The scalar Pion Form Factor with Wilson fermions

Vera Gülpers

G. von Hippel H. Wittig

Helmholtz-Institut Mainz and Institut für Kernphysik
Johannes Gutenberg-Universität Mainz

June 28, 2012

HELMHOLTZ GEMEINSCHAFT
Helmholtz-Institut Mainz

Lattice 2012
Cairns, Australia

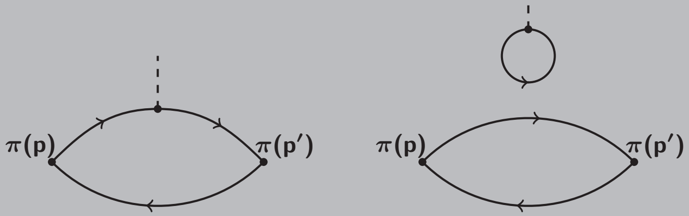
JG|U
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Outline

Outline

Introduction - The scalar Form Factor of the Pion

- describes the coupling of a charged pion to a scalar particle
- related to the low energy constant $\mathbf{l_4}$
- $\mathbf{F_s\left(Q^2\right) \equiv \langle \pi^+ (p') | m_d \bar{d} d + m_u \bar{u} u | \pi^+ (p) \rangle}$



- disconnected loop $\sum_{\vec{x}} \mathrm{Tr}\left(\mathbf{D^{-1}(x,x)}\right)$ requires all-to-all propagator
- method of stochastic sources very noisy
- stochastic estimate can be improved using a generalised hopping parameter expansion

Outline

The $\mathcal{O}(a)$-improved Wilson-Dirac Operator

- $\mathcal{O}(\mathbf{a})$-improved Wilson-Dirac operator

$$\mathbf{D_{sw}} = \frac{1}{2\kappa}\mathbb{1} + \mathbf{c_{sw}}\,\mathbf{B} - \frac{1}{2}\mathbf{H}$$

- rewrite as

$$\mathbf{D_{sw}} = \mathbf{A} - \frac{1}{2}\mathbf{H} = \mathbf{A}\left(\mathbb{1} - \frac{1}{2}\mathbf{A}^{-1}\mathbf{H}\right) \quad \text{with} \quad \mathbf{A} = \frac{1}{2\kappa}\mathbb{1} + \mathbf{c_{sw}}\,\mathbf{B}$$

The generalised Hopping Parameter Expansion

- geometric series

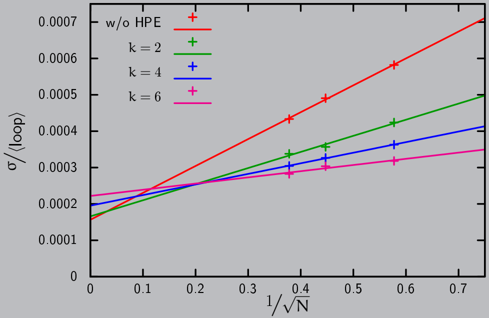
$$\mathbf{D_{sw}^{-1}} = \sum_{i=0}^{k-1}\left(\frac{1}{2}\mathbf{A}^{-1}\mathbf{H}\right)^{i}\mathbf{A}^{-1} + \left(\frac{1}{2}\mathbf{A}^{-1}\mathbf{H}\right)^{k}\mathbf{D_{sw}^{-1}}$$

- $\mathbf{D_{sw}^{-1}}$ on the right hand side estimated using stochastic sources
- cf. [Bali et al. arXiv:0910.3970]

The generalised Hopping Parameter Expansion

$$D_{sw}^{-1} = \sum_{i=0}^{k-1} \left(\frac{1}{2} A^{-1} H\right)^{i} A^{-1} + \left(\frac{1}{2} A^{-1} H\right)^{k} D_{sw}^{-1}$$

- $\langle \text{loop} \rangle = \left\langle \sum_{\vec{x}} \text{Tr} \left(D^{-1}(x,x) \right) \right\rangle_G$
- D_{sw}^{-1} on the right calculated with $N = 3, 5, 7$ stochastic sources
- Standard deviation without HPE and with $k = 2, 4, 6$ terms of HPE

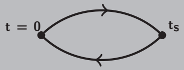


Outline

Building Ratios ($q^2 = 0$)

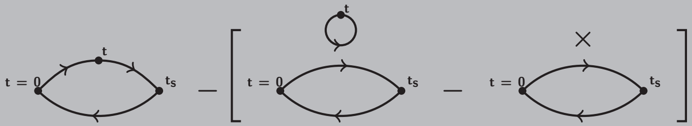
- 2pt-function:

$$C_{2pt} \sim |\langle 1|\phi(0)|0\rangle|^2 \left[e^{-m_\pi t_s} + e^{-m_\pi (T-t_s)} \right]$$



- 3pt-function with subtracted vacuum ($0 < t < t_s$)

$$C_{3pt} \sim |\langle 1|\phi(0)|0\rangle|^2 \, e^{-m_\pi t_s} \langle \pi|\mathcal{O}(0)|\pi\rangle$$



- Ratio of 3pt- and 2pt- function

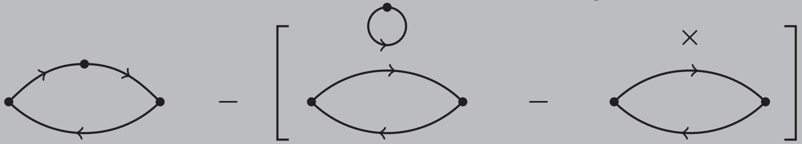
$$R = \frac{C_{3pt}}{C_{2pt}} \sim \langle \pi|\mathcal{O}(0)|\pi\rangle \frac{e^{-m_\pi t_s}}{e^{-m_\pi t_s} + e^{-m_\pi (T-t_s)}}$$

A few words on Renormalization

- chiral symmetry explicitly broken by Wilson fermions
- multiplicative and additive renormalization for the scalar operator

$$\left\langle \mathcal{O}^{\mathbf{R}} \right\rangle = \mathbf{Z_s} \left\langle \mathcal{O} - \mathbf{b_0} \right\rangle$$

- additive renormalization is cancelled when subtracting the vacuum



- for all form factor data shown in this talk the multiplicative renormalization is not taken into account
- the scalar radius is renormalization independent

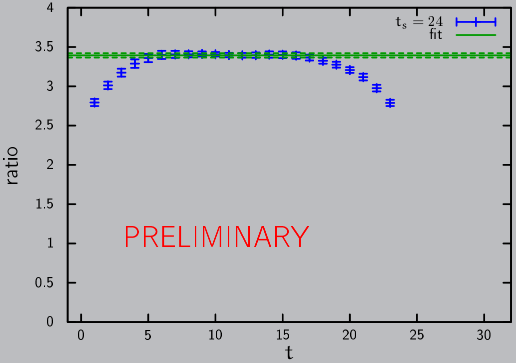
Connected part: Ratio

- CLS ensembles: $\mathcal{O}(\mathbf{a})$-improved Wilson-Dirac operator and Wilson glue $\mathbf{N_f = 2}$
- E4 lattice: $\mathbf{64 \times 32^3}$ lattice, $\mathbf{m_\pi = 554}$ MeV, $\mathbf{a = 0.0689}$ fm, 162 configurations

Connected part: Ratio

- CLS ensembles: $\mathcal{O}(\mathbf{a})$-improved Wilson-Dirac operator and Wilson glue $\mathbf{N_f = 2}$
- E4 lattice: $\mathbf{64} \times \mathbf{32^3}$ lattice, $\mathbf{m_\pi = 554}$ MeV, $\mathbf{a = 0.0689}$ fm, 162 configurations

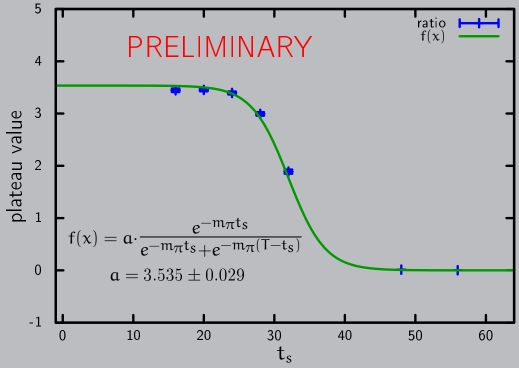
- ratios for fixed $\mathbf{t_s = 24}$ as an example
- data show a plateau
- constant fit to the plateau region



Connected part: Ratio vs t_s

$$R \sim \langle \pi | \mathcal{O}(0) | \pi \rangle \frac{e^{-m_\pi t_s}}{e^{-m_\pi t_s} + e^{-m_\pi (T - t_s)}}$$

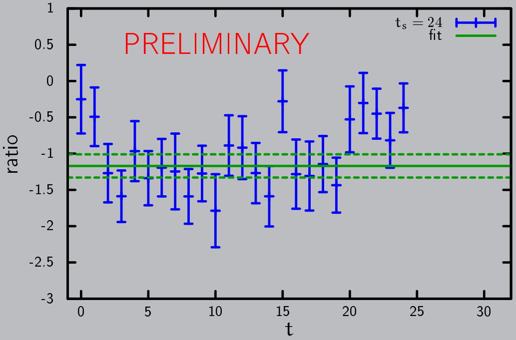
- plateau values for different t_s plotted against t_s
- data show the expected behaviour



$$\langle \pi | \mathcal{O} | \pi \rangle_{\text{con}}^{\text{bare}} (q^2 = 0) = 3.535 \pm 0.029$$

Disconnected part: Ratio

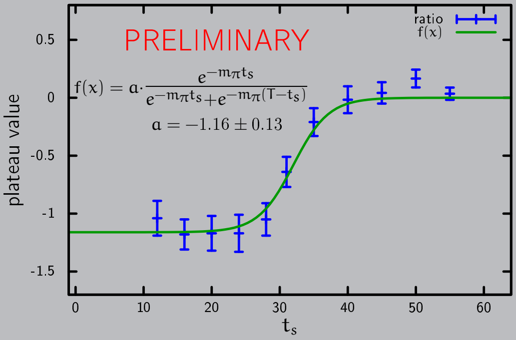
- statistic increased using 4 source positions for the 2pt-function
- ratios for fixed $\mathbf{t_s = 24}$ as an example
- a plateau is obtained
- constant fit to the plateau region



Disonnected part: Ratio vs $\mathbf{t_s}$

$$\mathbf{R} \sim \langle \pi | \mathcal{O}(0) | \pi \rangle \frac{\mathbf{e^{-m_\pi t_s}}}{\mathbf{e^{-m_\pi t_s} + e^{-m_\pi (T - t_s)}}}$$

- plateau values for different $\mathbf{t_s}$ plotted against $\mathbf{t_s}$
- data show the expected behaviour
- disconnected part of the scalar form factor $\approx \mathbf{9\sigma}$ away from zero



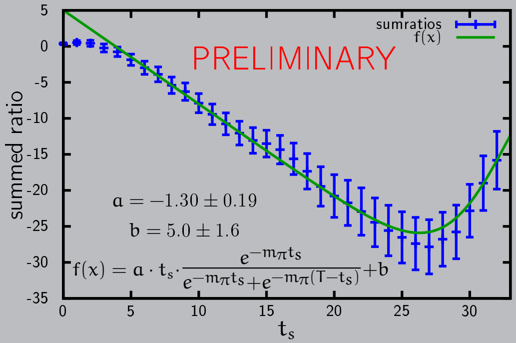
$$\langle \pi | \mathcal{O} | \pi \rangle^{\text{bare}}_{\text{disc}} (\mathbf{q^2 = 0}) = \mathbf{1.16 \pm 0.13}$$

Disconnected part: Summation method

summed operator insertion (cf. [Maiani et al. 1987]):

$$\sum_{t=0}^{t_s} R \sim b + t_s \cdot \frac{e^{-m_\pi t_s}}{e^{-m_\pi t_s} + e^{-m_\pi (T - t_s)}} \langle \pi | \mathcal{O} | \pi \rangle$$

- data for summed operator insertions plotted against $\mathbf{t_s}$
- expected behaviour
- value for the form factor compatible with value from plateau method
- statistical error slightly larger than error from plateau method



$$\langle \pi | \mathcal{O} | \pi \rangle^{\text{bare}}_{\text{disc}} (q^2 = 0) = 1.30 \pm 0.19$$

Outline

Building Ratios ($q^2 \neq 0$)

- 2pt-function:

$$C_{2pt}(p, t_s) \sim Z_p^2 \left[e^{-E_\pi t_s} + e^{-E_\pi (T - t_s)} \right]$$

- 3pt-function:

$$C_{3pt}(p, p', t, t_s) \sim Z_p Z_{p'} \langle \pi(p) | \mathcal{O}(q^2) | \pi(p') \rangle e^{-E_\pi (t_s - t)} e^{-E'_\pi t}$$

with $q^2 = (p - p')^2$

- Ratio

$$R = \frac{C_3(p, p', t, t_s)}{C_2(p, t_s)} \cdot \sqrt{\frac{C_2(p, t_s) C_2(p, t) C_2(p', (t_s - t))}{C_2(p', t_s) C_2(p', t) C_2(p, (t_s - t))}}$$

$$R \to \langle \pi(p) | \mathcal{O}(q^2) | \pi(p') \rangle \qquad \text{for} \quad 0 \ll t \ll t_s \ll T/2$$

Connected part: Ratio

- momentum insertion via Fourier transformation
- momentum insertion at the operator:

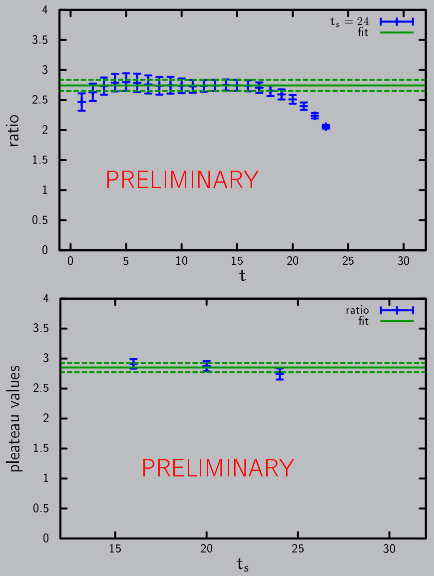
$$\vec{q} = (0,0,1)\frac{2\pi}{L \cdot a}$$

- momentum transfer:

$$Q^2 = -q^2 = 0.259\,\mathrm{GeV}^2$$

- constant fit to three different plateau values:

$$\langle \pi | \mathcal{O} | \pi \rangle_{\mathrm{con}}^{\mathrm{bare}} = 2.851 \pm 0.076$$



Disconnected part: Ratio

- momentum insertion via Fourier transformation
- momentum insertion at the operator:

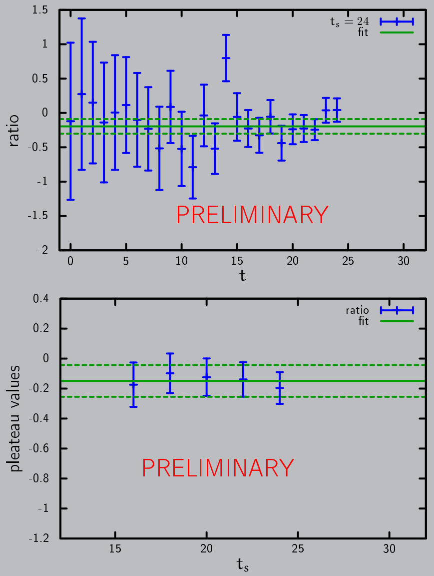
$$\vec{q} = (0,0,1)\frac{2\pi}{L\cdot a}$$

- momentum transfer:

$$Q^2 = -q^2 = 0.259\,\mathrm{GeV}^2$$

- constant fit to three different plateau values:

$$\langle\pi|\mathcal{O}|\pi\rangle^{\mathrm{bare}}_{\mathrm{disc}} = 0.149 \pm 0.106$$



Outline

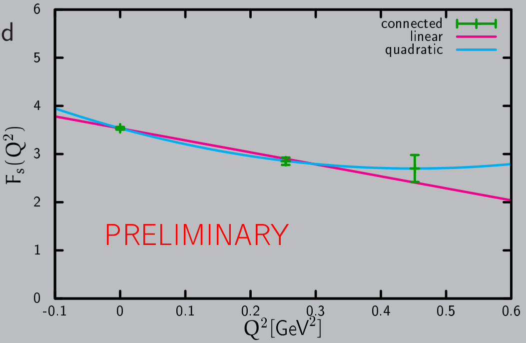
The $\mathbf{Q^2}$ dependence - connected form factor

$$\mathbf{F_s(Q^2) = F_s(0)\left(1 - \frac{1}{6}\left\langle r^2\right\rangle_s Q^2 + \mathcal{O}(Q^4)\right)}$$

The $\mathbf{Q}^2$ dependence - connected form factor

$$F_s(Q^2) = F_s(0)\left(1 - \frac{1}{6}\left\langle r^2\right\rangle_s Q^2 + \mathcal{O}(Q^4)\right)$$

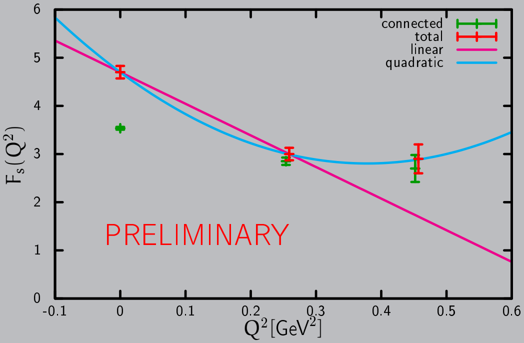
- considering only the connected part
- fitting a linear curve to the three data points:
  $\left\langle r^2\right\rangle_s = 0.164 \pm 0.020\,\mathrm{fm}^2$
- quadratic:
  $\left\langle r^2\right\rangle_s = 0.244 \pm 0.056\,\mathrm{fm}^2$



The Q^2 dependence

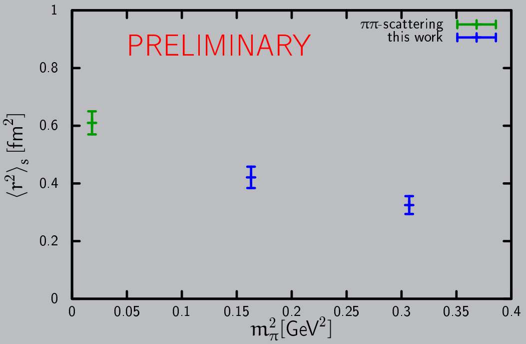
$$F_s(Q^2) = F_s(0)\left(1 - \frac{1}{6}\left\langle r^2\right\rangle_s Q^2 + \mathcal{O}(Q^4)\right)$$

- disconnected contribution to the scalar radius of same magnitude
  cf. [talk by A. Jüttner]
- linear:
  $\left\langle r^2\right\rangle_s = 0.325 \pm 0.031\,\mathrm{fm}^2$
- quadratic:
  $\left\langle r^2\right\rangle_s = 0.494 \pm 0.068\,\mathrm{fm}^2$



The scalar Radius - $\mathbf{m}_\pi^2$ dependence

- two different ensembles
- point at physical pion mass from $\pi\pi$-scattering data [Colangelo et al. arXiv:hep-ph/0103088]



Outline

Summary

- calculation of scalar form factor works with Wilson fermions
- disconnected contribution can be calculated precisely using the generalised hopping parameter expansion
- contribution of the disconnected part to the scalar radius is of same order as the connected

Outlook

- more ensembles
- calculate the connected part using partially twisted boundary conditions
- chiral extrapolation for the scalar radius
- study systematic errors
- calculation of other quantities including disconnected contributions