Two-Photon Decay of Neutral Pion in Lattice QCD

[arXiv:1206.1375]

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Motivation

- PrimEx@JLab: $\Gamma_{\pi^0\gamma\gamma} = 7.82(22)$ eV [PrimEx, PRL106, 2011]
- Precision: 2.8% $\to$ 1.4% (projected goal)
- Benchmark test of axial $U(1)$ anomaly in QCD
- $\Gamma_{\pi^0\gamma\gamma} = (\pi/4)\alpha_e^2 m_\pi^3 \mathcal{F}^2_{\pi^0\gamma\gamma}(m_\pi^2, 0, 0)$
  - $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$: $\pi^0 \to \gamma^*\gamma^*$ transition form factor
- $m_\pi^2 = p_{1,2}^2 = 0$, ABJ anmoaly valid to all orders [Adler, Bardeen, 1969]

$$\mathcal{F}_{\pi^0\gamma\gamma}(0,0,0) = \mathcal{F}^{\mathrm{ABJ}}_{\pi^0\gamma\gamma} = \frac{1}{4\pi^2 F_\pi}$$

- $m_\pi^2 \neq 0$, $p_{1,2}^2 \neq 0$, corrections need to be calculated nonperturbatively
- overlap fermion: exact chiral symmetry $\Rightarrow$ clean test of chiral anomaly

Lattice setup

- $\pi^0 \to \gamma\gamma$ is nontrivial
  - γ is not an asympototic state of QCD
  - conventional method to extract the eigenstate fails
  - 1^{--} interpolating operator yields vector meson rather than γ
- new method is needed
- all-to-all propagators is useful $\Rightarrow \langle J_\mu(t_1) J_\nu(t_2) \pi^0(t_\pi) \rangle$
  - calculate correlator at any time slice of $t_1,\ t_2,\ t_\pi$
  - disconnected diagram

Ensemble information

- four $m_{u,d}$: $m_\pi = 540 \to 290$ MeV $\Rightarrow$ chiral extrapolation
- m_s fixed to be close to its physical value $\Rightarrow$ dynamical s-quark effects
- $L/a = 16$ and 24 $\Rightarrow$ finite-size effects
- $Q = 0$ and 1 $\Rightarrow$ fixing-topology effects
- $a = 0.11$ fm $\Rightarrow$ study possible lattice artifacts

Theoretical setup

- starting point: S-matrix

$$\langle\gamma(p_1,\lambda_1)\gamma(p_2,\lambda_2)|\pi^0(q)\rangle$$

- transition form factor is defined by matrix element

$$\int d^4x e^{ip_1x}\langle\Omega|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle = \epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta F_{\pi^0\gamma\gamma}(m_\pi^2,p_1^2,p_2^2)$$

  - $\epsilon_{\mu\nu\alpha\beta}p_1^\alpha p_2^\beta$: induced by the negative parity of the π^0
- analytical continuation from Minkowski to Euclidean space-time [Ji, Jung, 2001; Dudek, Edwards, 2006]

$$\int dt e^{\omega t}\int d^3\vec{x} e^{-i\vec{p}_1\cdot\vec{x}}\langle\Omega|T\{J_\mu(x)J_\nu(0)\}|\pi^0(q)\rangle$$

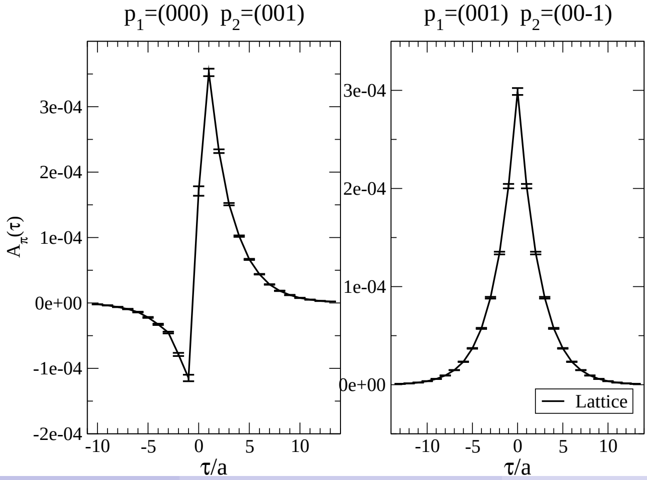
- Euclidean space-time, 3-point correlation function

$$\langle\Omega|J_\mu(\vec{p}_1,t_1)J_\nu(\vec{p}_2,t_2)\pi^0(-\vec{q},t_\pi)|\Omega\rangle$$

Amplitude $A_\pi(\tau)$

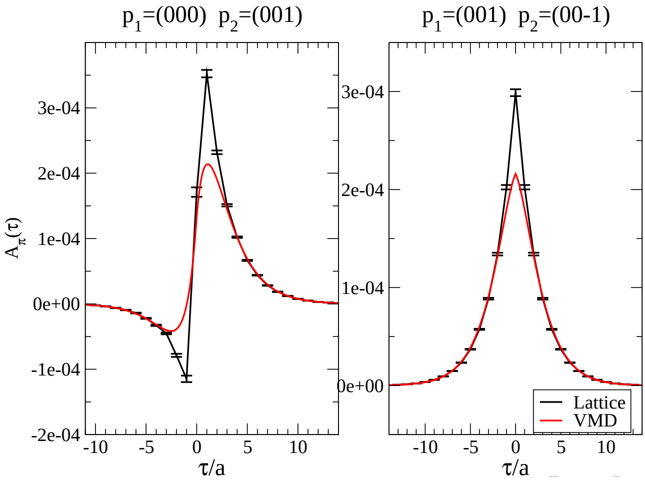
- extract $|\pi^0(q)\rangle$: set $\tau = t_1 - t_2$ and $t = \min\{t_1, t_2\}$

$$A_\pi(\tau) \equiv \lim_{t-t_\pi\to\infty} \langle\Omega|J_\mu(\vec{p}_1, t_1)J_\nu(\vec{p}_2, t_2)\pi^0(-\vec{q}, t_\pi)|\Omega\rangle / e^{-E_\pi(t-t_\pi)}$$



Time dependence of $A_\pi(\tau)$

- VMD model: $\mathcal{F}^{\mathrm{VMD}}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2)$
- $G_V(p^2) = M_V^2/(M_V^2 - p^2)$ is the (lightest) vector meson propagator

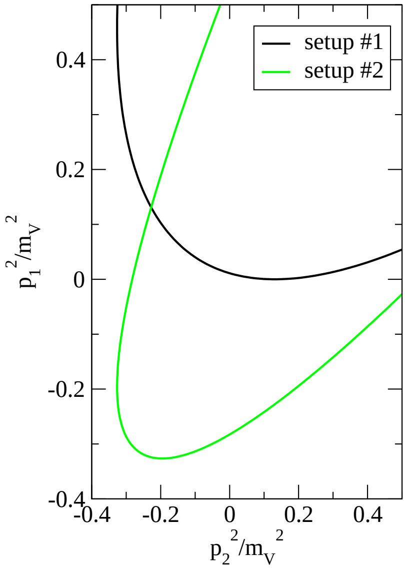


Fit ansatz

- lowest vector meson effects should be accounted for first
- corrected by including excited-state effects
- includes contributions from excited states as a polynomial of $p_{1,2}^2$

$$
\begin{aligned}
\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) &= c_V G_V(p_1^2) G_V(p_2^2) \\
&+ \sum_m c_m \left((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)\right) \\
&+ \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n
\end{aligned}
$$

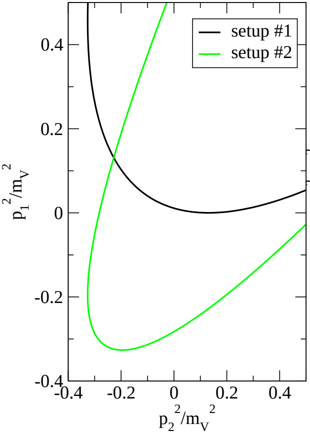
Momentum domain

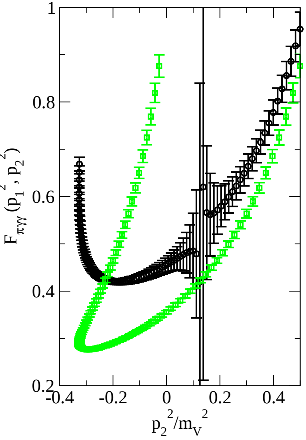


- $A_\pi(\tau) \Rightarrow$
  $\langle\Omega|T\{J_\mu(\vec{p}_1,t)J_\nu(\vec{p}_2,0)\}|\pi^0(q)\rangle$
- perform integration:
  $\int dt\ e^{\omega t}\langle\Omega|T\{J_\mu(t)J_\nu(0)\}|\pi^0\rangle$
  $\Rightarrow \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$
  - ω is input by hand
  - form factor relies on p_1^2 & p_2^2
- tunning ω, fixing $\vec{p}_{1,2}$:
  $p_1^2 = \omega^2 - \vec{p}_1^2$
  $p_2^2 = (E_\pi - \omega)^2 - \vec{p}_2^2$
- diff. spatial momentum setup,
  diff. contour of (p_1^2, p_2^2)

Form factor

- along the contour of (p_1^2, p_2^2) (left), $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2)$ (right) is determined

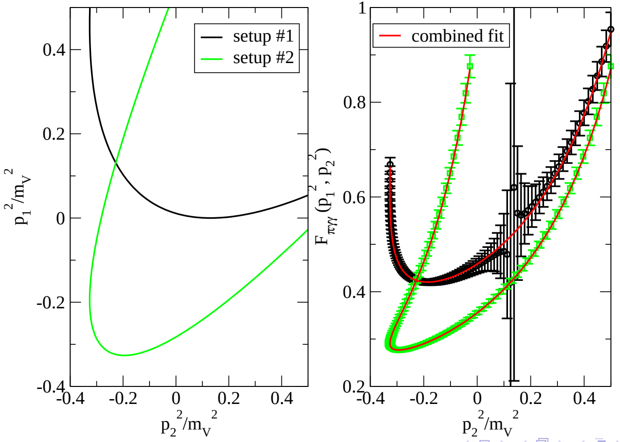




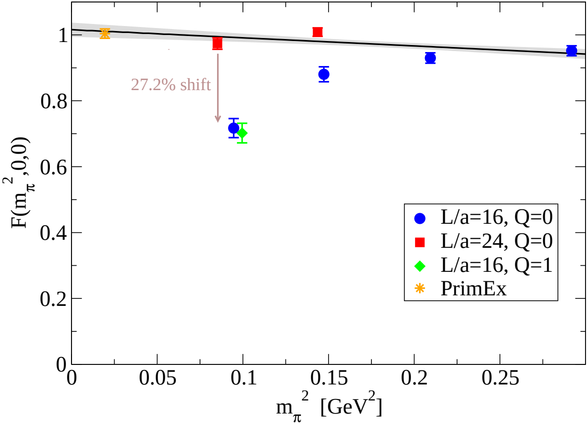
Combined fit

- fit ansatz $\mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2, p_1^2, p_2^2) = c_V G_V(p_1^2) G_V(p_2^2) +$

$$\sum_m c_m \left((p_2^2)^m G_V(p_1^2) + (p_1^2)^m G_V(p_2^2)\right) + \sum_{m,n} c_{m,n} (p_1^2)^m (p_2^2)^n$$



On-shell photon limit



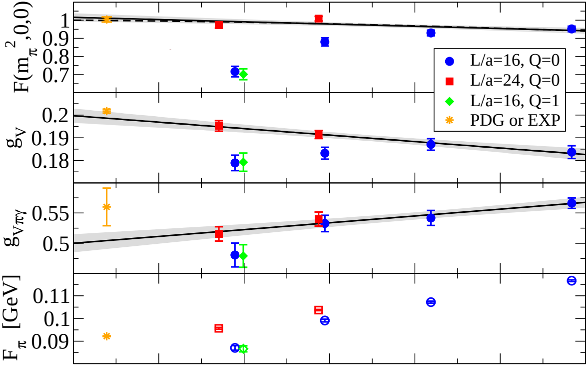
- $F(m_\pi^2,0,0) \equiv \mathcal{F}_{\pi^0\gamma\gamma}(m_\pi^2,0,0)/\mathcal{F}_{\pi^0\gamma\gamma}^{\mathrm{ABJ}}$
- data with $m_\pi L \geq 4$: consistent with ABJ and PrimEx
- $L = 16$: smallest two quark mass, $m_\pi L < 4$, big FS effects
- FS effects checked at topological sector $Q = 0$ and 1

Finite-size effects

- expand the correlator into three hadronic matrix elements:

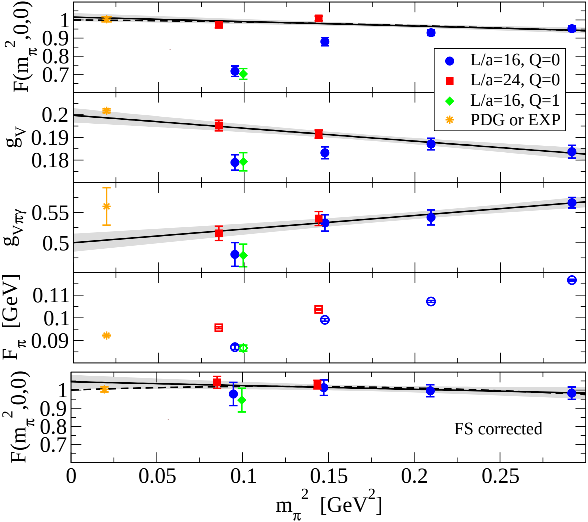
$$\langle J_\mu J_\nu \pi^0 \rangle \to \langle \Omega | J_\mu | V \rangle \langle V | J_\nu | \pi^0 \rangle \langle \pi^0 | \pi^0 | \Omega \rangle$$

$g_V \qquad g_{V\pi\gamma} \qquad F_\pi$



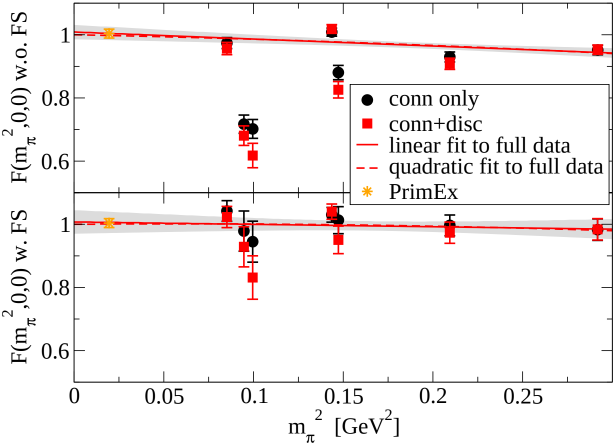
- FS effects in g_V, $g_{V\pi\gamma}$, F_π accumulate and add up to a large effect

Finite-size corrections



- FS corrections $R_{\mathcal{O}} \equiv \mathcal{O}(\infty)/\mathcal{O}(L)$, assume $R_{F(m_\pi^2,0,0)} = R_{g_\rho} R_{g_{\rho\pi\gamma}} R_{F_\pi}$

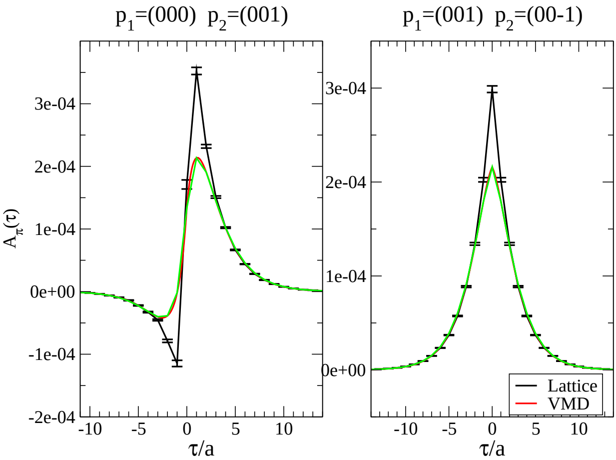
Disconnected-diagram effects



- all-to-all propagator: control error of disc. contribution
- although not significant, conn+disc systematically shift down
- precision level (3% for form factor): disc. diagram should be included

Lattice artifacts

- discrete data v.s. continuum case?



- disc. effects in VMD model: less than 5×10^{-4}, neglegiable

Numerical results

- after examining possible systematic effects

$$
\begin{aligned}
F(0,0,0) &= 1.009(22)(29) \\
F(m^2_{\pi,\mathrm{phy}},0,0) &= 1.005(20)(30) \\
\Gamma_{\pi^0\gamma\gamma} &= 7.83(31)(49)\ \mathrm{eV}
\end{aligned}
$$

- ABJ anomaly and PrimEx measurement

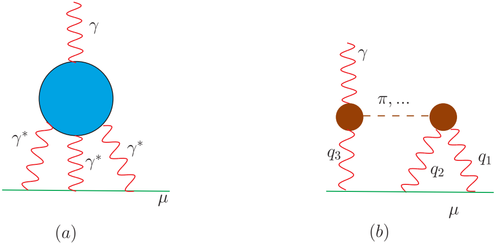
$$
\begin{aligned}
F(0,0,0) &= 1 \\
F(m^2_{\pi,\mathrm{phy}},0,0) &= 1.004(14) \\
\Gamma_{\pi^0\gamma\gamma} &= 7.82(22)\ \mathrm{eV}
\end{aligned}
$$

Conclusions

- $\pi^0 \to \gamma\gamma$ calculation is done successfully
  - by analytic continuation
  - using all-to-all propagators
- ABJ anomaly confirmed in the chiral limit
  - with the overlap fermion, it is satisfied at finite lattice spacing
- correction to ABJ can also be calculated precisely
- remaining major effect would be isospin breaking
- open a possibility to calculate amplitudes with no QCD asymptotic state
  - $\pi^+ \to l^+ \nu_l \gamma$
  - $g - 2$, light-by-light

Motivated by muon $g_\mu - 2$

- Exp. determination of $g_\mu - 2$ to 0.54 ppm [E821@BNL, PRD73, 2006]
- S.M. prediction of $g_\mu - 2$ to 0.51 ppm [Jegerlehner, EPJC71, 2011]
- Discrepancy: $3.3\sigma \Rightarrow$ New Physics ??
- HLbL is predicted to be dominant error in the next round



- Difficult: HLbL involves $\langle J_\mu J_\nu J_\rho J_\sigma \rangle$
- Better to start with $\pi^0(\eta,\ \eta') \to \gamma^*\gamma^*$

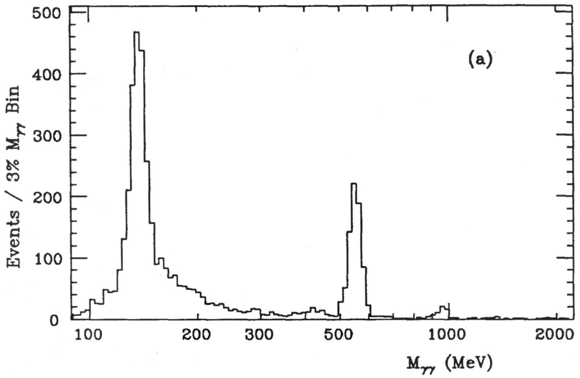
π^0 contribution

Contribution	BPP	HKS	KN	MV	BP	PdRV	N/JN
π^0, η, η'	85±13	82.7±6.4	83±12	114±10	–	114±13	99±16
π, K loops	−19±13	−4.5±8.1	–	–	–	−19±19	−19±13
π, K loops + other subleading in N_c	–	–	–	0±10	–	–	–
axial vectors	2.5±1.0	1.7±1.7	–	22± 5	–	15±10	22± 5
scalars	−6.8±2.0	–	–	–	–	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	–	–	–	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	110±40	105±26	116±39

- summary table [Jegerlehner, Nyffeler, Phys.Rept.477:1-110,2009]
  - $\pi^0(\eta,\ \eta') \rightarrow \gamma^*\gamma^*$ are consistent to total contributions
  - Among three PS mesons, π^0 takes about $\tilde{7}0\%$
  - calulation on the $\pi^0 \rightarrow \gamma^*\gamma^*$ can be duplicated to the η, η' sector

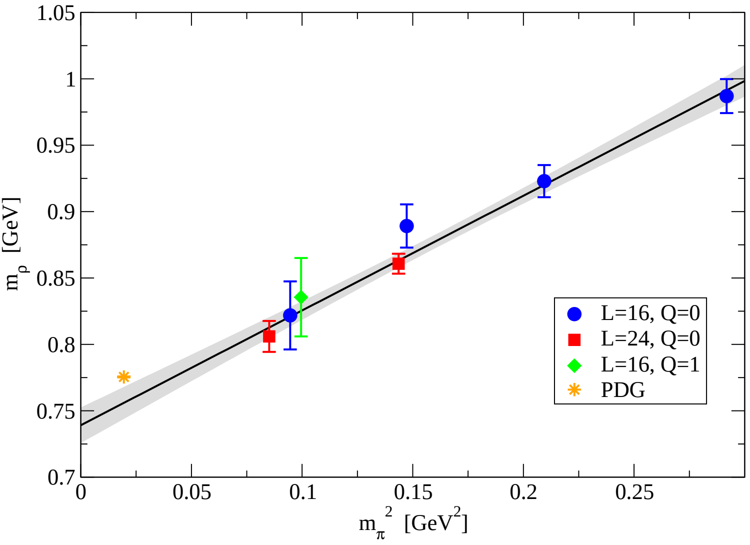
Non-perturbative nature

- Invariant mass spectrum for two-photon



- Three spikes presents three bound states: π^0, η, η'
- Bound states → confinement → LQCD

Rho mass


1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0
0.05
0.1
0.15
0.2
0.25
m_ρ [GeV]
m_π^2 [GeV^2]
L=16, Q=0
L=24, Q=0
L=16, Q=1
PDG