Exploring QCD Thermodynamics Using Möbius Fermions

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General Möbius Action

$$D_{\rm DW}(m) =$$

$$\begin{pmatrix} D_+^1 & D_-^1 P_- & & & & -mD_-^1 P_+ \\ D_-^2 P_+ & D_+^2 & D_-^2 P_- & & & \\ & D_-^3 P_+ & \cdots & \cdots & & \\ & & \cdots & \cdots & \cdots & \\ & & & \cdots & \cdots & D_-^{2N-1} P_- \\ -mD_-^{2N} P_- & & & & D_-^{2N} P_+ & D_+^{2N} \end{pmatrix}$$

- $D_+^i = 1 + b_i D_W$
- $D_-^i = -1 + c_i D_W$
- plain DWF is a special case: $b_i = 1$, $c_i = 0$ for all i.

Reference: R.C. Brower, H. Neff and K. Orginos, Nucl. Phys. B(Proc. Suppl.) 153(2006) 191-198.

4D Effective Overlap Operator

$$D_{OV}(m) = \frac{1}{2}\left(1 + m + (1-m)\gamma_5 \frac{(\lambda x+1)^{2N} - (\lambda x-1)^{2N}}{(\lambda x+1)^{2N} + (\lambda x+1)^{2N}}\right)$$

with

$$\begin{aligned} \lambda =& b + c \\ x =& \gamma_5 D_W (2 + (b-c) D_W)^{-1} \end{aligned}$$

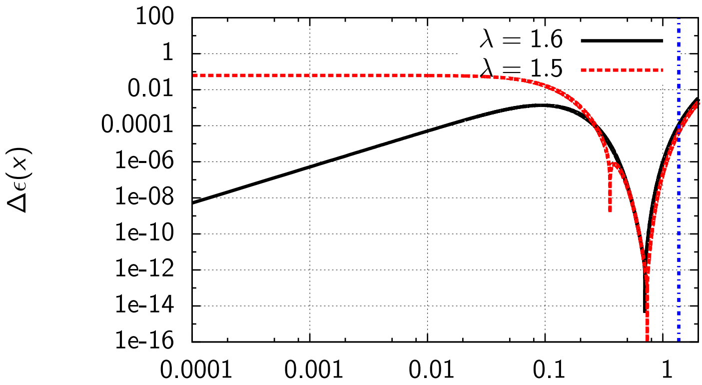
To have $(b, c, 2N)$ Möbius approximate $(b', c', 2N')$ Möbius, roughly we need

$$\begin{aligned} 2N(b+c) \approx& 2N'(b'+c') \\ b - c =& b' - c' \end{aligned}$$

4D Effective Overlap Operator

$$\epsilon_1(x) = \frac{(1+x)^{16} - (1-x)^{16}}{(1+x)^{16} + (1-x)^{16}}, \quad \epsilon_2(x) = \frac{(1+\lambda x)^{10} - (1-\lambda x)^{10}}{(1+\lambda x)^{10} + (1-\lambda x)^{10}}$$

$$\Delta\epsilon(x) = \left| \frac{\epsilon_1(x) - \epsilon_2(x)}{\epsilon_1(x)} \right|$$



Möbius Residual Mass

Starting from the path integral,

$$Z = \int \mathcal{D}U \mathcal{D}\overline{\psi} \mathcal{D}\psi \exp\left(-S_G(U) - \sum_{xs;ys'} \overline{\psi}_{xs} D_{xs;ys'} \psi_{ys'} \right),$$

do the following variation

$$\begin{cases} \psi_{xs} & \longrightarrow & \exp\left(\varepsilon^a_{xs}\lambda^a\right)\psi_{xs} \\ \overline{\psi}_{xs} & \longrightarrow & \overline{\psi}_{xs}\exp\left(-\varepsilon^a_{xs}\lambda^a\right) \end{cases},$$

... and we have a conserved 5D current from $\delta Z = 0$

$$-\sum_{ys'} \langle \overline{\psi}_{xs} \lambda^a D_{xs;ys'} \psi_{ys'} \rangle + \sum_{ys'} \langle \overline{\psi}_{ys'} D_{ys';xs} \lambda^a \psi_{xs} \rangle = 0.$$

Möbius Residual Mass

$$\begin{aligned}\Delta_\mu V_\mu^a(x) =& 0\\ \Delta_\mu A_\mu^a(x) =& 2mJ_5^a(x) + 2J_{5q}^a(x)\end{aligned}$$

$$J_5^a(x) = \sum_y \left(\overline{\psi}_1(y)\lambda^a D_-^1(y,x)P_+\psi_{2N}(x) - \overline{\psi}_{2N}(y)\lambda^a D_-^{2N}(y,x)P_-\psi_1(x)\right)$$

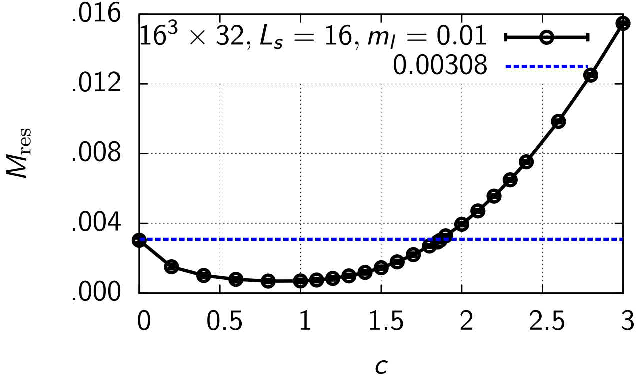
$$J_{5q}^a(x) = \sum_y \left(\overline{\psi}_{N+1}(y)\lambda^a D_-^{N+1}(y,x)P_+\psi_N(x) - \overline{\psi}_N(y)\lambda^a D_-^N(y,x)P_-\psi_{N+1}(x)\right)$$

Implication on switching from plain DWF to Möbius action

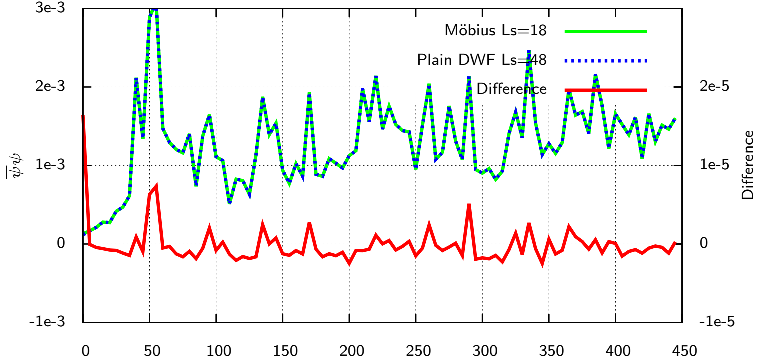
- Replace D_{DW}^{-1} by $D_{\mathrm{DW}}^{-1}D_-$ ($D_- = \mathrm{diag}\left\{D_-^1, D_-^2, \cdots, D_-^{2N}\right\}$).
- $\gamma_5 R_5 D_-^{-1} D_{\mathrm{DW}}$ is Hermitian (not $\gamma_5 R_5 D_{\mathrm{DW}}$) if $b_i = b_{2N+1-i}$ and $c_i = c_{2N+1-i}$ (i.e., the coefficients are symmetric w.r.t. s).

Möbius Residual Mass

$L_s = 10$ Möbius action measured on $L_s = 16$ DWF ensemble. All $b_i = c + 1$, $c_i = c$.

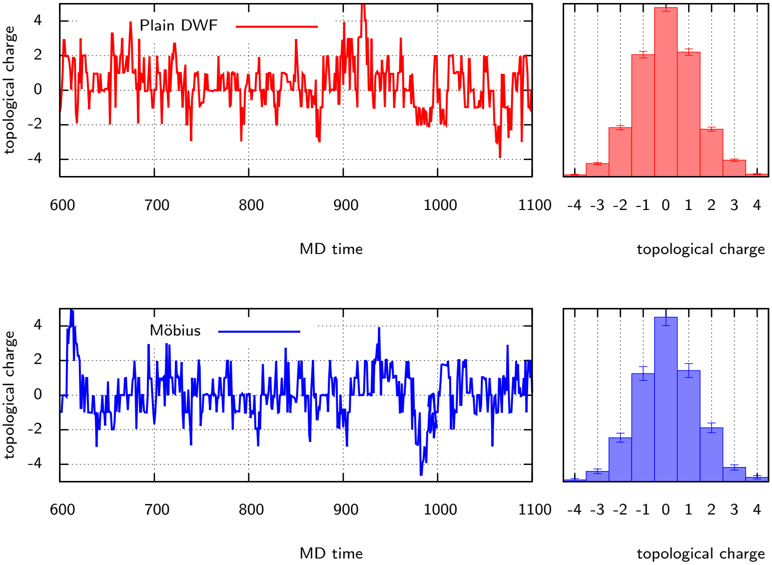


$\langle \overline{\psi}\psi \rangle$ on Dynamic Möbius Ensembles

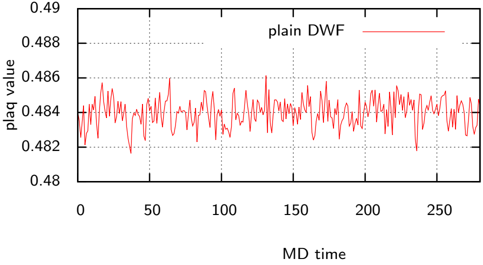


- $16^3 \times 8$, $L_s = 48$ ensemble used in QCD thermodynamics. Use $L_s = 18$ Möbius to approximate it. $b = 1.832$, $c = 0.832$.
- The relative difference between Möbius and plain DWF in $\langle \overline{\psi}\psi \rangle$ is around 0.1% in this case (yes, the difference is there).

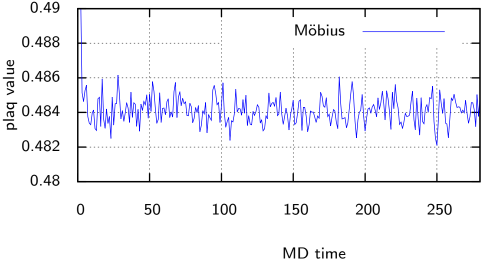
Topological charge on Dynamic Möbius Ensembles



Plaquette on Dynamic Möbius Ensembles



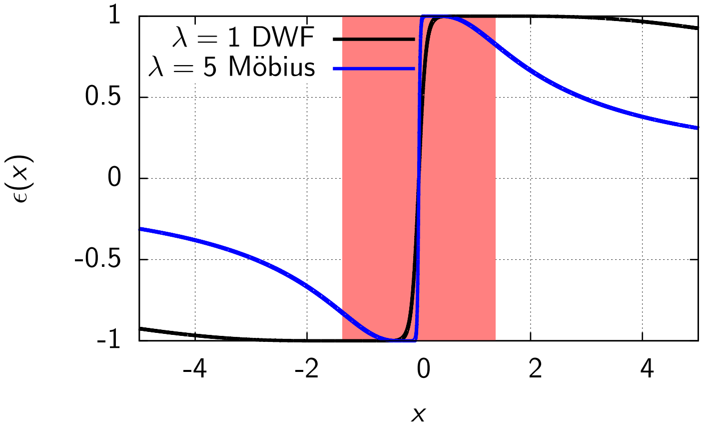
0.484065(24) from 7000 trajectories.



0.484048(50) from 900 trajectories.

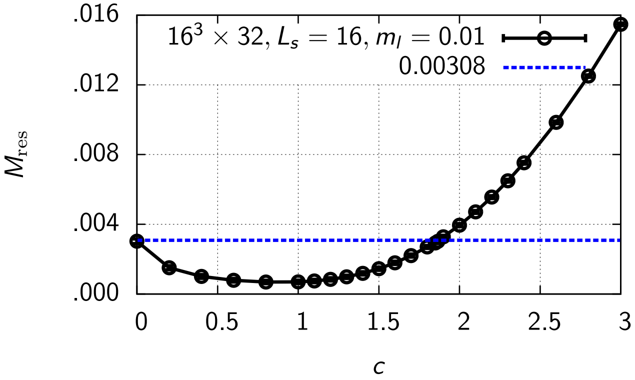
What if c is too large ...

$$\epsilon(x) = \frac{(\lambda x + 1)^L - (\lambda x - 1)^L}{(\lambda x + 1)^L + (\lambda x + 1)^L}$$



Reference: R.C. Brower, H. Neff and K. Orginos, Nucl. Phys. B(Proc. Suppl.) 153(2006) 191-198.

What if c is too large ...



Compare 2 streams of evolutions at $c = 0$ and $c = 1.875$.

c	$n\Delta t$	Int. type	$\|\Delta H\|$	#traj	time/traj(s)
0	8*0.125	Omelyan	0.27 ± 0.01	320	310
1.875	10*0.1	Omelyan	8.15 ± 1.38	205	800
1.875	20*0.05	ForceGrad	0.16 ± 0.03	215	2000

outlook: parameters for $64^3 \times 8$ QCD thermodynamics

- Planning $64^3 \times 8$ QCD thermodynamics using DWF-like actions (HotQCD collaboration).
- Running on physical pion mass requires very small residual mass.
- Possible choices: DSDR, improved DWF-like actions (Möbius, optimal domain wall fermion, overlap, ...).
- We may use more agressive Möbius parameter settings (Zolotarev) in the future. For this project however, we decided to use the simplified case of the Möbius action (single parameter scaled Shamir).

outlook: parameters for $64^3 \times 8$ QCD thermodynamics

β	$T(\mathrm{MeV})$	L_s	c	$L_s(2c+1)$	m_l	M_{res}
1.633	141	24	1.5	96.00	0.00314	0.00187(19)
1.671	150	20	1.098	63.92	0.0	0.00095(38)
1.671	150	20	1.098	63.92	0.00173	0.00163(20)
1.707	160	16	0.995	47.84	0.00069	0.00148(31)
1.740	169	16	0.995	47.84	0.0	0.00078(41)
1.740	169	16	0.624	35.97	0.0	0.00207(56)
1.771	177	14	0.637	31.84	0.0	0.00119(35)
1.771	177	12	0.6624	27.90	0.0	0.00186(29)
1.771	177	12	0.4968	23.92	0.0	0.00298(24)
1.801	187	12	0.4965	23.92	0.0	0.00169(21)
1.829	195	12	0.4965	23.92	0.0	0.00132(27)
1.829	195	12	0.4965	23.92	0.001	0.00141(17)

- Running at $L_s = 24$ instead of $L_s = 96$, $L_s = 20$ instead of $L_s = 64$.