Determination of $\Lambda^{\overline{\mathrm{MS}}}$ from the gluon and ghost propagators in Landau gauge

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<u>Outline</u>

the MM coupling
lattice data, systematic effects
conclusion

in collaboration with
K. Maltman, M. Müller-Preussker, L. von Smekal

Lattice2012, Cairns, Australia

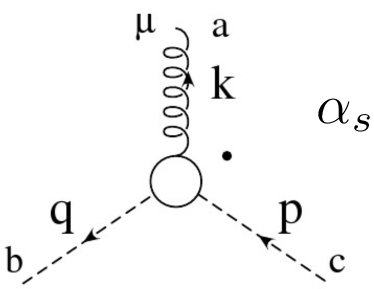
The MM coupling

[L. von Smekal et al (1995)]

$$\boxed{\alpha_s^{\mathrm{MM}}(p^2) = \frac{g_0^2}{4\pi} Z(p^2) G^2(p^2)}$$

Minimal MOM coupling

- aka ghost-gluon coupling, because it is based on ghost-ghost-gluon vertex
- defined in a particular **MOM scheme** in **general covariant** gauges
- defined beyond lattice QCD



$$\alpha_s(\mu) = \frac{Z_3 \widetilde{Z}_3^2}{\widetilde{Z}_1^2} \frac{g_0^2}{4\pi}$$

Propagators (Landau gauge)

- Ghost propagator function:

$$G^{ab}(q) = -\delta^{ab} \frac{G(q^2)}{q^2}$$

- Gluon propagator function:

$$D^{ab}_{\mu\nu}(q) = \delta^{ab} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \frac{Z(q^2)}{q^2}$$

The MM coupling

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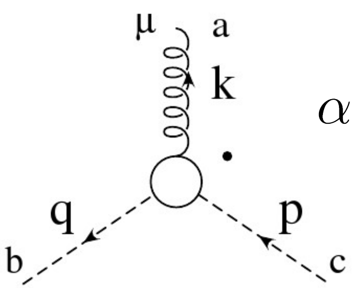

$$\alpha_s(\mu) = \frac{Z_3 \widetilde{Z}_3^2}{\widetilde{Z}_1^2} \frac{g_0^2}{4\pi}$$

Propagators

- renormalised as in MOM scheme

$$Z(p^2)\big|_{p^2=\mu^2} = 1 \quad \to Z_3$$

$$G(p^2)\big|_{p^2=\mu^2} = 1 \quad \to \tilde{Z}_3$$

Vertex

$$\boxed{\widetilde{Z}_1 = \widetilde{Z}_1^{\overline{\mathsf{MS}}}}$$

Landau gauge
$\widetilde{Z}_1 = 1$

Minimal MOM subtraction scheme =
modified **Minimal** subtraction scheme + **MOM** subtraction scheme

Relating a coupling to the $\overline{\text{MS}}$ scheme

At large enough scales $\left(a \equiv \alpha_s^{\overline{\text{MS}}}/4\pi\right)$

$$\beta(\alpha_s) = \mu^2 \frac{\partial \alpha_s(\mu^2)}{\partial \mu^2}$$

$$\frac{\alpha_s^S}{\alpha_s^{\overline{\text{MS}}}} = 1 + D_1 a + D_2 a^2 + D_3 a^3 + \mathcal{O}(a^4)$$

$$\boxed{\alpha_{s,\overline{\text{MS}}}^{n_f=5}(M_Z)}$$

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Alternatively, "$\mathbf{\Lambda_{QCD}}$"

$$\ln \frac{\mu^2}{\Lambda_S^2} = \int \frac{da}{\beta^S(a)}$$

Exact relation between schemes

$$\frac{\Lambda^S}{\Lambda^{\overline{\text{MS}}}} = \exp\left(\frac{D_1}{2\beta_0}\right)$$

Relating a coupling to the $\overline{\text{MS}}$ scheme

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$$\Lambda_{n_f}^{S_1},\ \Lambda_{n_f}^{S_2},\ \vdots,\ \Lambda_{n_f}^{S_n} \rightarrow \Lambda_{n_f}^{\overline{\text{MS}}}$$

The MM coupling

$$\alpha_s^{\text{MM}}(p^2) = \frac{g_0^2}{4\pi} Z(p^2) G^2(p^2)$$

Relation to $\overline{\text{MS}}$

[L. von Smekal, K. Maltman, *A.S.*, PLB681, 336 (2009)]

up to **4-loop** order

$(a \equiv \alpha_s^{\overline{\text{MS}}}/\pi)$

$$\frac{\alpha_s^{\text{MM}}}{\alpha_s^{\overline{\text{MS}}}} = Z(\mu^2)^{\overline{\text{MS}}} G^2(\mu^2)^{\overline{\text{MS}}} = 1 + D_1 a + D_2 a^2 + D_3 a^3 + \mathcal{O}(a^4)$$

β^{MM} function

$$\mu^2 \frac{da(\mu^2)}{d\mu^2} = \beta(a) := -\sum_{i=0} \beta_i a^{i+2}$$

$$\beta_0^{\text{MM}} = 2.25 \qquad (N_c = N_f = 3)$$

$$\beta_1^{\text{MM}} = 4.0 - 0.422\xi - 0.281\xi^2 + 0.141\xi^3$$

$$\beta_2^{\text{MM}} = \ldots$$

refer to [PLB681:336 (2009)]

$$\beta_3^{\text{MM}} = \ldots$$

[for ξ=0 (i.e., Landau gauge) MM scheme = *Taylor* scheme of Boucaud et al. (2009-12)]

Minimal MOM coupling on the lattice

$$\alpha_s^{\mathsf{MM}}(p) = \frac{g_0^2(a)}{4\pi} Z(p,a) G^2(p,a)$$

$$g_0^2(a) = \frac{2N_c}{\beta}$$

Recipe

1) generate / download gauge configurations (any flavor)
2) gauge-fix these to Landau gauge
3) calculate gluon and ghost propagator in momentum space (diagonal momenta only)
4) extract dressing functions using the correct lattice tree-level form

→ for plain Wilson action:

$$D_{\mu\nu}^{ab}(p) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{\hat{p}_\mu \hat{p}_\nu}{\hat{p}^2}\right)\frac{Z(p^2)}{\hat{p}^2}$$

$$G^{ab}(p) = -\delta^{ab}\frac{G(p^2)}{\hat{p}^2}$$

Advantages

- no vertex, **only 2-pt function** (much less noise)
- β-function (and relation to $\overline{\text{MS}}$) known to **4-loop** [PLB681, 336 (2009)]
- good understanding of lattice spacing effects → better use of data at $a^2p^2 \gg 1$
- could be used to determine $a(\beta)$

Minimal MOM coupling on the lattice

$$\boldsymbol{\alpha_s^{\mathsf{MM}}(r_0 p) = \frac{g_0^2(a)}{4\pi} Z(r_0 p, a) G^2(r_0 p, a)}$$

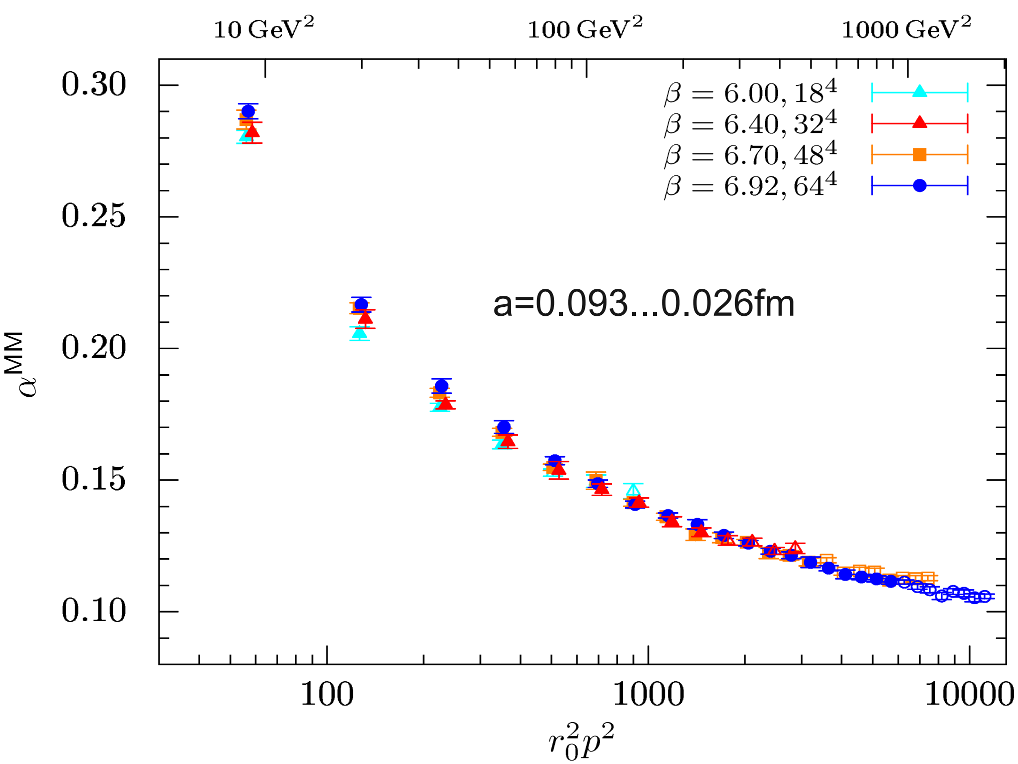
$$g_0^2(a) = \frac{2N_c}{\beta}$$

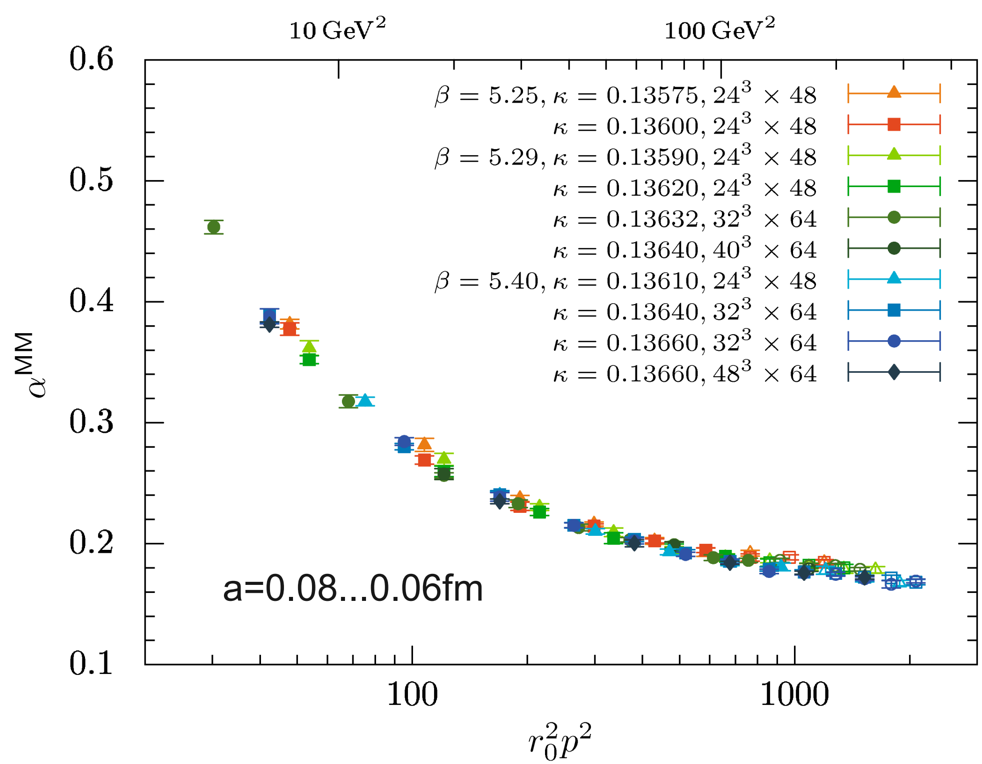
Momentum / Scale

- Express all lattice momenta in units of r_0: $ap \to r_0 p = \frac{r_0}{a}(\beta) \cdot ap$
- Use as input $r_0/a(\beta)$ values from
  - $N_f = 0$: Necco-Sommer interpolation formula [Necco&Sommer (2002)]
  - $N_f = 2$: new QCDSF values for chirally extrapolated $r_0/a(\beta)$ [Bali&Najjar (2012)]
  - $N_f = 2+1$: (currently) QCDSF values for $r_0/a(\beta)$ [Bali&Najjar (2012)]
- Seems ok: no deviation seen in the data for different β
- Why QCDSF values?
  - we use their $N_f = 2, 2+1$ configurations available via ILDG

Minimal MOM coupling on the lattice

$$\alpha_s^{\mathrm{MM}}(r_0 p) = \frac{g_0^2(a)}{4\pi} Z(r_0 p, a) G^2(r_0 p, a) \qquad g_0^2(a) = \frac{2N_c}{\beta}$$





- data for different β overlaps almost completely, as it should
- understanding the small deviations is the subject of the talk
- also, no visible quark mass dependence

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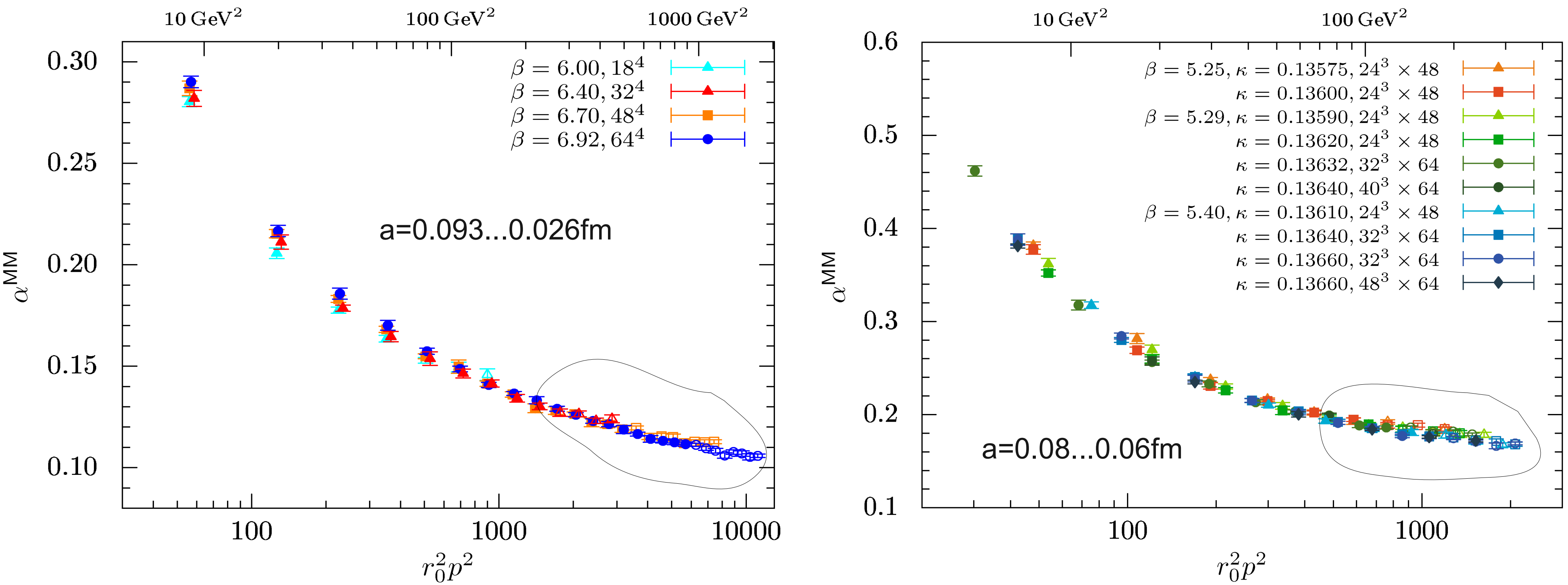

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Why should I care about these deviations?

Minimal MOM coupling on the lattice

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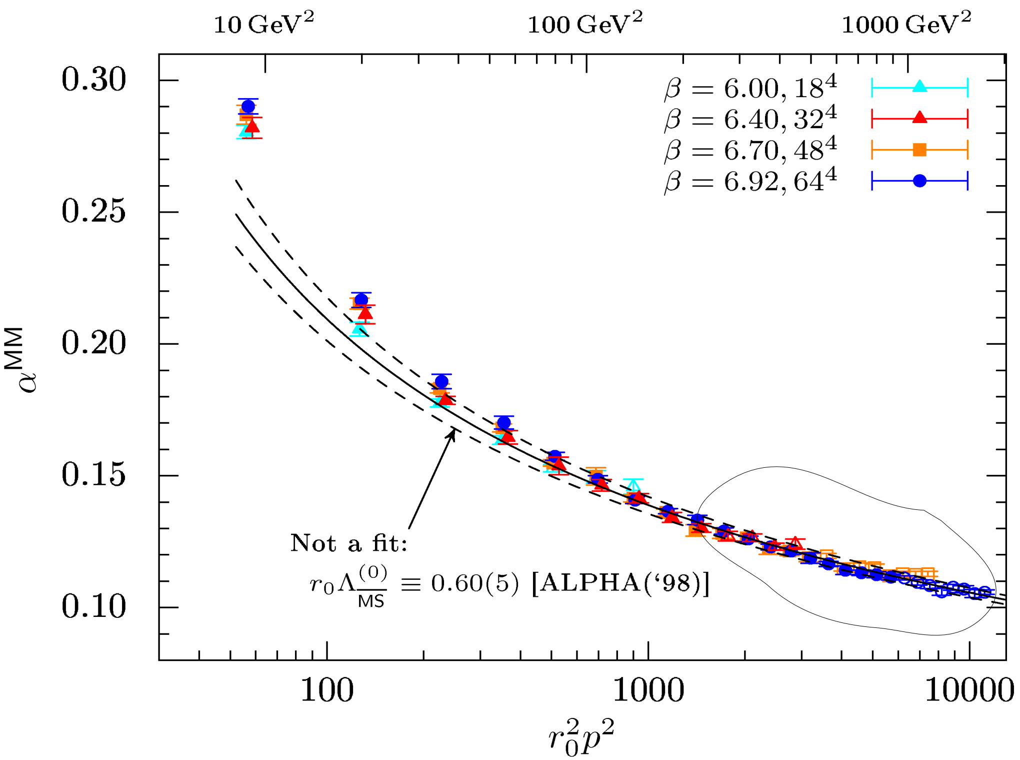
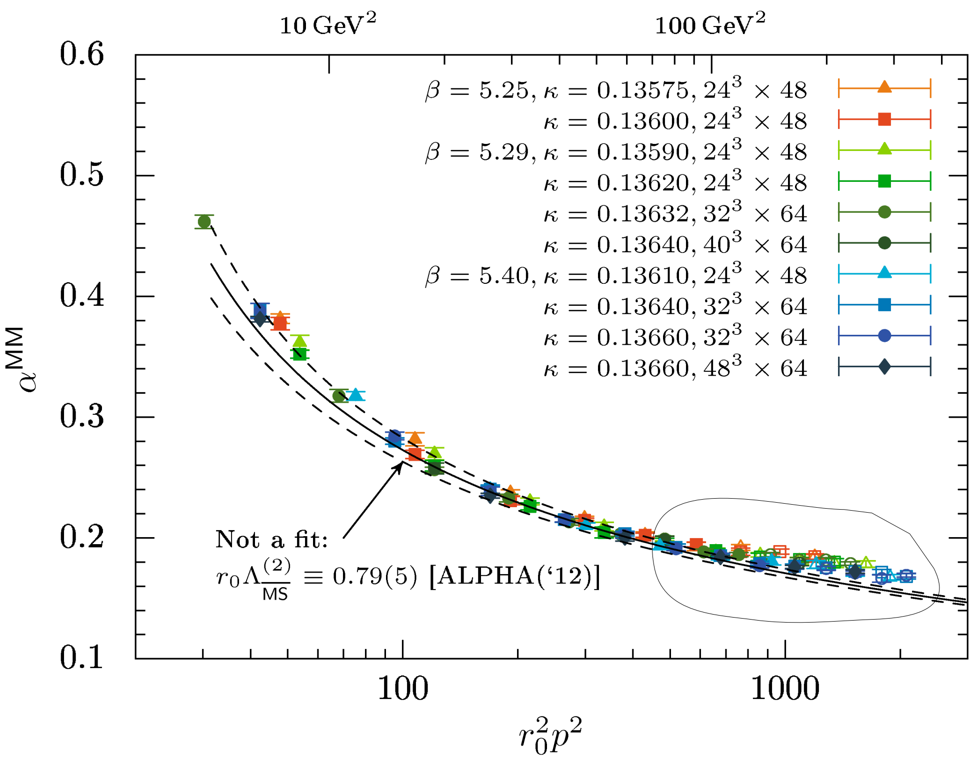




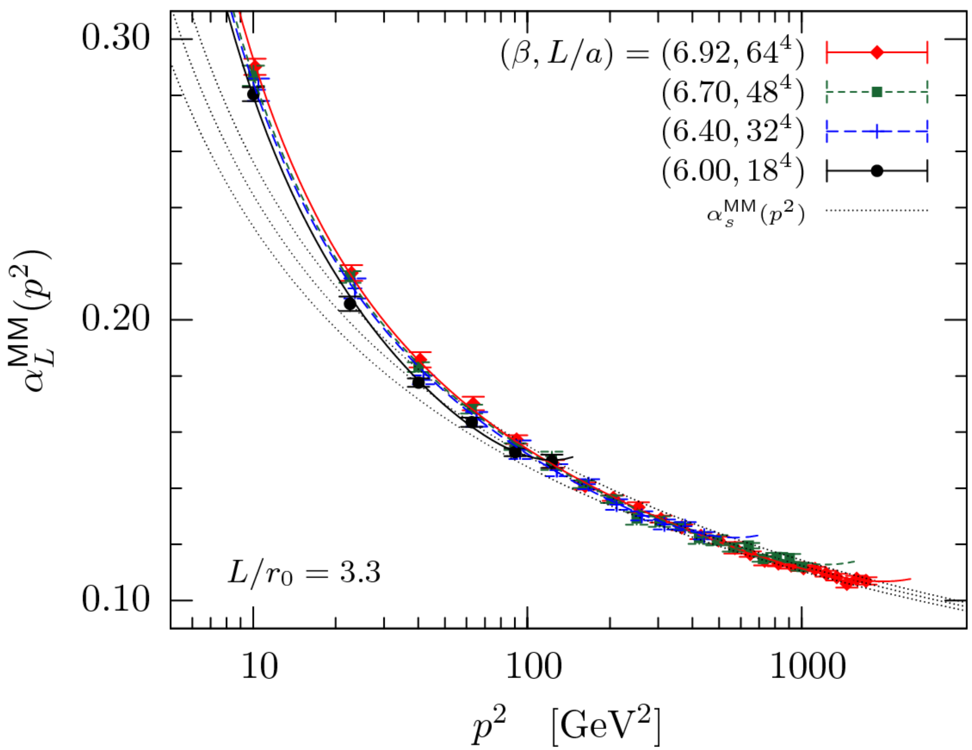
- for unquenched configurations: access to large momenta is restricted
- nonperturbative and other effects sneak in

Why should I care about these deviations?

Effects

At smaller scales

- higher-loop contributions
- condensates
  - dim=4 gluon condensate
  - dim=2 condensate (maybe)
- non-perturbative corrections



At larger lattice momenta

- hyper-cubic artifacts due to finite a

Effective description of data

$\langle A^2 \rangle$? Boucaud et al., see Petrov's talk

$$\alpha_L^{\mathsf{MM}} = \alpha_{4-\mathrm{loop}}^{\mathsf{MM}}(p^2)\left[1 + \frac{c_0}{p^2}\right](1 + c_1 a^2 p^2 + c_2 a^4 p^4)$$

- Correlation of different fit parameters: have to carefully entangle this

Let's play safe: 4-loop regime

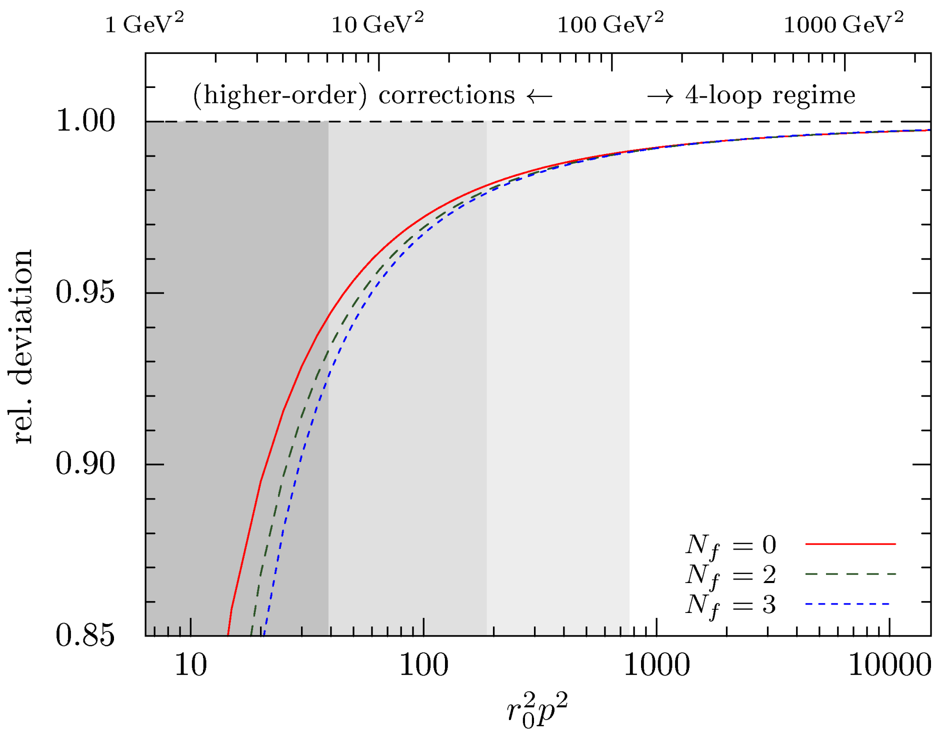
(introduced above)

4-loop relation: MM $\leftrightarrow$ $\overline{\text{MS}}$

$$\alpha_{\text{exp}}^{\text{MM}}[\alpha_s^{\overline{\text{MS}}}] = \alpha_s^{\overline{\text{MS}}}\left(1 + d_1\alpha_s^{\overline{\text{MS}}} + d_2\left(\alpha_s^{\overline{\text{MS}}}\right)^2 + d_3\left(\alpha_s^{\overline{\text{MS}}}\right)^3 + \mathcal{O}\left(\left(\alpha_s^{\overline{\text{MS}}}\right)^4\right)\right)$$

"Back-and-forth" ratio

$$\frac{\alpha_{\text{exp}}^{\overline{\text{MS}}}\left[\alpha_{\text{exp}}^{\text{MM}}\left[\alpha_s^{\overline{\text{MS}}}\left(r_0^2p^2\right)\right]\right]}{\alpha_s^{\overline{\text{MS}}}\left(r_0^2p^2\right)}$$



Fits to pure 4-loop running

- stay above ~100GeV^2, if possible

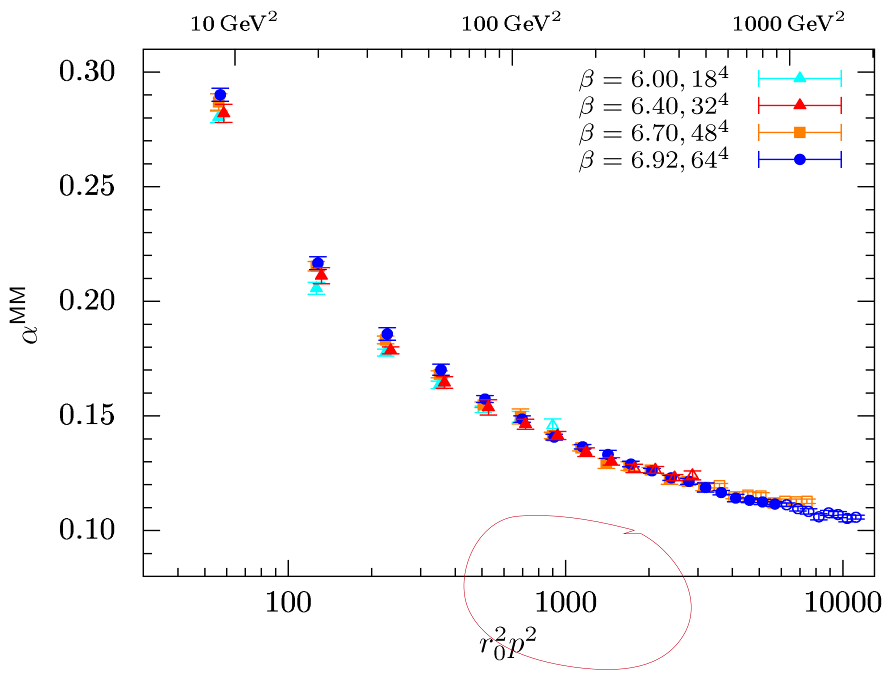
Data at sufficiently large momentum

Fits to 4-loop running

- have to stay in perturbative regime $r_0^2 p^2 \geq 500$

$N_f\!=\!0$ calculation

- access to large momenta **easier** (reasonable large volumes)
- β=6.5 – 7.20 on 48^4, 64^4 feasible



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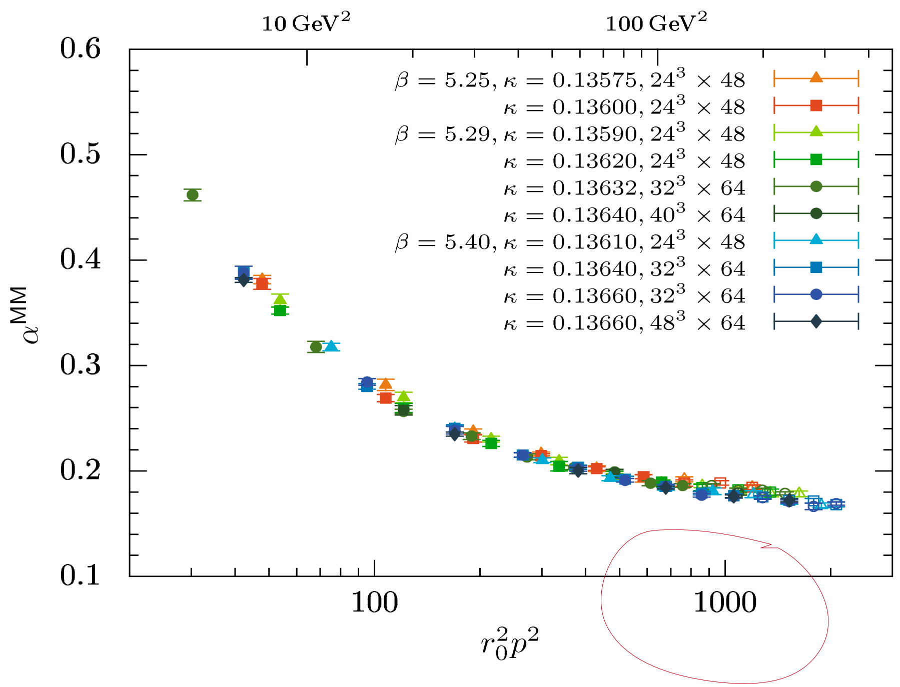
$N_f\!=\!2$ and $N_f\!=\!2\!+\!1$

- access to large momenta **limited**
- e.g., QCDSF
  - $N_f\!=\!2$: β=5.29, 5.40, 48^3x64
  - $N_f\!=\!2\!+\!1$: β=5.50, 5.80, 32^3x64



Have to get most out of the data at larger lattice momentum

→ need to understand lattice discretization effects

Ghost dressing function

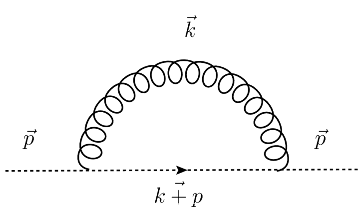
At 1-loop order LPT

$$Z_G(ap) = 1 + \frac{\Pi(ap)}{a^2\hat{p}^2} = 1 + g_0^2\left[F(a^2p^2) + \Delta_G(ap)\right] \stackrel{a\to 0}{=} 1 + g_0^2 F(a^2p^2)$$

$$F(a^2p^2) = 1 + g_0^2\left(c_1 \log(a^2p^2) + c_2\right)$$

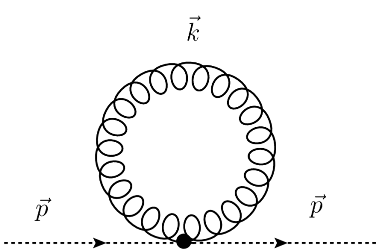
[Kawai et al. (1981)]

Ghost self-energy at 1-loop LPT



$$\Pi^{(a)} = g_0^2 N\delta_{ab} \sum_{\mu,\nu} \int \frac{d^4k}{(2\pi)^4} \frac{\delta_{\mu\nu} - (1-\alpha)\hat{k}_\mu\hat{k}_\nu/\hat{k}^2}{\hat{k}^2\,(\widehat{p+k})^2} (\widehat{p+k})_\mu$$

$$\times \cos\frac{p_\mu a}{2}\,\hat{p}_\nu \cos\frac{(p+k)_\nu a}{2},$$



$$\Pi^{(b)} = \tfrac{1}{12} g_0^2 a^2 N\delta_{ab} \sum_{\mu,\nu} \hat{p}_\mu\hat{p}_\nu\delta_{\mu\nu} \int \frac{d^4k}{(2\pi)^4} \frac{\delta_{\mu\nu} - (1-\alpha)\hat{k}_\mu\hat{k}_\nu/\hat{k}^2}{\hat{k}^2}$$

Ghost dressing function

Hypercubic corrections

At 1-loop order LPT

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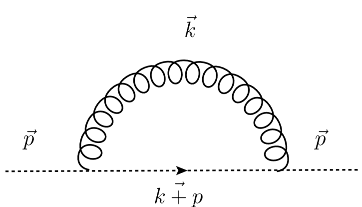

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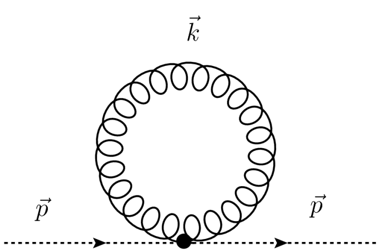

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Ghost dressing function – hypercubic corr.

(exact)

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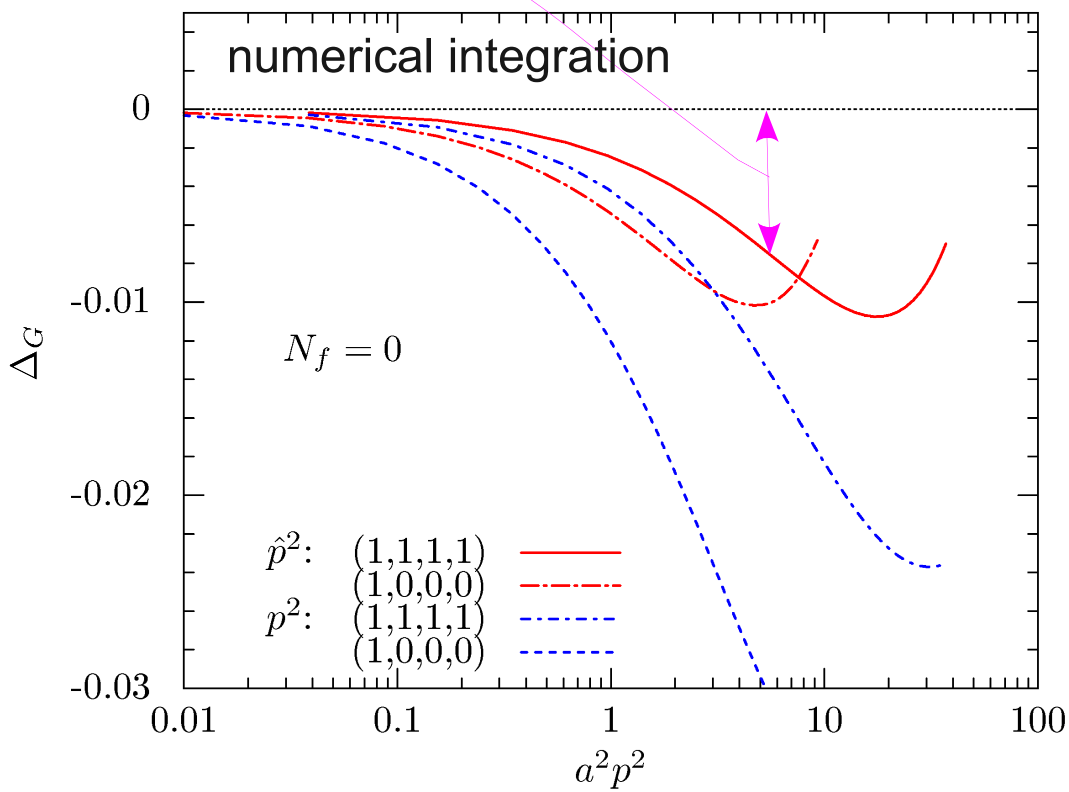
Lessons

- diagonal momenta come with smallest deviations (not zero!)
- use always correct lattice tree-level structure
- Remember: ghost propagator at lattice tree-level

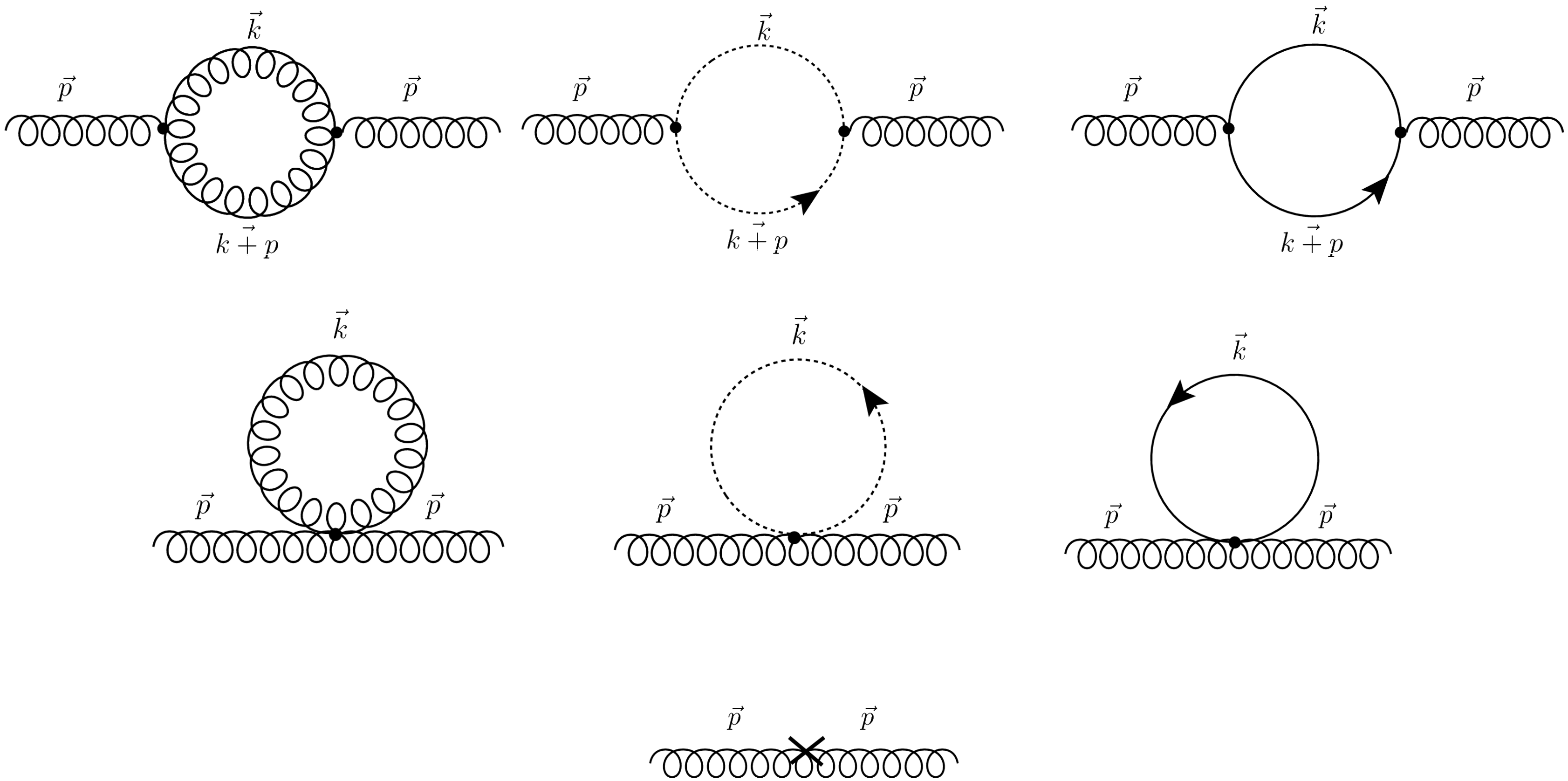
$$G^0(ap) = \frac{1}{a^2\hat{p}^2}$$

$$p_\mu = \frac{2\pi k_\mu}{L_\mu} \qquad \hat{p}_\mu = \frac{2}{a}\sin\left(\frac{p_\mu}{2}\right)$$



Gluon self-energy

At 1-loop order LPT



Thanks to H.Perlt for the 3gl and 4gl-vertex functions

Gluon dressing function – hypercubic corr.

(exact)

At 1-loop order LPT

$$Z_D(ap) = 1 + \frac{\Pi(ap)}{3a^2\hat{p}^2} = 1 + g_0^2\left[F(a^2p^2) + \Delta_D(ap)\right] \overset{a\to 0}{=} 1 + g_0^2 F(a^2p^2)$$

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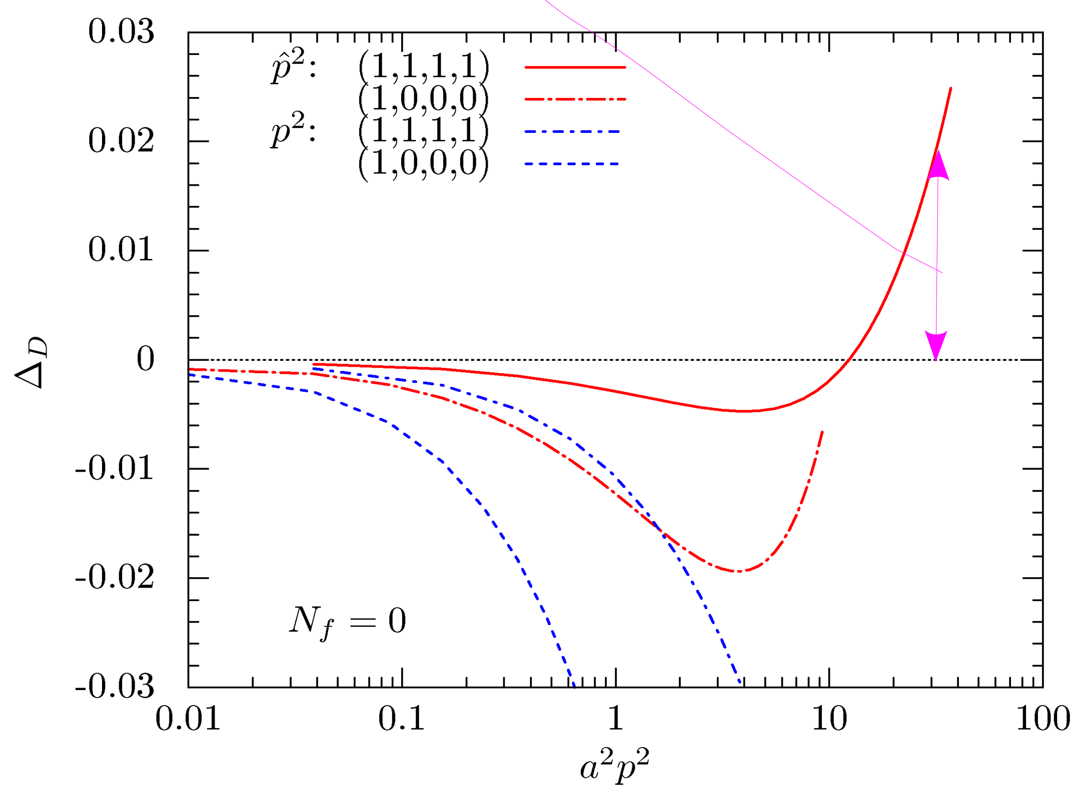

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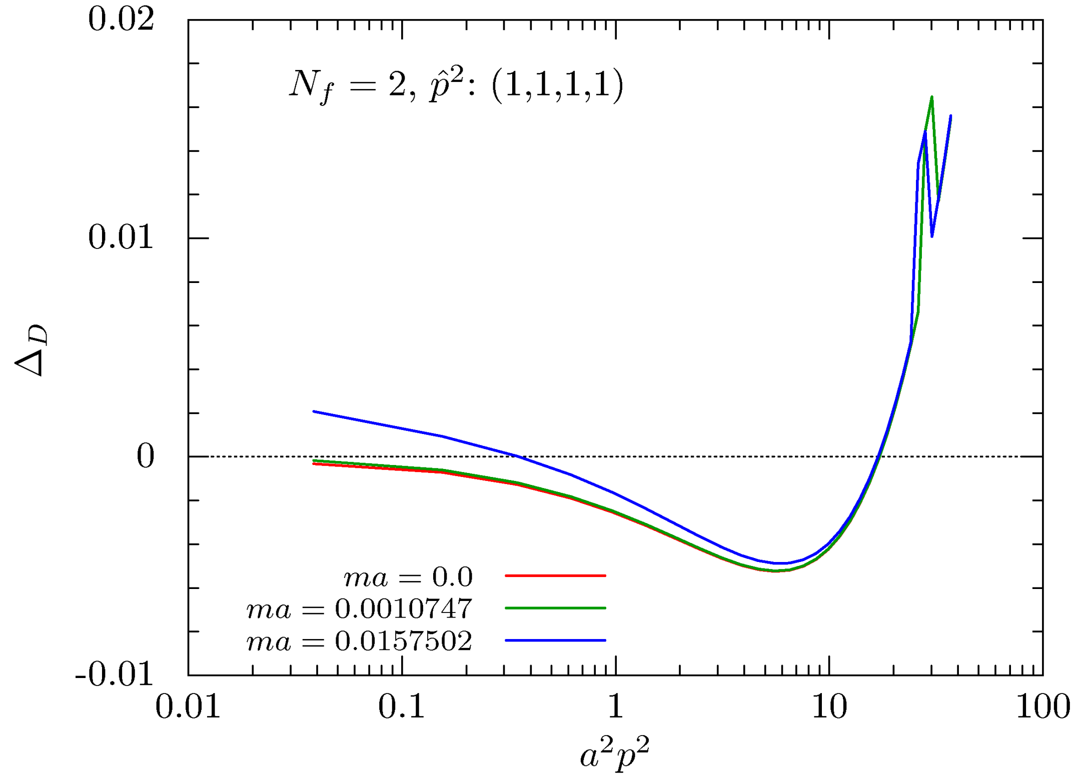


Subtracting corrections

In perturbative regime

$$\alpha_{\mathrm{MC}}^{\mathsf{MM}}(r_0 p, ap) = \alpha_s^{\mathsf{MM}}(r_0^2 p^2) + \underbrace{g_0^4\,(\Delta_Z + 2\Delta_G)}_{1-\mathrm{loop\ corrections}} + O(g_0^6)$$

Subtracting exact 1-loop corrections

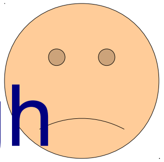
$$\alpha_{\mathrm{MC,sub}}^{\mathsf{MM}}(r_0 p, ap) = \alpha_s^{\mathsf{MM}}(r_0^2 p^2) + O(g_0^6)$$

Hyper-cubic Taylor expansion of higher order corrections

$$\alpha_{\mathrm{MC,sub}}^{\mathsf{MM}}(r_0 p, ap) = \alpha_s^{\mathsf{MM}}(r_0^2 p^2) + g_0^6\left(c_2 \cdot (ap)^2 + c_4 \cdot (ap)^4 + \ldots\right) + O(g_0^8)$$

(for diagonal momenta)

Subtracting corrections – 1-loop is not enough



It turns out

- 1-loop corrections remove only a minor part of the effects
- Most of the effect can be described by a **global fit** to the data (simultaneously for different β)
- 3 Parameters (in total)

$$\alpha_{\mathrm{ref}}^{\overline{\mathrm{MS}}}, c_2, c_4$$

Fits, make sure that

1) stay in perturbative regime

$$r_0^2 p^2 > 600 \qquad \left(p^2 > 90\mathrm{GeV}^2\right)$$

2) avoid too large lattice momenta, but still sufficient enough

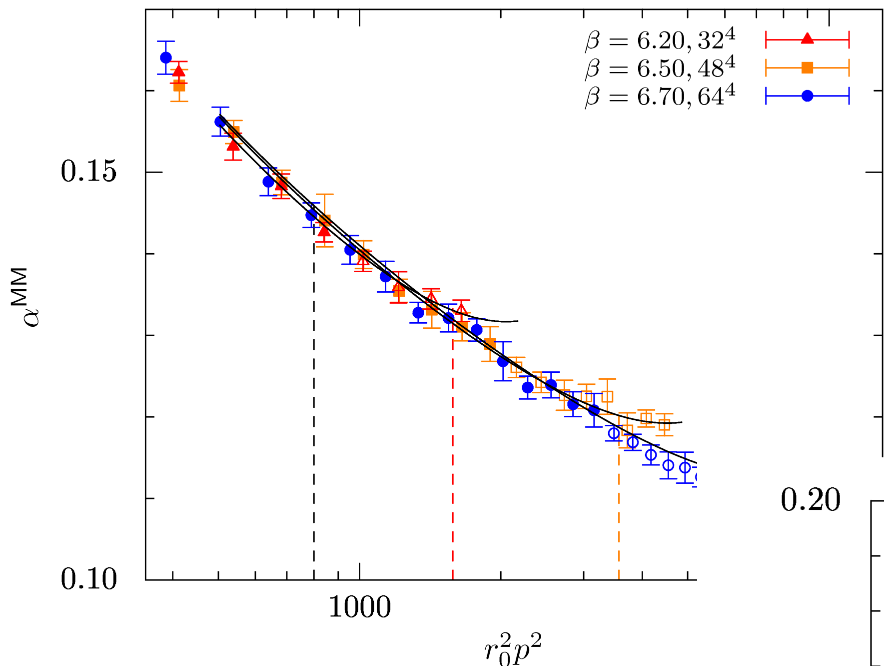
$$17 \leq a^2 p^2 \leq 34$$

Global fit function

$$g_0^2(a) = \frac{2N_c}{\beta}$$

$$\alpha_{\mathrm{MC,sub}}^{\mathrm{MM}}(r_0 p, \beta) = \underbrace{\alpha_s^{\mathrm{MM}}(r_0^2 p^2, \alpha_{\mathrm{ref}}^{\overline{\mathrm{MS}}})}_{\text{cont. running}} + \underbrace{g_0^6 \left(c_2 \cdot (ap)^2 + c_4 \cdot (ap)^4\right)}_{\text{leading corrections}}$$

One global fit



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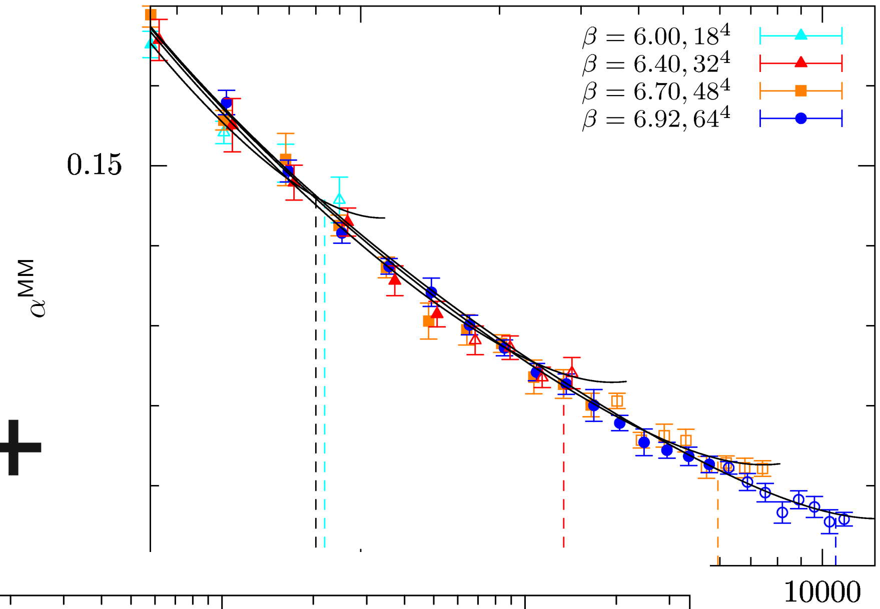
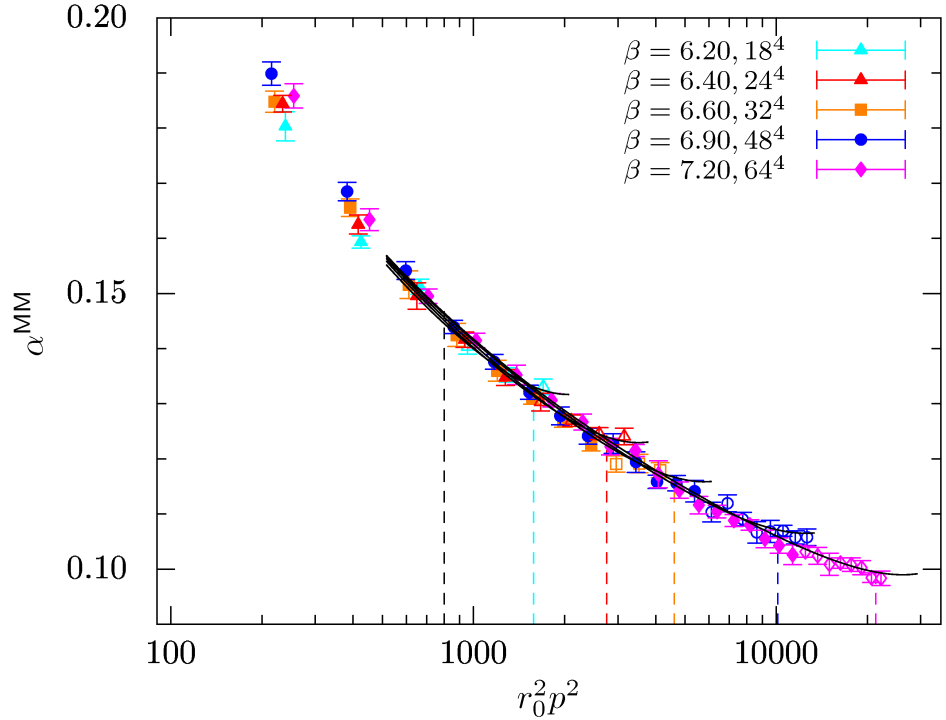
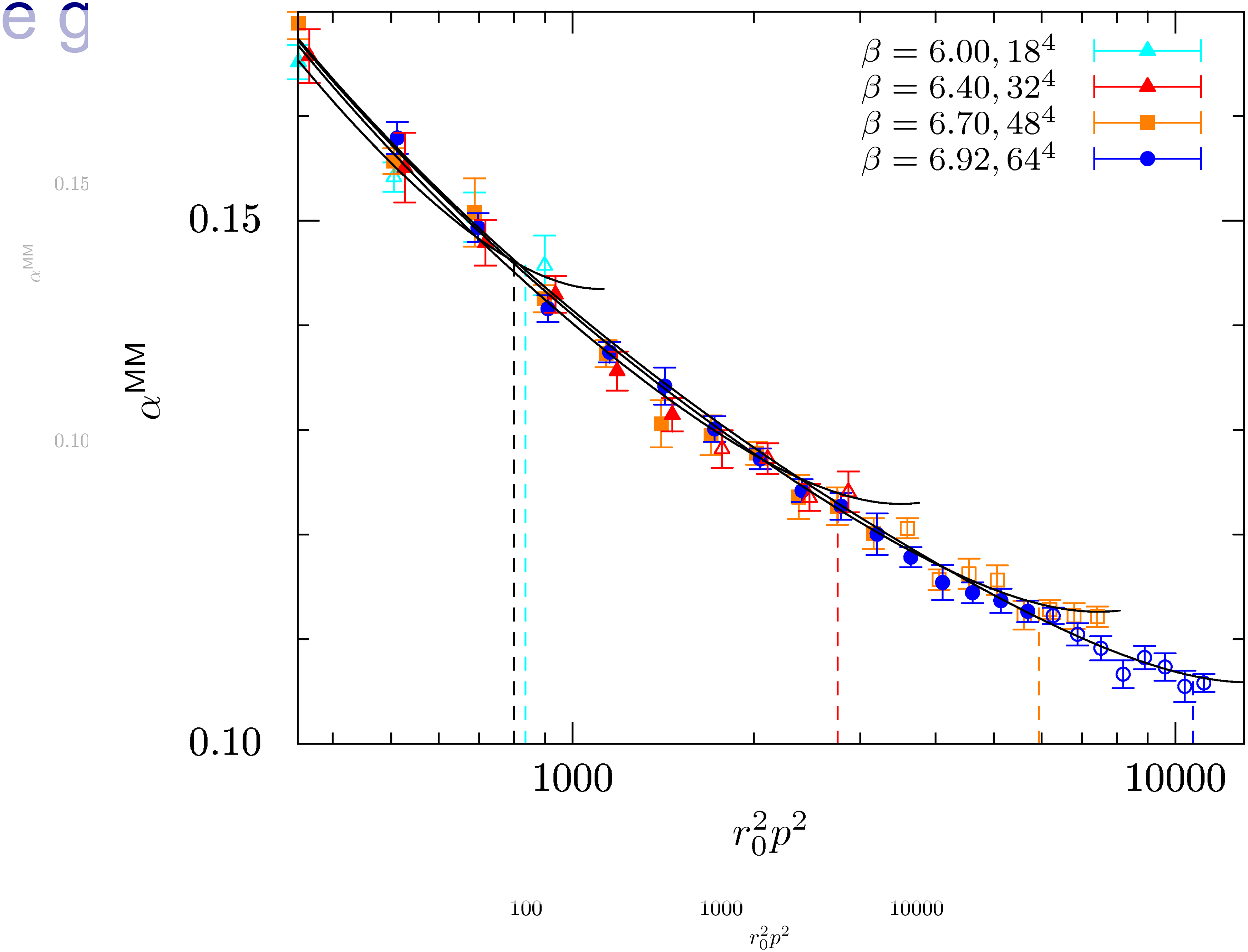




Parameters:

$\alpha_{\mathrm{ref}}^{\overline{\mathrm{MS}}}, c_2, c_4$

One g



$\beta = 6.00, 18^4$
$\beta = 6.40, 32^4$
$\beta = 6.70, 48^4$
$\beta = 6.92, 64^4$
0.15
0.10
α^{MM}
1000
10000
$r_0^2 p^2$
100

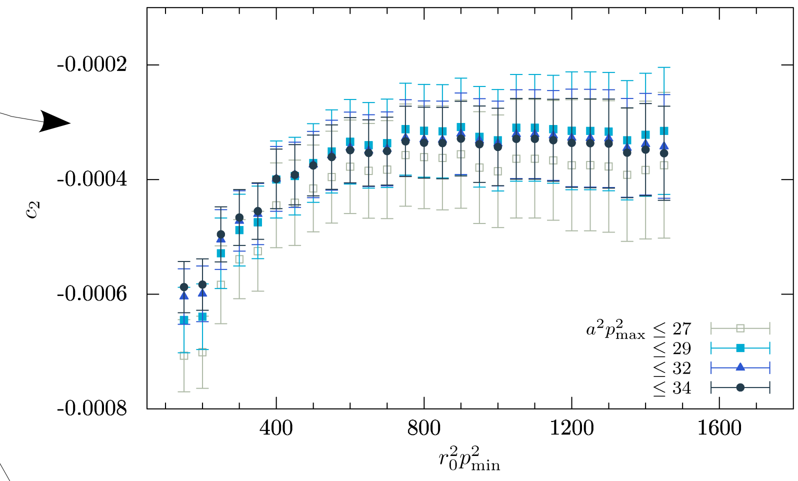
Global fits

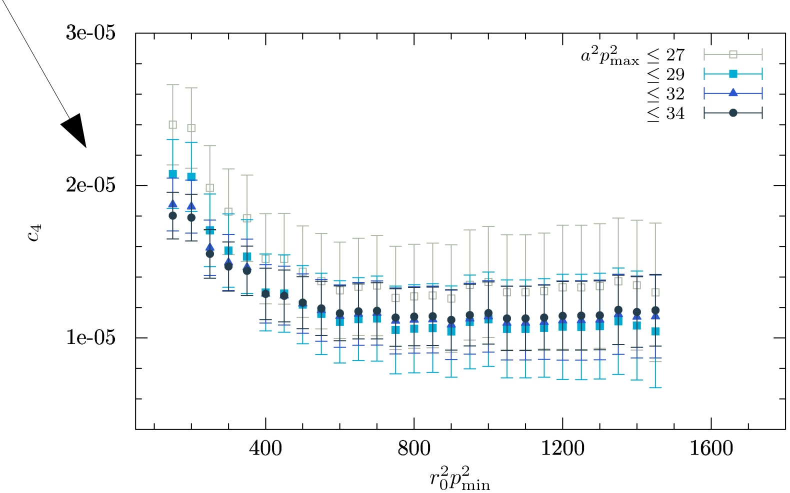
Parameters:

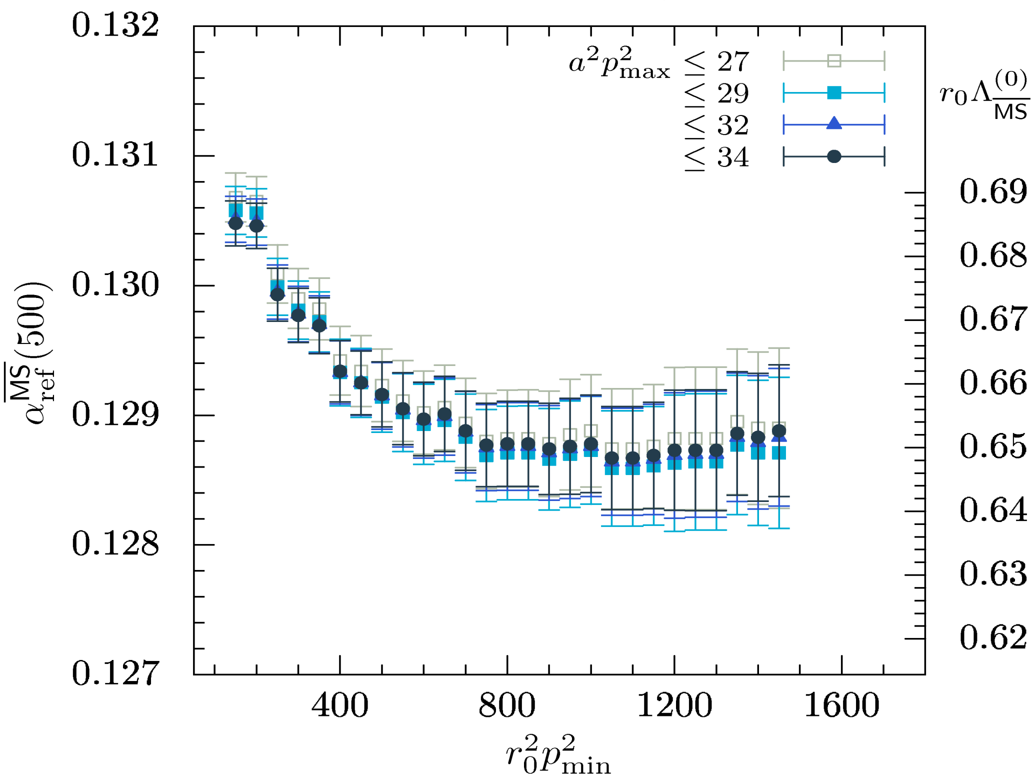
$\alpha_{\mathrm{ref}}^{\overline{\mathrm{MS}}}, c_2, c_4$

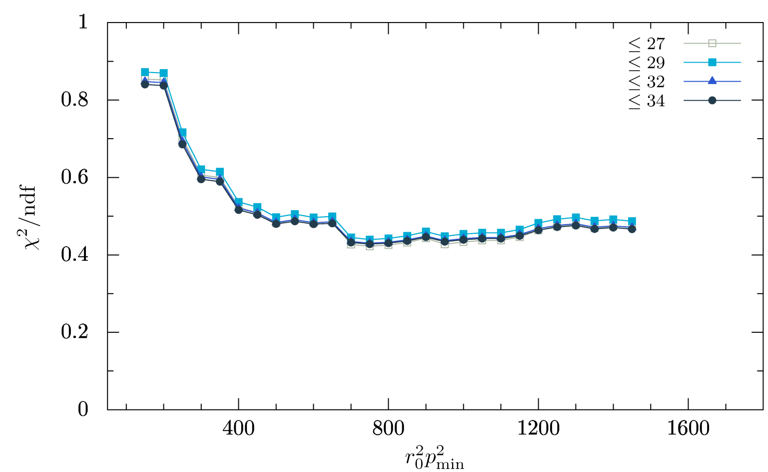
Varying the fit window

- Lower cutoff $\quad r_0^2 p^2 > 600 \quad (p^2 > 90\mathrm{GeV}^2)$
- Upper cutoff $\quad 17 \leq a^2 p^2 \leq 34$

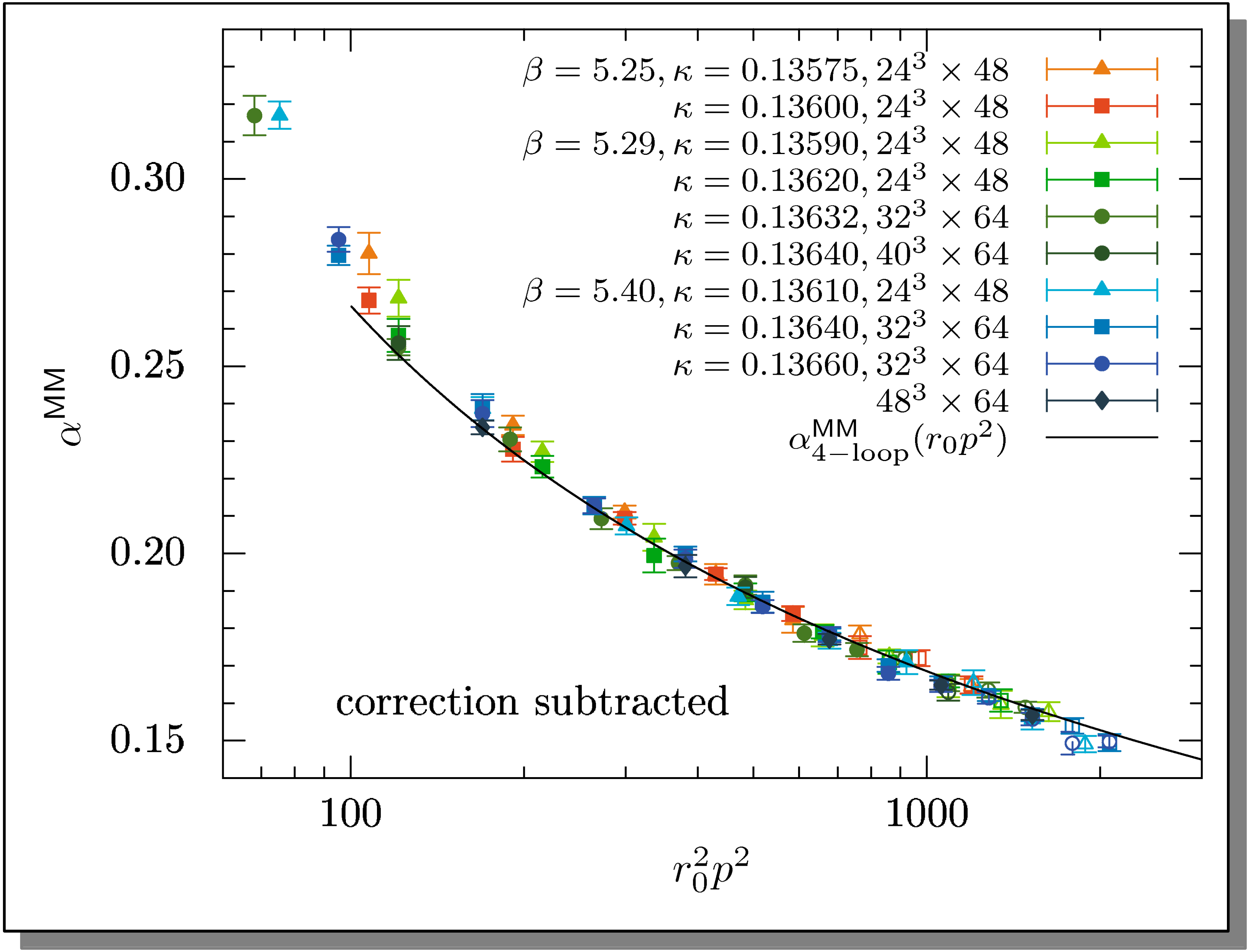




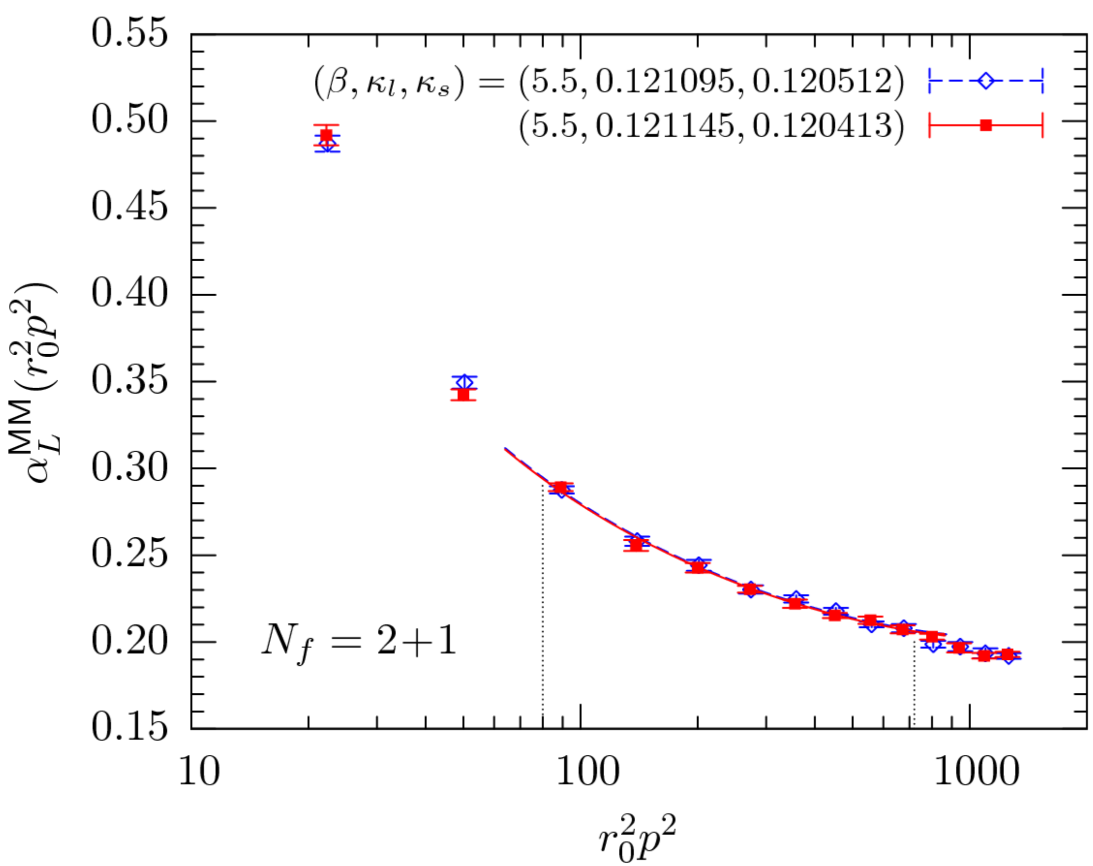


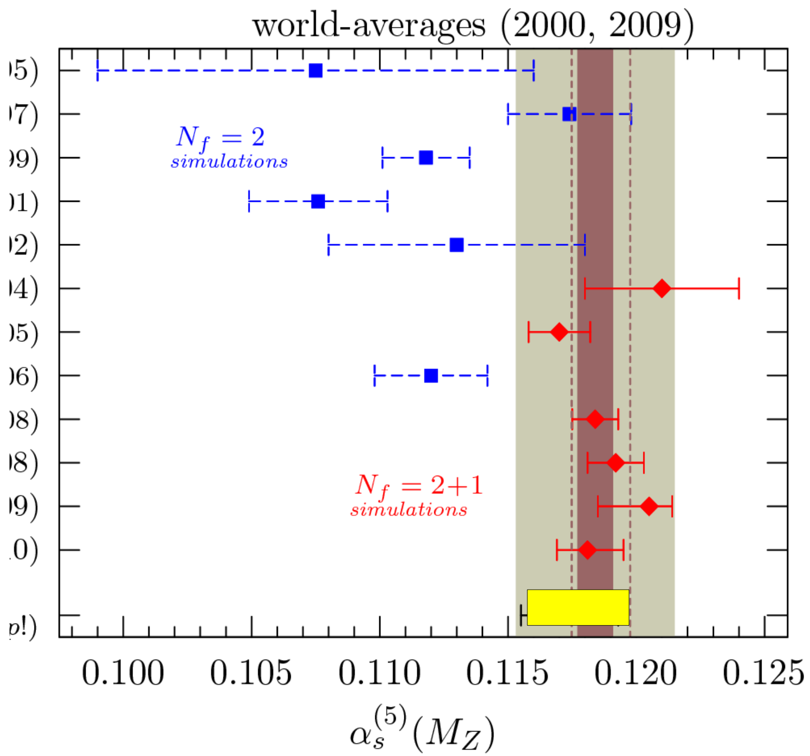


Unquenched data after (fitted) subtraction



First three-flavour results





Currently

- Only data sets for one β = 5.50, more to come
- Encouraging: Data is in the right ball park

Conclusions

Minimal MOM coupling

- data at sufficiently large scales allows for high-precision determination of $\Lambda^{\overline{\mathrm{MS}}}$
- if scales to small, nonperturbative / condensate / perturbative effects mix up
  - Effectively $1/p^2$ corrections to 4-loop running at lower scales $\rightarrow \langle A^2 \rangle$?

  $\rightarrow$ fix $\Lambda^{\overline{\mathrm{MS}}}$ first at large scales, then check behavior at lower scales

Problem and solution

- access to higher scales restricted (in particular for Nf=2,2+1)
- Understanding of lattice artefacts important (mainly hypercubic corrections)
- Have calculated exact 1-loop corrections (unfortunately only minor fraction)
- Remaining hypercubic corrections are fitted (global 3-parameter fit)

Conclusions

Values for $\Lambda^{\overline{\text{MS}}}$

- Still investigating systematic error (3 fit parameters are anti-correlated)
- **Preliminary values** are in agreement with other studies (please **do not copy yet**)

$$r_0\Lambda^{(0)}_{\overline{\text{MS}}} = 0.61\ldots0.65 \quad \stackrel{r_0=0.5\text{fm}}{=} \quad 240\ldots256\text{MeV}$$

$$r_0\Lambda^{(2)}_{\overline{\text{MS}}} = 0.72\ldots0.82 \quad \stackrel{r_0=0.5\text{fm}}{=} \quad 284\ldots323\text{MeV}$$

$$\left(r_0\Lambda^{(3)}_{\overline{\text{MS}}} = 0.79\ldots0.85 \quad \stackrel{r_0=0.5\text{fm}}{=} \quad 312\ldots335\text{MeV}\right)$$

Aim of talk was to show

- method is reliable
- precision is a matter of understanding finite lattice spacing corrections

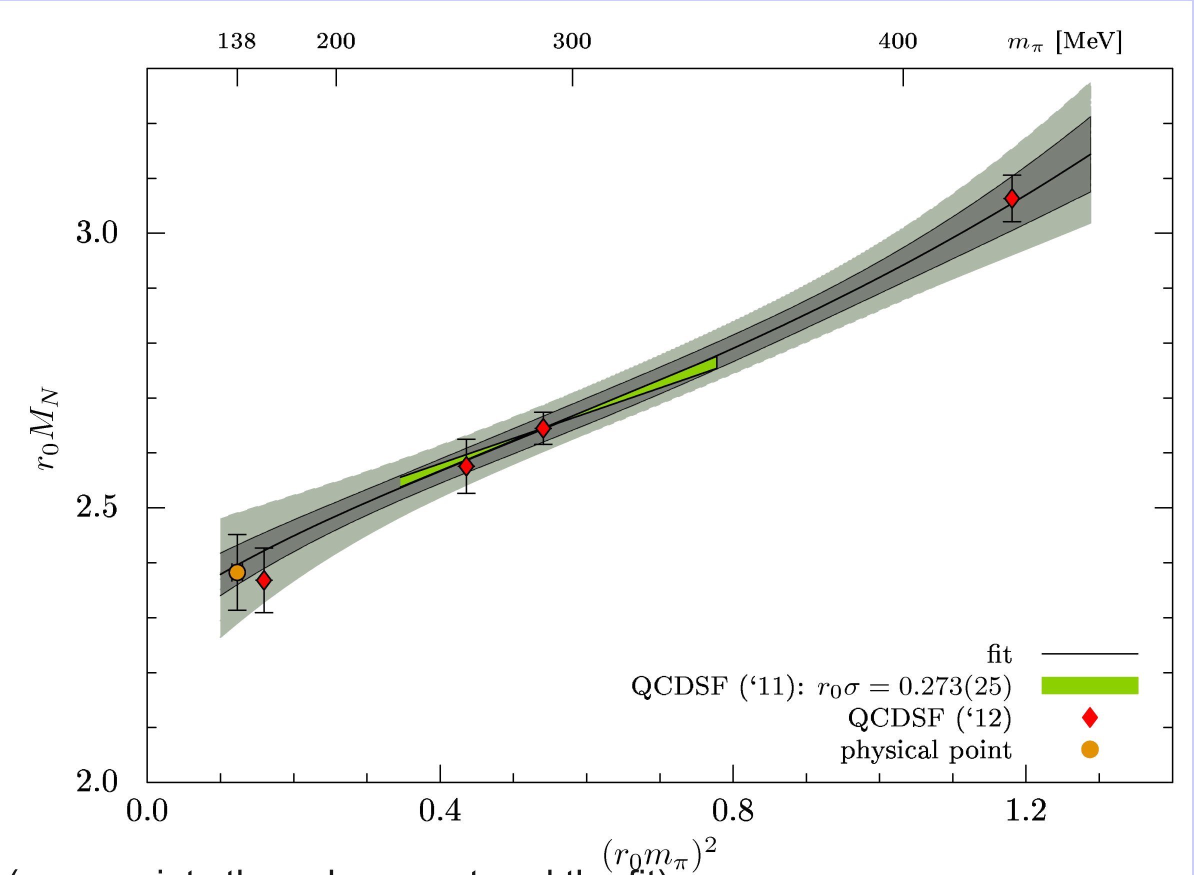
– Advertisement –

New analysis of nucleon mass data (QCDSF)

- for N_f = 2 we find

$$r_0 = 0.501(10)(11)\,\mathrm{fm}$$

$$\sigma_{\pi N} = 37(8)(6)\,\mathrm{MeV}$$



(more points than shown entered the fit)

- more details will appear on the arXiv the next days
- or visit LHP IV

Thank you for your attention!