Non-Perturbative Renormalization for Staggered Fermions

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SWME Collaboration

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Introduction [PRL 2012 SWME]

Error Budget for B_K using SU(2) SChPT fitting		
cause	Error(%)	Memo
statistics	0.6	
matching factor	4.4	$\Delta B_K^{(2)}$(U1)
discretization	1.9	diff. of constant and constrained fits
X-fits	0.33	varying Bayesian priors (S1)
Y-fits	0.07	diff. of linear and quadratic (C3)
am_l extrap	1.5	diff. of (C3) and linear extrap
am_s extrap	1.3	diff. of (C3) and linear extrap
finite volume	0.5	diff. of V=inf fit and FV fit
r_1	0.14	r_1 error propagation (C3)
f_π	0.4	132 MeV vs. 124.4 MeV

The biggest error comes from the matching factor.
NPR can reduce the matching factor error.

Acceleration of Landau gauge fixing using GPUs

- Landau gauge fixing is needed to do the non-perturbative renormalization.
- Landau gauge fixing is very slow.
- It needs about 10 days for one $28^3 \times 96$ gauge configuration using 2 CPUs.
- Using GPUs, Landau gauge fixing speed is about 80 times faster than CPUs.
- It needs only 3 hours for one $28^3 \times 96$ gauge configuration using 2 GPUs.

Mass Renormalization

- We generate staggered fermion propagators using momentum source.
- Momentum source : $h = e^{-i(\tilde{p}+\pi_B)x}\delta_{cc'}$
- $\tilde{p}$ is the momentum in reduced Brillouin zone.

$$p \in \left(-\frac{\pi}{a}, \frac{\pi}{a}\right]^4, \qquad \tilde{p} \in \left(-\frac{\pi}{2a}, \frac{\pi}{2a}\right]^4, \qquad p = \tilde{p} + \pi_B$$

  where $\pi_B (\equiv \frac{\pi}{a} B)$ is cut-off momentum in hypercube.
- a : lattice spacing.
- B : vector in hypercube. Each element is 0 or 1
- c, c' : color index.
- We can obtain the propagator in momentum space after Fourier transform.

The inverse free propagator is

$$[\widetilde{S}^f(\tilde{p})]^{-1;free}_{AB;cc'} = [\sum_\mu \frac{i}{a}\sin(\tilde{p}_\mu a)\overline{\overline{(\gamma_\mu \otimes 1)}}_{AB} + m_0^f \overline{\overline{(1 \otimes 1)}}_{AB}]_{cc'}$$

where m_0^f is bare mass.
The inverse bare propagator is

$$[\widetilde{S}^f(\tilde{p})]^{-1}_{AB;cc'} = [(1+\Sigma_S)m_0^f \overline{\overline{(1 \otimes 1)}}_{AB} + (1+\Sigma_V)\sum_\mu \sin(\tilde{p}_\mu a)\overline{\overline{(\gamma_\mu \otimes 1)}}_{AB}$$
$$+ \Sigma_T m_0^f \sum_{\mu\neq\nu}\sin(\tilde{p}_\mu a)(\sin(\tilde{p}_\nu a))^3\overline{\overline{(\gamma_{\mu\nu} \otimes 1)}}_{AB}$$
$$+ \Sigma_A \sum_{\mu\neq\nu\neq\rho}\sin(\tilde{p}_\mu a)(\sin(\tilde{p}_\nu a))^3(\sin(\tilde{p}_\rho a))^5\overline{\overline{(\gamma_{\mu\nu\rho} \otimes 1)}}_{AB}$$
$$+\Sigma_P m_0^f \sum_{\mu\neq\nu\neq\rho\neq\sigma}\sin(\tilde{p}_\mu a)(\sin(\tilde{p}_\nu a))^3(\sin(\tilde{p}_\rho a))^5(\sin(\tilde{p}_\sigma a))^7\overline{\overline{(\gamma_{\mu\nu\rho\sigma} \otimes 1)}}_{AB}]_{cc'}$$

[NPR for Improved Staggered Bilinears, Lytle and Sharpe]

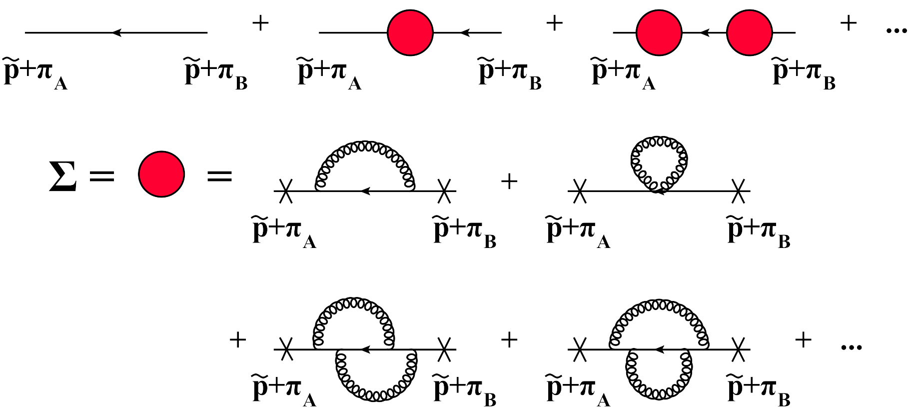


Figure : self-energy diagrams

The renormalized quark propagator is

$$\widetilde{S}_R^f(\tilde{p}) = Z_q \widetilde{S}_0^f(\tilde{p})$$

and the renormalized quark mass is

$$m_R = Z_m m_0$$

The RI-MOM scheme prescription is

$$\widetilde{S}_R^f(\tilde{p}) = \widetilde{S}_{tree}^f(\tilde{p})$$

The renormalized propagator is

$$\widetilde{S}_R^f(\tilde{p}) = \frac{1}{\sum_\mu \frac{i}{a}\sin(\tilde{p}_\mu a)\overline{\overline{(\gamma_\mu \otimes 1)}}_{AB} + m_R^f \overline{\overline{(1 \otimes 1)}}_{AB}}$$

$$= \frac{Z_q}{(1+\Sigma_V)}\left(\frac{1}{\sum_\mu \frac{i}{a}\sin(\tilde{p}_\mu a)\overline{\overline{(\gamma_\mu \otimes 1)}}_{AB} + \frac{(1+\Sigma_S)}{(1+\Sigma_V)}\frac{m_R^f}{Z_m}\overline{\overline{(1 \otimes 1)}}_{AB} + \ldots}\right)$$

Therefore

$$Z_q = (1+\Sigma_V)$$

$$Z_m = \frac{(1+\Sigma_S)}{(1+\Sigma_V)}$$

If we use the vector projection operator $\hat{\mathbb{P}}^w = \overline{\overline{(\gamma_\nu \otimes 1)}}_{BA}\delta_{c'c}$ and sum over A and B, trace on color indices.

$$\sum_{A,B} Tr[[\widetilde{S}^f(\tilde{p})]^{-1}_{AB;cc'}\hat{\mathbb{P}}^w_{BA;c'c}]$$
$$= 48(1+\Sigma_V)\frac{i}{a}\sin(\tilde{p}_\nu a)$$

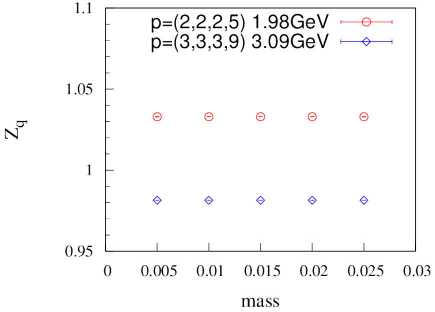
We know the values of all terms except Σ_V, so we can obtain Σ_V.

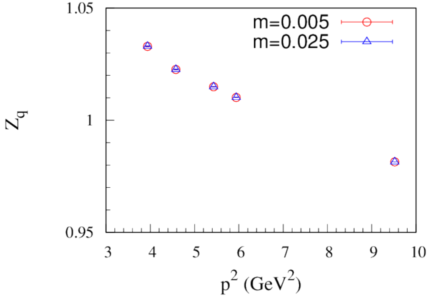
If we use the scalar projection operator $\hat{\mathbb{P}}^m = \overline{\overline{(1 \otimes 1)}}_{BA}\delta_{c'c}$, we can obtain Σ_S, which is related to Z_m.

$$\sum_{A,B} Tr[[\widetilde{S}^f(\tilde{p})]^{-1}_{AB;cc'}\hat{\mathbb{P}}^m_{BA;c'c}]$$
$$= 48(1+\Sigma_S)m_0^f$$

Wave function renormalization factor (Preliminary)

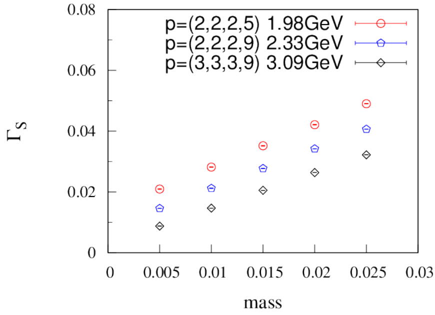
- $20^3 \times 64$ MILC coarse lattice ($am_\ell/am_s = 0.01/0.05$)
- We are using mixed action (asqtad sea + HYP valence).
- We checked that all the results are the same using Dirac and Weyl conventions.

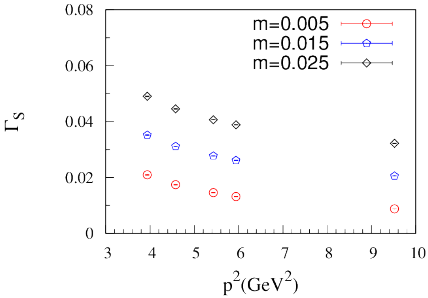




Analysis of Γ_S (Preliminary)

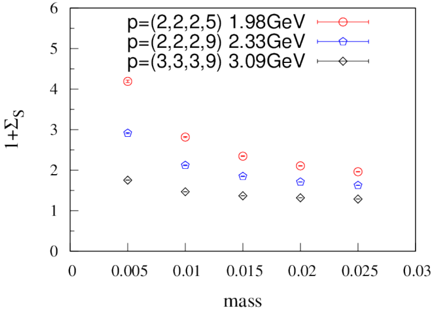
- $\Gamma_S = \frac{1}{48} Tr(S^{-1}(\tilde{p})\hat{\mathbb{P}}^m), \qquad \hat{\mathbb{P}}^m = \overline{\overline{(1 \otimes 1)}}_{BA}\delta_{c'c}$
- $\Gamma_S = Z_q(Z_m m_0 + C_1 \frac{\langle\chi\overline{\chi}\rangle}{\tilde{p}^2}) + O(\tilde{p}^2)$

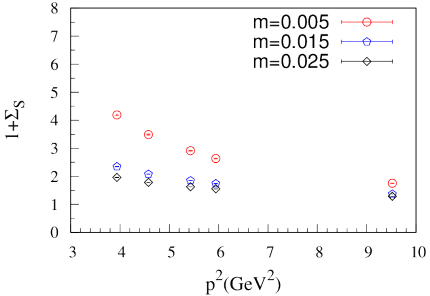




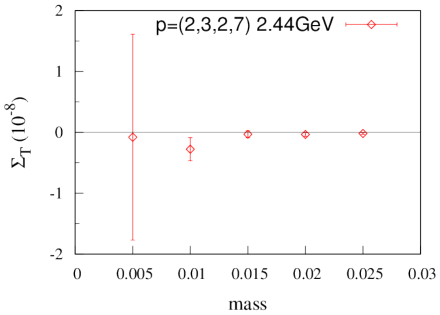
Analysis of Σ_S (Preliminary)

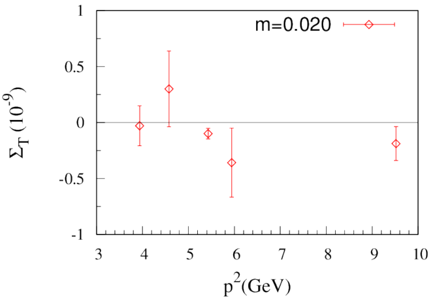
- $1 + \Sigma_S = \dfrac{\Gamma_S}{m_0}$
- We are in the middle of analysis.





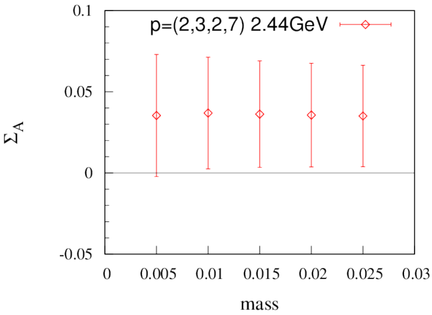
Analysis of Σ_T (Preliminary)





Analysis of Σ_A (Preliminary)

- We are working on the analysis of Σ_A.



Conclusion

- Using GPUs, we performed Landau gauge fixing over 500 gauge configurations in a week.
- We are in the middle of analysis of wave function renormalization factor Z_q, mass renormalization factor Z_m and **the matching factors of bilinear and four-fermion operators** relavent for B_K and its BSM operators.