[
\text{Necessary to improve the statistics significantly.}
\]
Low-Mode Averaging (LMA)

- Good for low-mode-dominant observables.
- Use low eigenmodes to approximate the observable.
  \[ O = O_l + O_{rest} \]
- Can improve statistics by averaging over covariant symmetry transformations, e.g., lattice translation \( g \).
  \[ O = \frac{1}{N_g} \sum_g O^g_l + O_{rest} \equiv O_{appx} + O_{rest} \]
- Correct for the bias by computing \( O \) regularly (but less frequently), and
  \[ O_{rest} = O - O_{appx}. \]

Cheap with low-mode deflation.

For details, see poster by Eigo Shintani.
All-Mode Averaging (AMA)

- Necessary for observables with significant high-mode contributions.
- For each $g$ transformation, use sloppy CG (loose stopping condition, $O(10^{-3})$) to correct for the bias from the low modes.

$$O_{\text{appx}} = \frac{1}{N_g} \sum_g (O^g_l + O^g_h),$$

$$O^g_h = O^g_{\text{sloppy}} - O^g_l.$$

- Again, correct for the bias by computing $O$ regularly (but less frequently), and

$$O_{\text{rest}} = O - O_{\text{appx}}.$$

Cheap with low-mode deflation.

For details, see poster by Eigo Shintani.
Tests on $24^3 \times 64$ Lattices

- $24^3 \times 64$ lattices, $N_f = 2 + 1$ DWF, $a^{-1} \approx 1.73$ GeV
- $am_l = 0.01 \rightarrow m_{\pi} \approx 420$ MeV
- # of configurations = 80.
- LMA: 180 low eigenmodes, $N_g = 32$ translations ($2^3 \times 4$)
- AMA: Sloppy CG with stop. cond. 0.003. (further speedup with low-mode deflation)
- Full calculations as in Yamazaki et al., PRD79, 114505 (2009): # of configurations = 356, with 4 sources / config.
- Cost in units of full propagators:

<p>| | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>LMA</td>
<td>80 props</td>
</tr>
<tr>
<td>AMA</td>
<td>138 props</td>
</tr>
<tr>
<td>full stat.</td>
<td>1424 props</td>
</tr>
</tbody>
</table>
Comparison of LMA, AMA and Original

\[24^3 \times 64, \ am_l = 0.01 \ [m_\pi \approx 420 \text{ MeV}]\]

Points are shifted for clarity.

[Eigo Shintani]

- LMA is not enough to reduce the errors → high-mode contributions are important.
- Errors from AMA comparable to “full stat.”, but with 1/10 the cost.
Conclusions

- It is pricey to go to lighter pion masses with a sufficiently large source-sink separations.
- Current results for the nucleon isovector vector form factors and their associated radii suffer from large statistical errors.
- Improved error reduction techniques are essential.

Plans

- Calculations with AMA are underway.
  → Expect to reduce the errors by a factor of 5.
- AMA makes it easier to change the source-sink separations.
  → multiple source-sink separations.
- Longer term:
  → Bigger volumes. Continuum extrapolations.
Backup Slides
Cost (in the case of m=0.01)

Use of unit of quark propagator “prop” in full CG w/o deflation

• Case of full statistics
  In $N_{\text{conf}} = 356$, $N_{\text{mes}} = 4$,
  \[ \text{Total: } 356 \times 4 = 1424 \text{ prop} \]

• Case of AMA w/o deflation
  Since calculation of $O^{\text{appx}}$ need 1/50 prop, then in $N_{\text{conf}} = 81$, $N'_\text{mes} = 32$
  \[ \text{Total: } 80 + 80 \times \frac{32}{50} = 131 \text{ prop } \Rightarrow \text{10 times fast} \]

• Case of AMA w/ deflation
  When using 180 eigenmode, calculation of $O^{\text{appx}}$ need 1/80 prop,
  but in this case the calculation of lowmode is $\sim$1 prop/configs.
  Deflated CG makes reduction of full CG to 1/3 prop, then
  \[ \text{Total: } \frac{80}{3} + 80 \times \frac{32}{80} + 80 = 138 \text{ prop } \Rightarrow \text{10 times fast} \]

Note that stored eigehmode is useful for other works.

[slide from E. Shintani’s poster]