# Sigma commutators from an SU(3) chiral extrapolation

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# Outline

# History

- Idea of symmetries and broken symmetries important in physics since the 50s
- Understand hadron structure within QCD in terms of symmetries
- Want to understand / quantify small violations of fundamental symmetries
- Would like to measure matrix elements of symmetry breaking density

# History continued...

We consider

- Sigma terms
  - Matrix element of even number of charge commutators of the Hamiltonian
  - Modern definition: usually two commutators
  - e.g., $\sigma_N = \frac{1}{3}\langle N|[{}^5\mathcal{Q}_i, [{}^5\mathcal{Q}_i, H]]|N\rangle$
- Double commutator: picks out symmetry breaking part of $H$
  - For QCD, that is $m\overline{\psi}\psi$
  - Operator identity: $[{}^5\mathcal{Q}_i, [{}^5\mathcal{Q}_j, H]] = \delta_{ij} \int d\vec{r} m\overline{\psi}(\vec{r})\psi(\vec{r})$
  - So $\sigma_N = \overline{m}\langle N|\overline{u}u + \overline{d}d|N\rangle$

# Definitions

For a baryon $B$, the sigma terms are scalar form-factors evaluated in the limit of vanishing momentum transfer.
For each quark flavor $q$,

$$\sigma_{Bq} := m_q \langle B|\overline{q}q|B\rangle \tag{1}$$
$$\overline{\sigma}_{Bq} := \sigma_{Bq}/M_B \tag{2}$$

For the nucleon, define (traditional)

$$\sigma_{\pi N} := m_l \langle N|\overline{u}u + \overline{d}d|N\rangle \tag{3}$$
$$\sigma_s := m_s \langle N|\overline{s}s|N\rangle \tag{4}$$

# Motivation - why study $\sigma$ terms?

To understand

- Nature of explicit chiral symmetry breaking in QCD
- Decomposition of the mass of the nucleon
- Nucleon structure

Also important for

- Interpretation of experimental searches for dark matter
  - e.g., neutralino
  - Interactions with hadronic matter determined by couplings to $\sigma_l$, $\sigma_s$
  - See recent papers[1,2]

**Particularly topical: Dark Matter**

[1] *S. J. Underwood et al., [arXiv:1203.1092 [hep-ph]]*

[2] *R. J. Hill and M. P. Solon, Phys. Lett.* **B707**, *539 (2012)*

# Traditional evaluation

**$\boldsymbol{\sigma}_{\pi\mathbf{N}}$**

- Experimental: $\sigma_{\pi N}$ determined from $\pi N$ scattering data
  - not well known
- $\sigma_{\pi N} \sim$ 45-70 MeV
- Controversial, limited precision (but better than $\sigma_s$)

**$\boldsymbol{\sigma}_{\mathbf{s}}$**

- Indirect: $\sigma_s$ evaluated using $\sigma_{\pi N}$ and $\sigma_0 = m_l \langle N|\overline{u}u + \overline{d}d - 2\overline{s}s|N\rangle$
- $\sigma_s \sim$ 300 MeV
  - up to $\frac{1}{3}M_N$ from non-valence quarks
  - incompatible with constituent quark models
- Determination has limited precision
- $\sigma_s$ has uncertainty $\sim$ 90 MeV, even with perfect $\sigma_{\pi N}$

**$\boldsymbol{\sigma}_{\mathbf{s}}$ difficult to pin down experimentally**

# Lattice QCD results

Need to reduce uncertainty on $\sigma_s$ - **lattice QCD**

Many calculations in recent years, give $\sigma_s$ from the lattice

- Global: 20-50 MeV
- This work: $21 \pm 6$ MeV (See later)

Common methods:

1. Direct calculation
2. Feynman-Hellman Theorem (e.g., this work)

# Feynman-Hellman Relation

$$\sigma_{Bq} = m_q \frac{\partial M_B}{\partial m_q}$$

Gell-Mann-Oakes-Renner:

$$m_l \to m_\pi^2/2 \quad (5)$$

$$m_s \to m_K^2 - m_\pi^2/2 \quad (6)$$

Lattice QCD:

- Baryon masses evaluated at different $(m_\pi, m_K)$
- Most simulations not yet at physical point

# Our method

- **Have:** PACS-CS[1] lattice data for octet baryon masses at different $(m_\pi, m_K)$ values
- **Fit:** Chiral perturbation theory baryon mass function to data
  - Extrapolate to physical point
  - Fit constrains slope at physical point
  - Correlated error analysis
- **Differentiate:** Use Feynman-Hellman relation to evaluate sigma terms

[1] *S. Aoki et al. [PACS-CS Collaboration], Phys. Rev.* **D79**, *034503 (2009)*

# Chiral perturbation theory ($\chi$PT) with finite-range regularisation (FRR)

**Idea:** Write baryon masses as a function of quark mass

**But:** Traditional low-oder polynomial fits not consistent with symmetries of QCD

- Chiral symmetry broken $\rightarrow$ pion as Goldstone boson with small mass
- Goldstone boson loops contribute to hadron properties
- Give terms $\propto$ odd powers or logs of pion mass
- GMOR: $m_\pi^2 \propto m_q$
- Therefore need non-analytic functions of quark mass

**Solution:** Chiral perturbation theory ($\chi$PT)

# Chiral perturbation theory ($\chi$PT)

An **effective field theory** for QCD

- Goldstone bosons (pions) become the fundamental degrees of freedom
- Built on the symmetries of QCD
- Naturally generates observed non-analyticity
- i.e., Preserves correct chiral behavior of QCD

# Formulae

- Obtain effective Lagrangian at leading order in $\chi$PT
- Power series expansion in powers of derivatives and (chiral-symmetry breaking) quark mass matrix
- Chiral loops account for non-analytic behavior

**Expansion of baryon mass:**

$$M_B = \{\text{terms analytic in } m_q\} + \{\text{chiral loop corrections}\}$$

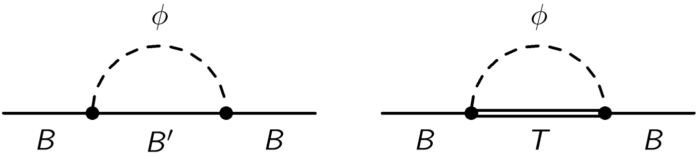


Figure: One-loop graphs at order $m_q^{(3/2)}$

# Finite-range regularisation (FRR)

- Need to regularize infinities arising from loop diagram integrals
- QCD: Goldstone bosons emitted/absorbed by composite objects made from quarks, gluons
  - Form factors suppress these processes for momenta greater than $R^{-1}$
- FRR: Introduce finite ultra-violet cutoff into loop integrals
- Physical results independent of shape of regulator

**Summary:**

- Physically motivated way to regularize infinities
- Shown to be effective over a large range of quark masses

# Our method - Reminder

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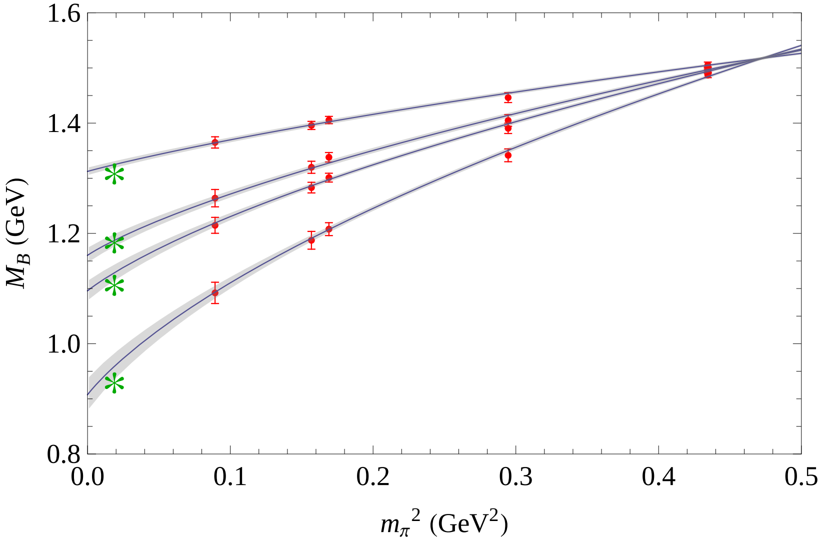
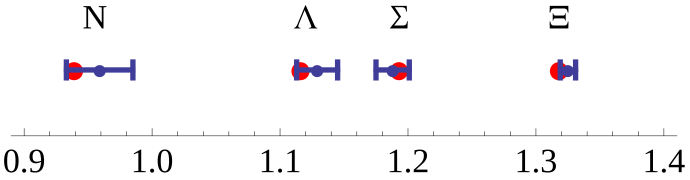
# Fit to PACS-CS lattice data

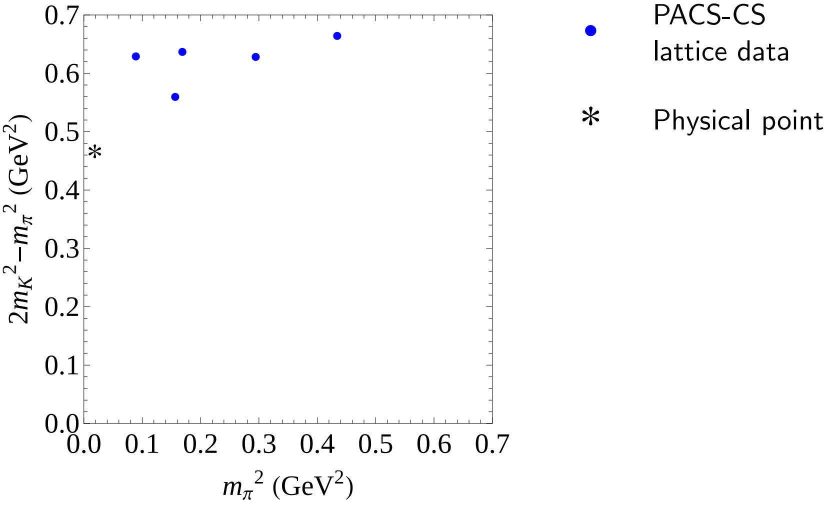


# Comparison with physical values




| $B$ | Mass (GeV) | Experimental |
|---|---|---|
| $N$ | 0.959(24)(9) | 0.939 |
| $\Lambda$ | 1.129(15)(6) | 1.116 |
| $\Sigma$ | 1.188(11)(6) | 1.193 |
| $\Xi$ | 1.325(6)(2) | 1.318 |

# Lattice data locations in $m_l - m_s$ plane



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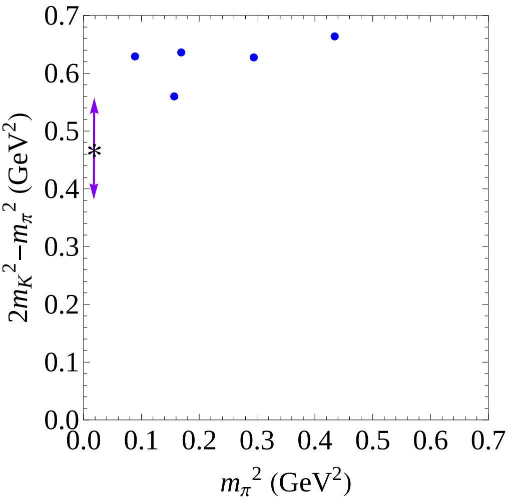


● PACS-CS lattice data

$*$ Physical point

**derivative along purple arrow $\leftrightarrow$ $\sigma_s$**

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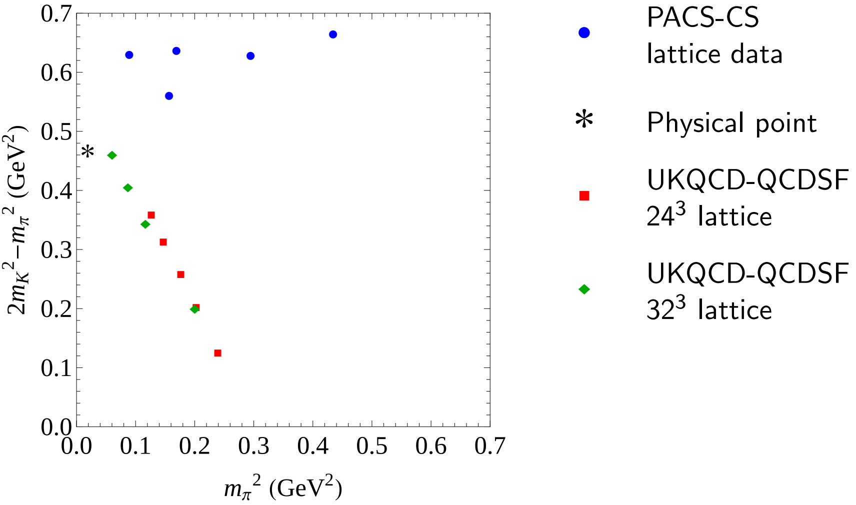

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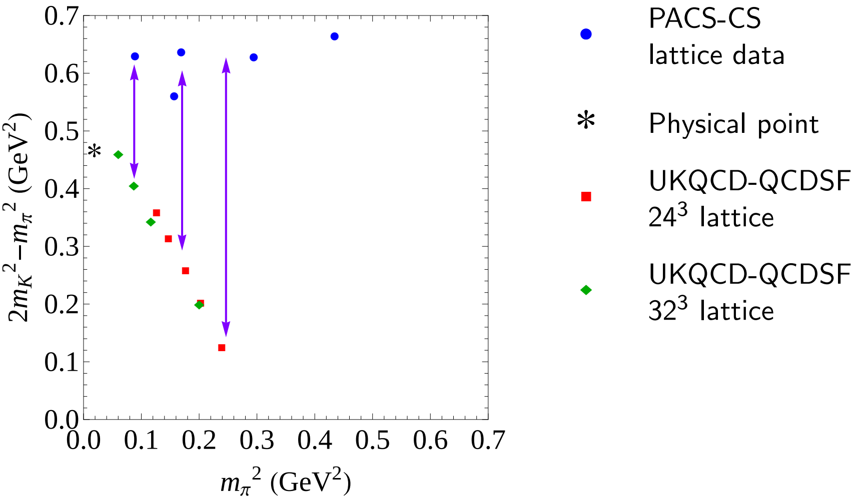
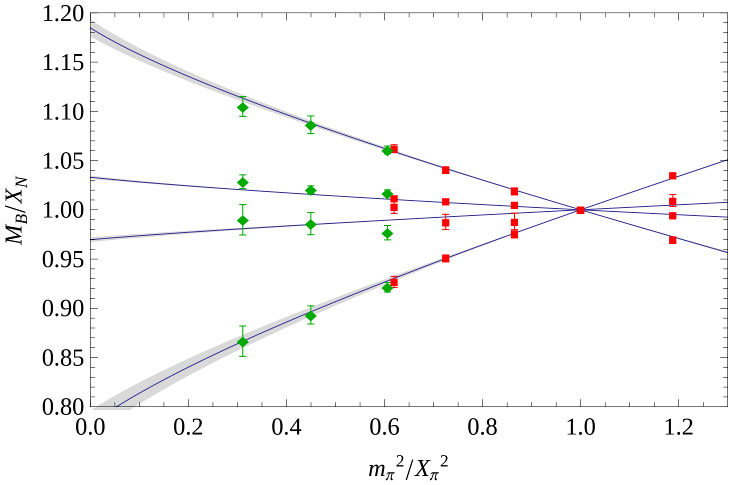


# This is NOT a fit to the data shown...




$X_\pi^2 = \frac{1}{3}(2m_K^2 + m_\pi^2)$ $\qquad$ $X_N = \frac{1}{3}(M_N + M_\Sigma + M_\Xi)$

*W. Bietenholz et al. [QCDFS-UKQCD Collaboration], [arXiv:1102.5300[hep-lat]]*

# Results

| $B$ | $\overline{\sigma}_{Bl}$ | $\overline{\sigma}_{Bs}$ |
|---|---|---|
| $N$ | 0.047(6)(5) | 0.022(6)(0) |
| $\Lambda$ | 0.026(3)(2) | 0.141(8)(1) |
| $\Sigma$ | 0.020(2)(2) | 0.172(8)(1) |
| $\Xi$ | 0.0089(7)(4) | 0.239(8)(1) |

For the nucleon, dimensionful results

$$\sigma_{\pi N} = 45 \pm 6 \text{ MeV}$$

$$\sigma_s = 21 \pm 6 \text{ MeV}$$

# Comparison with direct results from lattice QCD

- Our method = easy to evaluate $\sigma_{Bq}$ at any $(m_\pi, m_K)$
- QCDSF Collaboration[1]: precise direct calculations of $\sigma_{\pi N}$ and $\sigma_s$ from lattice QCD
  - at $(m_\pi, m_K) = (281, 547)$ MeV
  - $\sigma_{\pi N}^{\mathrm{QCDSF}} = 106(11)(3)$ MeV
  - $\sigma_s^{\mathrm{QCDSF}} = 12^{+23}_{-16}$ MeV
- We find
  - at $(m_\pi, m_K) = (281, 547)$ MeV
  - $\sigma_{\pi N} = 131(11)(5)$ MeV
  - $\sigma_s = 16(5)(1)$ MeV
- Another consistent check!

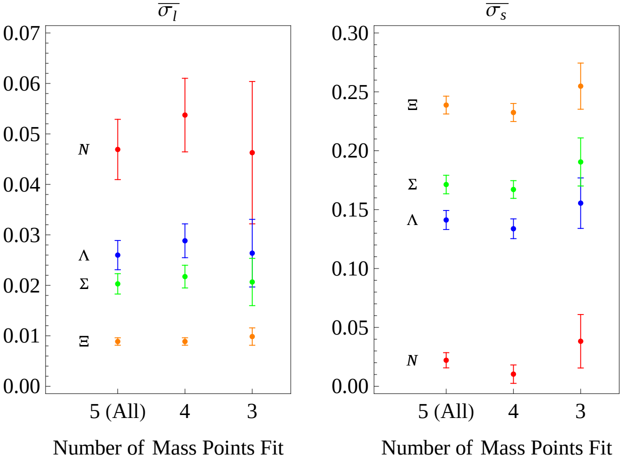
[1] *G. S. Bali et al. [QCDSF Collaboration], Phys. Rev.* **D85**, *054502 (2012)*

# Conclusion

- Success: Precise determination of $\sigma_s$ and $\sigma_{\pi N}$ from lattice QCD

$$\sigma_s = 21 \pm 6 \text{ MeV}$$
$$\sigma_{\pi N} = 45 \pm 6 \text{ MeV}$$

# Backup - Check not leaving PCR



# Backup - Sigma terms as input to dark matter cross-sections

Note: Write sigma commutator as $M_N f_{Tq} := \langle N | m_q \overline{q} q | N \rangle$

- e.g., Neutralino
  - Particle candidate for dark matter
  - Weakly interacting fermion
  - Mass $\sim$ 100 GeV or more
  - Density: a few per Liter
- Neutralino-Hadron cross-section (spin-indep)
  - $\sigma_{SI} = 4(N_N^2/\pi) f^2$
  - $\frac{f}{M_N} = \sum_{q=u,d,s} \frac{\alpha_{3q} f_{Tq}}{m_q} + \sum_{Q=c,b,t} \frac{\alpha_{3Q} f_{TQ}}{m_Q}$

*For details see: S. J. Underwood et al., [arXiv:1203.1092 [hep-ph]]*