Neutron Electric Dipole Moment in the Standard Model and beyond from Lattice QCD

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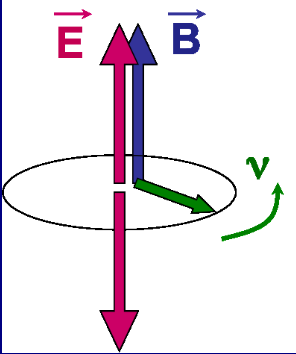
Introduction
Dipole Moments



$$H = -d\,\vec{E}\cdot\vec{S} - \mu\,\vec{B}\cdot\vec{S}$$

- Unaligned spin precesses in Electric and Magnetic Fields.
- Precession Frequency depends on E through EDM d.
- Change in Precession Frequency on flipping E measures EDM.

$\mu\,\vec{B}\cdot\vec{S}$ is even under C, P, and T

$\vec{B}$, $\vec{S}$ are parity even:	$\vec{B} \longleftrightarrow +\vec{B}$	$\vec{S} \longleftrightarrow +\vec{S}$
$\vec{B}$, $\vec{S}$ are time reversal odd:	$\vec{B} \longleftrightarrow -\vec{B}$	$\vec{S} \longleftrightarrow -\vec{S}$
$\vec{B}$, $\vec{S}$ are charge conjugation even:	$\mu\,\vec{B} \longleftrightarrow +\mu\,\vec{B}$	$\vec{S} \longleftrightarrow +\vec{S}$

$d\,\vec{E}\cdot\vec{S}$ term violates P, T, and CP

violates Parity:	$\vec{E} \longleftrightarrow -\vec{E}$	$\vec{S} \longleftrightarrow +\vec{S}$
violates Time reversal:	$\vec{E} \longleftrightarrow +\vec{E}$	$\vec{S} \longleftrightarrow -\vec{S}$
conserves Charge conjugation:	$d\,\vec{E} \longleftrightarrow +d\,\vec{E}$	$\vec{S} \longleftrightarrow +\vec{S}$

Introduction
Sakharov Conditions for Baryogenesis

CP violation needed in the universe.
Observed baryon asymmetry: $n_B/n_\gamma = 6.1^{+0.3}_{-0.2} \times 10^{-10}$.

WMAP + COBE 2003

Without CP violation, freezeout ratio: $n_B/n_\gamma \approx 10^{-20}$.

Kolb and Turner, *Front. Phys.* **69** (1990) 1.

Either asymmetric initial conditions or baryogenesis!

Sakharov Conditions

Sakharov, *Pisma Zh. Eksp. Teor. Fiz.* **5** (1967) 32.

- Baryon Number violation
- C, CP and T violation
- Out of equilibrium evolution

Introduction
Standard Model CP Violation

Two sources of CP violation in the Standard Model.

- Complex phase in CKM quark mixing matrix.
  - Too small to explain baryon asymmetry
  - Gives a tiny ($\sim 10^{-32}$ e-cm) contribution to nEDM

Dar arXiv:hep-ph/0008248.

- Effective $\Theta G\tilde{G}$ interaction from QCD instantons
  - Effects suppressed at high energies
  - nEDM limits constrain $\Theta \lesssim 10^{-10}$

Crewther *et al.*, *Phys. Lett.* **B88** (1979) 123.

Contributions from beyond the standard model

- Needed to explain baryogenesis
- May have large contribution to EDM

Introduction

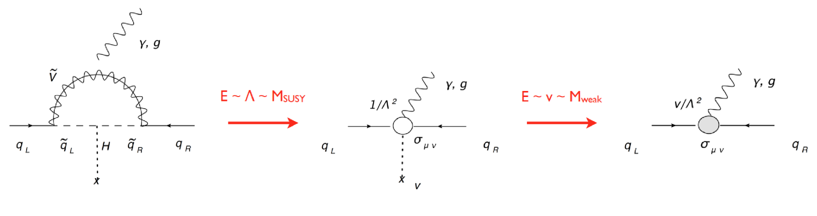
Effective Field Theory

Parameterize BSM contributions using an effective field theory at the weak scale. Two important dimension six operators are the Electric and Chromoelectric dipole moments of the quark.

$$\begin{aligned}
\mathcal{S} &= \mathcal{S}_{QCD}^{\text{CP Even}} - i\,\Theta\frac{g^2}{16\pi^2}\int d^4x\, G^{\mu\nu}\tilde{G}_{\mu\nu} \\
&+ \frac{i\,e\,d_u^\gamma}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}\tilde{H}U + \frac{i\,e\,d_d^\gamma}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5 F^{\mu\nu}HD \\
&+ \frac{i\,g_3\,d_u^G}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5\lambda^A G^{\mu\nu\,A}\tilde{H}U + \frac{i\,g_3\,d_d^G}{\Lambda_{\text{BSM}}^2}\bar{Q}\sigma_{\mu\nu}\gamma_5\lambda^A G^{\mu\nu\,A}HD \\
&+ \cdots .
\end{aligned}$$

The two quark dipole moments are generated at 3-loops in the standard model and give tiny nEDM ($\sim 10^{-34}$ e-cm).

They are generated at one loop in BSM.



Expected contribution is around experimental limit $\sim 2.9 \times 10^{-26}$e-cm.

Baker *et al.*, *Phys. Rec. Lett.* **97** (2006) 131801.

Matrix Elements
Model expectations

Model analysis estimate of the neutron electric dipole moment:

$$d_n \approx \frac{8\pi^2}{M_n^3}\Bigg[-\frac{2m_*}{3}\frac{\partial\langle\bar{q}\sigma q\rangle_F}{\partial F}\left(\bar{\Theta} + g_s\frac{\langle\bar{q}G\sigma q\rangle}{2\langle\bar{q}q\rangle}\sum\frac{d_q^G}{m_q}\right)$$

$$+\frac{\langle\bar{q}q\rangle}{3}\left(4\,d_d^\gamma - d_u^\gamma\right)$$

$$+\,g_s\frac{\langle\bar{q}G\sigma q\rangle}{6\langle\bar{q}q\rangle}\left(4\,d_d^G\frac{\partial\langle\bar{d}\sigma d\rangle_F}{\partial F} - d_u^G\frac{\partial\langle\bar{u}\sigma u\rangle_F}{\partial F}\right)\Bigg]$$

$$\approx \left(\frac{4}{3}d_d^\gamma - \frac{1}{3}d_u^\gamma\right) - \frac{2e\langle\bar{q}q\rangle}{M_n f_\pi^2}\left(\frac{2}{3}d_d^G + \frac{1}{3}d_u^G\right),$$

assuming the first term vanishes by Peccei-Quinn mechanism.

Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119

Numerically,

$$d_n(\bar\Theta) \approx (1 \pm 0.5)\frac{|\langle \bar q q\rangle|}{(225\mathrm{MeV})^3}\,\bar\Theta\,(2.5\times 10^{-16}\ \text{e-cm})$$

$$d_n(d_q^{\gamma,G}) \approx -d_n(\bar\Theta = \Theta_{\rm ind}) + (1 \pm 0.5)\frac{|\langle \bar q q\rangle|}{(225\mathrm{MeV})^3}\left[1.1\,(d_d^G + 0.5\,d_u^G)\,e + 1.4\,(d_d^\gamma - 0.25\,d_u^\gamma)\right],$$

where

$$\Theta_{\rm ind} \approx (3.1\times 10^{-17}\mathrm{cm})^{-1}\sum \frac{\mathrm{d_q^G}}{\mathrm{m_q/MeV}}\frac{|\mathrm{m_0^2}|}{(0.8\mathrm{GeV})^2}$$

is the minimum of the PQ potential.

Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119

Note that the quark dipole moments violate chirality, and, hence, are expected to be of the scale

$$\kappa_q = \frac{m_q}{16\pi^2 M_\Lambda^2} = 1.3 \times 10^{-25}\text{e-cm}\,\frac{m_q}{1\text{MeV}}\left(\frac{1TeV}{M_\Lambda}\right)^2 .$$

Rough estimates can also be made of the other dimension 6 operators:

Weinberg Operator:

$$|d_n(w)| \approx (4.4 \times 10^{-22}\,\text{e-cm})\left(\frac{w(1\text{GeV})}{1\text{TeV}^{-2}}\right)$$

Four-quark Operators:

$$|d_n(C)| \approx (1.2 \times 10^{-24}\,\text{e-cm})\left(\frac{C_{bd}(m_b) + C_{db}(m_b)}{1\text{TeV}^{-2}}\right)$$

Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119

Matrix Elements
Lattice Basics

We can extract nEDM in two ways.

- As the difference of the energies of spin-aligned and anti-aligned neutron states:

$$d_n = \frac{1}{2}\left(M_{n\downarrow} - M_{n\uparrow}\right)\Big|_{E=E\uparrow}$$

- By extracting the CP violating form factor of the electromagnetic current.

$$\begin{aligned} \langle n|J_\mu^{\mathrm{EM}}|n\rangle &\sim \frac{F_3(q^2)}{2M_n}\bar{n}\, q_\nu \Sigma^{\mu\nu}\gamma_5\, n \\ d_n &= \lim_{q^2\to 0}\frac{F_3(q^2)}{2M_n} \end{aligned}$$

Difficult to perform simulations with complex $\not{CP}$ action

Expand and calculate correlators of the $\not{CP}$ operator:

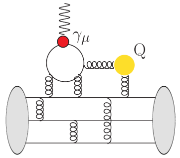
$$
\begin{aligned}
\langle\, C^{\not{CP}}(x,y,\ldots)\,\rangle_{\text{CP}+\not{CP}} &= \int [\mathcal{DA}] \exp\left[-\int d^4x(\mathcal{L}^{\text{CP}} + \mathcal{L}^{\not{CP}})\right] \\
&\qquad \times C^{\not{CP}}(x,y,\ldots) \\
&\approx \int [\mathcal{DA}] \exp\left[-\int d^4x \mathcal{L}^{\text{CP}}\right] \\
&\qquad \times \left(-\int d^4x \mathcal{L}^{\not{CP}}\right) C^{\not{CP}}(x,y,\ldots) \\
&= \langle\, C^{\not{CP}}(x,y,\ldots)\, \mathcal{L}^{\not{CP}}(p_\mu = 0)\,\rangle_{\text{CP}}
\end{aligned}
$$

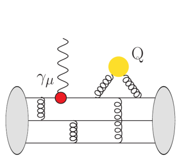
Matrix Elements

Topological charge

To find the contribution of $\bar{\Theta}$, we note that $\int d^4x G\tilde{G} = Q$, the topological charge. So, we need the correlation between the electric current and the topological charge.

$$\left\langle n \left| (\frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d) Q \right| n \right\rangle = \frac{1}{2}\left\langle n \left| (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d) Q \right| n \right\rangle + \frac{1}{6}\left\langle n \left| (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d) Q \right| n \right\rangle$$





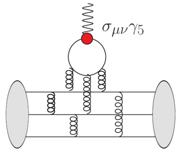
Matrix Elements

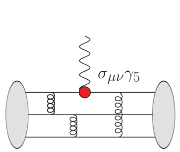
Quark Electric Dipole Moment

Since the quark electric dipole moment directly couples to the electric field, we just need to calculate its matrix elements in the neutron state.

$$\langle n \left| d_u^\gamma \, \bar{u}\Sigma^{\mu\nu} u + d_d^\gamma \, \bar{d}\Sigma^{\mu\nu} d \right| \rangle =$$

$$\frac{d_u^\gamma + d_d^\gamma}{2} \langle n \left| \bar{u}\Sigma^{\mu\nu} u + \bar{d}\Sigma^{\mu\nu} d \right| n \rangle + \frac{d_u^\gamma - d_d^\gamma}{2} \langle n \left| \bar{u}\Sigma^{\mu\nu} u - \bar{d}\Sigma^{\mu\nu} d \right| n \rangle$$





Matrix Elements
Quark Chromoelectric Moment

The effect of the quark chromoelectric moment can be simplified using the Feynman-Hellmann Theorem.

$$\left\langle n \middle| J_\mu \int d^4x (d_u^G\, \bar{u}\Sigma^{\nu\kappa} u + d_d^G\, \bar{d}\Sigma^{\nu\kappa} d)\, \tilde{G}_{\nu\kappa} \middle| n \right\rangle$$
$$= \frac{\partial}{\partial A_\mu} \left\langle n \middle| \int d^4x (d_u^G\, \bar{u}\Sigma^{\nu\kappa} u + d_d^G\, \bar{d}\Sigma^{\nu\kappa} d)\, \tilde{G}_{\nu\kappa} \middle| n \right\rangle_E$$

where the subscript E refers to the correlator calculated in the presence of a background electric field.

Lattice QCD
Renormalization

Renormalization of the lattice operators can be performed non-perturbatively.

- Topological charge is well studied and understood.
- Electric current and Quark Elecric Dipole moment operators are quark bilinears: well understood renormalization procedure.
- Quark Chromoelectric Moment operator mixes with the Topological charge; need to disentangle.

Also need to calculate the influence of Chromoelectric moment of the quark on the potential for Θ.

Lattice QCD
State of the Art

Neutron electric dipole moment from

- Topological charge:
  - Limits exist from lattie calculations
- Quark Electric Dipole Moment:
  - Same as the tensor charge of the nucleon
  - Connected diagrams calculated
- Quark Chromoelectric Dipole Moment: not yet calculated

Lattice QCD
Needed Calculations

Preliminary calculations needed before one can estimate errors and resource requirements.
For preliminary calculations

- Use previously generated staggerred lattices
- Use Clover valence quarks
- Study
  - Statistical signal
  - Chiral behavior
  - Dependence on lattice spacing
  - Excited state contamination

Lattice QCD
Outlook

The connected diagram for Quark Electric Dipole Moment will soon reach 20% precision.
The calculation of the disconnected diagrams, and the other operators need more study.
Remaining systematic errors not expected to be major.

- nEDM not overly sensitive to neglected EM and isospin-breaking
- Modern calculations include dynamical charm