Electromagnetic Form-Factors of the $\Lambda(1405)$ in (2+1)-flavour Lattice QCD





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First Results

Outline

Background

Analysis Techniques

Correlation Matrix Method Extracting Form Factors

First Results

Electric Form Factors Magnetic Form Factor Effect of Partial Quenching

Conclusion

The $\Lambda(1405)$

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- It has a mass of 1405.1 MeV.
 - This is lower than the lowest odd-parity nucleon state, even though it has has a valence strange quark.
- What is special about this state? Why does it lie so low?

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• Our recent work has successfully isolated three low-lying states.

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- We used a correlation matrix analysis together with source and sink smearing.
- Using the same technique, we can investigate the electromagnetic structure of these states.
 - $\hfill\square$ Negative-parity need to be careful with formalism.

Simulation Details

- We use the PACS-CS (2 + 1)-flavour lattices, available through the ILDG.
 - S. Aoki, et al (PACS-CS Collaboration), Phys. Rev. D 79, 034503 (2009)
 - $\hfill\square$ Lattice size of $32^3\times 64$ with $\beta=1.90.$
 - □ Physical lattice spacing of a = 0.0907(33) fm.
 - □ 5 pion masses, ranging from 640 MeV down to 156 MeV.
 - \Box Fixed strange quark $\kappa_s = 0.13640$.

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 - This gives a kaon that's slightly too heavy, so we partially quench the strange quark sector by using κ_s = 0.13665 for the valence quarks.
- We focus on the heaviest quark mass, with $\kappa_{u,d} = 0.13700$.
 - □ There are 400 independent configurations available.
 - $\hfill\square$ We have analysed 128 configurations so far first look.

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- Consider a set of N operators \u03c6_i(x) that couple to the baryon we are interested in.
- We calculate the N × N matrix of cross-correlation functions from these operators,

$$\begin{split} G_{ij}(t,\mathbf{p}) &= \sum_{\mathbf{x}} \mathrm{e}^{-\mathrm{i}\mathbf{p}\cdot\mathbf{x}} \operatorname{tr} \left(\Gamma \left\langle \Omega | \chi_i(x,t) \overline{\chi}_j(0) | \Omega \right\rangle \right) \\ &= \sum_{\alpha=1}^N Z_i^{\alpha}(\mathbf{p}) Z_j^{\alpha\dagger}(\mathbf{p}) \mathrm{e}^{-E_{\alpha}(\mathbf{p})t} \operatorname{tr} \left(\Gamma \sum_{s} u^{\alpha}(p,s) \overline{u}^{\alpha}(p,s) \right), \end{split}$$

where Z_i^{α} and $Z_i^{\alpha\dagger}$ are the couplings of the operator χ_i to the state α at the source and sink, respectively.

Correlation Matrix Analysis

Construct a set of N "perfect" operators φ_α(x) that completely isolate the N lowest states, so that

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Using the linearity of the operator space, we write

$$\phi_{lpha} = \sum_{i} v_{i}^{lpha}(\mathbf{p})\chi_{i}, \text{ and}$$

 $\overline{\phi}_{lpha} = \sum_{i} u_{i}^{lpha}(\mathbf{p})\overline{\chi}_{i}.$

Correlation Matrix Analysis

The coefficient-vectors u^α and v^α form the left and right generalised eigenvectors of the matrices G(t₀ + Δt) and G(t₀):

$$G(t_0 + \Delta t) \mathbf{u}^{\alpha} = e^{-m_{\alpha}\Delta t} G(t_0) \mathbf{u}^{\alpha}$$
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Using this, we can define eigenstate-projected correlation functions

$$G^{lpha}(t) := \mathbf{v}^{lpha op} G(t) \mathbf{u}^{lpha}$$
,

which contain correlation functions for a single energy eigenstate.

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- There are quite a few operators that couple to the Λ baryon.
 - $\hfill\square$ We can exploit the possible flavour symmetry structures:

$$\begin{split} \chi_i^8 &= \epsilon^{abc} (2(u_a^\top A_i d_b) B_i s_c + (u_a^\top A_i s_b) B_i d_c - (d_a^\top A_i s_b) B_i u_c) / \sqrt{6}, \\ \chi_i^1 &= -2 \epsilon^{abc} (-(u_a^\top A_i d_b) B_i s_c + (u_a^\top A_i s_b) B_i d_c - (d_a^\top A_i s_b) B_i u_c), \\ \chi_i^c &= \epsilon^{abc} ((u_a^\top A_i s_b) B_i d_c - (d_a^\top A_i s_b) B_i u_c) / \sqrt{2}, \end{split}$$

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□ We can also use different Dirac structures:

$$\begin{array}{ll} A_1=C\gamma_5, & A_2=C, & A_4=C\gamma_5\gamma_4\\ B_1=\mathbb{I}, & B_2=\gamma_5, & B_4=\mathbb{I} \end{array}$$

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 - □ Not enough signal to isolate 28 states, but we can use smaller subsets and compare to ensure we have completely isolated the states.
 - □ These results use χ_1^1 , χ_1^8 , and χ_2^8 , with 16 and 100 sweeps, giving a 6 × 6 matrix.

 To extract form factors we calculate the both the two-point and three-point correlation matrices,

$$\mathcal{G}_{ij}(\Gamma; t_2; \mathbf{p}') = \sum_{\mathbf{x}_2} e^{-i\mathbf{p}' \cdot \mathbf{x}_2} \operatorname{tr}(\Gamma \langle \Omega | \chi_i(x_2) \overline{\chi}_j(0) | \Omega \rangle)$$

and

$$\begin{aligned} \mathcal{G}_{ij}^{\mu}(\Gamma;t_{2},t_{1};\mathbf{p}',\mathbf{p}) \\ &= \sum_{\mathbf{x}_{1},\mathbf{x}_{2}} \mathrm{e}^{-\mathrm{i}\mathbf{p}'\cdot\mathbf{x}_{2}} \mathrm{e}^{i(\mathbf{p}'-\mathbf{p})\cdot\mathbf{x}_{1}} \operatorname{tr}(\Gamma \langle \Omega | \chi_{i}(x_{2}) j^{\mu}(x_{1}) \overline{\chi}_{j}(0) | \Omega \rangle), \end{aligned}$$

where the χ_i and χ_j are interpolating fields that couple to the state of interest, and $x_i := (\mathbf{x}_i, t_i)$.

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We use the correlation matrix analysis to project out both two- and three-point correlation functions for individual states:

$$egin{aligned} &\mathcal{G}_lpha(\mathsf{\Gamma};t_2;\mathbf{p}'):=v_i^lpha(\mathbf{p}')\mathcal{G}_{ij}(\mathsf{\Gamma};t_2;\mathbf{p}')u_j^lpha(\mathbf{p}'), \ &\mathcal{G}_lpha^\mu(\mathsf{\Gamma};t_2,t_1;\mathbf{p}',\mathbf{p}):=v_i^lpha(\mathbf{p}')\mathcal{G}_{ij}^\mu(\mathsf{\Gamma};t_2,t_1;\mathbf{p}',\mathbf{p})u_j^lpha(\mathbf{p}). \end{aligned}$$

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- The eigenvectors \mathbf{u}^{α} and \mathbf{v}^{α} are momentum dependent.
- We use the two-point eigenvectors to project the three-point correlation functions.

• For sufficiently large $t_2 - t_1$ and t_1 , $\mathcal{G}^{\mu}_{\alpha}$ takes the form

$$\sum_{s,s'} e^{-\mathcal{E}_{p'}(t_2-t_1)} e^{-\mathcal{E}_{p}t_1} \operatorname{tr}(\Gamma \langle \Omega | \chi_i | p', s' \rangle \langle p', s' | j^{\mu} | p, s \rangle \langle p, s | \overline{\chi}_j | \Omega \rangle),$$

where the current matrix element is

$$\langle p', s'|j^{\mu}|p, s\rangle = \left(\frac{M^2}{E_{\mathbf{p}}E_{\mathbf{p}'}}\right)^{1/2} \overline{u} \left(F_1(q^2)\gamma^{\mu} + \mathrm{i}F_2(q^2)\sigma^{\mu\nu}\frac{q_{\nu}}{2M}\right) u,$$

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• The F_1 and F_2 form factors are related to the Sachs forms through

$$\mathcal{G}_{\mathsf{E}}(q^2) = F_1(q^2) - rac{q^2}{(2M)^2}F_2(q^2)$$

 $\mathcal{G}_{\mathsf{M}}(q^2) = F_1(q^2) + F_2(q^2)$

• To eliminate the time dependence of the three-point correlation function, we construct the ratio

$$\begin{aligned} R^{\mu}_{\alpha}(\boldsymbol{\Gamma}_{2},\boldsymbol{\Gamma}_{1};t_{2},t_{1};\boldsymbol{p}',\boldsymbol{p}) \\ &:= \left(\frac{\mathcal{G}^{\mu}_{\alpha}(\boldsymbol{\Gamma}_{1};t_{2},t_{1};\boldsymbol{p}',\boldsymbol{p})\mathcal{G}^{\mu}_{\alpha}(\boldsymbol{\Gamma}_{1};t_{2},t_{1};\boldsymbol{p},\boldsymbol{p}')}{\mathcal{G}_{\alpha}(\boldsymbol{\Gamma}_{2};t_{2};\boldsymbol{p}')\mathcal{G}_{\alpha}(\boldsymbol{\Gamma}_{2};t_{2};\boldsymbol{p})}\right)^{1/2} \end{aligned}$$

We then define the reduced ratio

$$\overline{R}^{\mu}_{\alpha} := \left(\frac{2E_{\mathbf{p}}}{E_{\mathbf{p}} + M}\right)^{1/2} \left(\frac{2E_{\mathbf{p}'}}{E_{\mathbf{p}'} + M}\right)^{1/2} R^{\mu}_{\alpha}$$

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A suitable choice of Γ₁ and Γ₂ allows us to extract the Sachs form factors:

$$\mathcal{G}_{\mathsf{E}}^{\alpha\pm}(q^2) = \overline{R}^4_{\alpha}(\Gamma_4^{\pm}, \Gamma_4^{\pm}; \mathbf{q}, \mathbf{0})$$
$$|\epsilon_{ijk}q^i|\mathcal{G}_{\mathsf{M}}^{\alpha\pm}(q^2) = (E_{\mathbf{q}} + M)\overline{R}^k_{\alpha}(\Gamma_4^{\pm}, \Gamma_j^{\pm}; \mathbf{q}, \mathbf{0}),$$

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where for positive parity,

$$\Gamma_j^+ = rac{1}{2} egin{bmatrix} \sigma_j & 0 \ 0 & 0 \end{bmatrix}$$
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and for negative parity,

$$\Gamma_j^- = \gamma_5 \Gamma_j^+ \gamma_5 = \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \sigma_j \end{bmatrix}, \quad \Gamma_4^- = \gamma_5 \Gamma_4^+ \gamma_5 = \frac{1}{2} \begin{bmatrix} 0 & 0 \\ 0 & \mathbb{I} \end{bmatrix}.$$

State $1 - \Lambda(1405)$



State $1 - \Lambda(1405)$



Background 000

First Results

State 2



Background 000

First Results

State 3



State $1 - \Lambda(1405)$



Background 000

State 2



Background 000

State 3



















Remarks

- Conclusions
 - Demonstrated the concept of extracting negative-parity baryon EM form factors using correlation matrix techniques.
 - $\hfill\square$ First look at the EM form factors for negative-parity A states, particularly the A(1405).
- Further Work
 - $\hfill\square$ Complete the calculation for the remainder of this ensemble.
 - $\hfill\square$ Repeat the analysis for the other light quark $\kappa_{\rm u,d}.$
 - Include multi-particle operators to isolate and exclude any multi-particle states.