Excited state systematics in extracting nucleon electromagnetic form factors

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Form factors

- The matrix element of a nucleon interacting with an electro-magnetic current $V^\mu = \overline{\Psi}(x)\gamma^\mu\Psi(x)$ may be expressed as:

$$\langle N(p', s')|V_\mu|N(p, s)\rangle = \bar{u}(p', s')\left[\gamma_\mu F_1(Q^2) + i\frac{\sigma_{\mu\nu}q_\nu}{2m_N}F_2(Q^2)\right]u(p, s)$$

  where the matrix element is decomposed via the Dirac and Pauli form factors - $F_1(Q^2)$ and $F_2(Q^2)$ respectively
- These form factors are related to the Sachs form factors, G_E and G_M, that are measured in scattering experiments

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2}F_2(Q^2), \qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

- The form factors may be Taylor expanded in the momentum transfer Q^2

$$G(Q^2) = G(0)\left(1 + \frac{1}{6}\langle r^2\rangle Q^2 + \mathcal{O}(Q^4)\right)$$

- For the electromagnetic form factor
  - $G_E(0) = 1$ for the conserved current V_μ.
  - $G_M(0) = \mu$, measures the anomalous magnetic moment.
- The radii of the nucleon can be determined from:

$$\langle r_X^2\rangle = \frac{6}{G_X(Q^2)}\frac{\partial G_X(Q^2)}{\partial Q^2}\bigg|_{Q=0}$$

Simulation details

- $N_f = 2$ $\mathcal{O}(a)$ improved Wilson fermions.
- CLS ensembles

β	a[fm]	L/a	L[fm]	m_π[MeV]	no. meas
5.2	0.079	32	2.5	312, 363, 473	1696, 796, 1060
5.3	0.063	32	2.0	451	1344
		48	3.0	277, 324	1000, 796
5.5	0.050	48	2.4	430	600

Lattice formulation

- 3pt function

$$C_3 = e^{E'(ts-t)} e^{Et} Z_B^f Z_B^{*i} \sqrt{\frac{M^2}{EE'}} \frac{1}{EE'} \Gamma_{\beta\alpha} (-i\not{p}' + M)_{\alpha\gamma} O^{\mu}_{\gamma\gamma'} (-i\not{p} + M)_{\gamma'\beta}$$

- Want a ratio that cancels all pre-factors[Alexandrou et al.,2008]

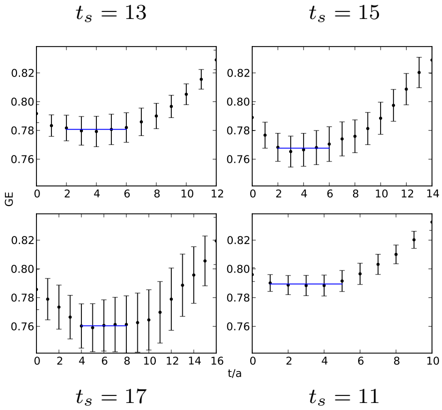
$$R(\overline{q}, t, t_s) = \frac{C_3(\overline{q}, t, t_s)}{C_2(\overline{0}, t_s)} \sqrt{\frac{C_2(\overline{q}, t_s - t) C_2(\overline{0}, t) C_2(\overline{0}, t_s)}{C_2(\overline{0}, t_s - t) C_2(\overline{q}, t_s) C_2(\overline{q}, t_s)}}$$

- for which G_E (in non-rel limit) and G_M may be extracted using:

$$R_{\gamma_0} = \sqrt{\frac{M+E}{2E}} G_E(Q^2) \qquad [R_{\gamma_i}]_{i=1,2} = \epsilon_{ij} p_j \sqrt{\frac{1}{2E(E+M)}} G_M(Q^2)$$

Systematics of extraction

- Form factors should be independent of time t and sink position t_s
- observe exponentially decaying excited states from source and sink
- Simple plateau fits show a trend of higher values for decreasing t_s



- Take excited states into account

Systematics of extraction

- Contributions to the ratio from the ground and excited states may be factorised as

$$R(\overline{q}, t, t_s) = R^0(\overline{q}, t, t_s) \left(1 + \mathcal{O}(e^{-\Delta t}) + \mathcal{O}(e^{-\Delta(t_s - t)}) + \mathcal{O}(e^{-\Delta t_s})\right)$$

- The fit function becomes

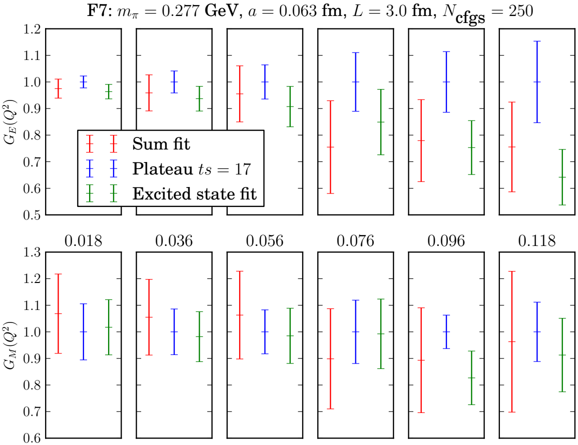
$$R(\overline{q}, t, t_s) = G_{E,M} + b_1 e^{-\Delta t} + b_2 e^{-\Delta(t_s - t)} + b_3 e^{-\Delta t_s}$$

- Summed operator insertion method [L.Maiani et al.,1987]

$$S(t_s) = \sum_{t=0}^{t_s} R(\overline{q}, t, t_s) \rightarrow c + t_s \left(G_{E,M} + \mathcal{O}(e^{-\Delta t_s})\right)$$

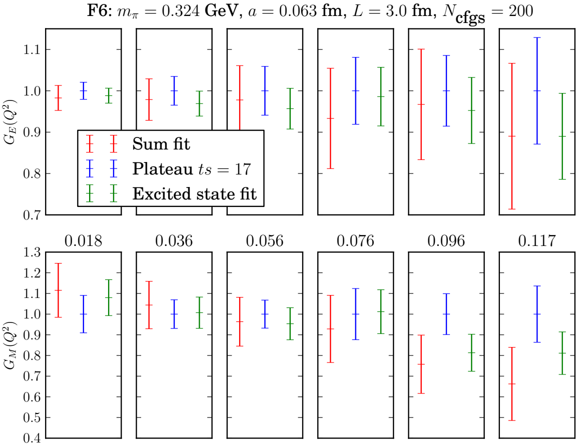
- Computing $S(t_s)$ for several t_s, form factors may be extracted from the slope
- Since $t_s > t, (t - t_s)$, excited state contr. should be more suppressed

Comparison of methods



- Plateau method: $t_s \sim 1\text{fm}$

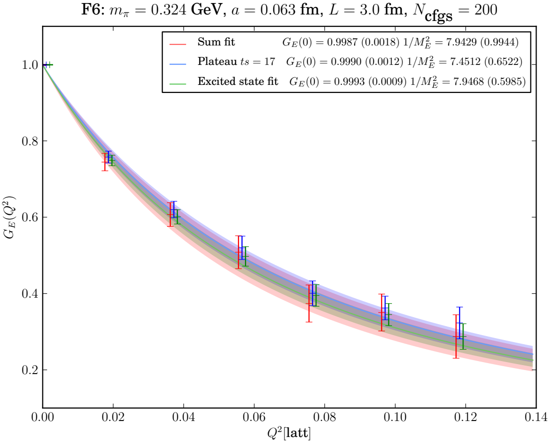
Comparison of methods



- Plateau method: $t_s \sim 1\text{fm}$

$G_E(Q^2)$ - preliminary

- using a dipole ansatz for the form factors $G_{E,M}(Q^2) = \frac{G_{E,M}(0)}{\left(1+\frac{Q^2}{M^2_{E,M}}\right)^2}$



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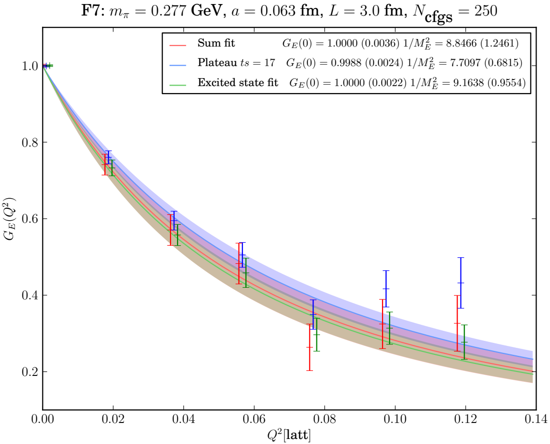
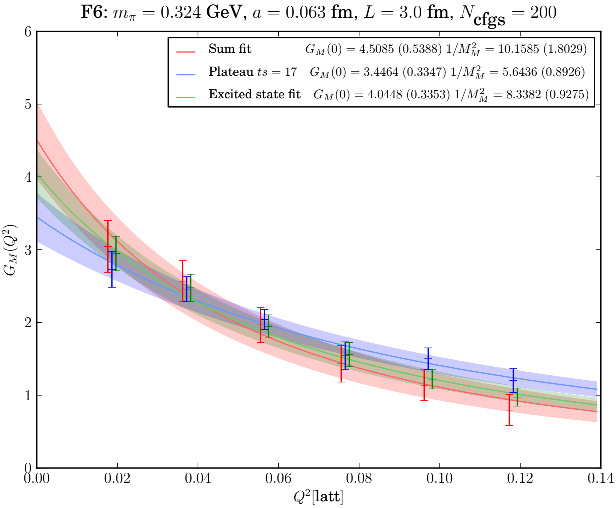
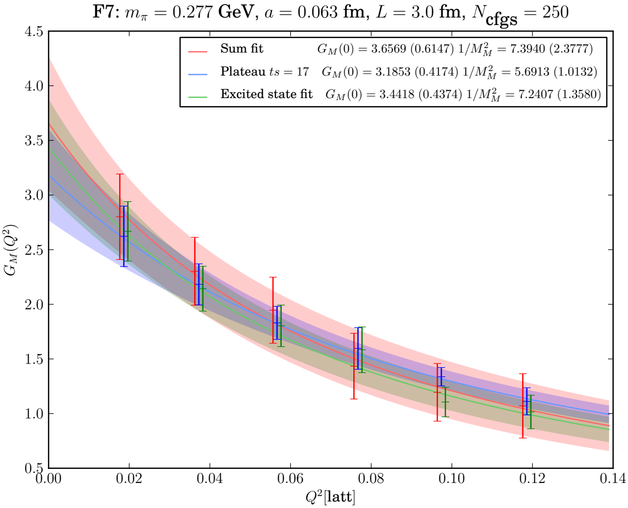


$G_M(Q^2)$ - preliminary

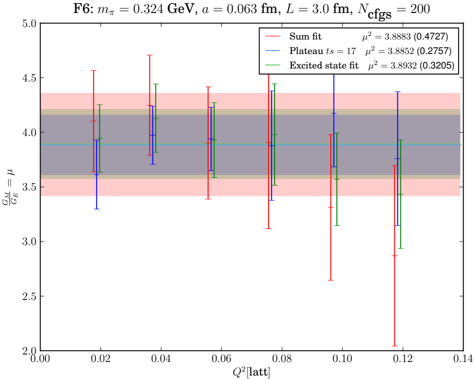


$G_M(Q^2)$ - preliminary



Magnetic moment extraction - preliminary

- assumed in phenomenology that electric and magnetic radii are very close to each other
- use this to get better handle on magnetic moment $\mu = G_M(0)$, using $\mu = \frac{G_M(Q^2)}{G_E(Q^2)}$



chiral behaviour $\langle r_1^2 \rangle$, κ, and $\langle r_2^2 \rangle$

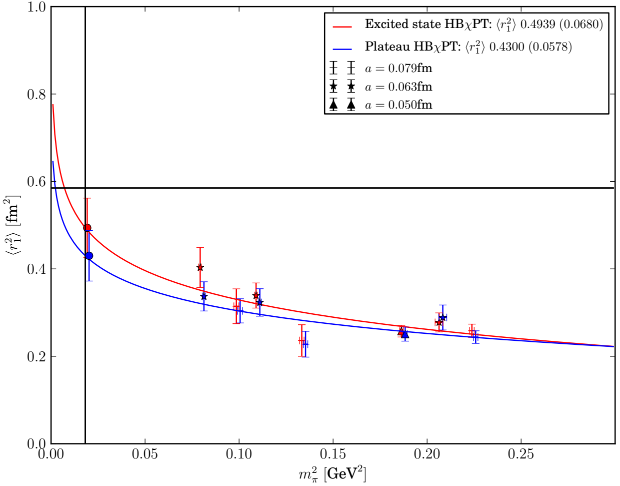
- we relate the Sachs form factors to the Dirac and Pauli form factors

$$\frac{1}{M_E^2} = \frac{r_1^2}{12} + \frac{\kappa}{8m_N^2}, \qquad \frac{1}{M_M^2} = \frac{r_1^2 + \kappa r_2^2}{12(1+\kappa)}$$

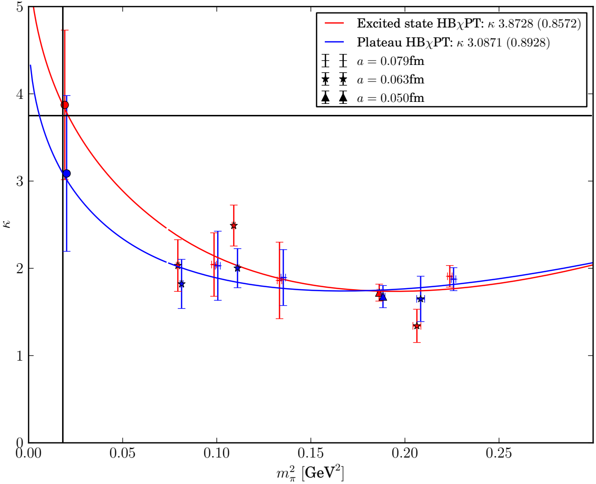
- where $\kappa = G_M(0) - 1 = \mu - 1$
- led by HBχPT we use the following fit forms to extrapolate our results to the chiral limit [M.Göckeler et al.,2005]

$$\begin{aligned}
\langle r_1^2 \rangle &= c_1 + c_2 \log(m_\pi^2) \\
\kappa \langle r_2^2 \rangle &= \frac{d_1}{m_\pi} + \frac{d_2}{\sqrt{\Delta^2 - m_\pi^2}} \log\left(\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1} \right) \\
\kappa &= \kappa_0 + K(m_\pi) - 8E_1^{(r)}(\lambda) m_N m_\pi^2
\end{aligned}$$

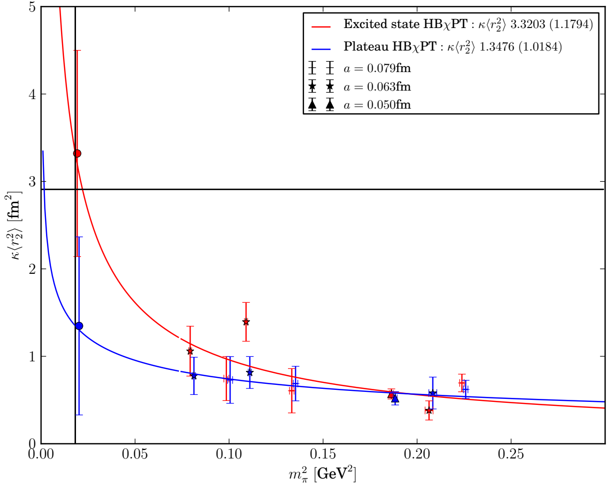
$\langle r_1^2 \rangle$ - preliminary



κ - preliminary



$\kappa\langle r_2^2\rangle$ - preliminary

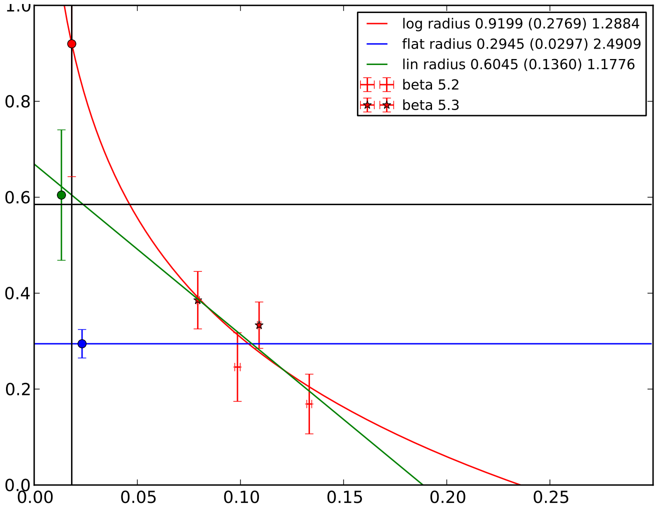


Summary and Outlook

- When applying the "Plateau" method, our results show a downwards trend for the form factors for increasing t_s.
- There is good agreement between methods (excited state and summed insertions) that account for excited state contributions.
- Furthermore there is a trend for a steeper slope to be extracted from these results.
- The statistical errors from the three methods highlight the need for greater statistics and for more chiral points, for which the Monte Carlo ensembles exist, but are yet to be analysed.
- The results indicate the importance of excited states and should be a consideration in studies of potential systematic effects.

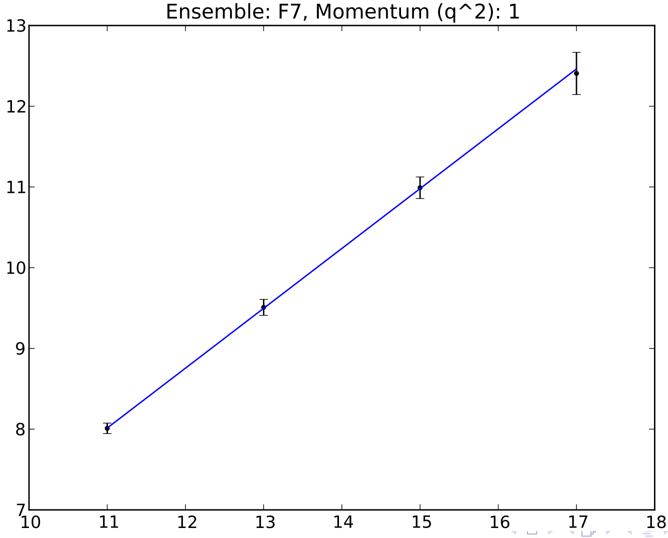
Backup

Backup



log radius 0.9199 (0.2769) 1.2884
flat radius 0.2945 (0.0297) 2.4909
lin radius 0.6045 (0.1360) 1.1776
beta 5.2
beta 5.3
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
0.20
0.25

Backup



Ensemble: F7, Momentum (q^2): 1
13
12
11
10
9
8
7
10
11
12
13
14
15
16
17
18

Backup - Preliminary Results

	Ex-state (preliminary)	Plateau (preliminary)	experiment
$\langle r_1^2 \rangle$	$0.49(7)$	$0.43(6)$	$0.585(17)$
$\langle r_2^2 \rangle$	$0.85(36)$	$0.44(35)$	$0.776(11)$
κ	$3.9(9)$	$3.1(9)$	3.701

Backup

$$
\begin{aligned}
K(m_\pi) &= -\frac{g_A^2 m_\pi m_N}{4\pi F_\pi^2} \\
&+ \frac{2c_A^2 \Delta m_N}{9\pi^2 F_\pi^2} \left\{ \sqrt{1 - \frac{m_\pi^2}{\Delta^2}} \log\left(\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi^2} - 1} \right) + \log\left(\frac{m_\pi}{2\Delta} \right) \right\} \\
&+ \frac{c_A c_V g_A m_N m_\pi^2}{9\pi^2 F_\pi^2} \log\left(\frac{2\Delta}{\lambda} \right) + \frac{4 c_A c_V g_A m_N m_\pi^3}{27\pi F_\pi^2 \Delta} \\
&- \frac{8 c_A c_V g_A \Delta^2 m_N}{27\pi^2 F_\pi^2} \\
&\cdot \left\{ \left(1 - \frac{m_\pi^2}{\Delta^2}\right)^{3/2} \log\left(\frac{\Delta}{m_\pi} + \sqrt{\frac{\Delta^2}{m_\pi} - 1} \right) + \left(1 - \frac{3m_\pi^2}{2\Delta^2}\right) \log\left(\frac{m_\pi}{2\Delta} \right) \right.
\end{aligned}
$$