Nucleon structure with pion mass down to 150 MeV

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Axial charge g_A

$$\langle p(p)|\bar{u}\gamma^\mu\gamma_5 d|n(p)\rangle = g_A \bar{u}_p(p)\gamma^\mu\gamma_5 u_n(p)$$

Benchmark nucleon structure observable for Lattice QCD:

- Forward matrix element.
- Isovector quantity (no disconnected diagrams).
- Well-measured from β decay of polarized neutrons.
  PDG: $g_A/g_V = 1.2701(25)$

Scalar and tensor charges, g_S and g_T

$$\langle p(p)|\bar{u}d|n(p)\rangle = g_S \bar{u}_p(p) u_n(p)$$
$$\langle p(p)|\bar{u}\sigma^{\mu\nu}d|n(p)\rangle = g_T \bar{u}_p(p)\sigma^{\mu\nu} u_n(p)$$

Recent interest because they are needed to know leading contributions to neutron β decay from BSM physics:
T. Bhattacharya *et al.*, Phys. Rev. D **85**, 054512 (2012) [1110.6448]

Dirac radius

Nucleon Dirac and Pauli form-factors:

$$\langle p', s' | \bar{q}\gamma^\mu q | p, s\rangle = \bar{u}(p', s') \left(\gamma^\mu F_1^q(Q^2) + i\sigma^{\mu\nu}\frac{\Delta_\nu}{2m_N} F_2^q(Q^2)\right) u(p, s),$$

where $\Delta = p' - p$, $Q^2 = -\Delta^2$.
Dirac and Pauli radii defined via slope at $Q^2 = 0$:

$$F_{1,2}(Q^2) = F_{1,2}(0)\left(1 - \tfrac{1}{6}(r_{1,2})^2 Q^2 + O(Q^4)\right).$$

Proton charge radius has 5σ discrepancy between measurements from e–p interactions and from Lamb shift in muonic Hydrogen.

BMW action

- Tree-level clover-improved Wilson fermions coupled to double-HEX-smeared gauge fields.
- Pion mass ranging from 150 MeV to 340 MeV.
- Mostly coarse lattices with $a = 0.116$ fm;
  one fine lattice with $a = 0.09$ fm.
- No disconnected diagrams, so we focus on isovector observables.
- Three source-sink separations to better handle excited states:
  $T \in \{0.93, 1.16, 1.39\}$ fm.

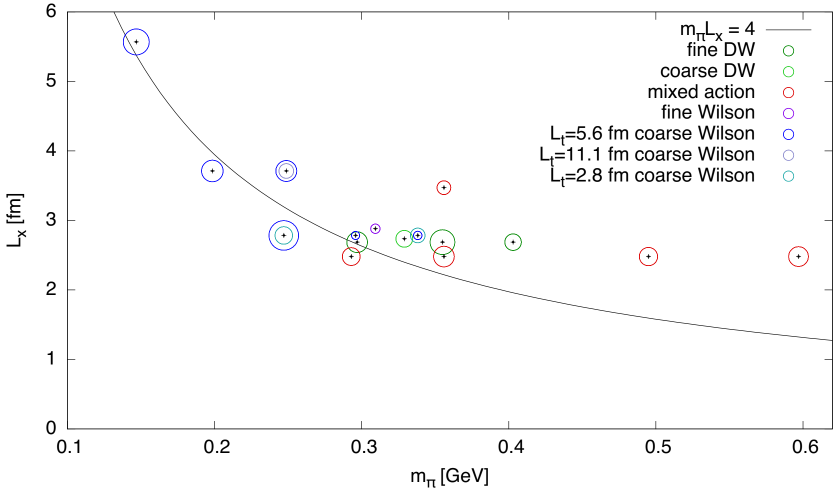
Older calculations

Scalar/tensor charge data were available but not extracted from previous calculations done by LHPC:

- Mixed-action: domain-wall valence quarks on lattices with Asqtad staggered sea quarks (MILC): 300 MeV $\leq m_\pi \leq$ 600 MeV, $a = 0.124$ fm.
- Unitary domain wall (RBC/UKQCD): 300 MeV $\leq m_\pi \leq$ 400 MeV, three fine $a = 0.084$ fm and one coarse $a = 0.114$ fm ensemble.

These were done using source-sink separations 1.0 fm $< T <$ 1.2 fm, so the intermediate T used on the Wilson ensembles is similar.

Ensemble summary



Areas of circles scale with number of measurements: largest is 10,000.

Renormalization

- Renormalize scalar and tensor bilinears in $\overline{MS}$ scheme at $\mu = 2$ GeV.
- For Wilson and unitary domain wall quarks: use nonperturbative Rome-Southampton method, matching via a momentum-subtraction scheme.
  - Z_S already computed by RBC and BMW collaborations for quark mass renormalization.
  - Z_T on coarse domain wall computed by RBC; we did new calculations for remaining ensembles.
- For mixed-action ensembles: use perturbative renormalization,

$$Z_{\mathcal{O}} = \frac{Z_{\mathcal{O}}^{\text{pert}}}{Z_A^{\text{pert}}} Z_A,$$

with nonperturbative calculation of Z_A.

Contributions to systematic error

- Quark masses: smallest pion mass is 150 MeV, so $m_\pi \to 135$ MeV chiral extrapolation is under control.
- Finite volume: effects expected to be small with $m_\pi L \approx 4$, but small range of $m_\pi L$ makes careful $L \to \infty$ extrapolation unlikely.
- Discretization: different actions and different lattice spacings give consistency check but no $a \to 0$ extrapolation.
- Excited states: use of multiple source-sink separations allows for clear identification of observables where excited states are a problem. We can also experiment with different analysis methods ...

Systematic error: excited states

Usual approach for extracting matrix elements (forward case):

$$C_{2\text{pt}}(t) = \langle N(t)\bar{N}(0)\rangle$$
$$C_{3\text{pt}}(T,\tau) = \langle N(T)\mathcal{O}(\tau)\bar{N}(0)\rangle$$

Take ratio:

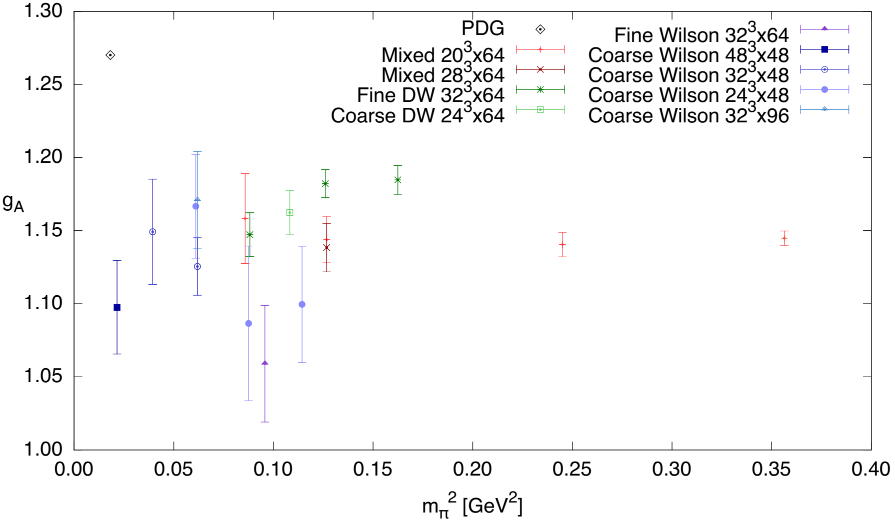
$$\begin{aligned} R(T,\tau) &= C_{3\text{pt}}(T,\tau)/C_{2\text{pt}}(T) \\ &= c_{00} + c_{10}e^{-\Delta E\tau} + c_{01}e^{-\Delta E(T-\tau)} + c_{11}e^{-\Delta ET} + \ldots, \end{aligned}$$

where c_{00} is the desired ground-state matrix element. Averaging a fixed number of points around $\tau = T/2$ yields asymptotic errors that fall off as $e^{-\Delta ET/2}$. Alternatively, use summation method: compute

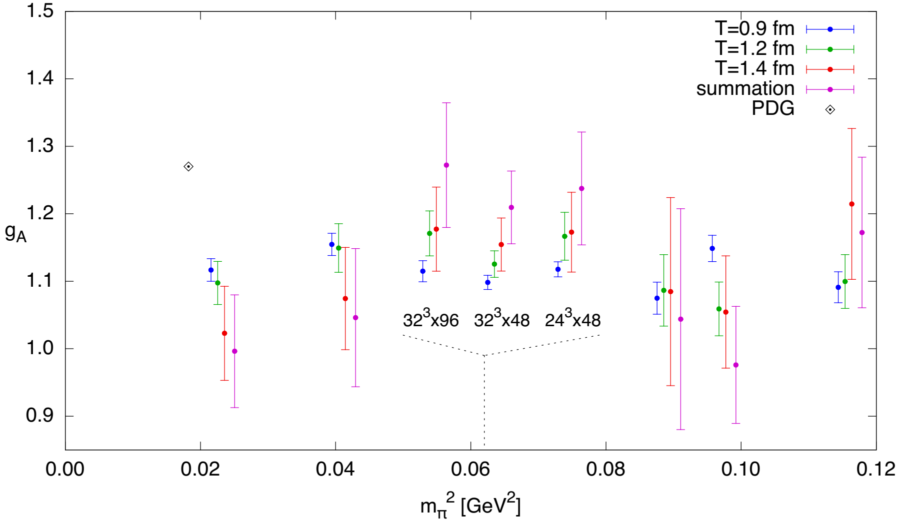
$$S(T) = \sum_\tau R(T,\tau) = b + c_{00}T + dTe^{-\Delta ET} + \ldots,$$

and then find its slope, which gives c_{00} with errors that fall off as $Te^{-\Delta ET}$.

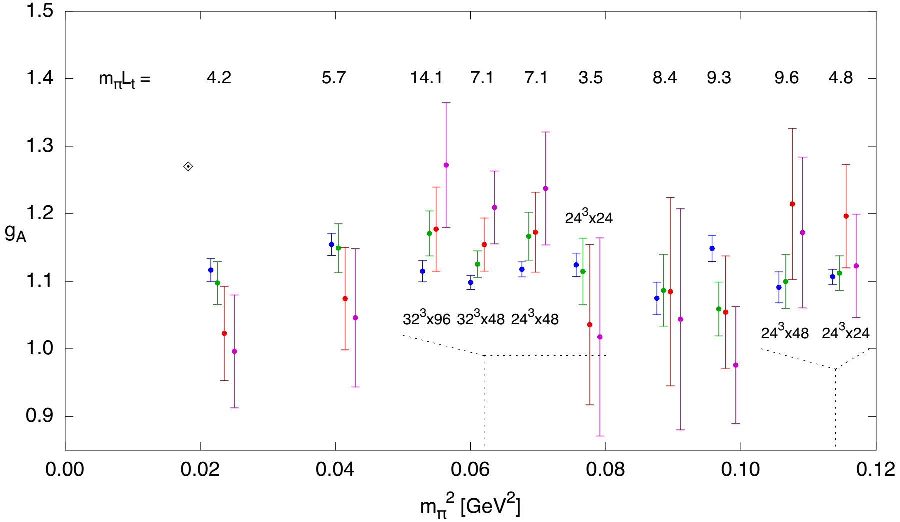
Axial charge



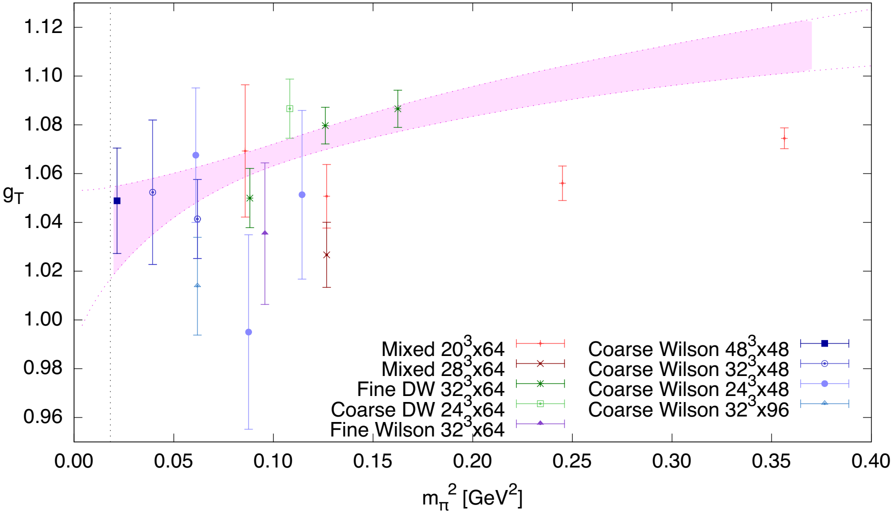
Axial charge: excited states?



Axial charge: thermal effects?

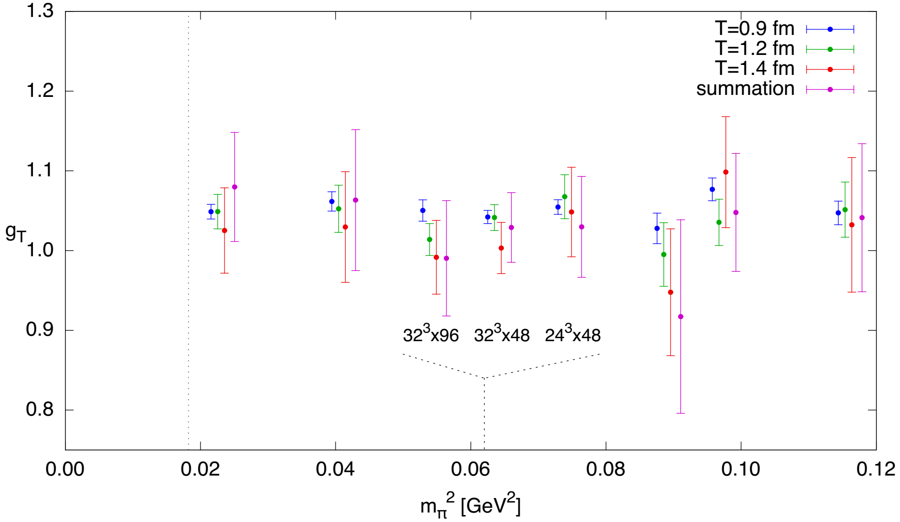


Tensor charge

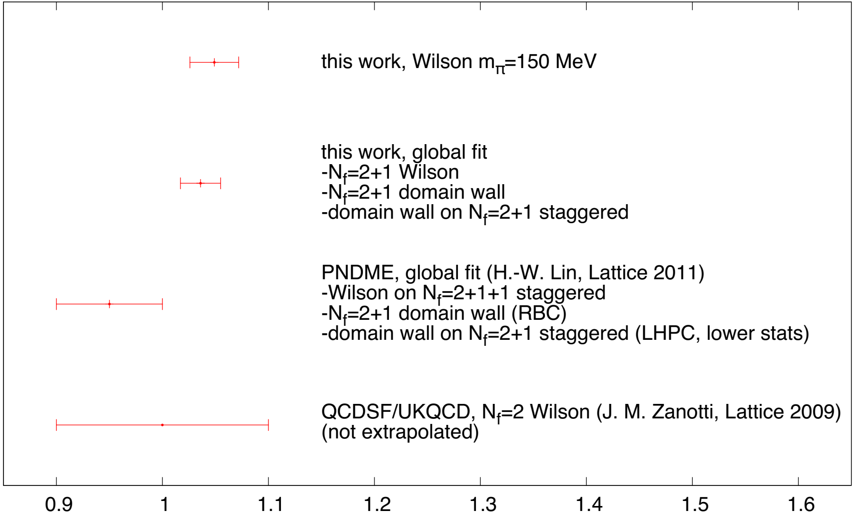


Three-parameter chiral fit to all data.

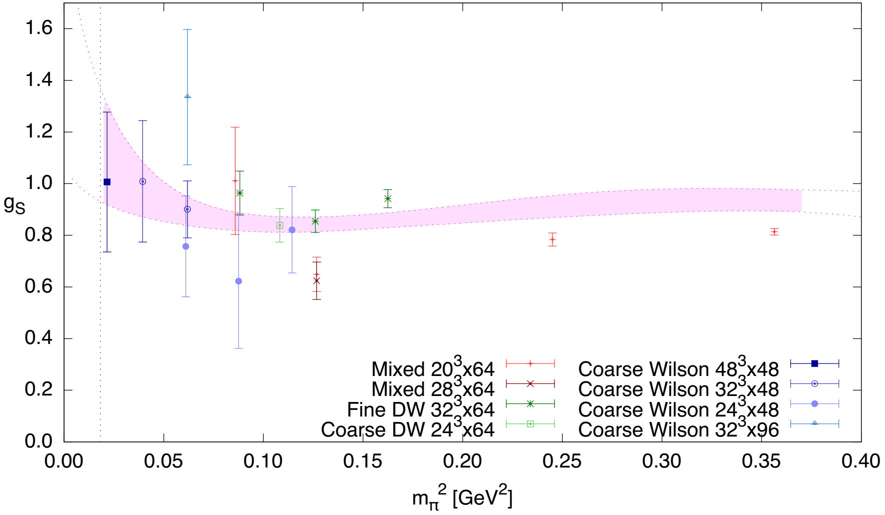
Tensor charge: excited states?



Tensor charge: other collaborations

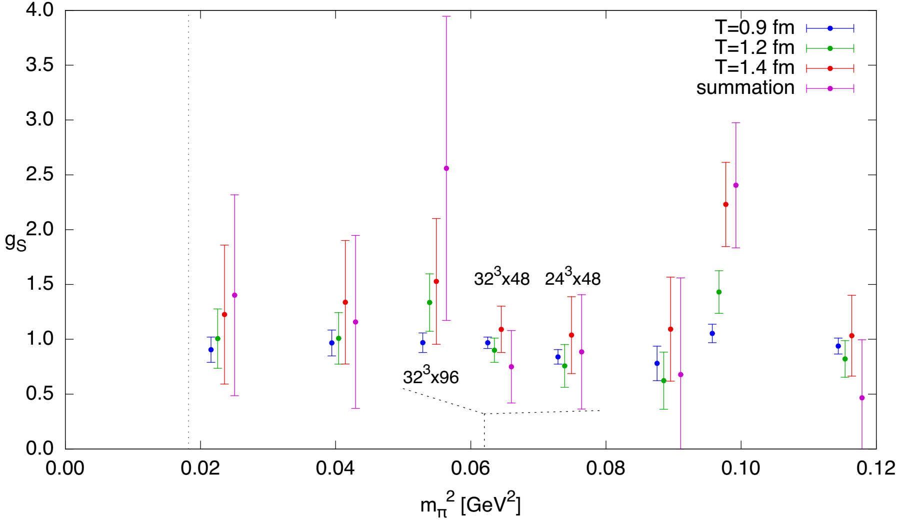


Scalar charge



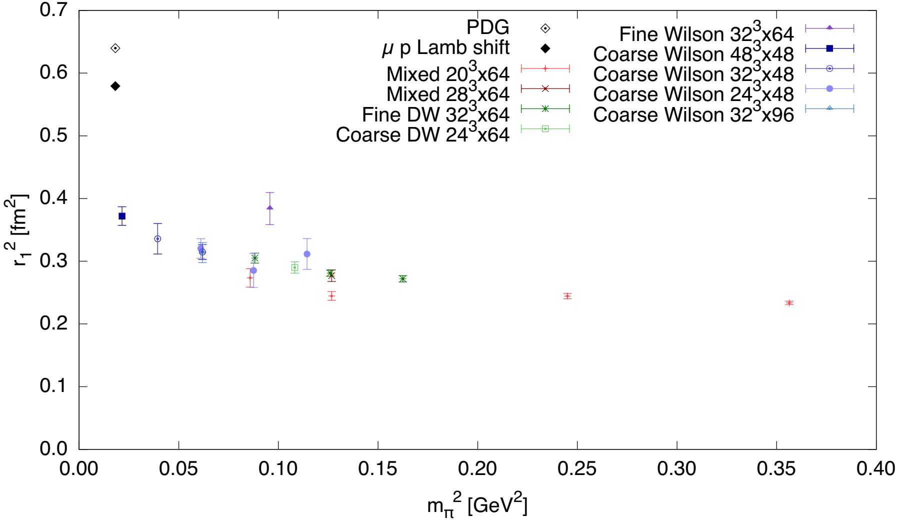
Four-parameter chiral fit to all shown data.

Scalar charge: excited states?



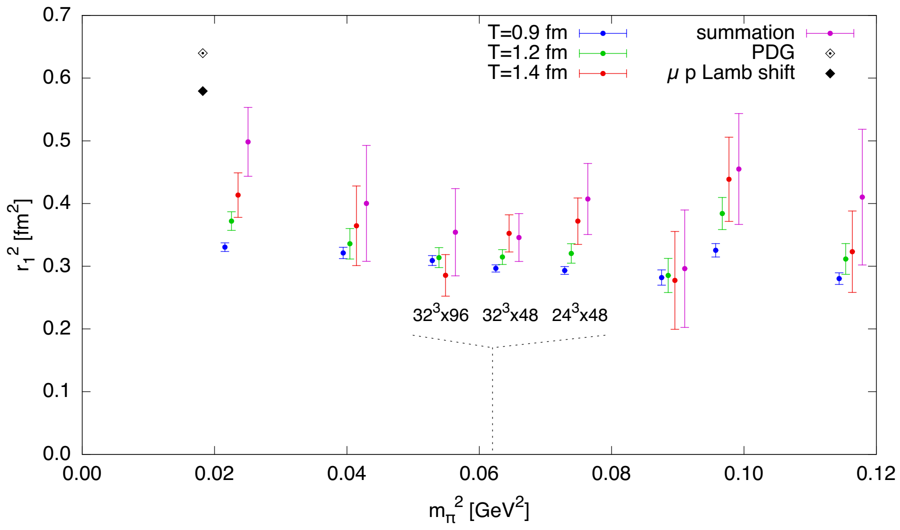
$m_\pi^2 \approx 0.1$ GeV^2 ensemble omitted from previous plot.

Isovector Dirac radius



From dipole fits to $F_1(Q^2)$.

Isovector Dirac radius: excited-state effects

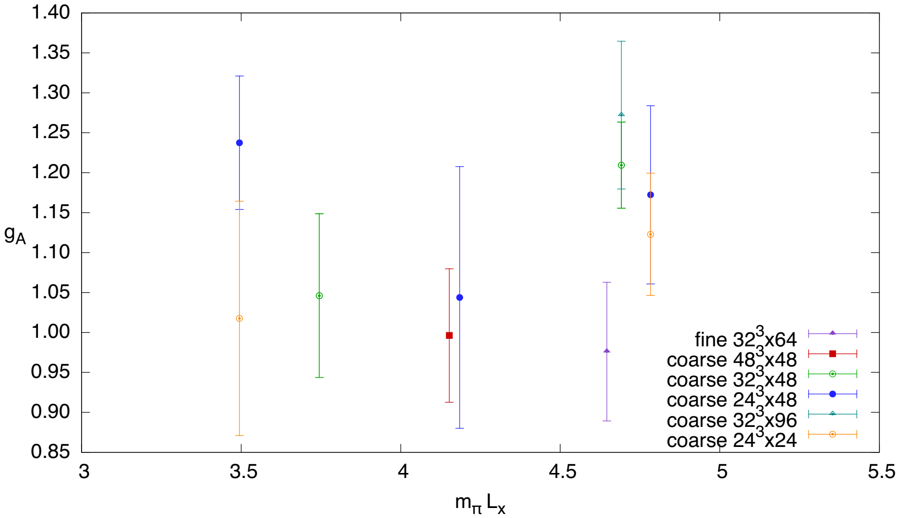


From dipole fits to $F_1(Q^2)$ for $0 \leq Q^2 < 0.5$ GeV^2.

Conclusions

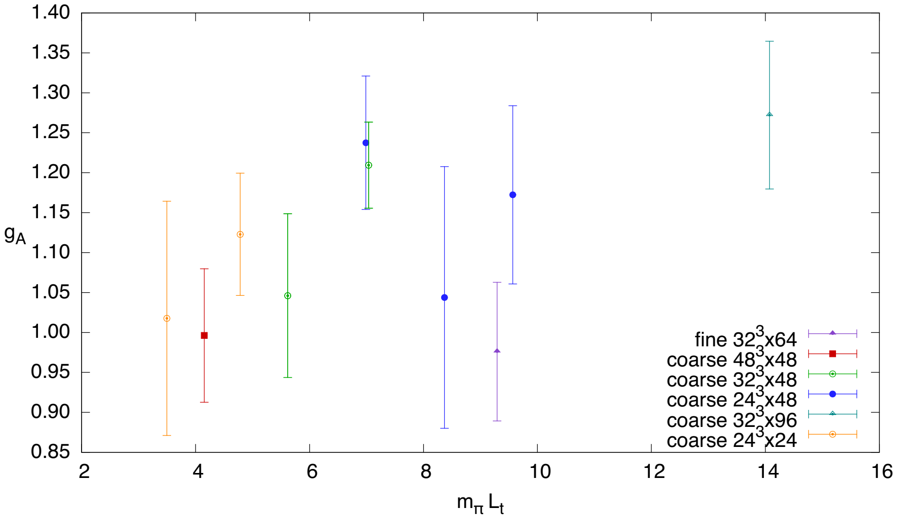
- Using a small pion mass close to the physical point is important for reducing errors from chiral extrapolation, but is not sufficient for agreement with experiment.
- Axial charge and isovector Dirac radius data are inconsistent with experiment, but excited-state effects are sufficiently large that establishing good control over them as well as volume effects may lead to reasonable agreement.
- Multiple source-sink separations including T greater than 1.4 fm, or alternative analysis methods like summation, are needed for good control over excited states.
- Scalar and tensor charges will provide useful input for new physics searches, but confident predictions can't be made before accurate postdictions of benchmark observables.

g_A versus $m_\pi L_x$



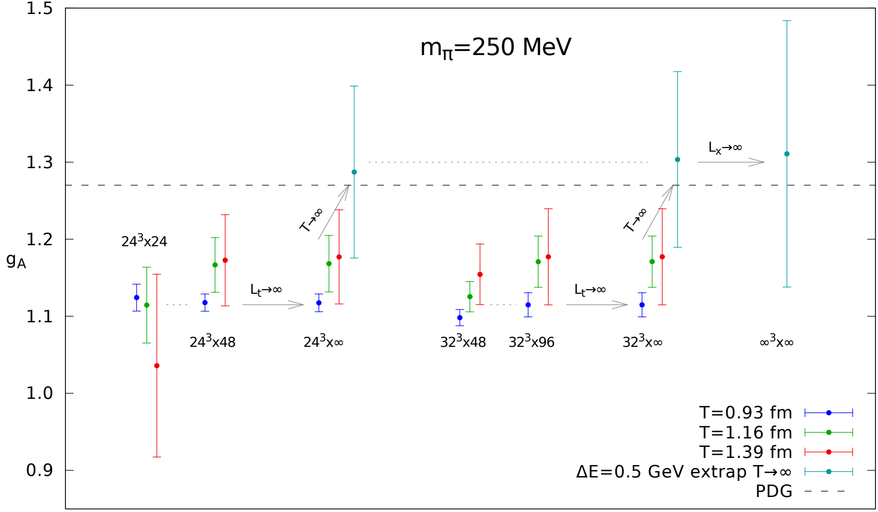
(summation values)

g_A versus $m_\pi L_t$

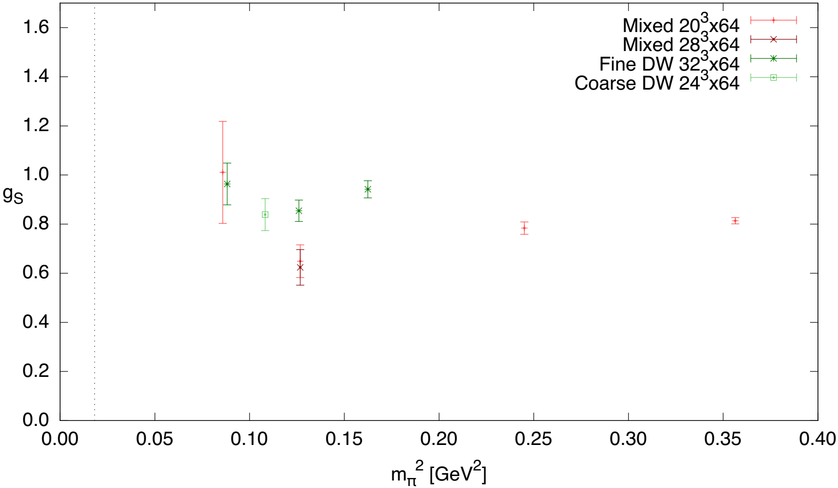


(summation values)

g_A: extrapolate to ground state in infinite volume

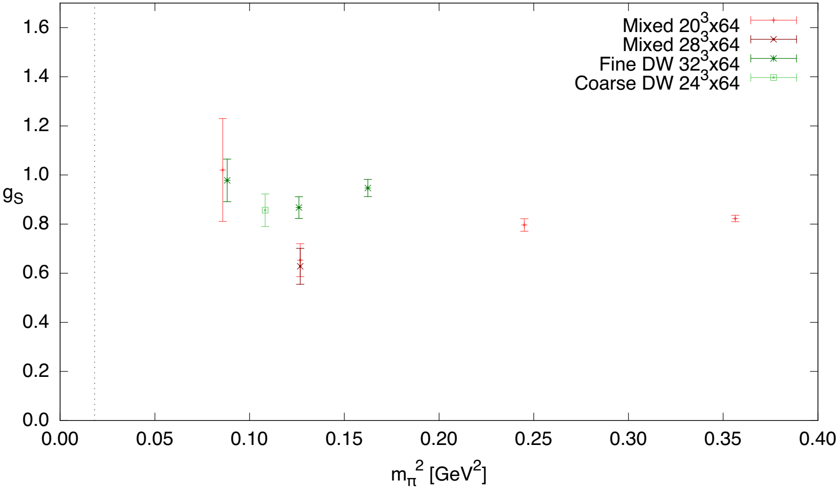


g_S renormalization: using Z_S



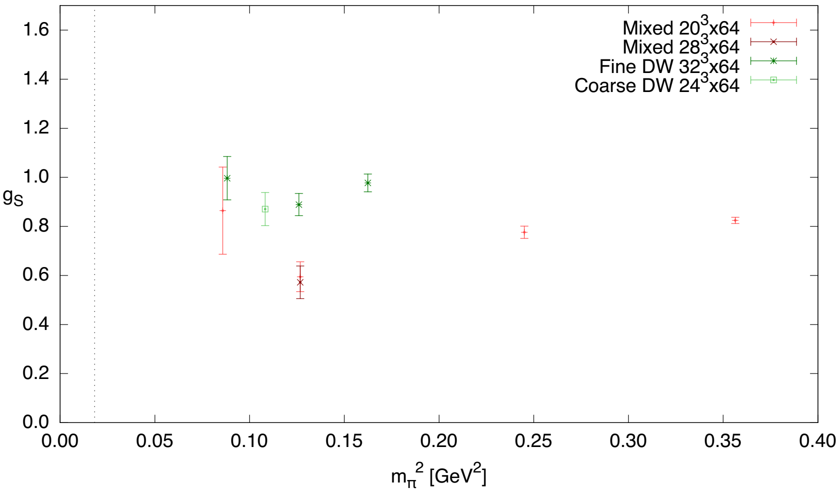
plotted: $Z_S g_S^{\text{bare}}$

g_S renormalization: using $m_s - m_{ud}$



plotted: $\frac{(m_s - m_{ud}) g_S}{m_s^{\text{phys}} - m_{ud}^{\text{phys}}} \times \frac{m_{K,\text{phys}}^2 - m_{\pi,\text{phys}}^2}{m_K^2 - m_\pi^2}$

g_S renormalization: using $m_{ud} + m_{\text{res}}$



plotted: $\frac{(m_{ud}+m_{\text{res}})g_S}{m_{ud}^{\text{phys}}} \times \frac{m^2_{\pi,\text{phys}}}{m^2_\pi}$