String Tension from gauge invariant magnetic monopoles

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- Recent Studies (Kondo, Shibata, Shinohara, Kato) have considered whether confinement is caused by Abelian Monopoles
- Using a non-Abelian Stokes' theorem, they relate the Wilson Loop to an Abelian field strength
- The Abelian field is the gauge invariant Abelian decomposition of Cho, Faddeev and Niemi
- The field strength can be decomposed into a Maxwell (electric) term and monopole (magnetic) term
- It is predicted that the static potential is dominated by the Abelian field and the monopole field
- This was confirmed in a lattice study
- Our project: to extend this analysis, demonstrate (or falsify) the dual-Meissner picture of confinement
- Start by examining the static potential

Non-Abelian Stokes theorem

Diakonov and Petrov, 1989; Cho, 2000; Kondo, 1998

- In SU(3), the Wilson Loop operator is

$$W_L = \frac{1}{3}\text{tr } P\left[e^{ig\oint_L dx_\mu A_\mu(x)}\right] = \frac{1}{3}\text{tr } P\left[e^{ig\oint_L ds\frac{dx_\mu}{ds}A_\mu(s)}\right]$$

- P represents path ordering;
- $0 \leq s(x) \leq L$ parametrises the position of the path;
- $A_\mu = A_\mu^a T^a$;
- T^a generators of SU(3).

- We insert an identity operator at each point along the path (at $s = 0, \epsilon, 2\epsilon \ldots$, $\epsilon \to 0$).
- Choose a basis in colour space $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$, $e_3 = (0, 0, 1)$
- Previous work has used the identity

$$I = \int d\theta \, \theta^\dagger \, |e_3\rangle \langle e_3| \, \theta \qquad\qquad \theta \in SU(3)/U(2)$$

- Can be used to convert the Path ordered Loop of non-Abelian links into an Loop of Abelian fields but with an integral over all θ fields.
- Note that this is invariant under $\theta \to \Lambda\theta$, $\Lambda = e^{i(\alpha_1\lambda_1+\alpha_2\lambda_2+\alpha_3\lambda_3+\alpha_8\lambda_8)} \in U(2)$

- An alternative is to use the identity operator

$$I = \theta^\dagger(s)(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| + |e_3\rangle\langle e_3|)\theta(s)$$
$$\theta \in SU(3)/(U(1)\times U(1))$$

- Choose θ so that along the Wilson loop $\theta(s)A_\mu\theta^\dagger(s+\epsilon)$ is diagonal.
- Invariance under $\theta \to \Lambda\theta$, $\Lambda = e^{i(\alpha_3\lambda_3+\alpha_8\lambda_8)} \in U(1)\times U(1)$
- This avoids the need for the path integral over θ
- Provides a rule to 'gauge-fix' θ
  - Numerically easy to solve
  - Avoids introducing an additional set of gauge variables
  - Unique solution for θ (baring exchange of eigenvectors and U(1) factor for each eigenvector)
  - Avoids a Gribov problem

- Using this identity operator gives

$$
\begin{aligned}
W_L =& \frac{1}{3}\sum_i \lim_{\epsilon\to 0} \mathrm{tr}\, \theta^\dagger(0)\, |e_i\rangle\langle e_i|\, \theta(0)\left(1 + ig\epsilon A_\mu\left(\frac{\epsilon}{2}\right)\frac{dx_\mu}{ds}\right)\theta^\dagger(\epsilon)\, |e_i\rangle\langle e_i|\, \theta(\epsilon)\ldots \\
=& \frac{1}{3}\sum_i \lim_{\epsilon\to 0} \mathrm{tr}\, \ldots \langle e_i|\, \theta\left(\frac{\epsilon}{2}\right)\left(1 + ig\epsilon A_\mu(\frac{\epsilon}{2})\frac{dx_\mu}{ds} + \epsilon\frac{dx_\mu}{ds}\partial_\mu\right)\theta^\dagger\left(\frac{\epsilon}{2}\right)|e_i\rangle\ldots \\
=& \frac{1}{3}\sum_i \lim_{\epsilon\to 0} \mathrm{tr}\, \ldots e^{ig\langle e_i|\theta A_\mu\theta^\dagger - \frac{i}{g}\theta\partial_\mu\theta^\dagger|e_i\rangle}\ldots \\
=& \frac{1}{3}\sum_i \mathrm{tr}\, e^{ig\oint \mathrm{tr}\left[\left(\frac{1}{3}+\Lambda_i^j H_j\right)\left(\theta A_\mu\left(\frac{\epsilon}{2}\right)\theta^\dagger - \frac{i}{g}\theta\partial_\mu\theta^\dagger\right)\right]}
\end{aligned}
$$

- $|e_i\rangle\langle e_i| = \frac{1}{3} + \Lambda_i^j H_j$
- H_j diagonal element of the Lie algebra ($\propto \lambda_3, \lambda_8$).
- We have removed the path ordering operation to give an Abelian integral
- The topological or Monopole term represents the number of times θ winds around the gauge group as we pass along the Wilson line

- Define $n^j = \theta^\dagger \lambda_j \theta$, normalised so that tr $(n^j)^2 = 1$

$$W_L = \frac{1}{3} \sum_i e^{ig \oint dx_\mu \Lambda^i_j \text{tr } (n^j A_\mu + i g^{-1} n^j \theta^\dagger \partial_\mu \theta)}$$

- As the integrand is Abelian, we can (in principle)
  - Continue θ across all space
  - Apply Stokes' theorem to give an integral over the surface

$$W_L = \frac{1}{3} \sum_i e^{ig \oint dS_{\mu'\nu'} \Lambda^i_j \epsilon^{\mu'\nu'\mu\nu} \text{tr } (n^j \hat{F}_{\mu\nu})}$$

with

$$\hat{F}^{\mu\nu} = \text{tr } (\partial_\mu (n^j A_\mu) - \partial_\nu (n^j A_\mu) + \frac{i}{g} n^j [\theta^\dagger \partial_\mu \theta, \theta^\dagger \partial_\nu \theta])$$

- We can decompose the gauge field using colour vectors n^j

$$A_\mu = \hat{A}_\mu + X_\mu,$$

where the Abelian field $\hat{A}_\mu$ contains the components of A parallel to n^j.

- If the defining equations are satisfied

$$D_\mu[\hat{A}]n^j = 0 \qquad \text{tr}\,(n^j A_\mu) = \text{tr}\,(n^j \hat{A}_\mu)$$

then the field strength $F_{\mu\nu}[\hat{A}] \propto n^j$

- Two choices in SU(3):
  - **U(2) decomposition** One n field, $n^8 = \frac{1}{\sqrt{2}}\theta^\dagger \lambda_8 \theta$
  - **U(1)×U(1) decomposition** Two n^j fields, $n^8 = \frac{1}{\sqrt{2}}\theta^\dagger \lambda_8 \theta$, $n^3 = \frac{1}{\sqrt{2}}\theta^\dagger \lambda_3 \theta$

- **U(1)$\times$ U(1) decomposition**

$$\hat{A}_\mu = \sum_{j=3,8} \left[n^j \text{tr } (n^j A_\mu) - \frac{i}{2g}[n^j, \partial_\mu n^j] \right]$$

$$F_{\mu\nu}[\hat{A}] = n^j \text{tr} \left[\partial_\mu \text{tr } (n^j A_\nu) - \partial_\nu \text{tr } (n^j A_\mu) + \frac{i}{2g} n^k [\partial_\mu n^k, \partial_\nu n^j] \right]$$

- The Wilson Loop is

$$W_L = \frac{1}{3} \sum_i \int \exp \left(ig \oint dS^{\mu'\nu'} \epsilon_{\mu'\nu'\mu\nu} \Lambda^i_j \text{tr } (n^j F^{\mu\nu}[\hat{A}]) \right)$$

- The Wilson Loop is dominated by the Abelian field strength

$$F_{\mu\nu}[\hat{A}] = n^j \text{tr}\left(\partial_\mu(n^j A_\nu) - \partial_\nu(n^j A_\mu) + \frac{i}{2g} n^j [\partial_\mu n^k, \partial_\nu n^k]\right)$$

- The first term is the contribution from an electric charge, with a current
  $\partial^\mu F_{\mu\nu}[\hat{A}] = j_\nu$.
- The second term is the contribution from a magnetic monopole, with a current
  $\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial^\nu F_{\rho\sigma}[\hat{A}] = k_\nu$.
- Expect Abelian Dominance of the string tension and Monopole dominance.
- Previous lattice implementations have chosen one particular n rather than integrating over all n, e.g. minimise tr X_μ^2, Maximum Abelian gauge, etc.

Lattice implementation

- Decompose the gauge link $U_\mu = V_\mu(x)\hat{X}_\mu(x+\hat{\mu})$.

$$U_\mu \Leftrightarrow P e^{ig\int dx_\mu A_\mu}$$

$$V_\mu \Leftrightarrow P e^{ig\int dx_\mu \hat{A}_\mu} \qquad \hat{X}_\mu \Leftrightarrow e^{igaX_\mu}$$

$$\begin{aligned} D_\mu[\hat{A}]n^j &= 0 & \Leftrightarrow && V_\mu n_{x+\hat{\mu}} V_\mu^\dagger - n_x &= 0 \\ \text{tr}\,(n^j X_\mu) &= 0 & \Leftrightarrow && \text{tr}\,(n^j(X_\mu - X_\mu^\dagger)) &= 0 \end{aligned}$$

- Solve these defining equations numerically.

- Gauge transformations which leave the action invariant ($\theta \to \theta\Omega^\dagger$)

$$\begin{aligned} U_\mu(x) \to & \Lambda^\dagger_x U_\mu(x) \Lambda_{x+\hat{\mu}} & n^j_x \to & \Omega_x n^j_x \Omega^\dagger_x \\ V_\mu(x) \to & \Lambda^\dagger_x V_\mu(x) \Lambda_{x+\hat{\mu}} & V_\mu(x) \to & \Omega_x V_\mu(x) \Omega^\dagger_{x+\hat{\mu}} \\ V_\mu(x) \to & \Lambda^\dagger_x V_\mu(x) \Lambda_{x+\hat{\mu}} & V_\mu(x) \to & \Omega_x V_\mu(x) \Omega^\dagger_{x+\hat{\mu}} \end{aligned}$$

  This represents a $SU(3) \times [SU(3)/(U(1) \times U(1))]$ gauge symmetry
- Fix it back to SU(3) by applying

$$[n^j, A_\mu - ig^{-1}\partial_\mu] = 0 \quad \Leftrightarrow \quad U_\mu(x) n^j_{x+\hat{\mu}} U^\dagger_\mu(x) - n_x = 0$$

  along each of the nested Wilson Loops.
- Forces $\Omega^\dagger = \Lambda$

- 30 quenched Configurations, $\beta = 8.0, 8.3, 8.52$, $16^3 32$ lattice, $a \sim 0.10, 0.12, 0.14$
- Applied 8 steps of improved stout smearing at $\rho = 0.025$
- Extract the monopole part of the Wilson Loop using the Laplacian operator

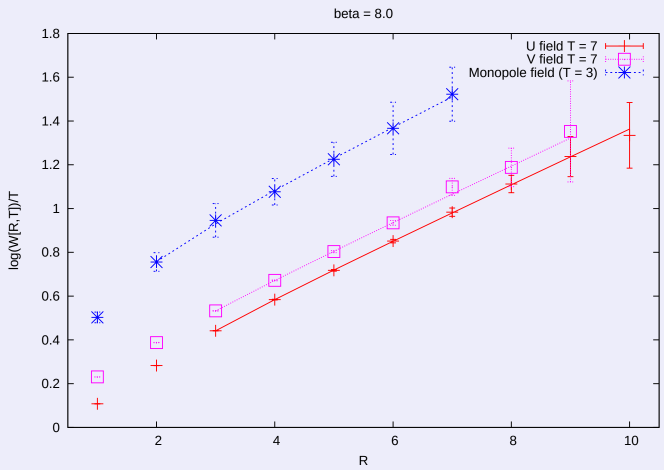
$$\Delta_{\mu\nu,\alpha\beta}(s) = \alpha\delta_{\mu\alpha}\delta_{\nu\beta} + \frac{\partial^2}{\partial x_\mu \partial x_{\sigma'}}\delta_{\nu\beta}\delta_{\sigma'\alpha} + \frac{1}{4}\epsilon_{\mu\nu\rho\sigma}\frac{\partial^2}{\partial x_\sigma \partial x_{\sigma'}}\epsilon_{\rho\sigma'\alpha\beta},$$

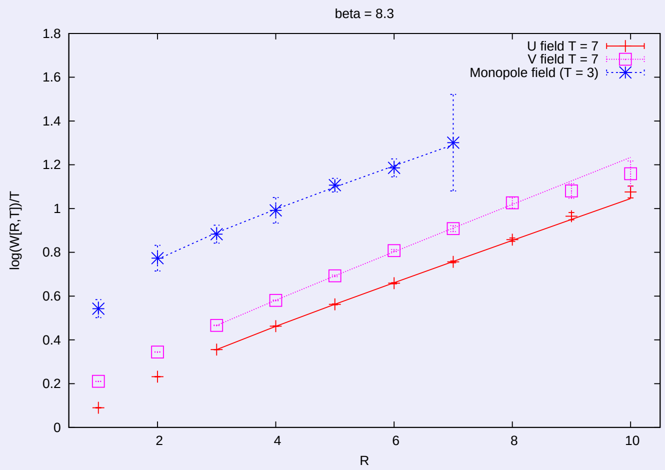
$$\int dS^{\mu\nu}F^j_{\mu\nu} = \lim_{s\to 0}\int d^4x\Theta^{\mu\nu}(x)\Delta(s)^{-1}{}_{\mu\nu\rho\sigma}\Delta(s)^{\rho\sigma\alpha\beta}F^j_{\alpha\beta}$$

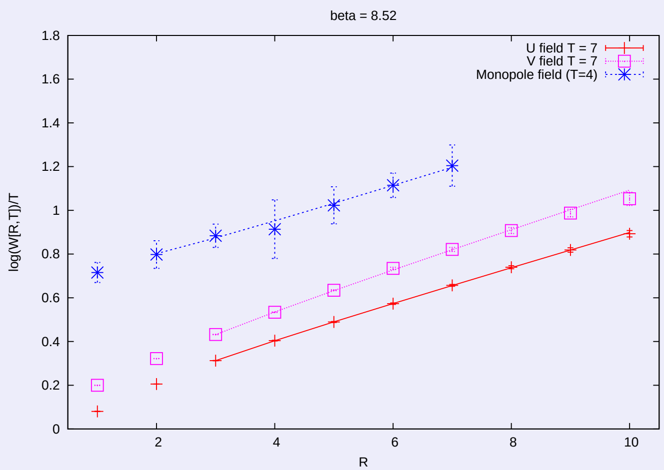
$$\Delta_{\mu\nu}{}^{\alpha\beta}(s)F^j_{\alpha\beta} = sF^j_{\mu\nu} + \partial_\mu j^j_\nu + \epsilon_{\mu\nu\rho}{}^{\sigma}\partial^\rho k^j_\sigma$$

- Discretization of this quantity is not gauge invariant, so fix to Landau gauge
- Measure the string tension using every planar Wilson Loop

Results


beta = 8.0
U field T = 7
V field T = 7
Monopole field (T = 3)
log(W[R,T])/T
R


beta = 8.3
U field T = 7
V field T = 7
Monopole field (T = 3)
log(W[R,T])/T
R

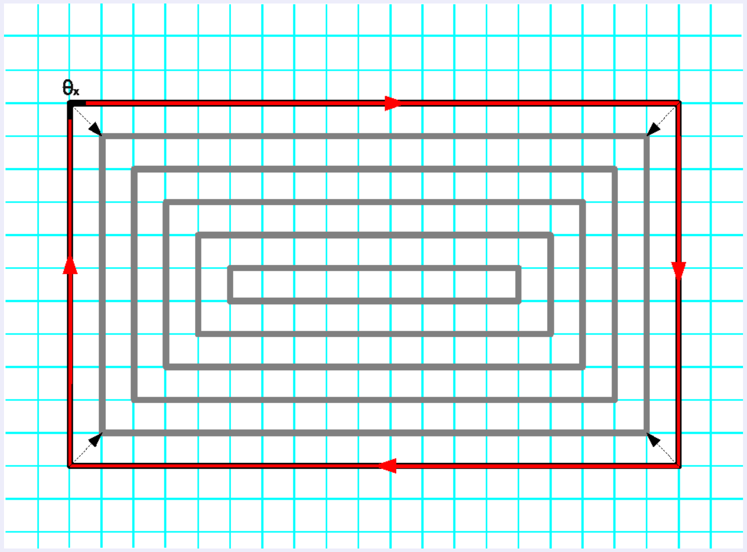

beta = 8.52
U field T = 7
V field T = 7
Monopole field (T=4)
log(W[R,T])/T
R

- Fits for string tension
- Preliminary (not yet extrapolated to $T = \infty$)

	$\beta = 8.0$	$\beta = 8.3$	$\beta = 8.52$
U	0.125(2)	0.095(2)	0.079(1)
V	0.126(1)	0.094(1)	0.081(1)
Monopole	0.134(17)	0.083(10)	0.084(14)

Conclusions

- We apply a modified form of the Non-Abelian Stokes theorem to the Wilson Loop
- Principle difference compared to previous work is study the $U(1) \times U(1)$ rather than U(2) monopole.
- No Longer have to integrate over all colour fields θ
- The result is related to the CFNS exact Abelian decomposition
- We see clear Abelian dominance of the string tension
- Our results are consistent with Monopole dominance, but need considerably more statistics to be sure
- In the process of generating more statistics to obtain more precise results and extend this study to other observables



- θ represents eigenvector of A_μ $[+\partial_\mu]$
- If A_μ analytic, with no degenerate eigenvalues, θ is continuous across space
  $\Rightarrow$ no winding number $\Rightarrow$ De-confinement
- Degeneracies or singularities in A_μ give discontinuities in θ
  $\Rightarrow$ possible non-zero winding number $\Rightarrow$ Confinement