Strangeness of the nucleon with $Nf = 2 + 1 + 1$ twisted mass fermions

V. Drach
NIC, DESY Zeuthen

in collaboration with C. Alexandrou, S. Dinter, K. Jansen, G. Koutsou, A. Vaquero
ETM Collaboration

Lattice 2012, Cairns, Australia,
June 25^{th}, 2012

Outline

Introduction

Motivations

- Experimental direct detection of dark matter put bounds on the WIMP-Nucleon cross section
- Results are interpreted using various models (including SUSY) : systematic uncertainty due to $\langle N(p)|\bar{q}q|N(p)\rangle$

  $\longrightarrow$ non-perturbative computation is required

  see Plenary talk [R. Young]

- sigma terms : $\sigma_{\pi N} \equiv m_l \langle N(p)|\bar{u}u + \bar{d}d|N(p)\rangle$ and $\sigma_{KN} = m_s \langle N(p)|\bar{s}s|N(p)\rangle$
- dimensionless ratio : $y_N \equiv \frac{2\langle N(p)|\bar{s}s|N(p)\rangle}{\langle N(p)|\bar{u}u+\bar{d}d|N(p)\rangle}$
- Twisted mass fermions offer two main advantages :
  - efficient noise reduction technique
  - multiplicative renormalization

  see [arXiv:1202.1480] for details

- In this talk : discussion of one systematic effects :

  excited states contamination

Lattice setup

$N_f = 2+1+1$ dynamical simulations

- $N_f = 2+1+1$ configurations generated by ETMC
- lattice spacing $a = 0.078$ fm
- $32^3 \times 64$ lattice : $L = 2.5$ fm, $m_{\mathrm{PS}} L > 3.5$
- fixed pion mass ≈ 380 MeV

Mixed action setup

- Mixed action setup : introduce a doublet of degenerate twisted mass fermions (χ_q, μ_q)
- $a\mu_s$ and $a\mu_c$ can be tuned to reproduce the K, D meson masses in the unitary setup
- Noise reduction techniques based on an exact property of the valence action see also talks [F. Zimmermann] and [A. Vaquero]

Twisted mass fermions and disconnected diagrams

- Twisted mass Wilson Dirac operator : $D_\pm[U] = D_{\mathrm{W}}[U] + am_0 \pm ia\mu_q\gamma_5$
- Noise reduction technique specific for twisted mass fermions :
  - Exact relation : [C. Michael, C. Urbach, 2007]

$$\frac{1}{D_-} - \frac{1}{D_+} = 2ia\mu_q \frac{1}{D_-}\gamma_5\frac{1}{D_+}$$

- r.h.s can be evaluated stochastically using “one-end-trick”
- Useful for correlators containing the insertion of the operator $\overline{\chi}(x)\Gamma\tau^3\chi(x)$ (twisted basis)

Correlators

- two-point function : (J : smeared nucleon interpolating field)

$$C_{\rm 2pts}(t) = \sum_{\vec{x}} \langle J(t,\vec{x}) J^{\dagger}(0) \rangle \propto e^{-M_X t} + \mathcal{O}(e^{-\Delta M_X t}), \qquad \Delta M_X = M_{X^*} - M_X$$

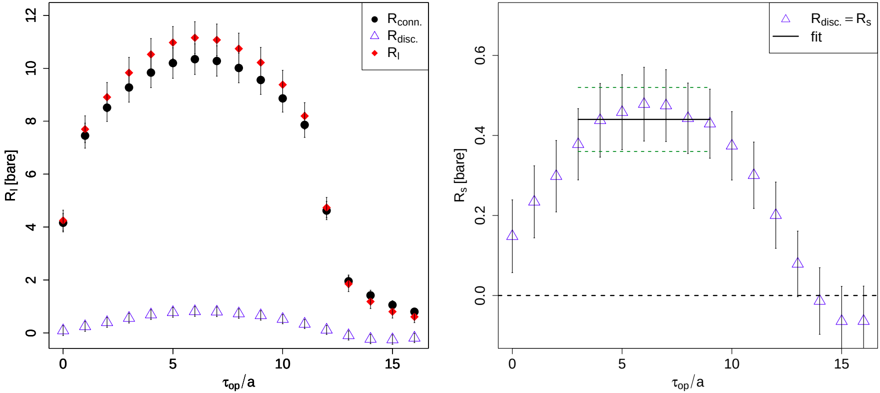
- Ratio :

$$R(t,t_s) = \frac{\sum_{\vec{x},\vec{y}} \langle J(t_s,\vec{x}) O(t,\vec{y}) J^{\dagger}(0) \rangle}{C^X_{\rm 2pts}(t_s)} = \langle X|O(0)|X\rangle + \mathcal{O}(e^{-\Delta M_X (t-t_s)}) + \mathcal{O}(e^{-\Delta M_X t_s})$$

- Plateau summation method :

$$P(t_s) = \sum_{t=0}^{t_s} R(t,t_s) = \sum_{t=0}^{t_s} \frac{\sum_{\vec{x},\vec{y}} \langle J(t_s,\vec{x}) O(t,\vec{y}) J^{\dagger}(0) \rangle}{C^X_{\rm 2pts}(t_s)} = A + \langle X|O(0)|X\rangle t_s + \mathcal{O}(e^{-\Delta M_X t_s})$$

Results

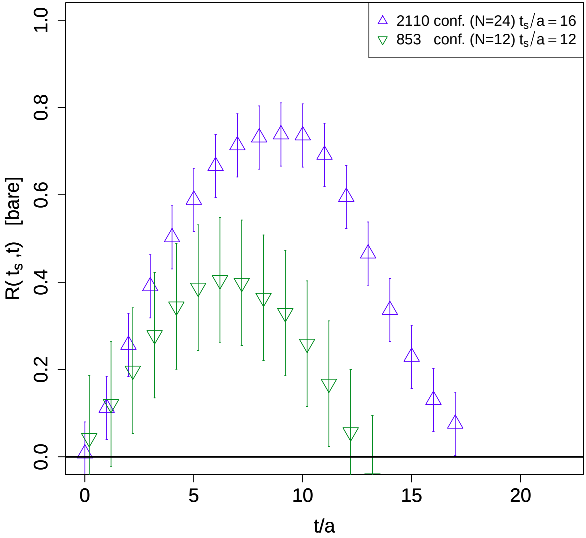


- $R(t, t_s = 12a)$ (bare) in the light (connected, disconnected and sum) and strange sector
- source-sink separation fixed to ~ 1 fm
- ~ 800 configurations used

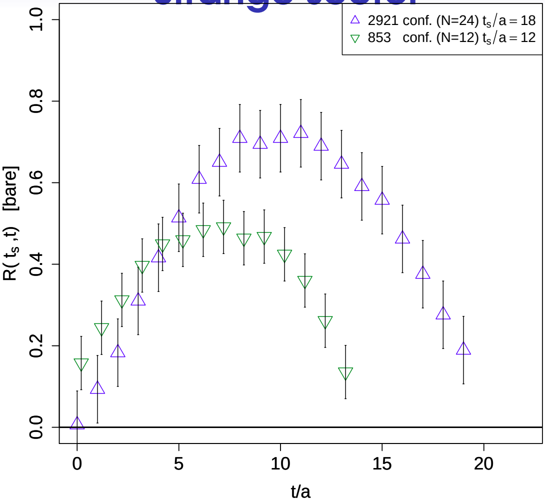
Excited states contamination

- Accurate estimation for a source-sink separation of about 1 fm BUT we need to take the limit $t, t_s \to \infty$
- Problem : the noise increases exponentially when t or t_s increases :
  $\rightsquigarrow$ High statistic is needed to check for a contamination of the results by excited states

Light sector

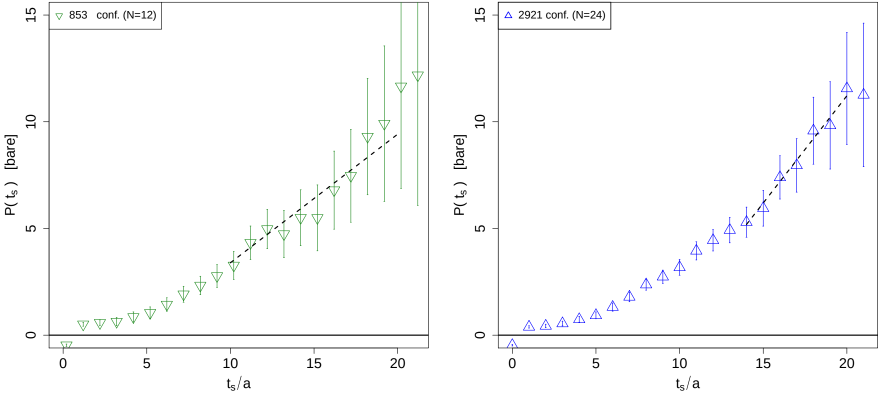


Strange sector



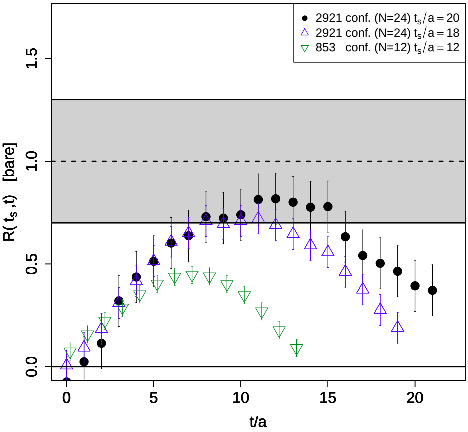
- small stat. $t_s/a = 12$ $\langle N|\bar{s}s|N\rangle = 0.41(4) \longrightarrow f_{T_s} = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N} \approx 0.13(2)$
- large stat. $t_s/a = 18$ $\langle N|\bar{s}s|N\rangle = 0.68(9)$

Summed plateau

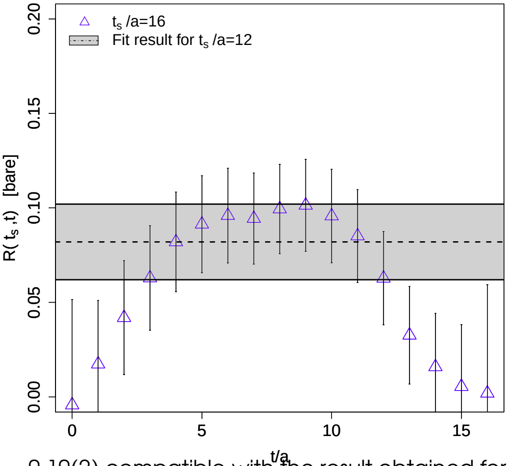


- small stat. [10, 20] : $\langle N|\bar{s}s|N\rangle = 0.60(27)$
- large stat. [14, 20] : $\langle N|\bar{s}s|N\rangle = 1.00(31)$

Comparison


2921 conf. (N=24) t_s/a = 20
2921 conf. (N=24) t_s/a = 18
853 conf. (N=12) t_s/a = 12
R(t_s ,t) [bare]
t/a
0.0
0.5
1.0
1.5
0
5
10
15
20

Strangeness of the nucleon



$t_s/a = 16: y_N = 0.10(2)$ compatible with the result obtained for $t_s/a = 12$

$\longrightarrow$ cancelation of the excited the excited states contribution

Conclusion

Twisted mass fermions

- Efficient noise reduction techniques (no eigenmode preconditionning)
- Multiplicative renormalization both in the unitary (light) and mixed action (strange) setup

Excited states contamination

- Large dependence in the source-sink separation indicates excited states contamination
- Is $t_s \sim 1.5$ fm enough for the σ-terms ?
- y_N parameter can be extracted safely with $t_s \sim 1$ fm

Future plans

- Larger statistic to increase even more the source-sink separtion
- Investigation at smaller a :
  $\longrightarrow$ confirm that there is no large unitary breaking
- light quark mass dependence