Status of Nucleon Structure Calculations with 2+1 Flavors of Domain Wall Fermions

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This work is done with

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The numerical calculations were performed on

- BG/P at ANL and U. Edinburgh [gauge configurations]
- TeraGrid/XSEDE supported by National Science Foundation grant number OCI-1053575 [propagators]
- RIKEN Cluster of Clusters at RIKEN [propagators]

Outline

Introduction

Calculation Details

Preliminary Results

Isovector Dirac and Pauli Form Factors

Error Reduction Techniques

Conclusions

Introduction

- Nucleon structure calculations suffer from various sources of systematic errors, among which
  - chiral extrapolation
  - finite volume effects
  - excited-state contaminations

  are the most actively researched topics in the past few years.

- Ideally we'd like to do the calculations at physical pion mass with infinitely large volume. Realistically, our goal is to
  - push the pion mass closer to the physical point.
  - simulate at a large box.
  - keep the excited-state contaminations under control: sufficiently large source-sink separation or extrapolation from multiple separations.
- Such calculations are very challenging:
  high numerical cost per propagator at small m_π,
  nucleon signal decreases exponentially with m_π.

Lattice Setup

- Gauge Ensembles: 2+1-flavor Domain Wall Fermion gauge ensembles generated by the RBC and UKQCD Collaborations.
  - Iwasaki gauge action, with Dislocation-Suppressing-Determinant Ratio (ID)
  - $\beta = 1.75 \rightarrow a^{-1} \approx 1.37$ GeV.

am_l	am_s	$L^3 \times T$	L_s	m_π [MeV]	$m_\pi L$	a [fm]	$am_{\rm res}$
0.001	0.042	$32^3 \times 64$	32	170	4.0	0.146	0.0018
0.0042	0.042	$32^3 \times 64$	32	250	5.8	0.146	0.0018

- Quark Propagators:
  - Gaussian-smeared source with APE-smeared gauge links
  - $(t_{snk} - t_{src})/a = 9 \Rightarrow t_{snk} - t_{src} \approx 1.3$ fm
  - 4 sources per configuration at $t/a = 0, 16, 32, 48$
  - Number of configurations analyzed:
    - $am_l = 0.001$: $103 \Rightarrow \underline{412}$ correlation functions
    - $am_l = 0.0042$: $165 \Rightarrow \underline{660}$ correlation functions

Nucleon Two and Three-Point Functions

- We use the standard proton interpolating operator, with smearing $S =$ Gaussian (G) or Local (L)

$$\chi_S(x) = \epsilon_{abc}\left([u_a^S(x)]^T C\gamma_5 d_b^S(x)\right) u_c^S(x)$$

- Nucleon two-point functions:

$$C_S(t - t_{src}, p) = \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \mathrm{Tr}\left[\mathcal{P}_4 \langle 0|\chi_S(\vec{x}, t)\overline{\chi}_G(\vec{0}, t_{src})|0\rangle\right]$$

- Nucleon three-point functions:

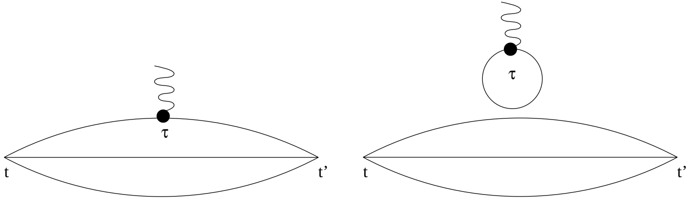
$$C_{J_\mu}^{\mathcal{P}_\alpha} = \sum_{\vec{x},\vec{z}} e^{i\vec{q}\cdot\vec{z}} \mathrm{Tr}[\mathcal{P}_\alpha \langle 0|\chi_G(\vec{x}, t_{snk}) J_\mu(\vec{z}, t)\overline{\chi}_G(\vec{0}, t_{src})|0\rangle]$$

  with the projection operators:

$$\begin{aligned} \mathcal{P}_4 &= (1 + \gamma_4)/2 \\ \mathcal{P}_{53} &= (1 + \gamma_4)\gamma_5\gamma_3/2 \end{aligned}$$

Connected vs. Disconnected

- Two types of contractions contribute to the three-point functions:



- We do not yet include disconnected digrams in our calculations.
- In the isovector case $(p - n)$, only connected diagrams contribute. [focus of the talk]

Determination of Form Factors

- Nucleon vector form factors:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[F_1(q^2) + \frac{\sigma_{\mu\lambda} q_\lambda}{2M_N} F_2(q^2) \right] u_n e^{iq\cdot x}$$

$F_1(q^2), F_2(q^2)$: Dirac and Pauli form factors.

- Nucleon Sachs form factors:

$$\begin{aligned} G_E(q^2) &= F_1(q^2) - \frac{q^2}{4M_N^2} F_2(q^2) \\ G_M(q^2) &= F_1(q^2) + F_2(q^2) \end{aligned}$$

- We define the following ratio

$$R_{J_\mu}^{\mathcal{P}_\alpha}(q,t) = K \cdot \frac{C_{J_\mu}^{\mathcal{P}_\alpha}(\vec{q},t)}{C_G(t_{\rm snk}-t_{\rm src},0)} \left[\frac{C_L(t_{\rm snk}-t,q) C_G(t-t_{\rm src},0) C_L(t_{\rm snk}-t_{\rm src},0)}{C_L(t_{\rm snk}-t,0) C_G(t-t_{\rm src},q) C_L(t_{\rm snk}-t_{\rm src},q)} \right]^{1/2},$$

with

$$K = M_N \sqrt{2E(q)(M_N + E(q))}$$

Determination of Form Factors

- The ratios conveniently defined to be directly related to the Sachs Form Factors:

$$G_E(q,t) = \frac{R_{V_4}^{\mathcal{P}_4}(q,t)}{M_N(M_N+E(q))},$$

$$G_M(q,t) = \frac{1}{2}\left(\frac{R_{V_1}^{\mathcal{P}_{53}}(q,t)}{q_2 M_N} - \frac{R_{V_2}^{\mathcal{P}_{53}}(q,t)}{q_1 M_N}\right),$$

- And the Dirac and Pauli form factors can be obtained by:

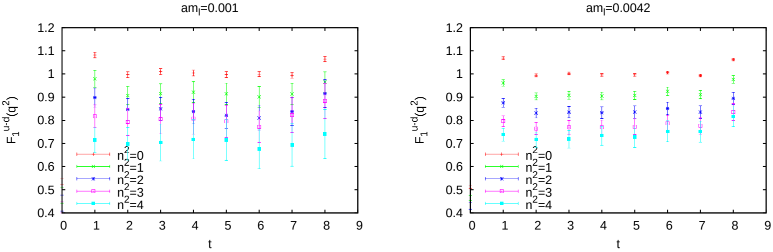
$$F_1(q^2) = \frac{G_E(q)+\tau G_M(q)}{1+\tau}, \text{ for all } q$$

$$F_2(q^2) = \frac{G_M(q)-G_E(q)}{1+\tau}, \text{ for } q \neq 0$$

where $\tau = q^2/(4M_N^2)$.

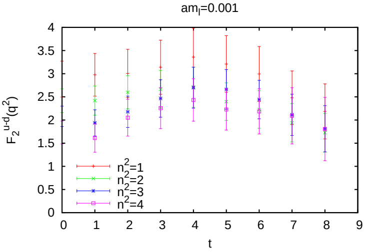
$F_1^{u-d}(q^2)$ Plateaus

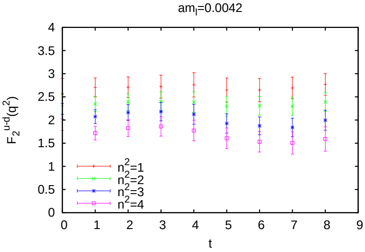
$$E(q) = \sqrt{n^2 \left(\frac{2\pi}{L}\right)^2 + M_N^2}$$



- Good plateaus for all values of n^2. No signs of excited-state contaminations.
- Choose fit range $t = [2, 7]$.

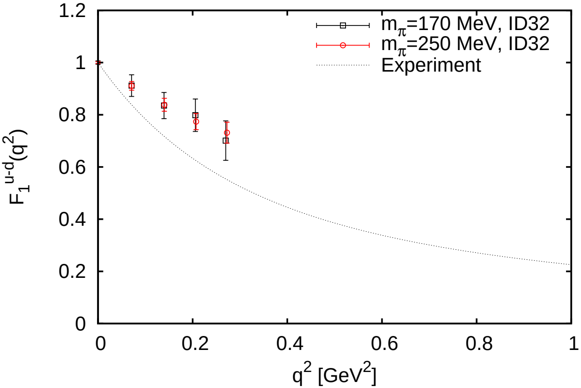
$F_2^{u-d}(q^2)$ Plateaus





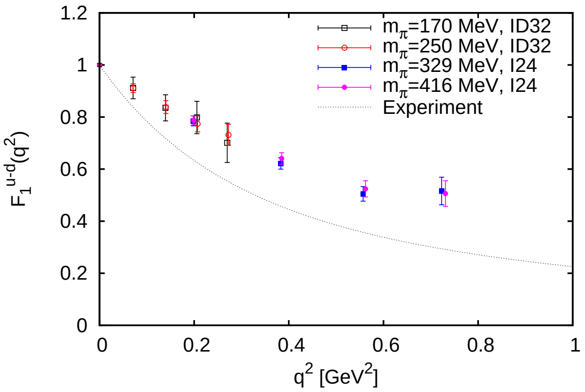
- Good plateaus for all values of n^2 at $am_l = 0.0042$.
- Signs of excited-state contaminations at $am_l = 0.001$? Statistical noise is still dominating.

$F_1^{u-d}(q^2)$



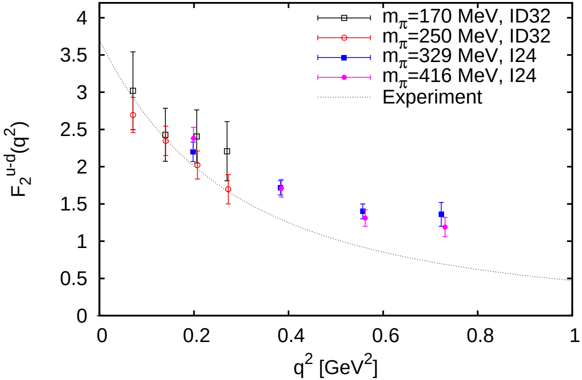
- Large volume $\rightarrow$ small q^2
- Results for two masses almost indistinguishable.

Comparison with Previous DWF Calculations



- Mild pion mass dependence
- Translates into mild mass dependence for the radii.

Similarly for $F_2^{u-d}(q^2)$

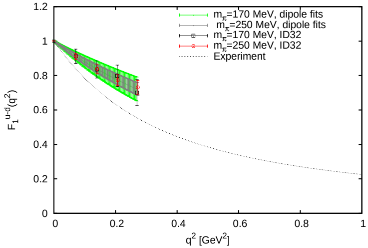


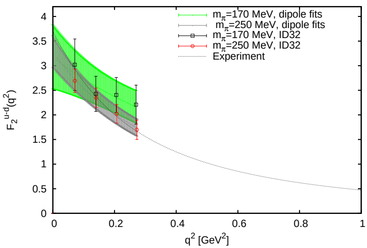
- Mild pion mass dependence
- Very noisy for lighter masses

Dirac and Pauli Radii

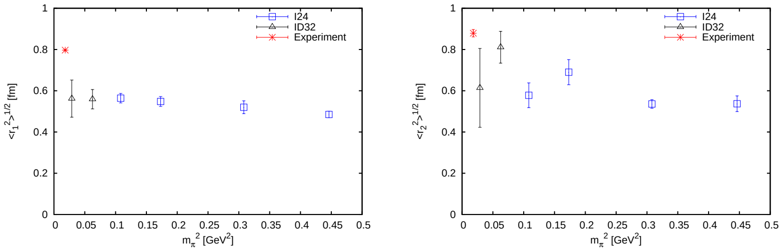
- Mean-squared radii are determined from dipole fits to the form factors:

$$F_i(q^2) = \frac{F_i(0)}{\left(1 + q^2/M_i^2\right)^2} \qquad \langle r_i^2 \rangle = \frac{12}{M_i^2}$$





Dirac and Pauli Radii



- $\langle r_1^2 \rangle^{1/2}$ undershoots the experiment by 25%.
- $\langle r_2^2 \rangle^{1/2}$ is approaching the experiment.
- For $m_\pi = 170$ MeV, we may need to worry about finite volume effects.

- Statistical errors are substantial for the ID32 data points.
- Necessary to improve the statistics significantly.

Low-Mode Averaging (LMA)

- Good for low-mode-dominant observables.
- Use low eigenmodes to approximate the observable.

$$O = O_l + O_{rest}$$

- Can improve statistics by averaging over covariant symmetry transformations, e.g., lattice translation g.

$$O = \frac{1}{N_g} \sum_g O_l^g + O_{rest} \equiv O_{appx} + O_{rest}$$

- Correct for the bias by computing O regularly (but less frequently), and

$$O_{rest} = O - O_{appx}.$$

  Cheap with low-mode deflation.

For details, see poster by Eigo Shintani.

All-Mode Averaging (AMA)

- Necessary for observables with significant high-mode contributions.
- For each g transformation, use sloppy CG (loose stopping condition, $O(10^{-3})$) to correct for the bias from the low modes.

$$O_{appx} = \frac{1}{N_g} \sum_g \left(O_l^g + O_h^g\right),$$
$$O_h^g = O_{sloppy}^g - O_l^g.$$

- Again, correct for the bias by computing O regularly (but less frequently), and

$$O_{rest} = O - O_{appx}.$$

  Cheap with low-mode deflation.

For details, see poster by Eigo Shintani.

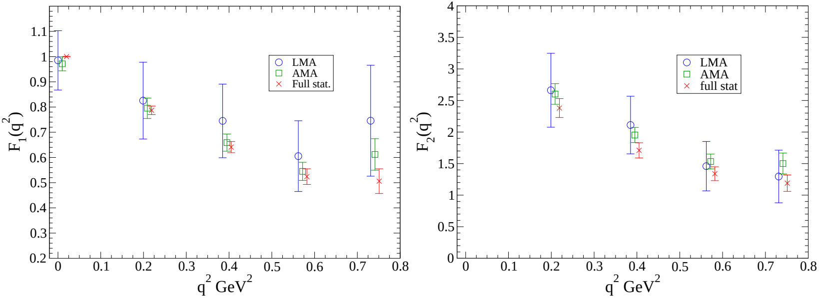
Tests on $24^3 \times 64$ Lattices

- $24^3 \times 64$ lattices, $N_f = 2+1$ DWF, $a^{-1} \approx 1.73$ GeV
- $am_l = 0.01 \rightarrow m_\pi \approx 420$ MeV
- # of configurations = 80.
- LMA: 180 low eigenmodes, $N_g = 32$ translations ($2^3 \times 4$)
- AMA: Sloppy CG with stop. cond. 0.003.
  (further speedup with low-mode deflation)
- Full calculations as in Yamazaki et al., PRD79, 114505 (2009):
  # of configurations $= 356$, with 4 sources / config.
- Cost in units of full propagators:

LMA	80 props
AMA	138 props
full stat.	1424 props

Comparison of LMA, AMA and Original

$24^3 \times 64$, $am_l = 0.01$ [$m_\pi \approx 420$ MeV]



Points are shifted for clarity.

[Eigo Shintani]

- LMA is not enough to reduce the errors → high-mode contributions are important.
- Errors from AMA comparable to “full stat.”, but with 1/10 the cost.

Conclusions

- It is pricey to go to lighter pion masses with a sufficiently large source-sink separations.
- Current results for the nucleon isovector vector form factors and their associated radii suffer from large statistical errors.
- Improved error reduction techniques are essential.

Plans

- Calculations with AMA are underway.
  $\rightarrow$ Expect to reduce the errors by a factor of 5.
- AMA makes it easier to change the source-sink separations.
  $\rightarrow$ multiple source-sink separations.
- Longer term:
  $\rightarrow$ Bigger volumes. Continuum extrapolations.

Backup Slides

Cost (in the case of m=0.01)

Use of unit of quark propagator “prop” in full CG w/o deflation

- Case of full statistics   Yamazaki et al., PRD79, 114505 (2009)

  In N_{conf} = 356, N_{mes}=4,

  Total : $356 \times 4 = 1424$ prop

- Case of AMA w/o deflation

  Since calculation of O^{appx} need 1/50 prop, then in N_{conf}=81, N'_{mes}=32

  Total : $80 + 80 \times 32/50 = 131$ prop $\Rightarrow$ 10 times fast

- Case of AMA w/ deflation

  When using 180 eigenmode, calculation of O^{appx} need 1/80 prop,

  but in this case the calculation of lowmode is ~1 prop/configs.

  Deflated CG makes reduction of full CG to 1/3 prop, then

  Total : $80/3 + 80 \times 32/80 + 80 = 138$ prop $\Rightarrow$ 10 times fast

  Note that stored eigehmode is useful for other works.

20

[slide from E. Shintani’s poster]