Stringy Excitation and Role of UV Gluons in Lattice QCD

H. Ueda (Kyoto U.),
in collaboration with T. Doi, S. Fujibayashi, S. Tsutsui,
T. Iritani, H. Suganuma (Kyoto U.)

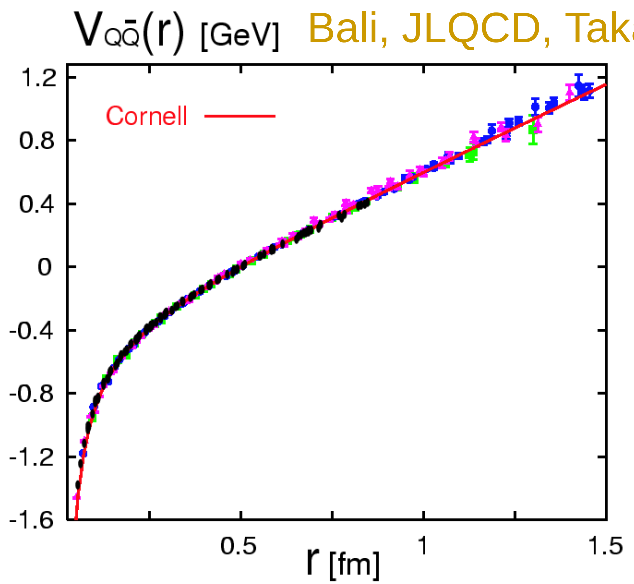
Abstract : Using SU(3) quenched lattice QCD, we study ground-state (GS) and low-lying even-parity excited-state (ES) potentials of quark-antiquark systems in terms of the gluon momentum component in the Coulomb gauge.
By introducing UV cut in the gluon-momentum space, we investigate the sensitivity of the GS and ES potentials, and the stringy excitation to the UV gluons quantitatively.

Lattice 2012 June 25-29, 2012, Cairns

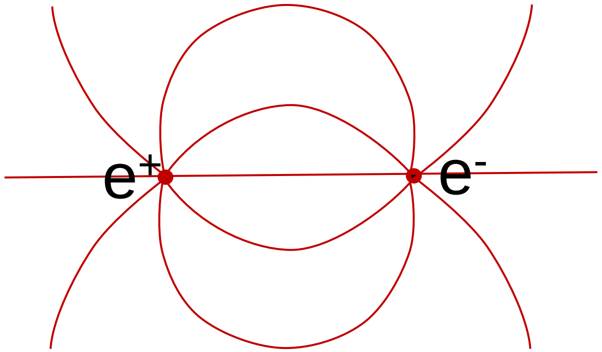
Introduction

One-dimensional squeezing of color electric field

Linear confinement potential



cf QED

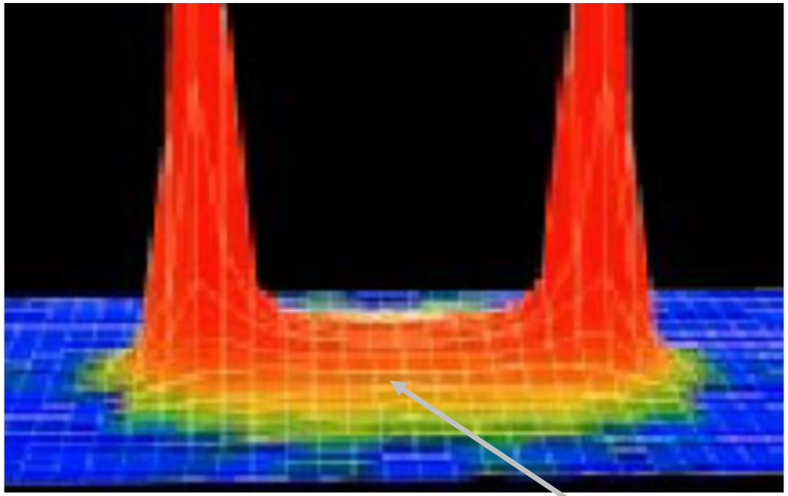


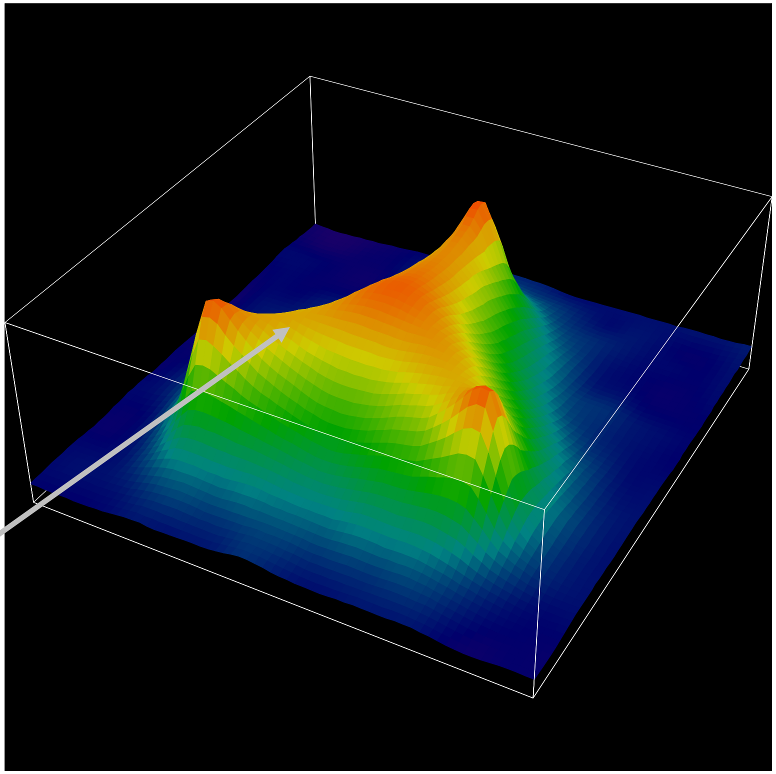
Unlike QED, QCD forms Color-Flux-Tube between quarks, and this One-dimensional Squeezing of color-electric field leads to Linear Confinement potential in the infrared region.

Flux tube formation for QQbar and 3Q systems in Lattice QCD

H. Ichie et al., Nucl. Phys. A721, 899 (2003)

G. S. Bali



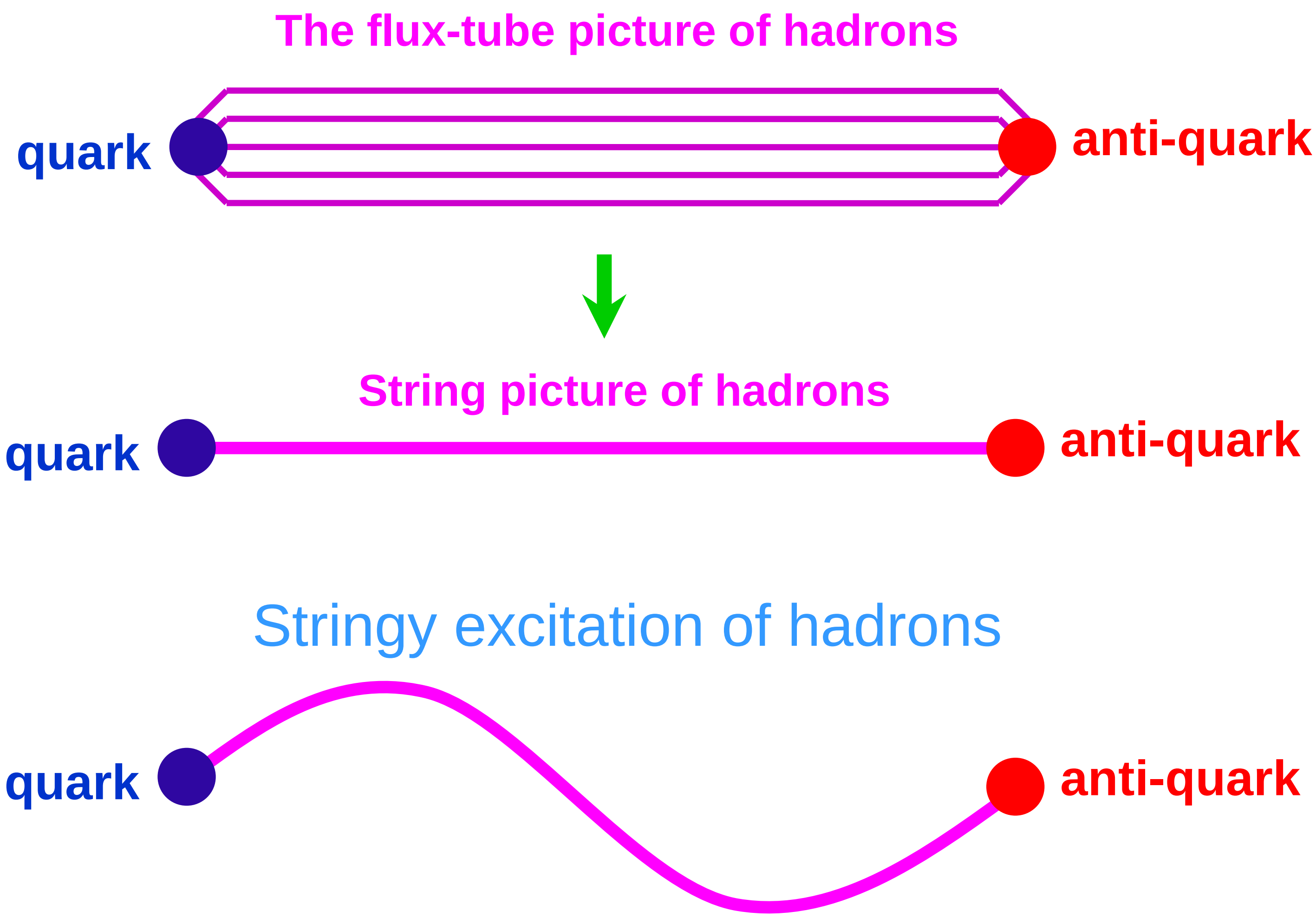


Color-Electric Flux Tube

Non-perturbative

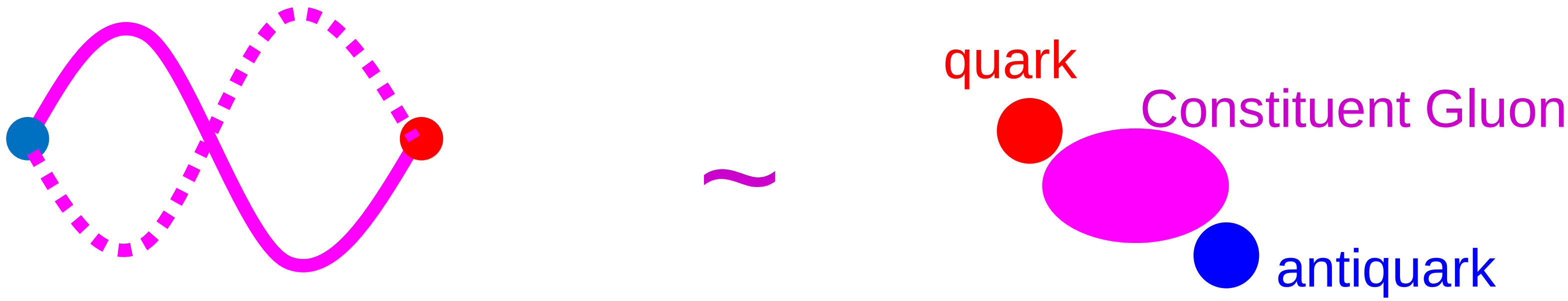
Actually, apart from the color-Coulomb energy around quarks,
Flux-Tube Formation has been observed in Lattice QCD
both for QQbar and 3Q systems

Stringy Excitation of Hadrons



In the Flux-Tube Picture of hadrons, which is idealized as the String Picture in the infrared region, we can expect "Stringy Excitations" of hadrons.

Gluonic Excitation and Hybrid Hadrons



Stringy Excitation as
a Gluonic Excitation

Gluonic Excitation
⇒ Hybrid meson

This **Stringy Mode** is non-quark-origin excitation, and therefore it can be regarded as **Gluonic Excitation**.

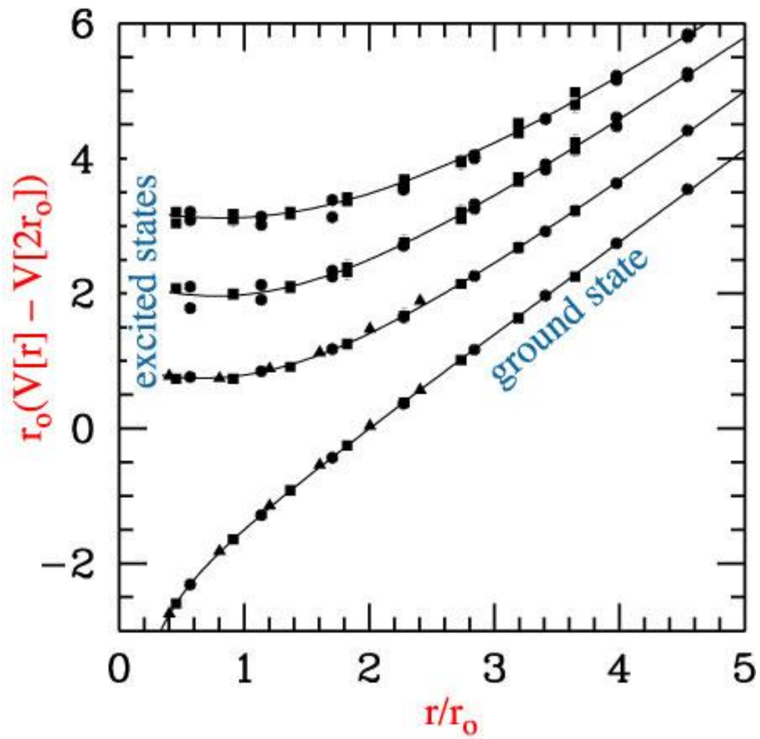
Such a gluonic-excited state may be interpreted as **Hybrid Hadrons** ($q\bar{q}G$ and $qqqG$), which are interesting hadrons beyond the Quark-Model framework.

In Lattice QCD, Gluonic Excitation has been studied using **Excited-State Potentials** in spatially-fixed QQbar systems.

Excited-state potential in Lattice QCD

Gluonic Excitation in $Q\bar{Q}$ System

K.J. Juge, J. Kuti, C. Morningstar,
Phys. Rev. Lett. 90, 161601 (2003),



An image of Gluonic Excitation
~ Excited String

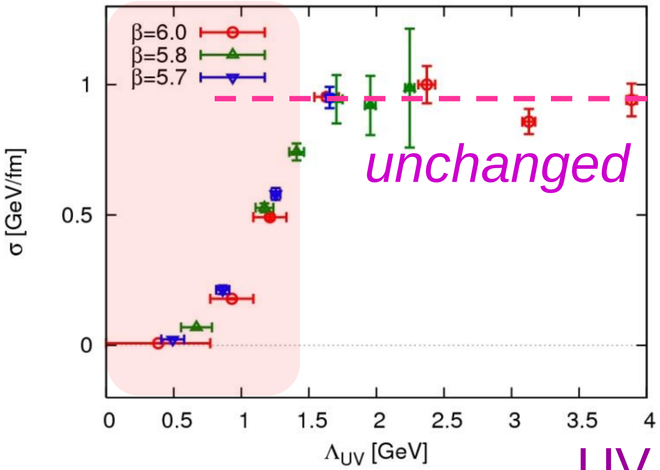
Ground-state String

In Lattice QCD, Excited-State Potentials have been calculated, and their behavior is almost consistent with string excitation in infrared region, in spite of significant difference at small distance.

In the previous work, IR/UV-Gluon Contribution to the Ground -State Potential or Confinement has been studied.

A. Yamamoto and H.Suganuma, PRL 101, 241601 (2008); PRD79, 054504 (2009).

String Tension



UV cutoff

The string tension is almost unchanged even after cutting off the high-momentum gluon component above 1.5GeV.

In this talk, we study not only ground-state potential but also low-lying even-parity Excited-State Potentials of QQbar systems in terms of Gluon Momentum Component in Coulomb gauge.

By introducing UV-cut in 3dim Gluon-Momentum space, we study the UV-gluon contribution to Excited-State Potentials and Stringy Excitations.

Formalism to extract Excited-State Potentials in lattice QCD

T.T.Takahashi and H.Suganuma, PRL 90 (2003), PRD70 (2004).

We present the formalism to extract the excited-state potential for the spatially-fixed Q-Qbar system.
We denote the n-th eigen-state of the QCD Hamiltonian by $|n\rangle$,

$$H|n\rangle = V_n|n\rangle \quad (n=0,1,2,\ldots)$$

Here, V_n denotes n-th Excited-State Potential, and 0th eigen-state means the ground-state.

Consider arbitrary independent Q-Qbar states $|\phi_k\rangle$ (k=0,1,2…).
Generally, each Q-Qbar state $|\phi_k\rangle$ can be expressed by a linear combination of the Q-Qbar physical eigen-states:

$$|\phi_k\rangle = c_0^k|0\rangle + c_1^k|1\rangle + c_2^k|2\rangle + \ldots$$

Formalism to extract excited-state potentials in lattice QCD

The Euclidean−time evolution of the QQbar state $|\phi_k(t)\rangle$ is expressed with the operator e^{-Ht}, which corresponds to the Transfer Matrix in Lattice QCD.
The overlap $\langle\phi_j(T)|\phi_k(0)\rangle$ is given by the Wilson loop W_T^{jk}, sandwiched by initial state ϕ_k at $t=0$ and final state ϕ_j at $t=T$, and is expressed in the Euclidean Heisenberg picture as

$$W_T^{jk} \equiv \langle \phi_j(T) | \phi_k(0) \rangle = \langle \phi_j | W(T) | \phi_k \rangle = \langle \phi_j | e^{-HT} | \phi_k \rangle$$

$$= \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \bar{c}_m^{\,j} c_n^k \langle m | e^{-HT} | n \rangle = \sum_{n=0}^{\infty} \bar{c}_n^{\,j} e^{-V_n T} c_n^k$$

This is a basic relation between Wilson loops and potentials.

By introducing the matrix $C^{nk} = c_n^k$ $\quad \Lambda_T^{mn} = e^{-V_n T} \delta^{mn}$

this relation can be rewritten as $\boxed{W_T = C^\dagger \Lambda_T C}$

Formalism to extract excited-state potentials in lattice QCD

$$\boxed{W_T = C^\dagger \Lambda_T C} \qquad \boxed{\Lambda_T = \mathrm{diag}(e^{-V_0 T}, e^{-V_1 T}, e^{-V_2 T}, \ldots)}$$

Using this relation, we extract the potentials V_n (n=0,1,2···) from the Wilson loop W_T. Consider the following combination:

$$\boxed{W_T^{-1} W_{T+1} = \left\{C^\dagger \Lambda_T C\right\}^{-1} C^\dagger \Lambda_{T+1} C = C^{-1} \mathrm{diag}(e^{-V_0}, e^{-V_1}, e^{-V_2}, \ldots) C}$$

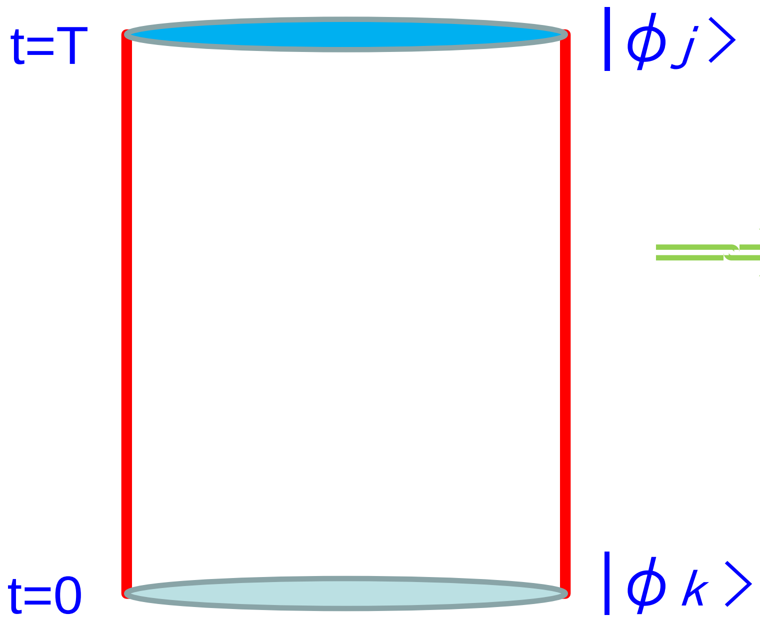
Then, e^{-V_n} can be obtained as eigen-values of matrix $W_T^{-1} W_{T+1}$.
In fact, they are the solutions of the secular equation,

$$\boxed{\det\left\{W_T^{-1} W_{T+1} - t1\right\} = \prod_n (e^{-V_n} - t) = 0}$$

In this way, the potentials V_n (n=0,1,2,···) can be obtained from the Wilson loop matrix, $W_T^{-1} W_{T+1}$.

Formalism to extract excited-state potentials in lattice QCD

In the practical calculation, we prepare gauge-invariant QQbar states $|\phi_k\rangle$ composed by fat-links obtained with smearing method, and calculate many Wilson loops sandwiched by various combination of initial state $|\phi_k\rangle$ and final state $|\phi_j\rangle$.



$$W_T^{jk} \equiv \langle \phi_j(T) | \phi_k(0) \rangle = \langle \phi_j | e^{-HT} | \phi_k \rangle$$

$$\det\left\{ W_T^{-1} W_{T+1} - t1 \right\} = \prod_n (e^{-V_n} - t) = 0$$

By solving the secular equation within a truncated dimension, ground-state and excited-state potentials can be obtained.

Fourier Transformation and Gluon Momentum Space

A. Yamamoto and H.Suganuma, PRL 101, 241601 (2008); PRD79, 054504 (2009).

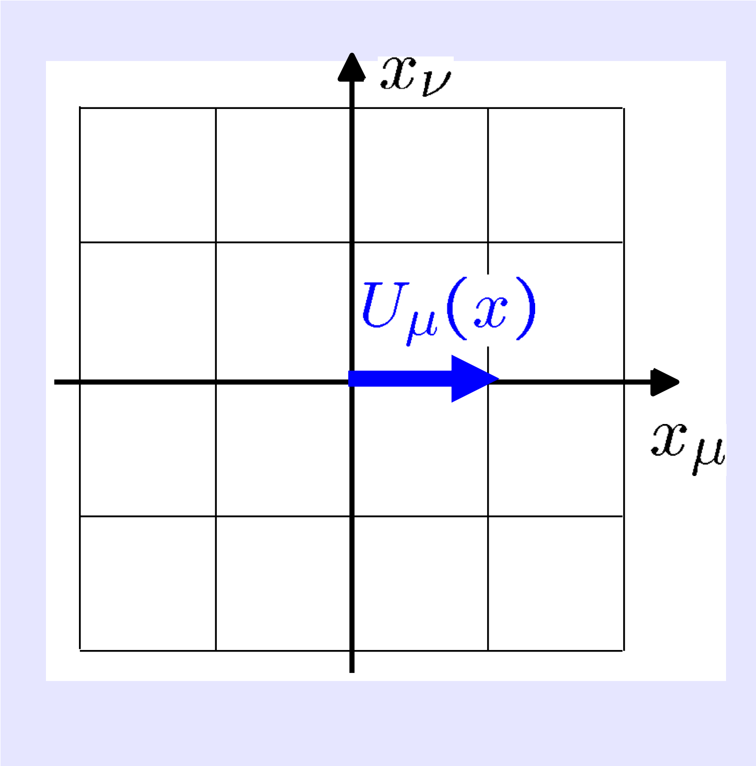
Next, we consider 3-dim Fourier Transformation of link-variable $U_\mu(s)$, and introduce UV-cut in 3-dim Momentum Space.

Step 1. Generation of link-variable in Coulomb gauge

Coordinate-space link-variable

$$U_\mu(x) = e^{iaA_\mu(x)} \in SU(3)$$

We consider link-variables fixed in Coulomb gauge, because spatial gauge-field fluctuation is strongly suppressed.

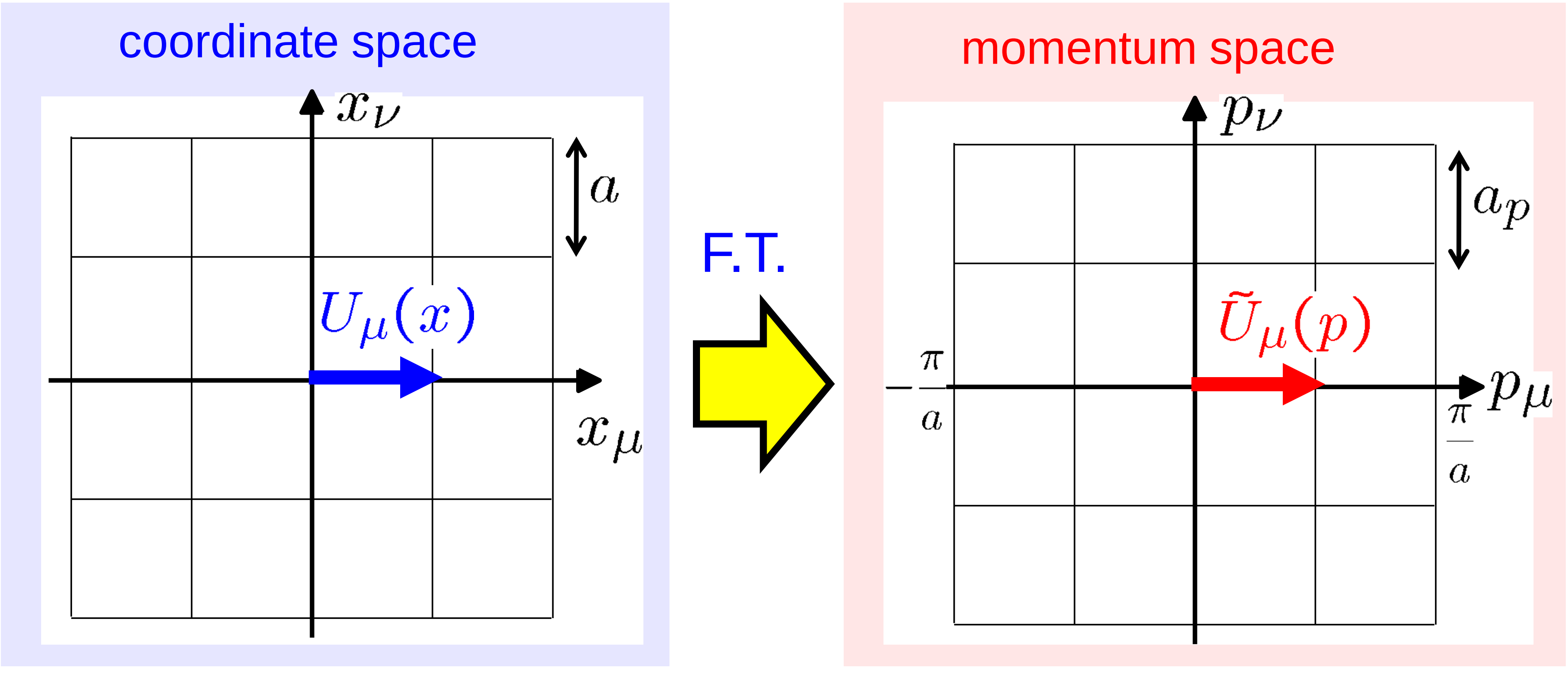


Step 2. *Discrete Fourier transformation*

By 3-dim discrete Fourier transformation,
we define momentum-space link-variable.

momentum-space link-variable

$$\tilde{U}_\mu(p) \equiv \frac{1}{L^3}\sum_{\vec{x}} U_\mu(x)\exp(i\vec{p}\cdot\vec{x})$$

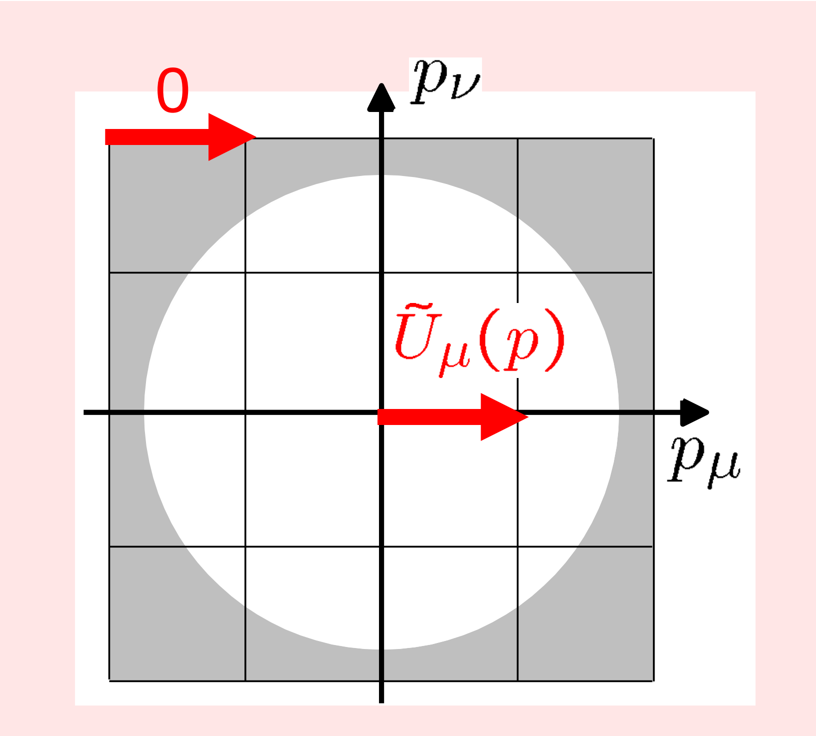


Step 3. *"UV-cut" in the momentum space*

We introduce "UV-cut" in the momentum space.
Outside the cut, $\tilde{U}_\mu(p)$ is replaced by 0 (free-field link-variable).

momentum-space link-variable with UV cut

$$\tilde{U}_\mu^\Lambda(p) \equiv \begin{cases} \tilde{U}_\mu(p) & \text{(inside UV-cut)} \\ 0 & \text{(outside UV-cut)} \end{cases}$$



free-field link variable

$$A_\mu^{\text{free}}(x) = 0$$

$$U_\mu^{\text{free}}(x) = e^{i0} = 1$$

$$\tilde{U}_\mu^{\text{free}}(p) = \delta_{p0}$$

Step 4. *Inverse Fourier transformation*

By the inverse Fourier transformation,

$$U'_\mu(x) \equiv \sum_{\vec{p}} \tilde{U}_\mu^\Lambda(p) \exp(-i\vec{p}\cdot\vec{x})$$

and SU(3) projection by maximizing

$$\mathrm{Re}\,Tr[U_\mu^\Lambda(x)^\dagger U'_\mu(x)]$$

we obtain *Coordinate-Space Link-variable with UV-cut*:

$$U_\mu^\Lambda(x) \in SU(3)$$

Step 5. *Calculation of Wilson Loops*

Using the "UV-cut" link-variable $U^\Lambda{}_\mu(x)$ instead of $U_\mu(x)$, we calculate many Wilson loops W_T^{jk} sandwiched by various combination of initial state $|\phi_k>$ and final state $|\phi_j>$

The calculation condition of Lattice QCD

- SU(3) quenched calculation
- $\beta = \frac{2N_C}{g^2} = 6.0$
- Lattice size : 16^4
- 100 gauge configurations

Here, we only consider
parity-even excited-state potentials.

We prepare $|\phi_k>$ (k=0,1,2,3)
composed by fat-links
obtained by smearing method
with smearing parameter α=2.3,
iteration = 0, 8, 16, 24.

$\beta = 6.0$

$L^4 = 16^4$

Coordinate space

coordinate lattice spacing

$a \simeq$ 0.104fm

$$V = (La)^4 \approx (16.6\mathrm{fm})^4$$

Momentum space

momentum lattice spacing

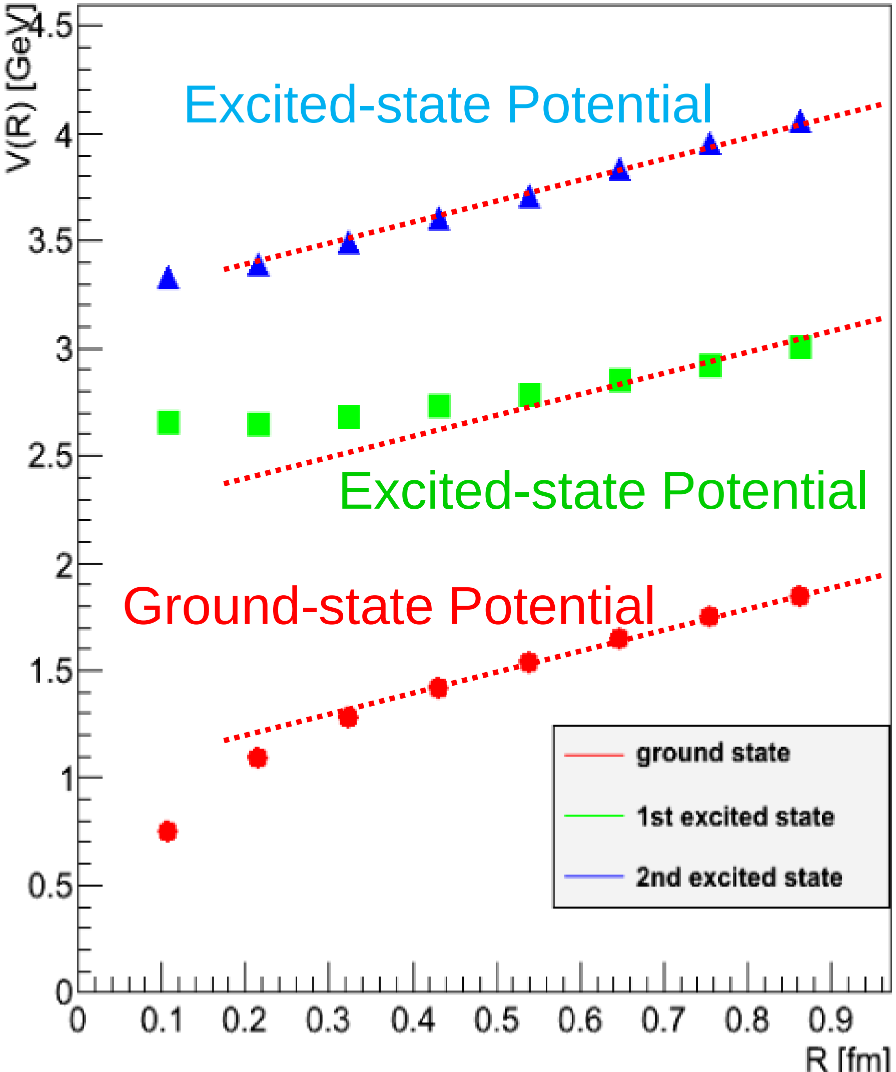
$$a_p \equiv \frac{2\pi}{La} \approx 0.74\mathrm{GeV}$$

$$\frac{\pi}{a} \approx 6\,\mathrm{GeV}$$

Coulomb gauge
3D Fourier transformation

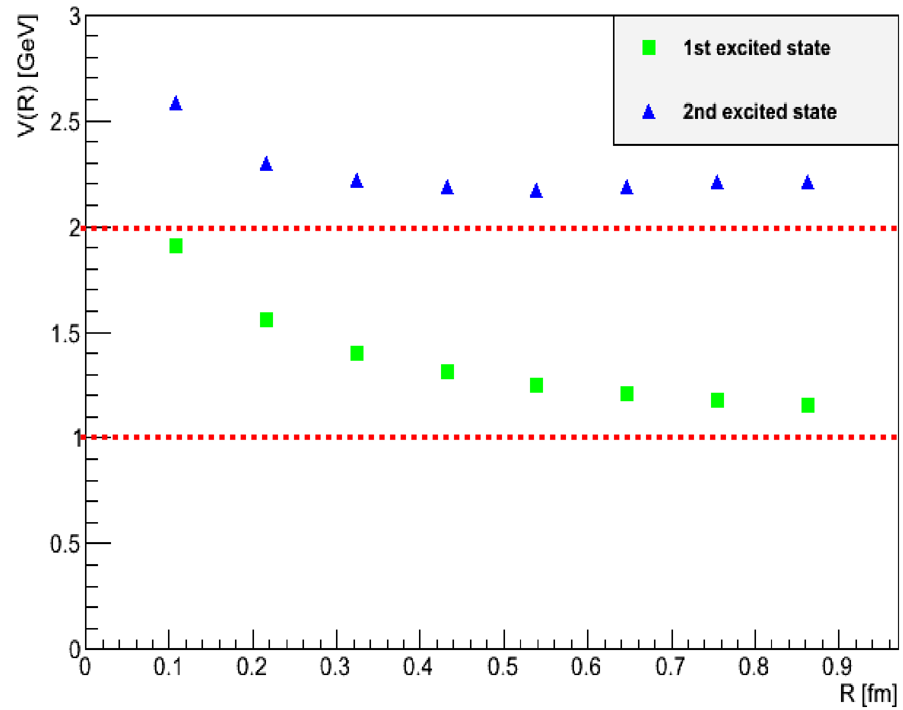
Ground-state/Excited-state Potentials and Gluonic Excitation in Q-Qbar Systems

No cut (original) data



Gluonic Excitation Energy

defined by the difference, $V_n - V_0$

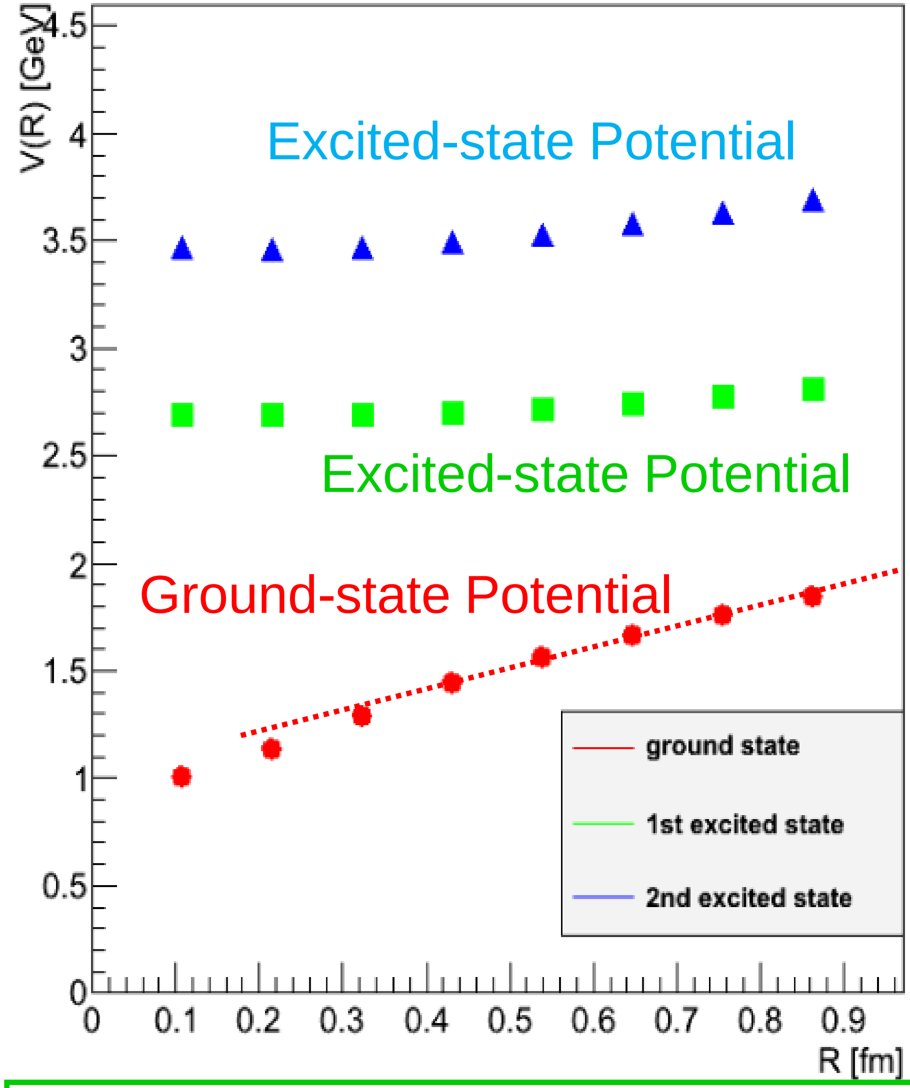


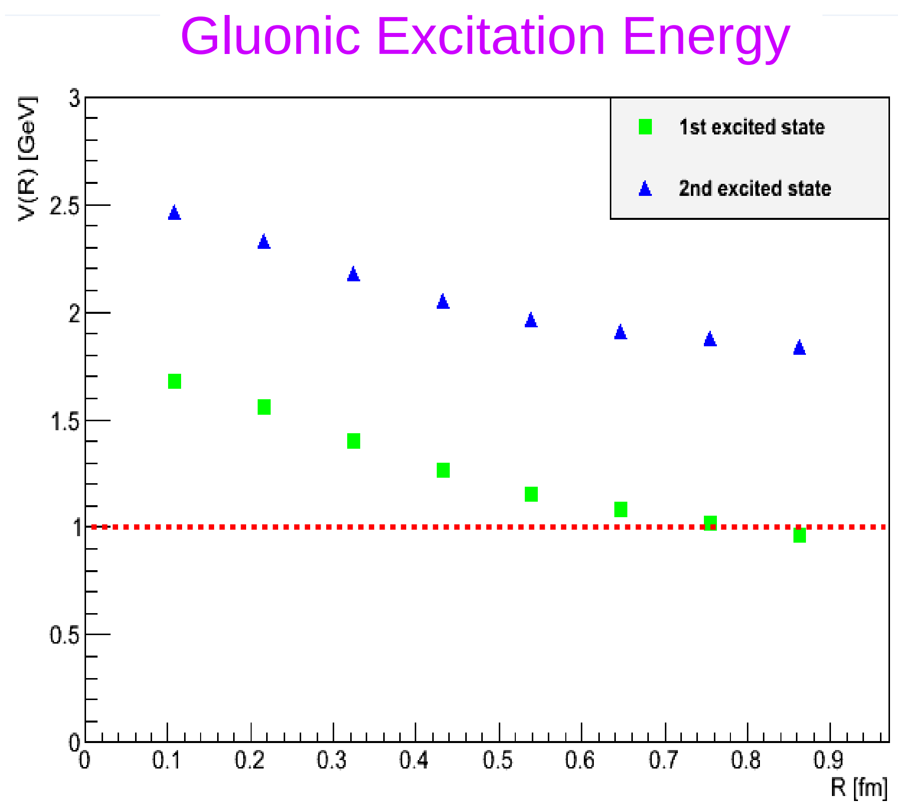
The Gluonic Excitation Energy is 1GeV order.

The IR slopes of Ground-state and Excited-state potentials are almost the same.

Ground-state/Excited-state Potentials and Gluonic Excitation in Q-Qbar Systems

UV-cut: 2.2GeV



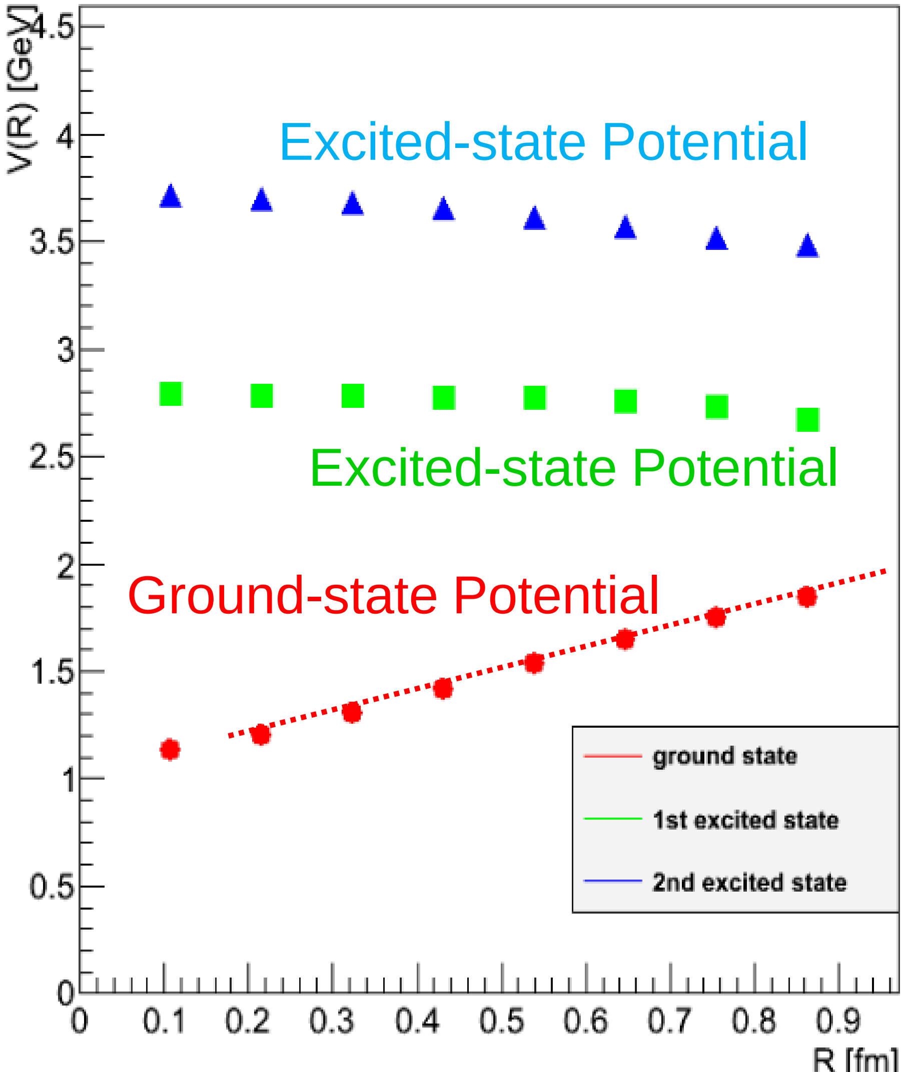


The Gluonic Excitation Energy remains to be 1GeV order.

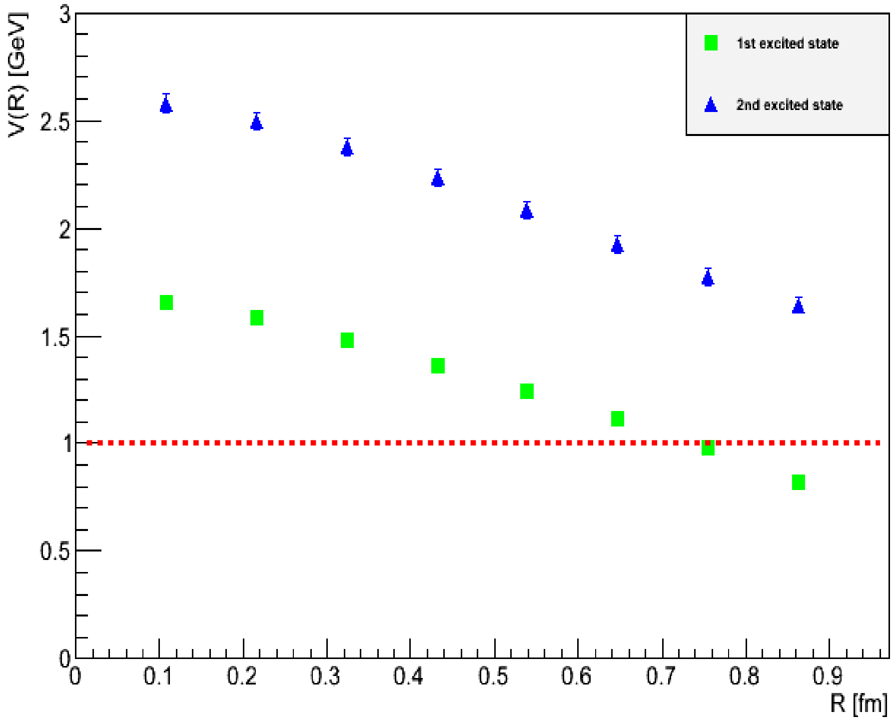
The short-distance Coulomb part (1/r) is reduced in Ground-state potential.
The shape of Excited-state potentials is changed.

Ground-state/Excited-state Potentials and Gluonic Excitation in Q-Qbar Systems

UV-cut: 1.5GeV



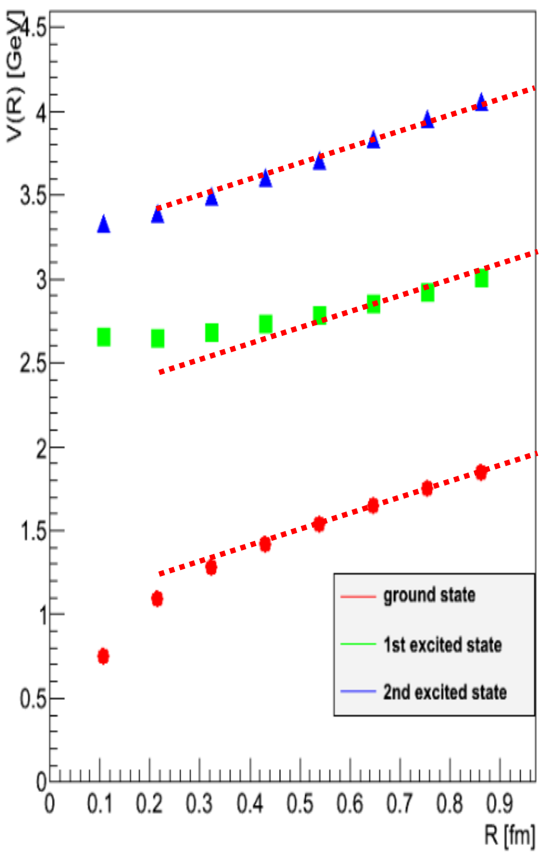
Gluonic Excitation Energy



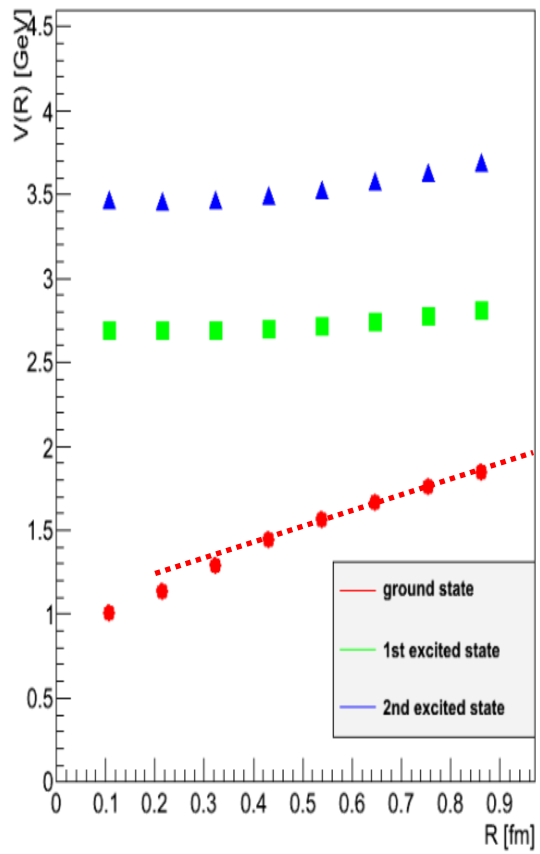
The Gluonic Excitation Energy remains to be 1GeV order.

The short-distance Coulomb part (1/r) disappears in Ground-state potential.
The shape of Excited-state potentials is changed.

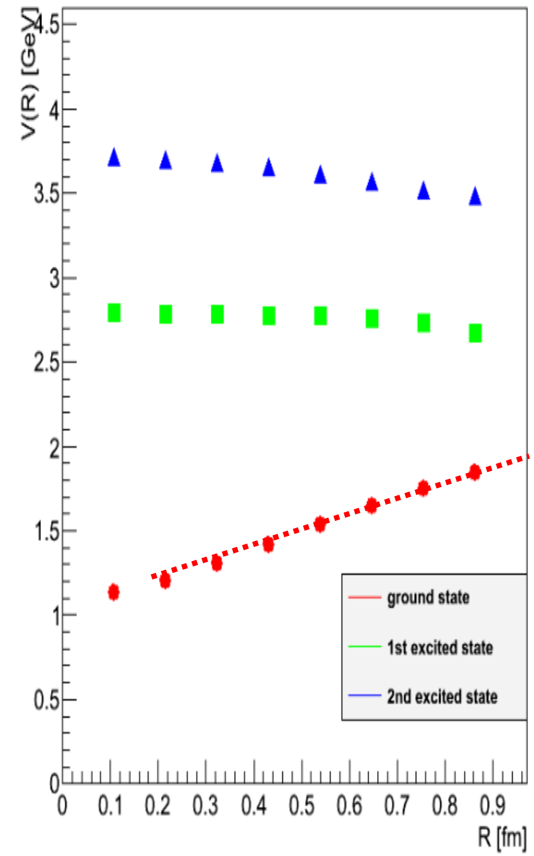
Ground-state and Excited-state Potentials in Lattice QCD



No cut (original)



UV-cut: 2.2GeV



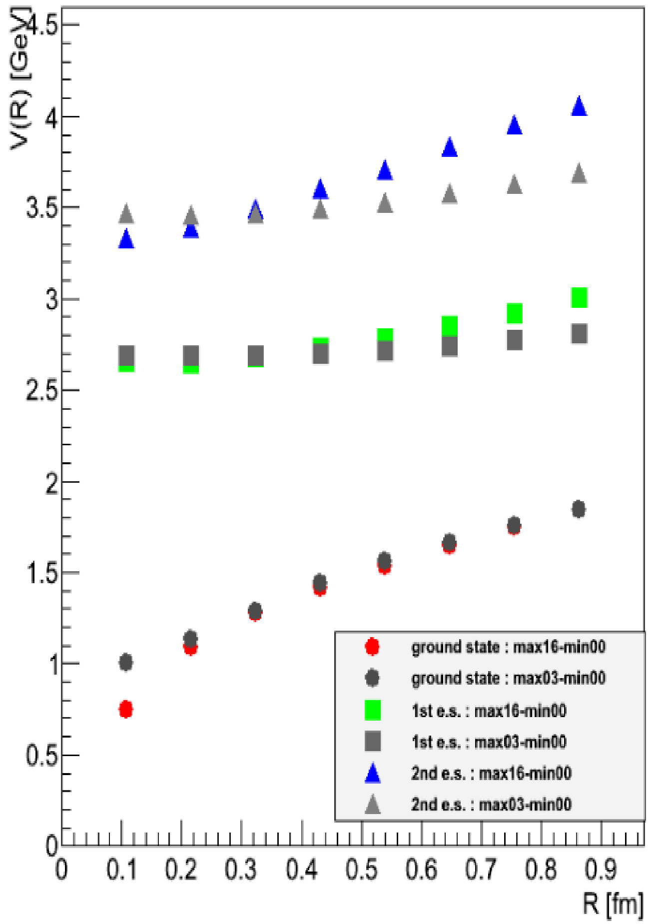
UV-cut: 1.5GeV

By the Cut of UV-gluons,

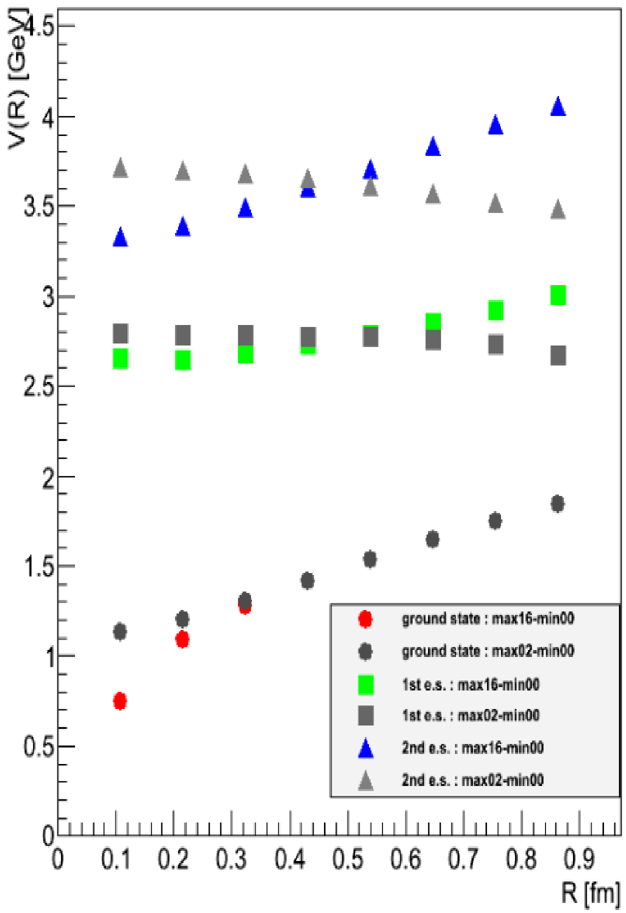
the IR part of Ground-State Potential is almost unchanged,

and the change of Excited-State Potential is more significant.

Effect of the UV-gluon removal from Ground-state and Excited-state Potentials



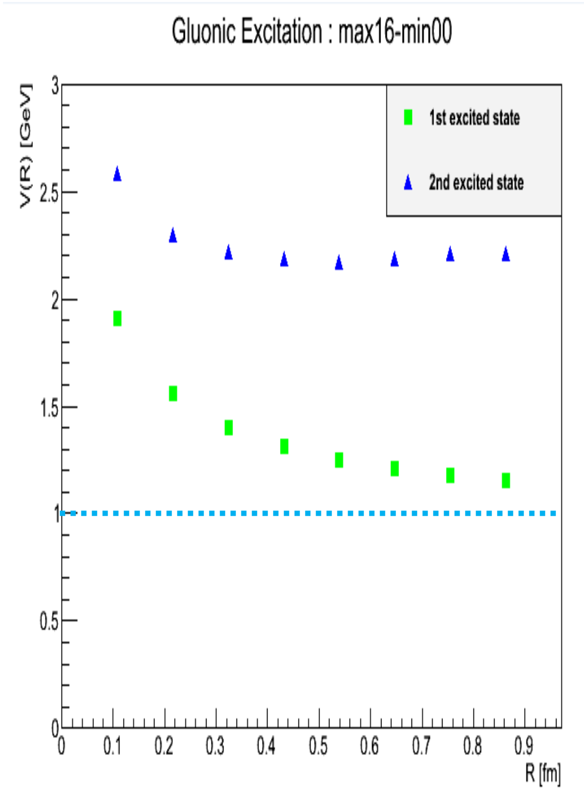
No cut vs UV-cut: 2.2GeV



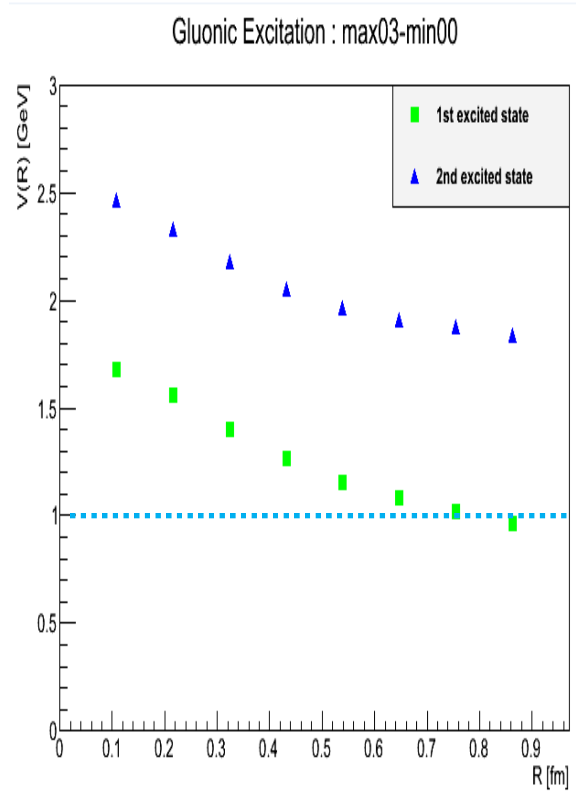
No cut vs UV-cut: 1.5GeV

The black symbols denote the UV-cut data. By the Cut of UV-gluons, the IR part of Ground-State Potential is almost unchanged, and the change of Excited-state potential is more significant.

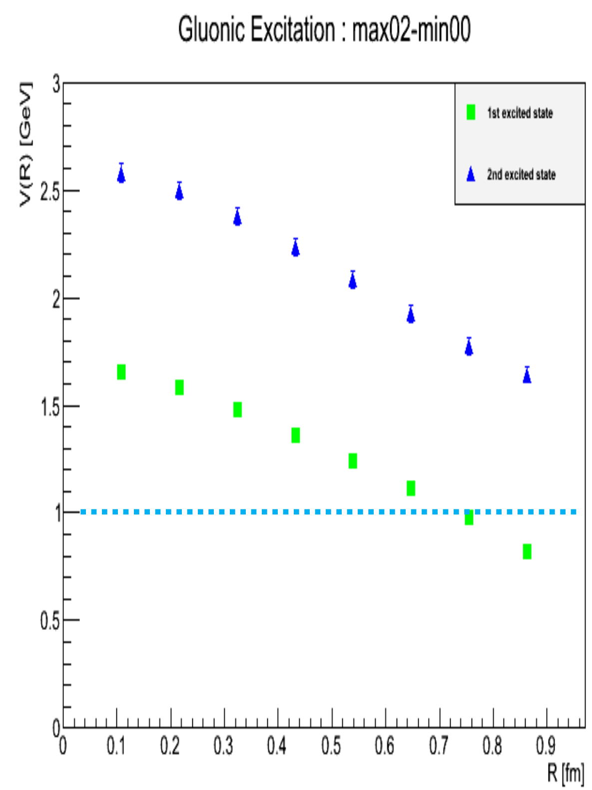
SU(3) Lattice QCD result for
Gluonic Excitation Energy defined by V_n-V_0



No cut (original)



UV-cut: 2.2GeV



UV-cut: 1.5GeV

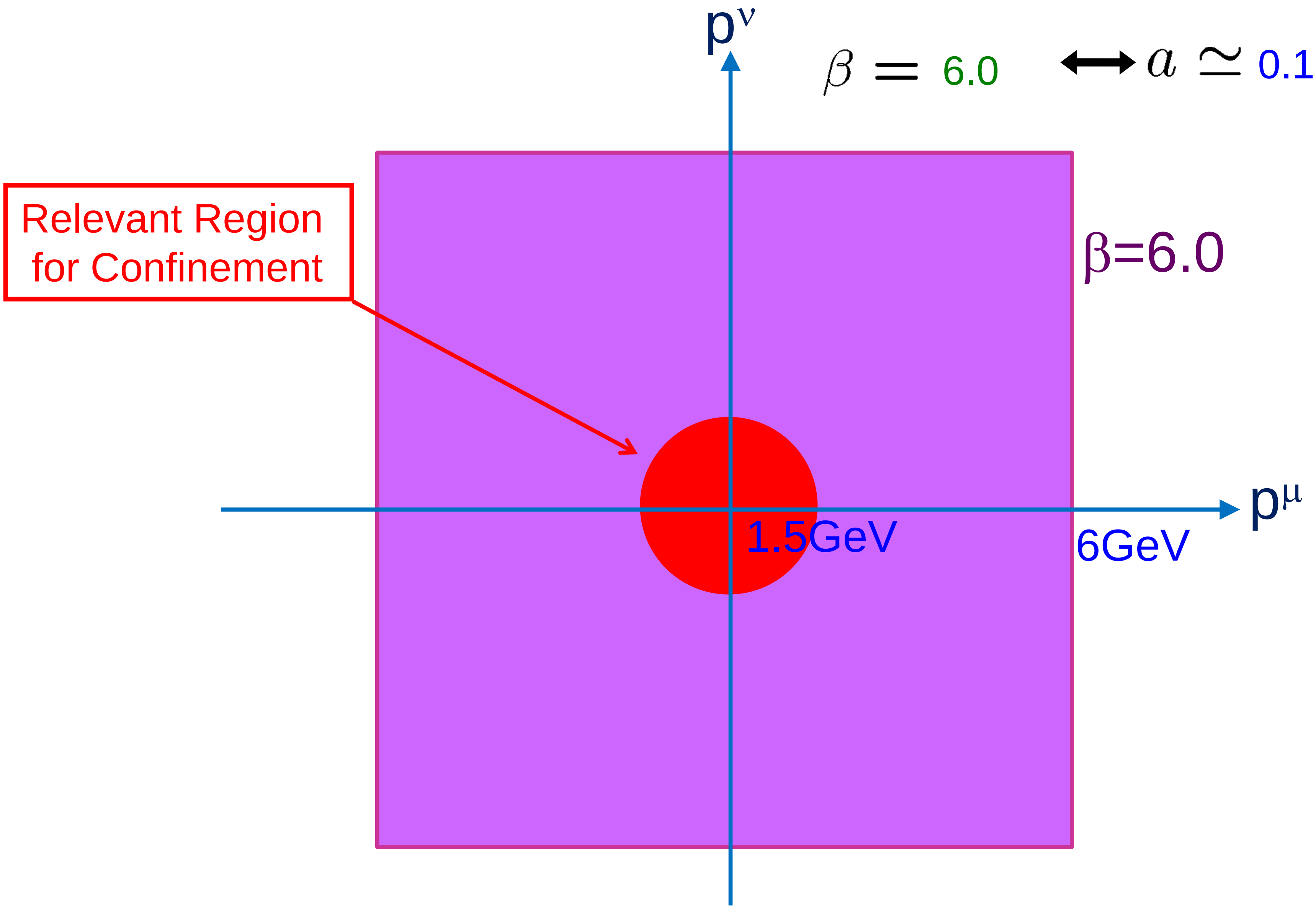
Roughly speaking, even after the removal of UV-gluons, the magnitude of Gluonic Excitation is approximately unchanged.
The Gluonic Excitation remains to be 1GeV order.

Summary and Concluding Remarks

- Using SU(3) quenched lattice QCD, we have studied Ground-state and low-lying even-parity Excited-state potentials of quark-antiquark systems in terms of the Gluon Momentum Component in Coulomb gauge.
- By introducing UV-Cut in the Gluon Momentum Space, we study the sensitivity of the potentials and the Stringy Excitation to the UV-gluons.
- Even after cutting off high-momentum gluon component above1.5 GeV, the IR part of Ground-State Potential are almost unchanged.
- The change of Excited-State Potential is more significant.
- Roughly, even after the removal of UV-gluons, the magnitude of the Gluonic Excitation is approximately unchanged, and remains to be 1GeV order.

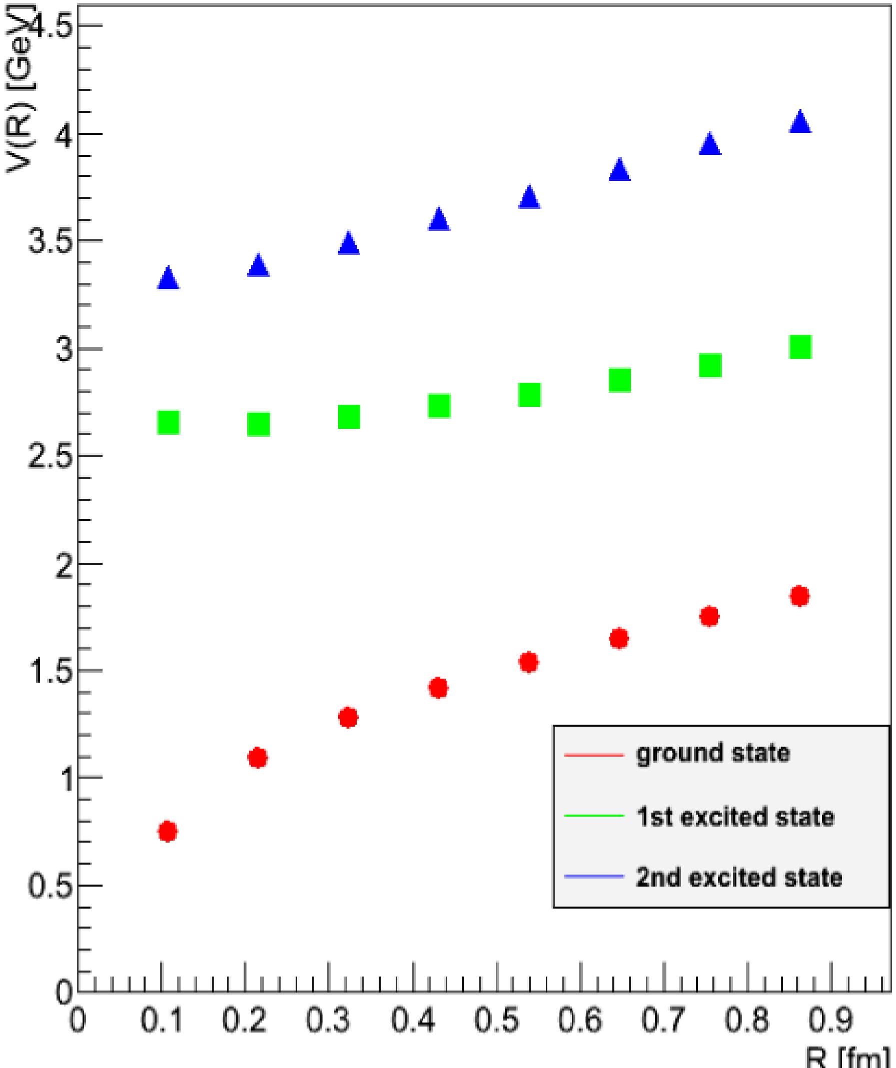
Lattice-QCD Brilliouin zone and Relevant Region for confinement

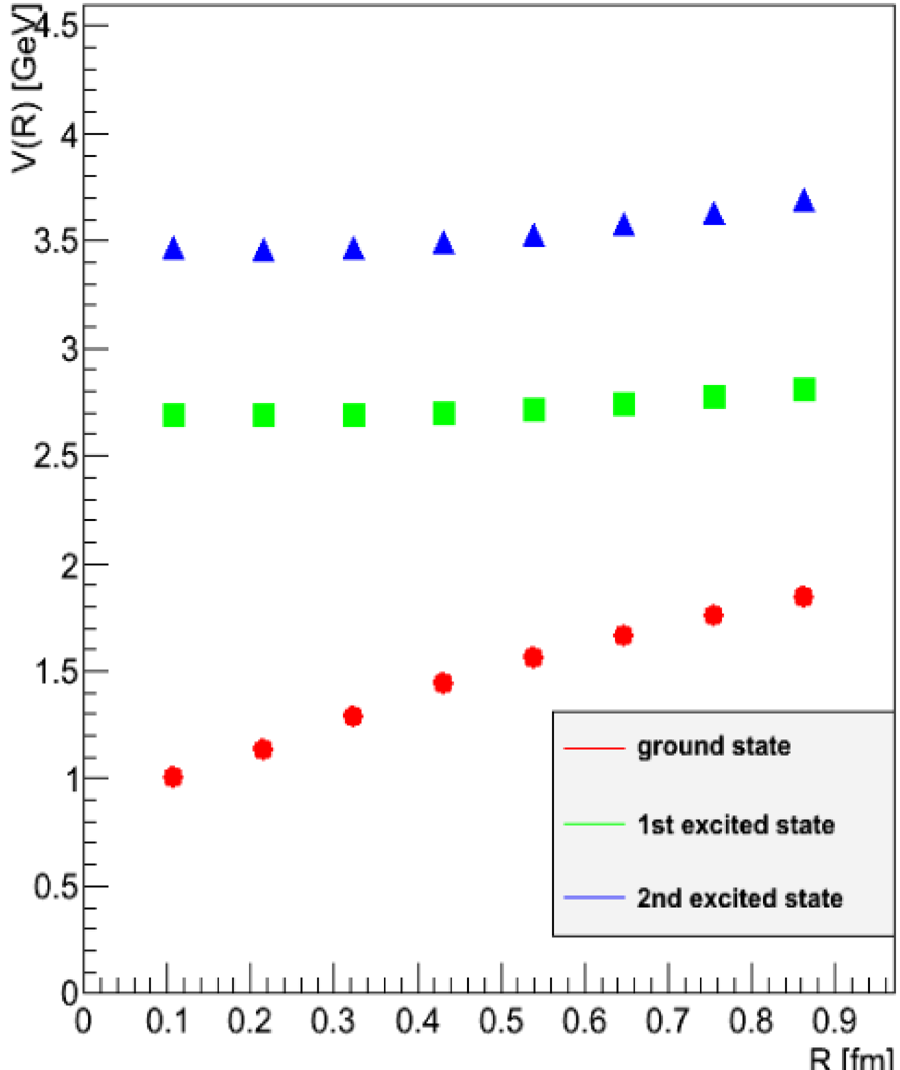
$\beta = 6.0 \longleftrightarrow a \simeq 0.104$ fm



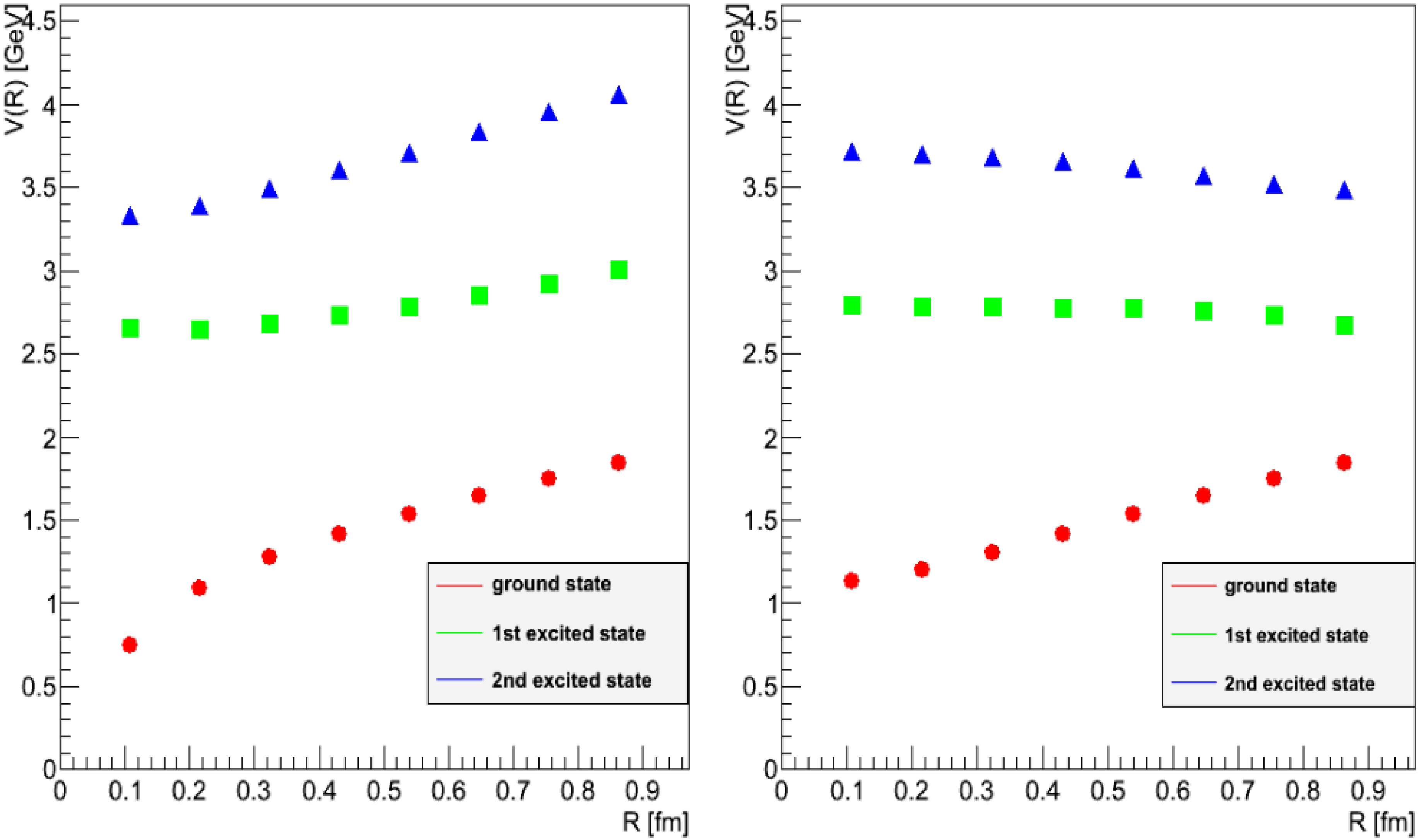
Only a small part of the total Brillioiun zone contributes to confinement.

Ground-state and Excited-state Potentials in Lattice QCD

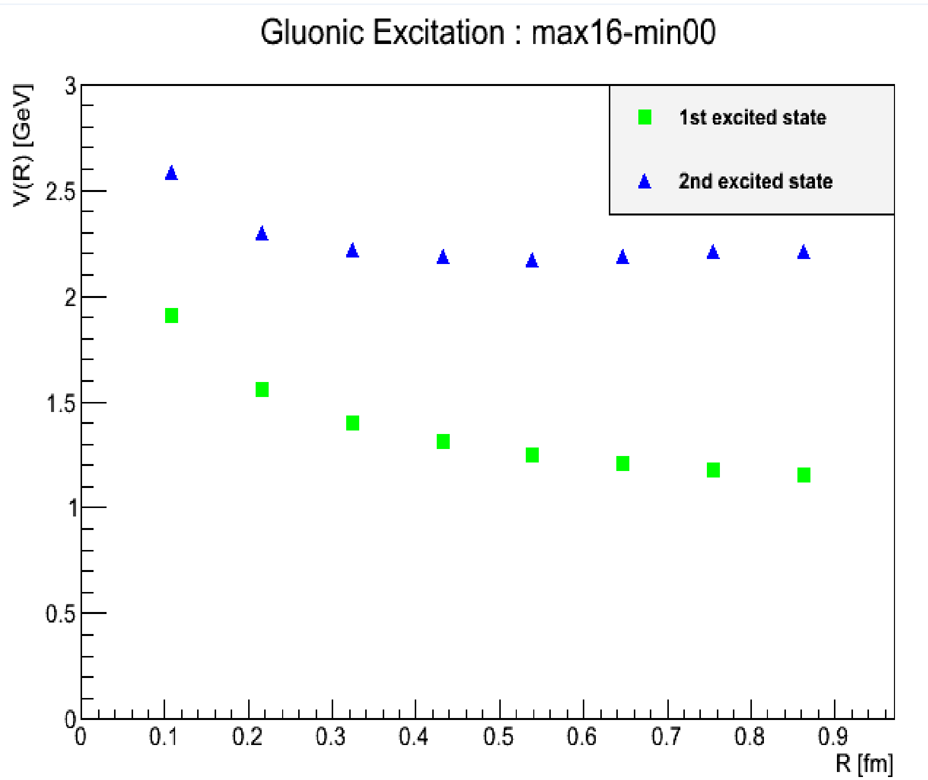


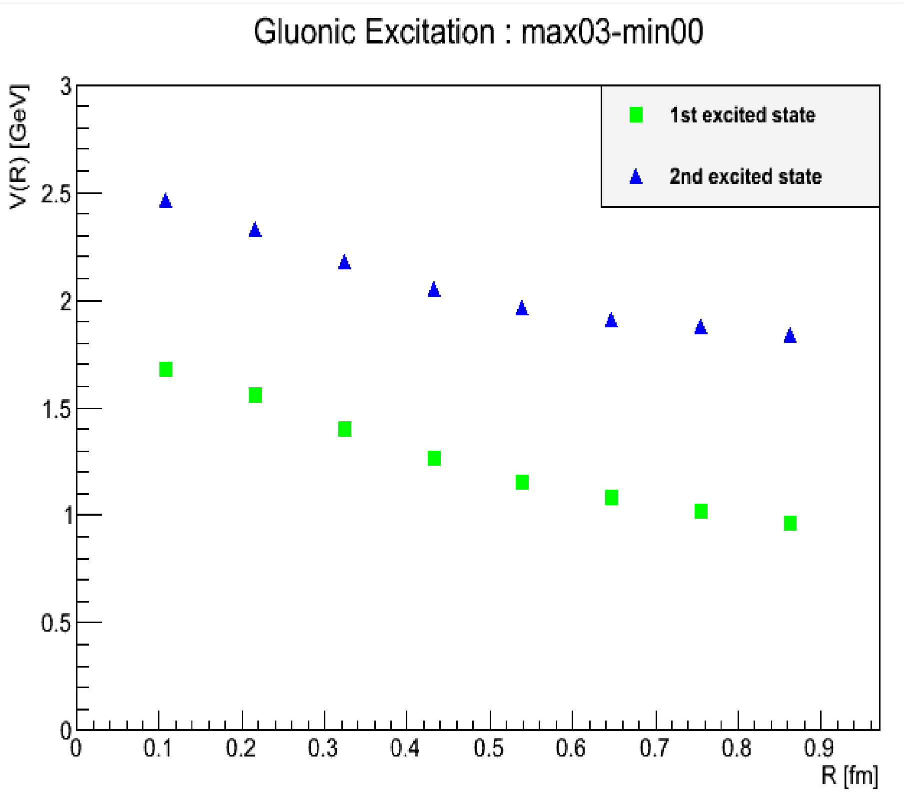


Ground-state and Excited-state Potentials in Lattice QCD

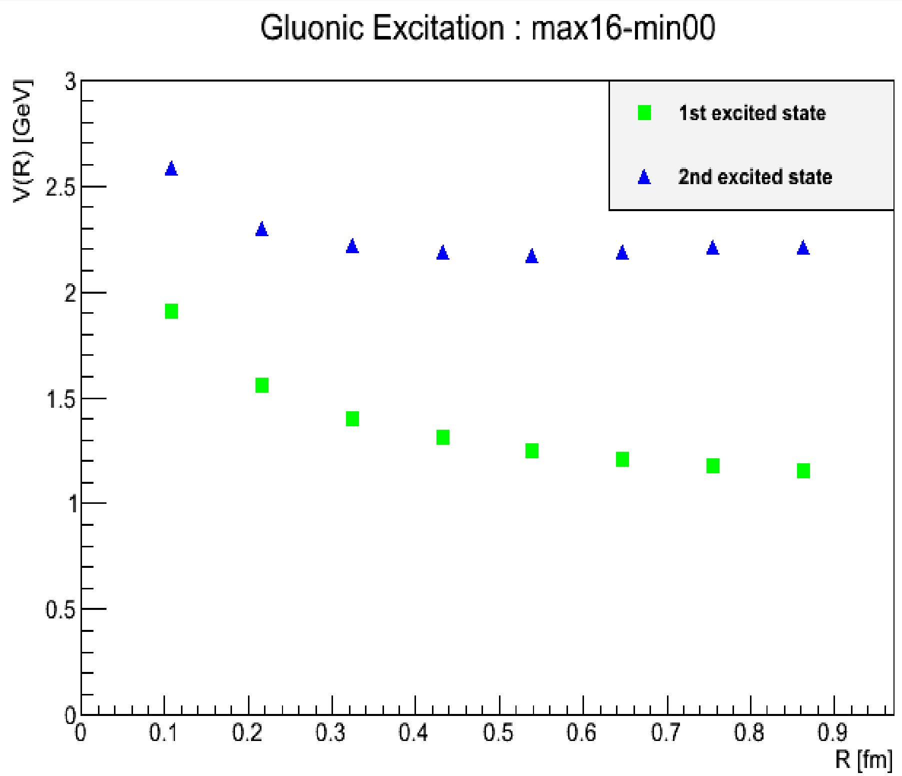


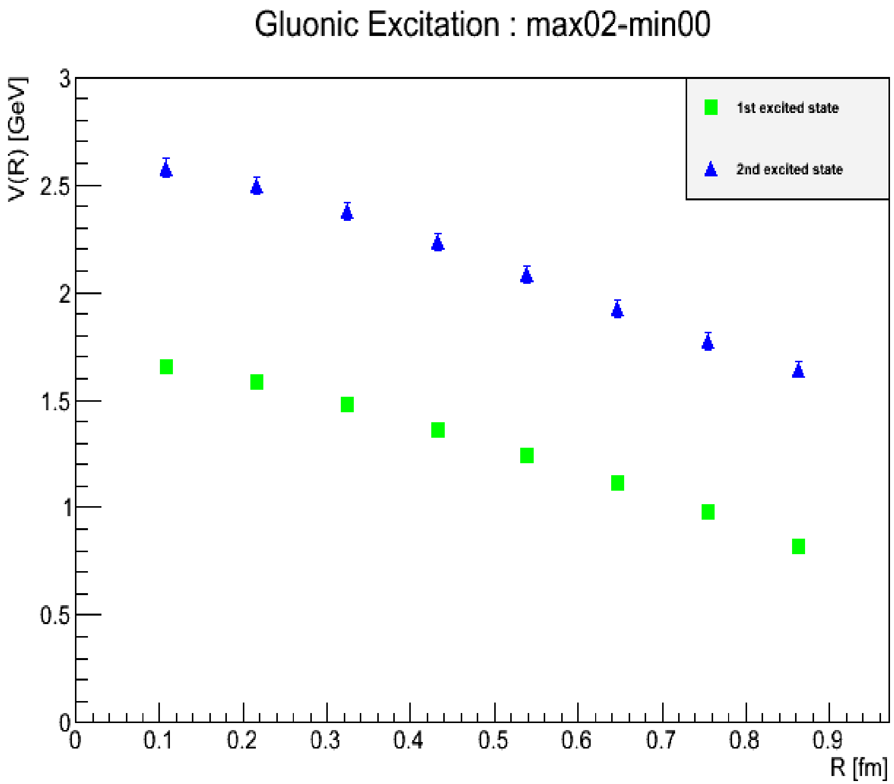
Gluonic Excitation in Q-Qbar Systems in SU(3) Lattice QCD



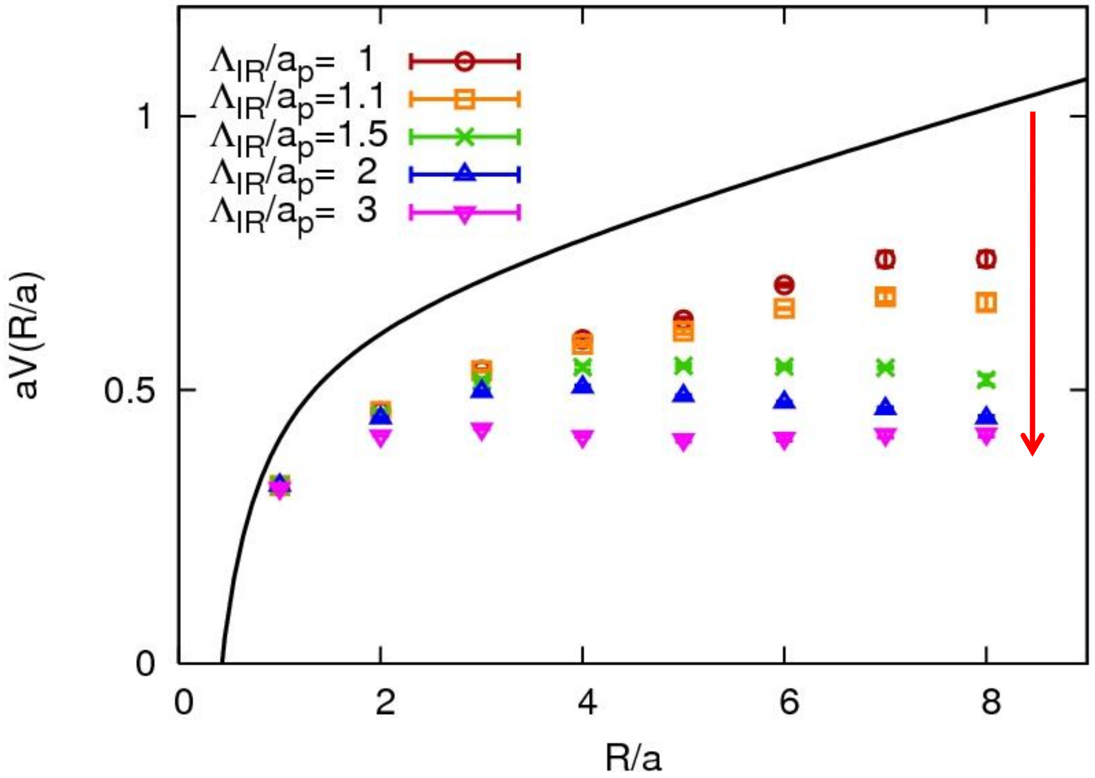


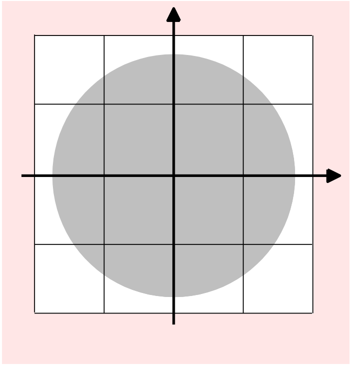
Gluonic Excitation in Q-Qbar Systems in SU(3) Lattice QCD





Lattice QCD result of IR cut for Quark-antiquark potential





$a \simeq 0.10$ fm
$a_p \simeq 0.77$ GeV

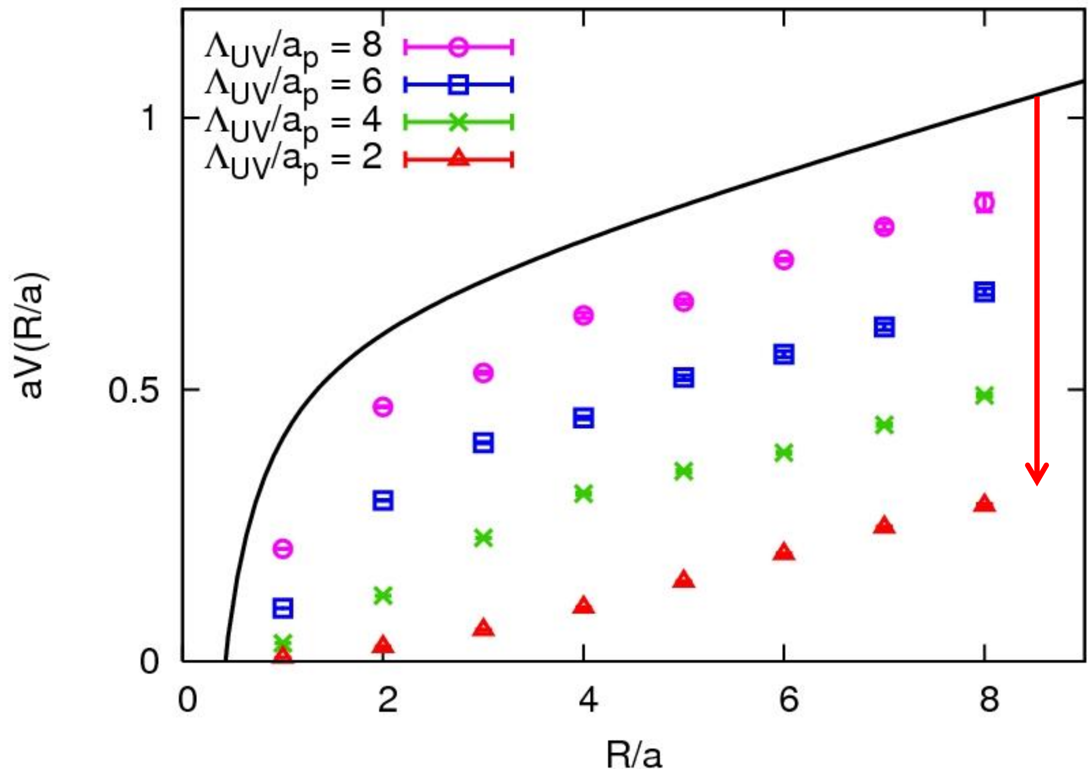
!

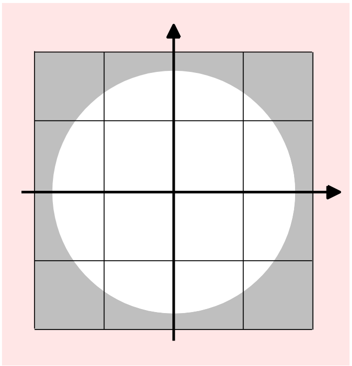
By the IR cutoff, the Coulomb potential seems to be unchanged, but the confinement potential is largely reduced.

Coulomb potential (UV) → unchanged

confinement potential (IR) → disappears

Lattice QCD result of UV cut for Quark-antiquark potential





$a \simeq 0.10$ fm

$a_p \simeq 0.77$ GeV

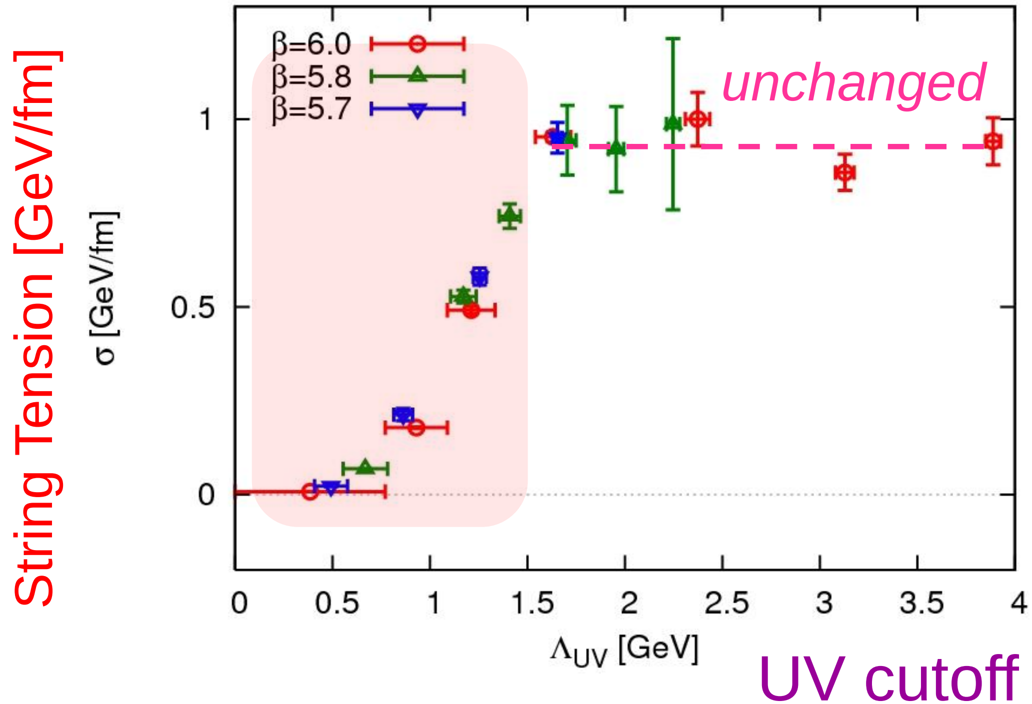
!

By the UV cutoff, the Coulomb potential is largely reduced, but the confinement potential is almost unchanged.

Coulomb potential (UV) → disappears

confinement potential (IR) → unchanged

Quark Confining Force (String Tension) with UV cut



!!

As a remarkable fact, the string tension is almost unchanged even after cutting off the high-momentum gluon component above 1.5 GeV. *When the UV cutoff is smaller than 1.5 GeV, the string tension is reduced.*

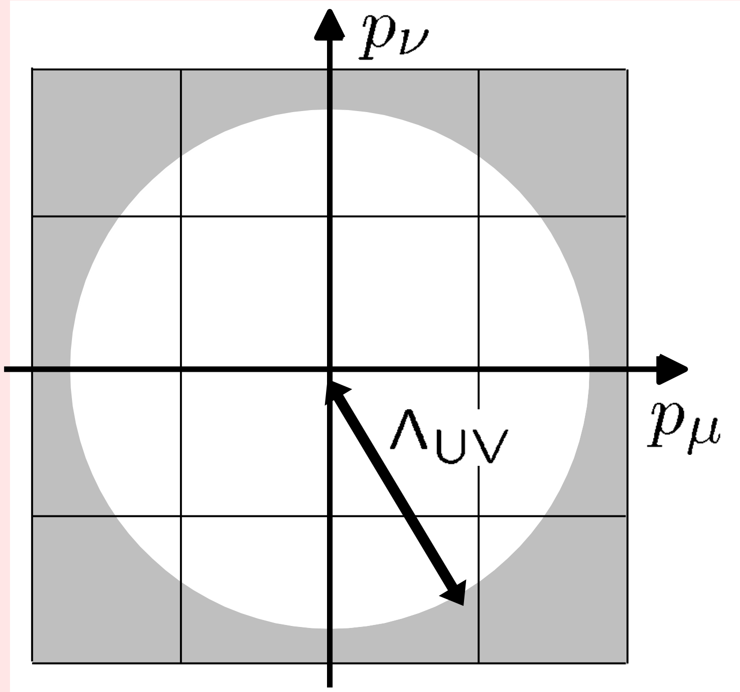
Only Low-Momentum Component of Gluons below 1.5GeV is relevant for Confinement.

Procedure to select Gluonic momentum-components in Lattice QCD

We mainly consider UV cut or IR cut in 3D momentum space.

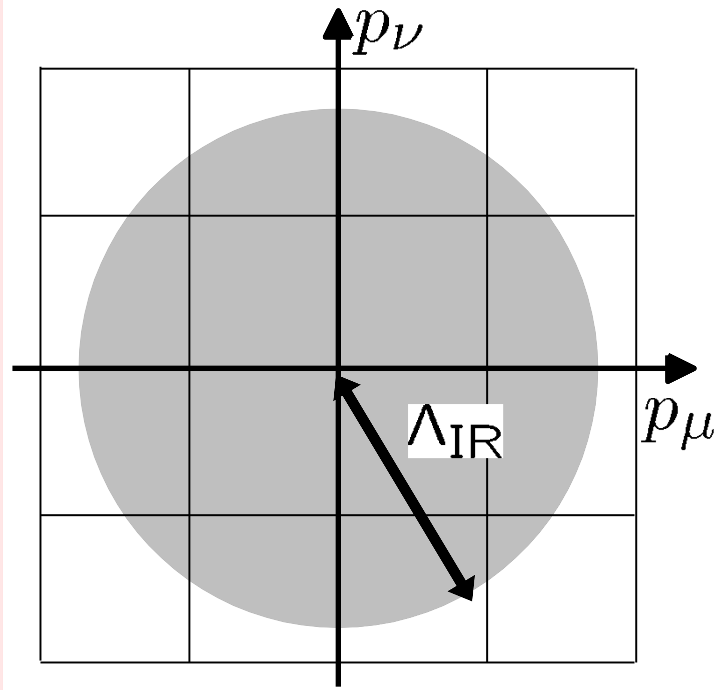
ultraviolet (UV) cut

$$\sqrt{p^2} > \Lambda_{UV}$$



infrared (IR) cut

$$\sqrt{p^2} < \Lambda_{IR}$$



Summary of procedure to select gluonic momentum-components in lattice QCD

Step 1. **Gauge Configuration Generation and Gauge Fixing**

coordinate-space link-variable $U_\mu(x)$

$\Downarrow$ F.T.

Step 2. Fourier Transformation

momentum-space link-variable $\tilde{U}_\mu(p)$

$\Downarrow$ "cut"

Step 3. Imposing a "cut"

momentum-space link-variable with a cut $\tilde{U}_\mu^\Lambda(p)$

$\Downarrow$ F.T.

Step 4. **Inverse Fourier Transformation and Projection**

coordinate-space link variable with a cut $U_\mu^\Lambda(x)$

Step 5. Calculation