Glueball Spectral Densities from the Lattice

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Motivation

- better understanding of the pure YM dynamics
- understand the physical spectrum

The Kallen-Lehmann Representation

$\mathcal{O}(p)$ euclidean momentum space operator

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle = \int d\mu \frac{\rho(\mu)}{p^2+\mu}$$

quantum associated with $\mathcal{O}(p)$ contributes to the S-matrix

$\rho(\mu)$ is real and positive defined

spectral density

$\rho(\mu) = \delta(\mu - m^2)$ free field theory

$\rho(\mu) = Z\delta(\mu - m^2) + \sigma(\mu)$

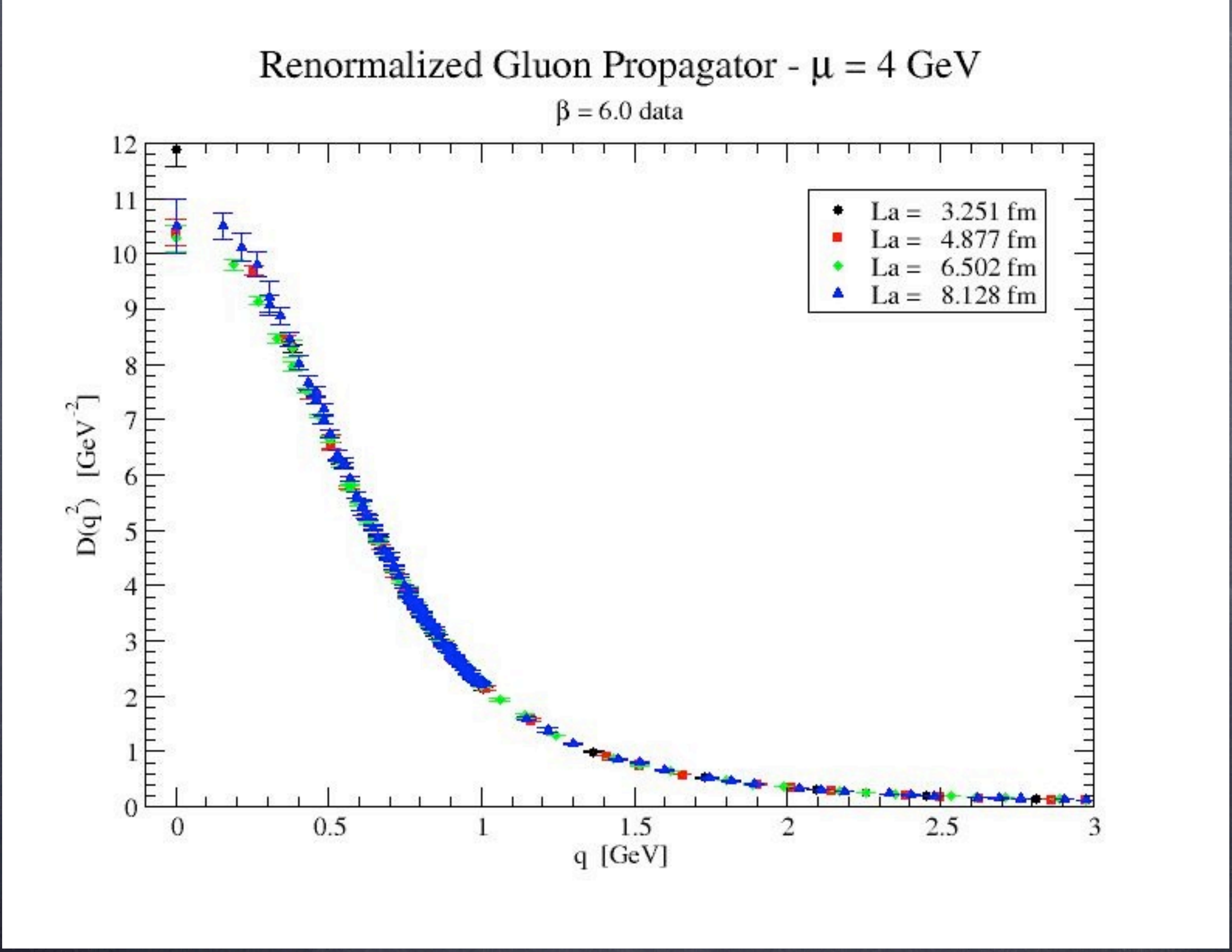
mass excitations, physical degrees of freedom

Landau Gauge Gluon Propagator

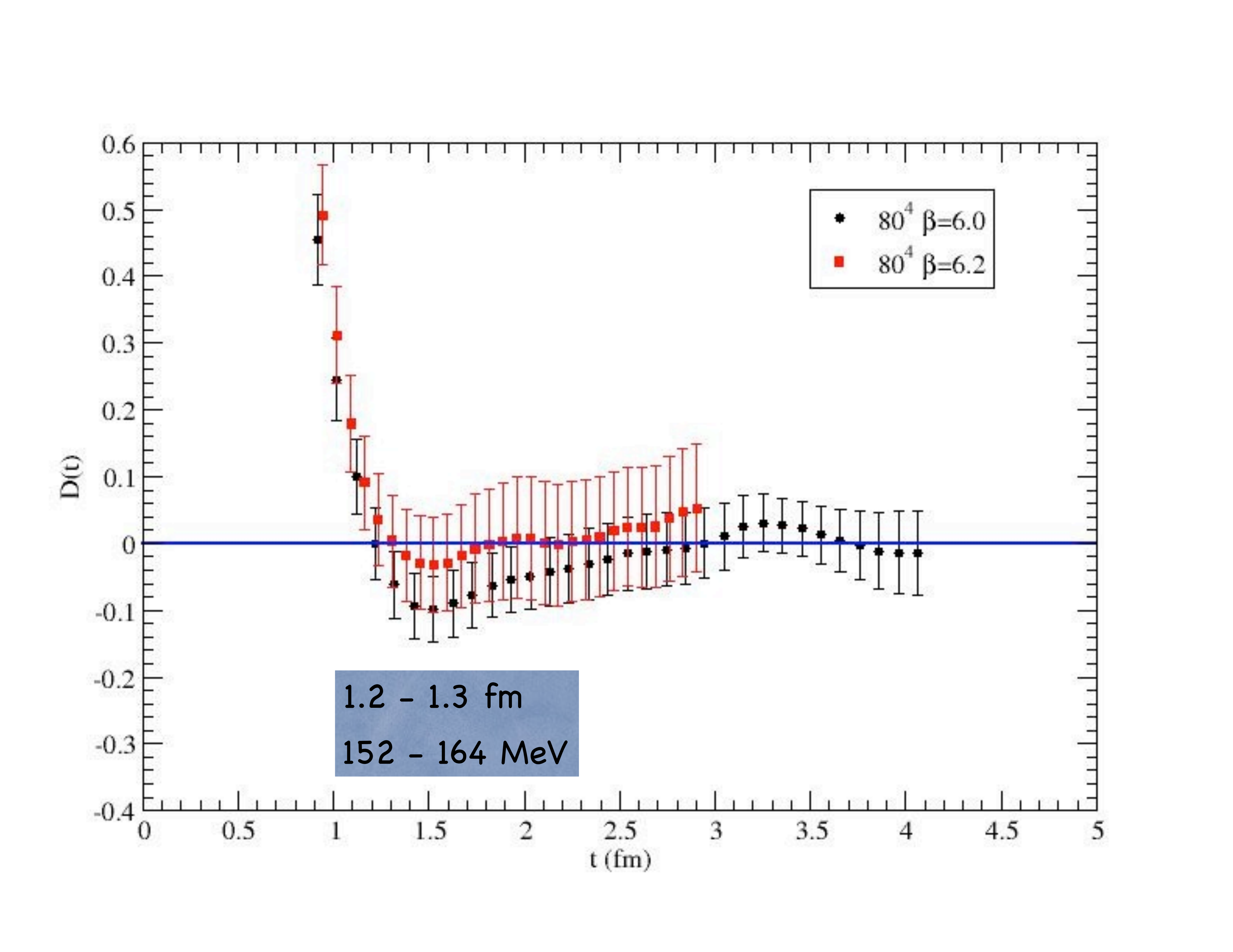
$$D^{ab}_{\mu\nu}(p) = \delta^{ab}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) D(p^2)$$

$$D(t) = \int dp_0 \; e^{-ip_0 t} D(p_0, \vec{p} = 0) = \int_0^{+\infty} d\mu^2 \; \rho(\mu^2) \, e^{-t\mu^2}$$

if the gluon is physical state $\rho(\mu), D(t) > 0$



Renormalized Gluon Propagator - μ = 4 GeV
β = 6.0 data
La = 3.251 fm
La = 4.877 fm
La = 6.502 fm
La = 8.128 fm
D(q²) [GeV⁻²]
q [GeV]
0
0.5
1
1.5
2
2.5
3
0
1
2
3
4
5
6
7
8
9
10
11
12



see also D. Dudal, O. Oliveira, N. Vandersickel, Phys Rev **D81**, 074505 (2010); D. Dudal, M. S. Guimarães, S. P. Sorella, Phys Rev Lett **106**, 062003 (2011)

Glueballs - pure Yang-Mills

Lattice calculations: $0^{++},\ 2^{++},\ 0^{-+},\ldots$

1710(130) MeV 0^{++}
2390(150) MeV 2^{++}
2560(155) MeV 0^{-+}
2670(310) MeV 0^{++}
3004(190) MeV 2^{-+}

C. J. Morningstar, M. J. Peardon, Phys Rev **D60**, 034509 (1999)

B. Lucini, M. Teper, JHEP **6**, 050 (2001)

Y. Chen et al., Phys Rev **D73**, 014516 (2006)

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Glueballs - pure Yang-Mills

Lattice (

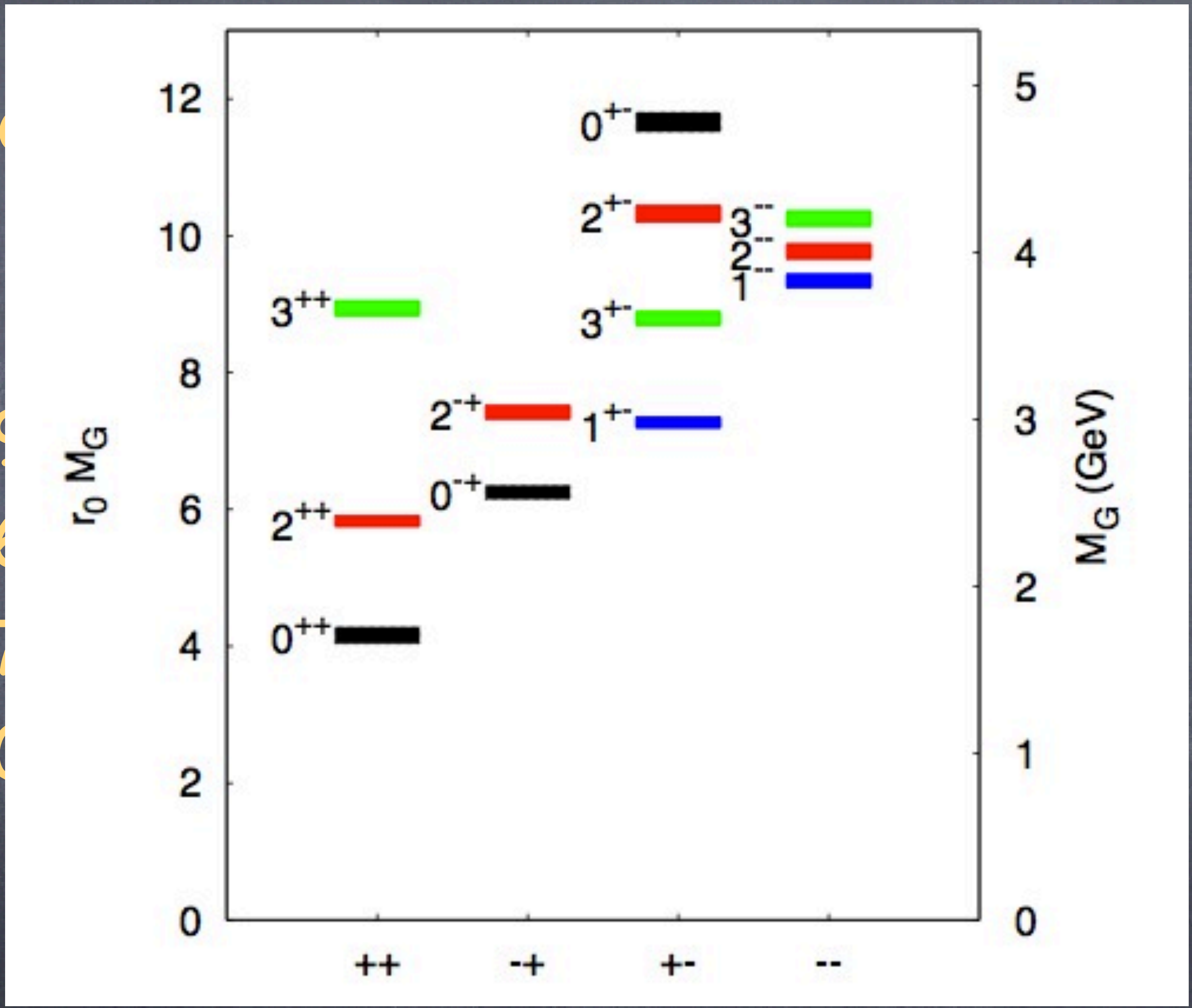
171

239

256

267

300



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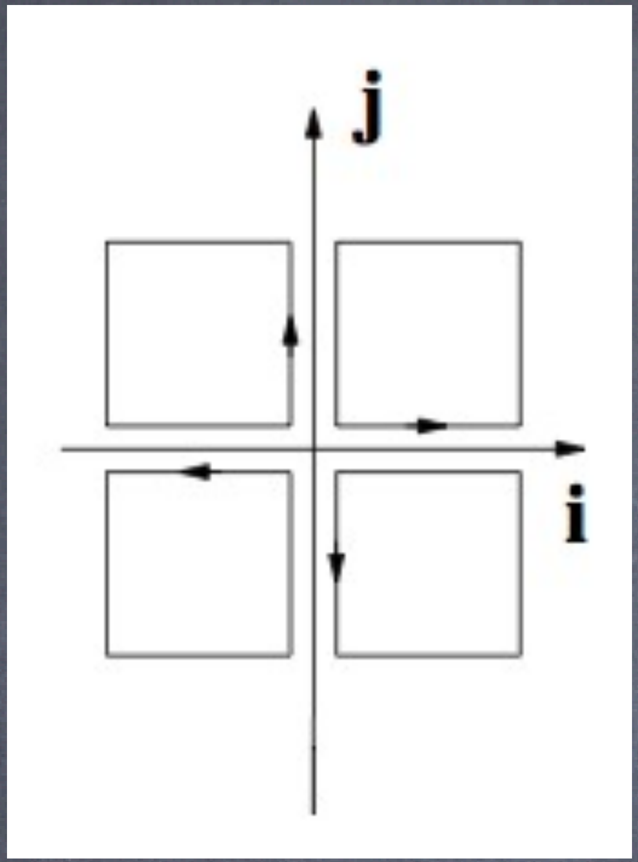
0^{++} Scalar Glueball

$\mathcal{O} = F_{\mu\nu} F_{\mu\nu}$ $\qquad \langle \mathcal{O}(p)\, \mathcal{O}(-p) \rangle = \int_0^{+\infty} d\mu \, \frac{\rho(\mu)}{p^2 + \mu}$

infinities polynomial in $p^2 + \langle \mathcal{O}(p) \mathcal{O}(-p) \rangle_{\text{finite}}$

$$a_0 + a_1 \left(p^2 - T\right) + a_2 \left(p^2 - T\right)^2$$
$$- \left(p^2 - T\right)^3 \int_0^{+\infty} d\mu \, \frac{\rho(\mu)}{p^2 + \mu} \, \frac{1}{(T + \mu)^3}$$

The glueball operator

Tr 

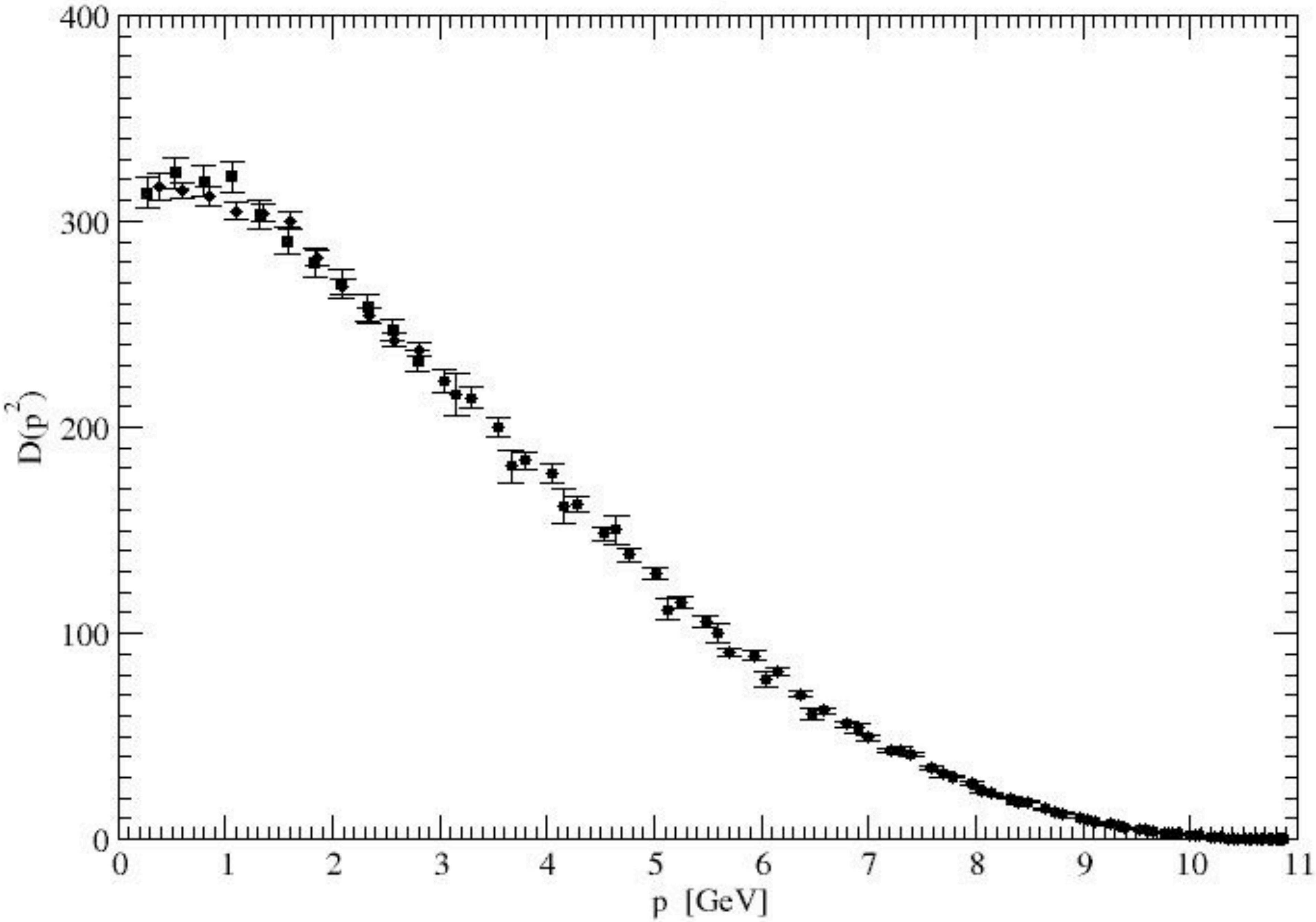
$$= 12 - g^2 a^4 F^2 + \mathcal{O}(a^6)$$

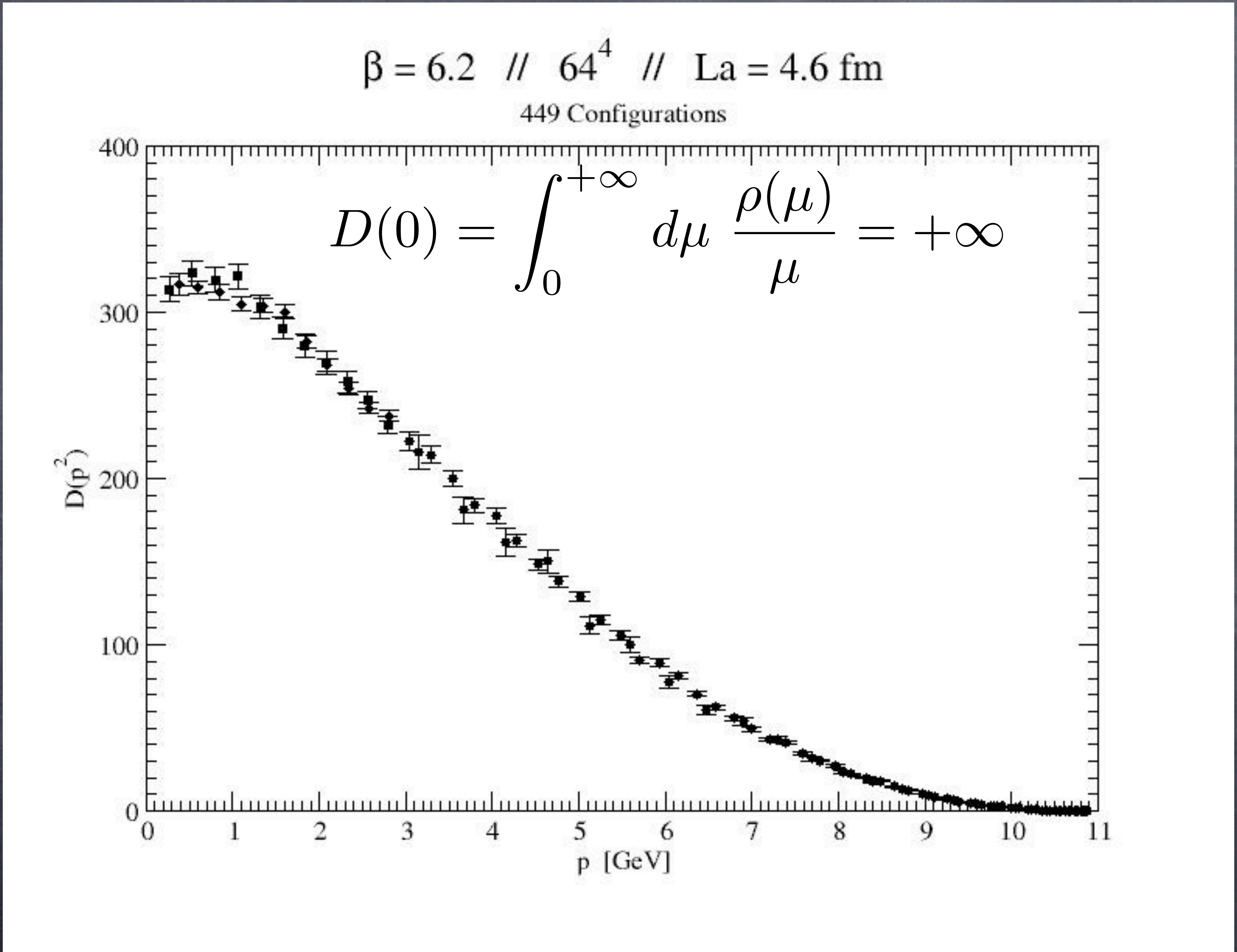
For the simulation we consider the Wilson action

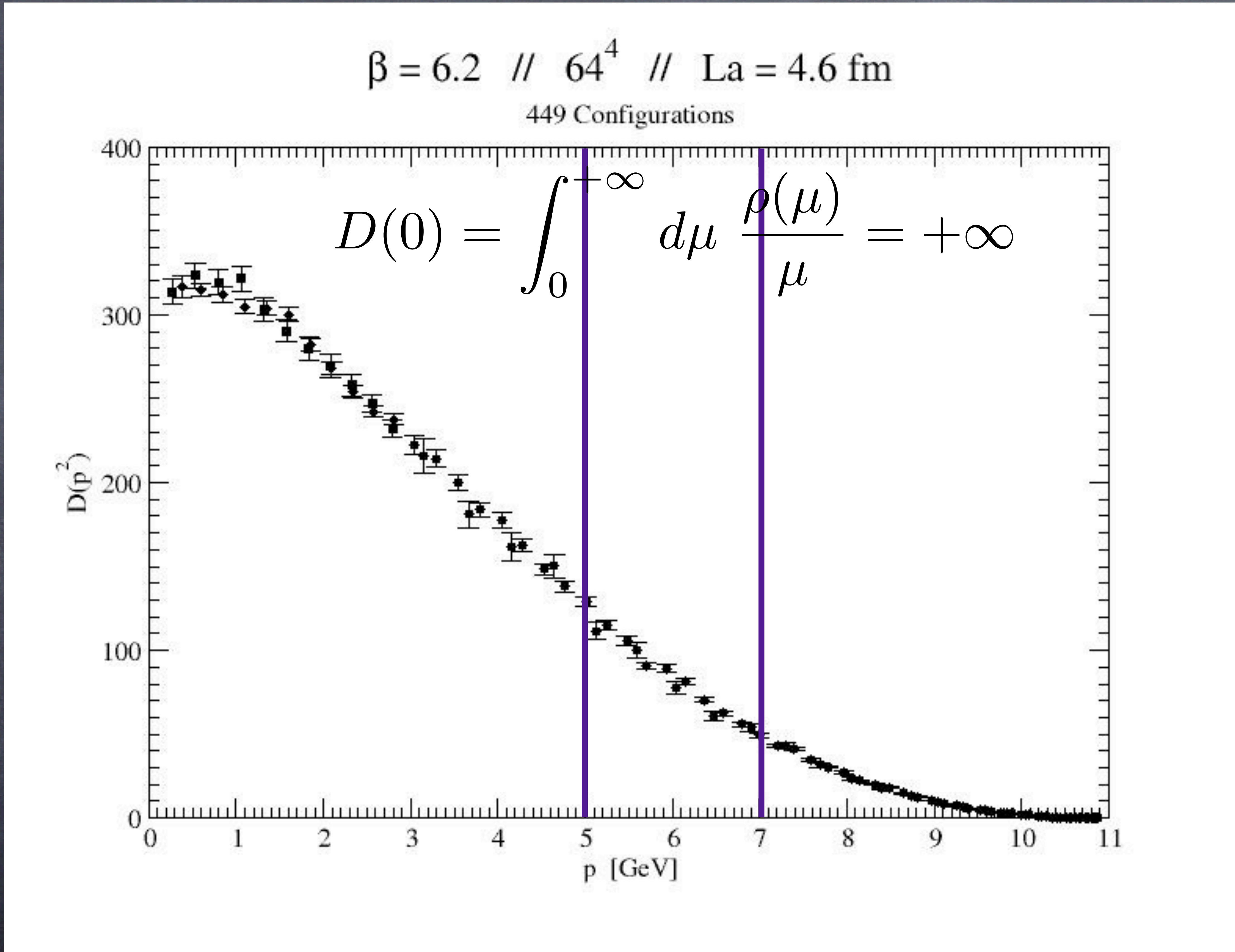
$$S_{\text{Wilson}} \longrightarrow S + \mathcal{O}(a^2)$$

$\beta = 6.2$ // 64^4 // La = 4.6 fm

449 Configurations




β = 6.2 // 64^4 // La = 4.6 fm
449 Configurations
$D(0) = \int_0^{+\infty} d\mu \; \frac{\rho(\mu)}{\mu} = +\infty$
D(p^2)
p [GeV]
0
100
200
300
400
0
1
2
3
4
5
6
7
8
9
10
11



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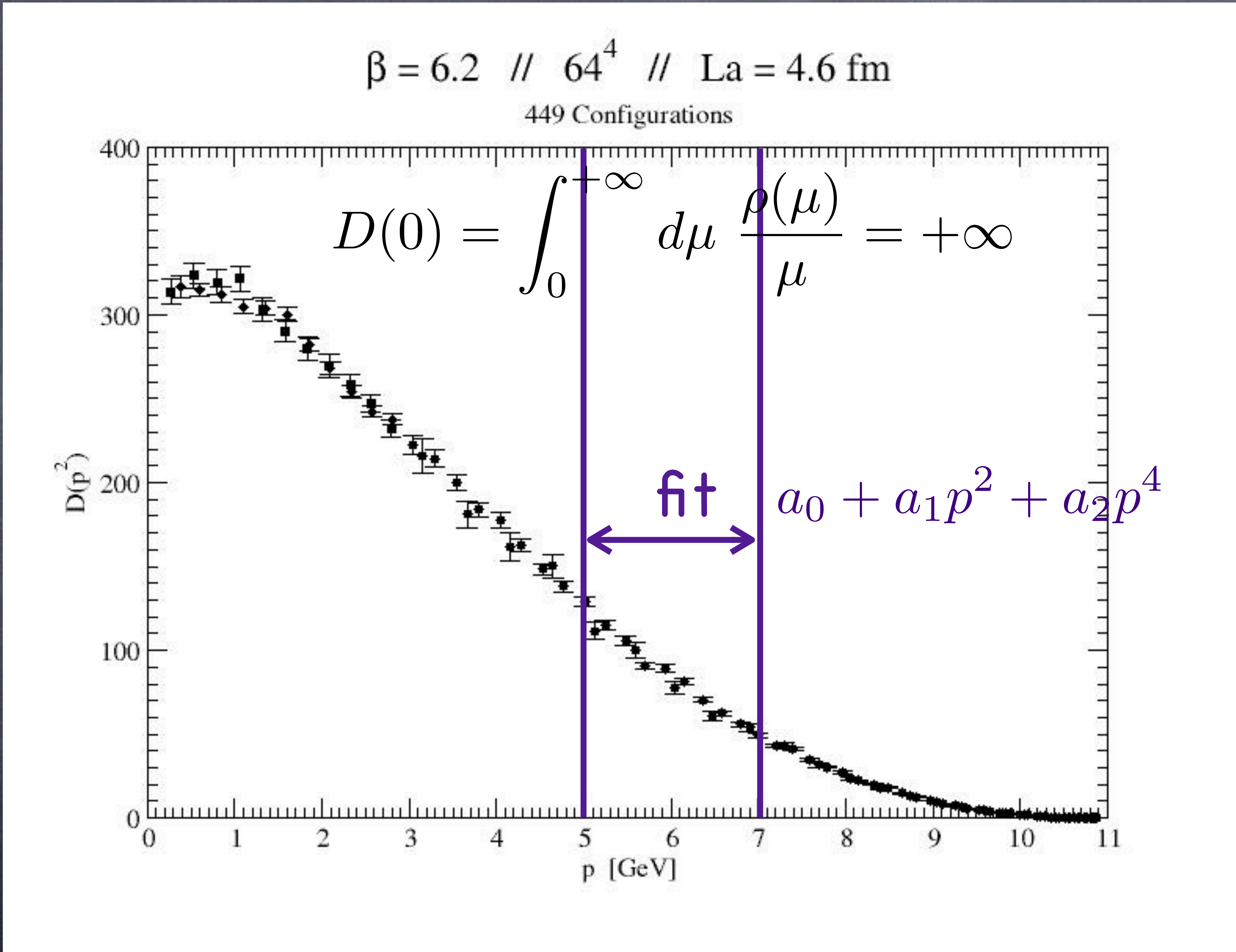
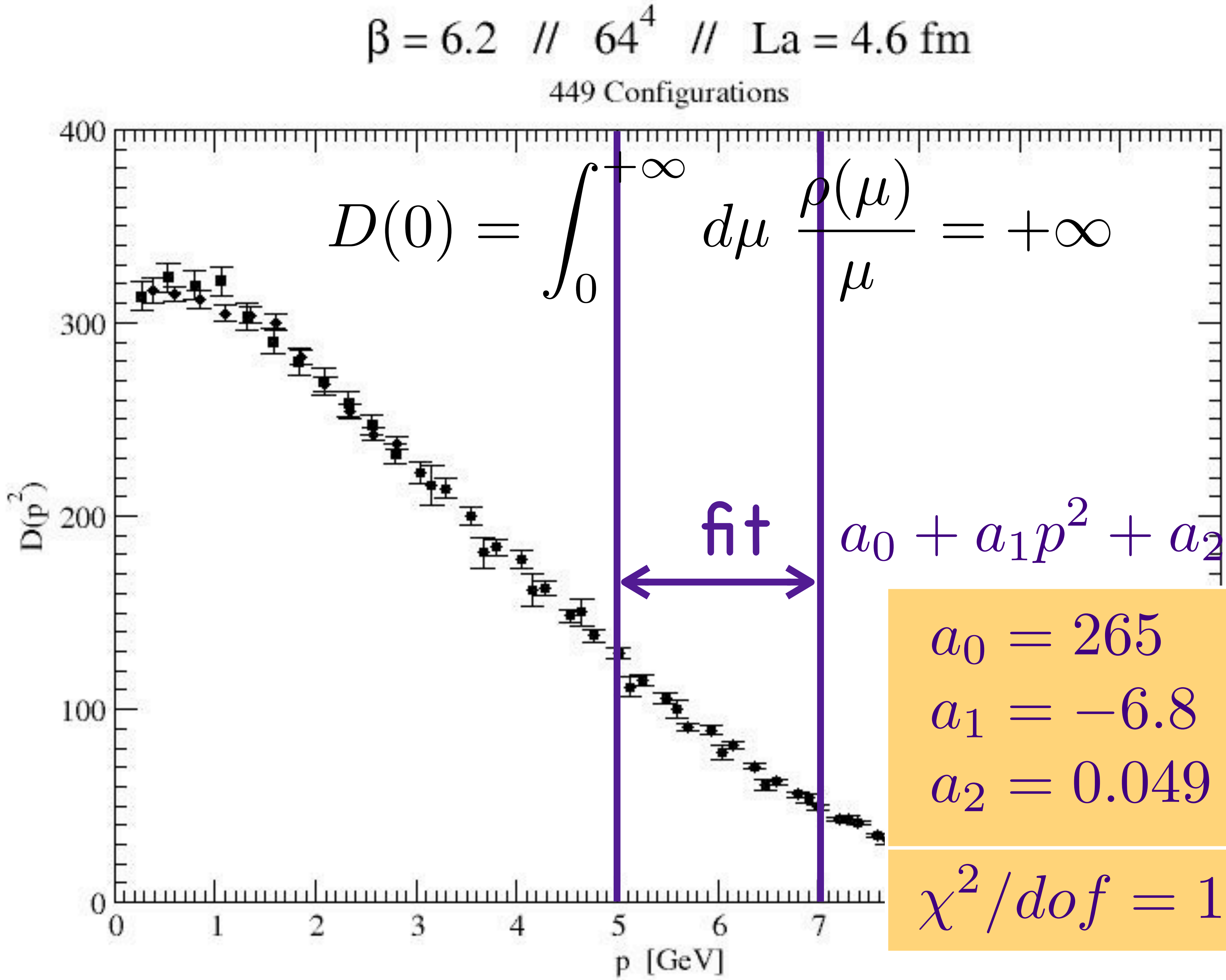


β = 6.2 // 64^4 // La = 4.6 fm
449 Configurations
$D(0) = \int_0^{+\infty} d\mu \, \frac{\rho(\mu)}{\mu} = +\infty$
fit
$a_0 + a_1 p^2 + a_2 p^4$
D(p^2)
p [GeV]

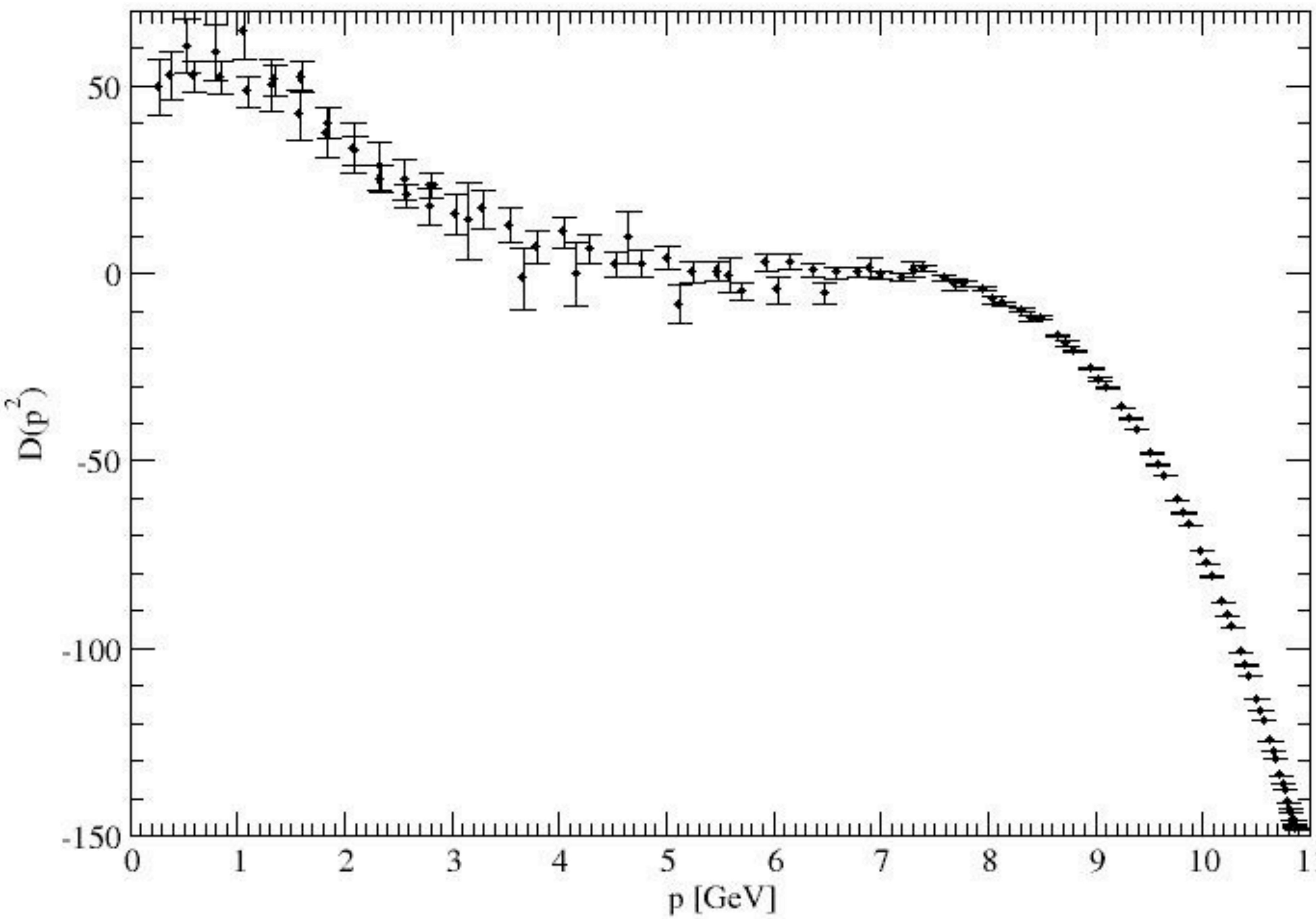


$$D(0) = \int_0^{+\infty} d\mu \, \frac{\rho(\mu)}{\mu} = +\infty$$

fit $a_0 + a_1 p^2 + a_2 p^4$

$a_0 = 265$	$\pm$	21
$a_1 = -6.8$	$\pm$	1.1
$a_2 = 0.049$	$\pm$	0.014

$\chi^2/dof = 1.69$



50
0
-50
-100
-150
D(p²)
0
1
2
3
4
5
6
7
8
9
10
11
p [GeV]

The Spectral Density

$$D(p^2) = -\left(p^2 - T\right)^3 \int_0^{+\infty} d\mu \frac{\rho(\mu)}{p^2+\mu} \frac{1}{(\mu+T)^3}$$

The Spectral Density

$$D(p^2) = -\left(p^2 - T\right)^3 \int_0^{+\infty} d\mu \frac{\rho(\mu)}{p^2+\mu} \frac{1}{\left(\mu + T\right)^3}$$

$$-\frac{D(p^2)}{\left(p^2 - T\right)^3} = \int_0^{+\infty} d\mu \frac{\hat{\rho}(\mu)}{p^2+\mu}$$

$$\hat{\rho}(\mu) = \frac{\rho(\mu)}{(\mu + T)^3}$$

The Spectral Density

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Double Laplace transform of

The Spectral Density

$$D(p^2) = -\left(p^2 - T\right)^3 \int_0^{+\infty} d\mu \frac{\rho(\mu)}{p^2+\mu} \frac{1}{\left(\mu + T\right)^3}$$

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$$\hat{\rho}(\mu) = \frac{\rho(\mu)}{(\mu + T)^3}$$

Double Laplace transform of

$$\mathcal{D} = \mathcal{L}^2 \hat{\rho}$$

noisy data

Need to solve a regularized linear system

Tikhonov-Morozov Regularization

$$Ax = B \qquad\qquad ||B - B^\delta|| < \delta$$

minimize $||Ax - B|| + \lambda ||x||^2 \qquad \lambda > 0$

solve normal equation

$$A^* A x^\lambda + \lambda x^\lambda = A^* B$$

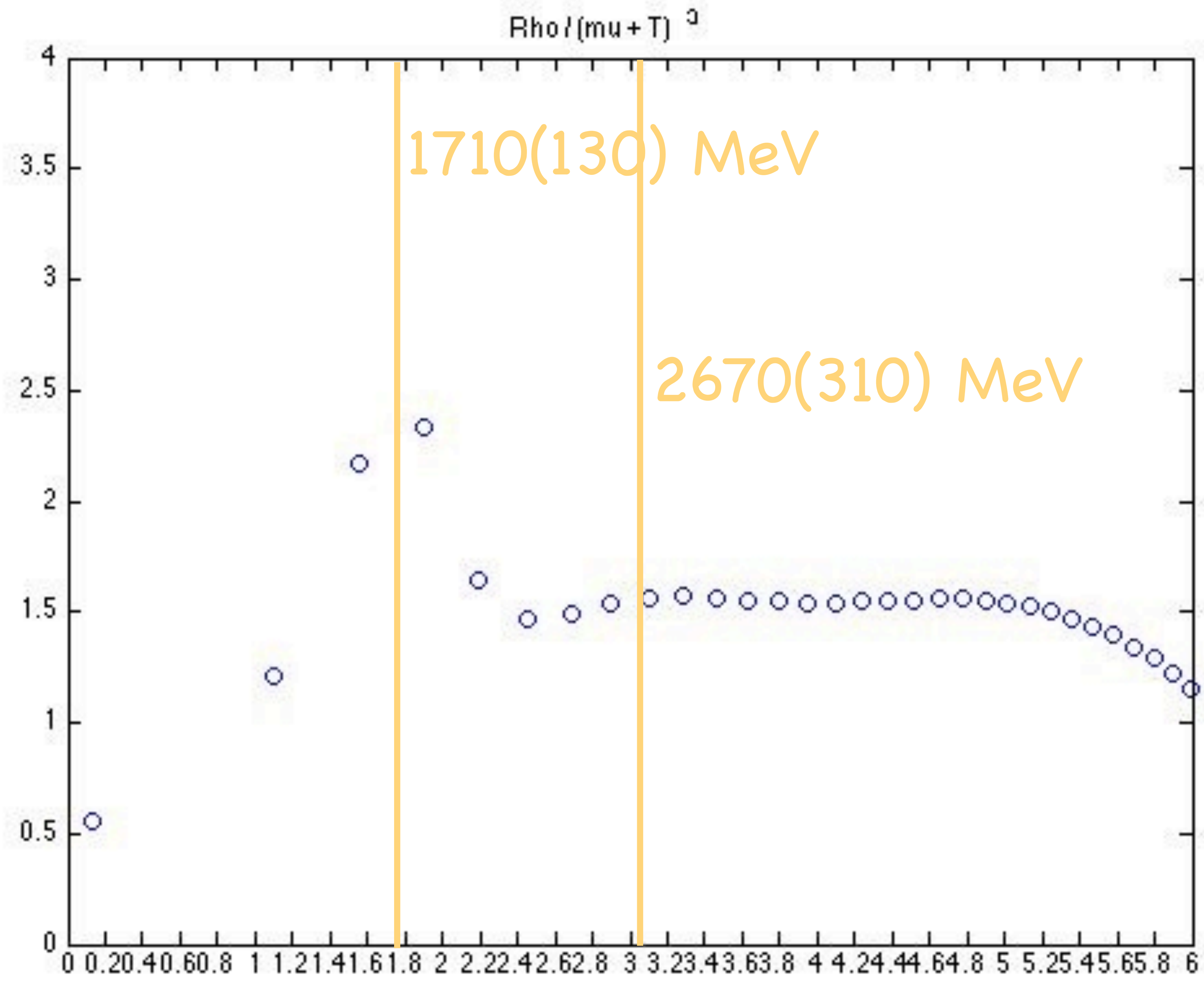
$$||Ax^\lambda - B|| = \delta$$

$$\mathcal{L}^4 \hat{\rho} + \lambda \hat{\rho} = \mathcal{L}^2 \mathcal{D}$$

$$\int_0^{+\infty} dt\, \hat{\rho}(t)\, \frac{\ln \frac{z}{t}}{z-t} + \lambda\, \hat{\rho}(z) = \int_0^{+\infty} dt\, \frac{\mathcal{G}(t)}{t+z}$$

integrals regularized via a cutoff $\Lambda = 11\ \mathrm{GeV}$

and discretized via a
half-open Newton-Cotes



Rho / (mu + T) ^3
1710(130) MeV
2670(310) MeV
0
0.5
1
1.5
2
2.5
3
3.5
4
0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8 2 2.2 2.4 2.6 2.8 3 3.2 3.4 3.6 3.8 4 4.2 4.4 4.6 4.8 5 5.2 5.4 5.6 5.8 6

Results and Conclusions

- preliminary results for the scalar glueball spectral density
- numerics needs to be better investigated
- spectral density shows good agreement with previous lattice spectrum calculations
- increase the statistics