Strange nucleon form factors on 2+1f anisotropic wilson clover lattices

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Lattice 2012
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Collaborators

Update of work in progress to calculate disconnected strange nucleon form factors.

Aim to calculate scalar, axial, electric, and magnetic. Only show ***preliminary results*** for scalar.

"Continuation" of study of nucleon strangeness begun in **Phys.Rev. D85 (2012) 054510**

Collaborators:

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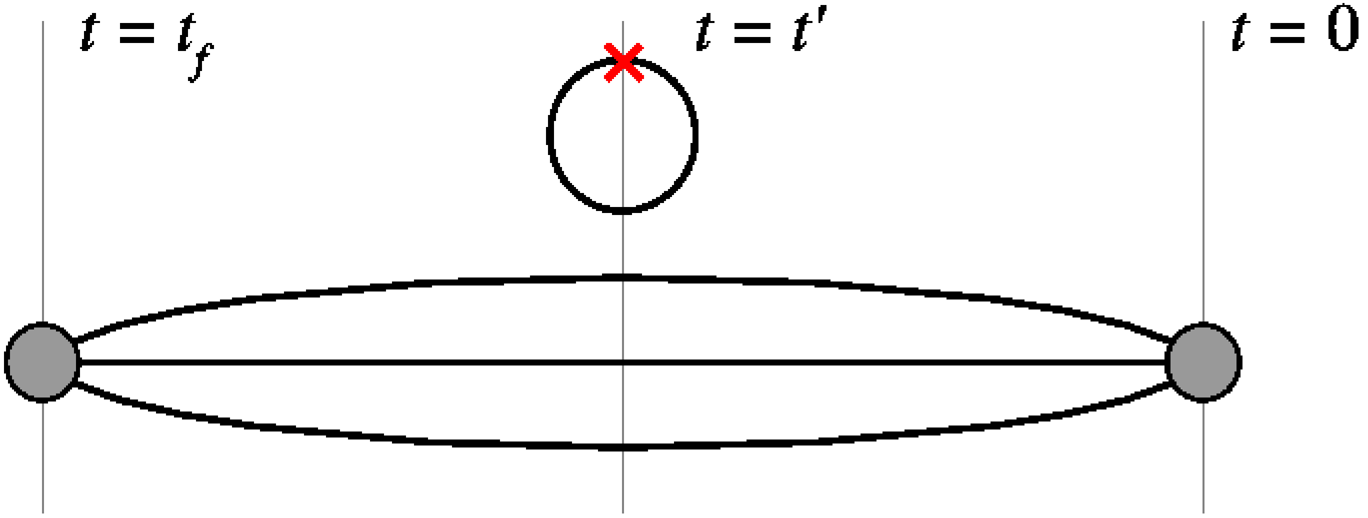
Strange form factors

- Strange form factors → understand the contribution of strange sea quarks to nucleon structure.
- Strange scalar form factor important in possible scattering of dark matter WIMPs via Higgs exchange.
- Cross section ~ m_q → sea quarks (strange, charm) may contribute because m_{ud} small.

Definition

$$f_{T_s} = \frac{m_s \langle N|\bar{s}s|N\rangle}{m_N}$$



Feynman-Hellman theorem:

$$\langle N|\bar{s}s|N\rangle = \frac{\partial m_N}{\partial m_s}$$

“Direct” method: Calculate three-point function with disconnected insertion explicitly.

- Need disconnected insertion

- Poor signal-to-noise ratio.

Three point functions

$$G_S^{(3)}(t_{sink}, t', t_{src}) = (1+\gamma_4)^{\alpha\beta} \sum_{\vec{x},\vec{x}'} \left\langle N^\beta(\vec{x}, t_{sink}) \left[\bar{s}s(\vec{x}', t') - \langle \bar{s}s(\vec{x}', t')\rangle\right] \overline{N}^\alpha(x_0, t_{src}) \right\rangle$$

- Signal depends on correlation between nucleon correlator and disconnected strange quark instertion
- Spectral decomposition:

$$G_S^{(3)}(t_{sink}, t', t_{src}) = \sum_{m,n} j_{mn} \exp(-m_n(t'-t_{src})) \exp(-m_m(t_{sink}-t'))$$

$$\begin{aligned} G^{(2)}(t_{sink}, t_{src}) &= (1+\gamma_4)^{\alpha\beta} \sum_{\vec{x}} \left\langle N^\beta(\vec{x}, t_{sink}) \overline{N}^\alpha(x_0, t_{src}=0) \right\rangle \\ &= \sum_n c_n exp(-m_n(t_{sink}-t_{src}) \end{aligned}$$

$$\langle N|\bar{s}s|N\rangle = G_s = \lim_{(t_{sink}-t_{src})\to\infty} \frac{G_s^{(3)}(t_{sink}, t', t_{src})}{G_s^{(2)}(t_{sink}, t_{src})} = \frac{j_{11}}{c_1}$$

Measuring the condensate

- Need estimate of trace of propagator on each time slice:
$$\sum_{x'} \langle \overline{s}s(x',t')\rangle = \sum_{x'} \mathrm{Tr}_{cs}\left[\overline{\psi}(x',t')D^{-1}\psi(x',t')\right]$$
- Use U(1) stochastic estimator to measure currents.
- Use dilution to estimate necessary propagator:
  - Diluted source is non-zero on only one site every $4^3 \times 16$
  - Off-diagonal contribution fall off with meson mass.
- 12288 sources/configuration
- Ideal for using GPUs: JLab GPU cluster, Bungee GPU cluster at BU.

Lattices

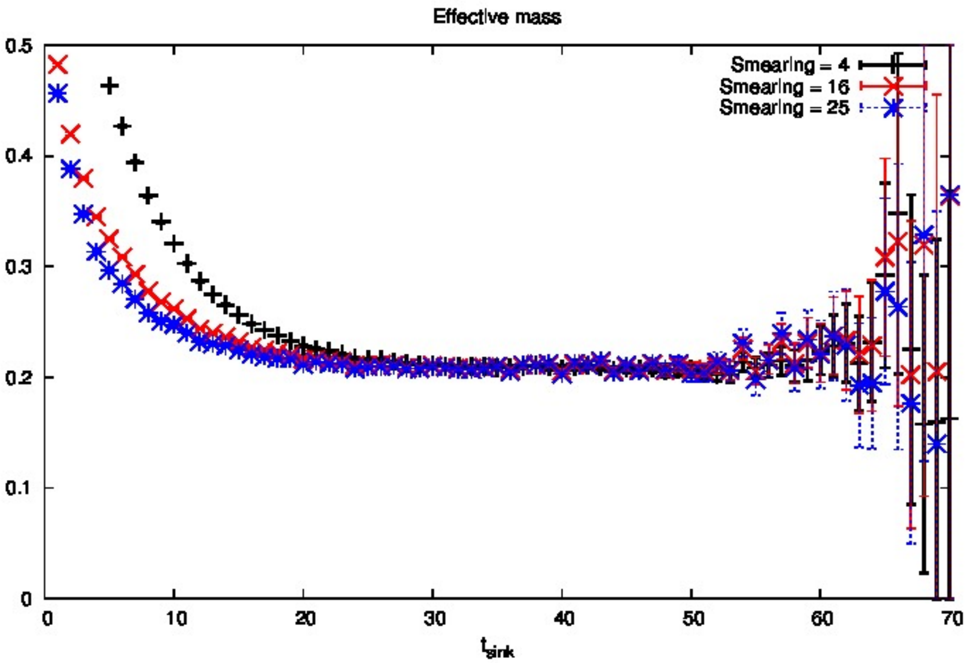
- Anisotropic Wilson clover lattices from Hadron Spectrum Collaboration (stout-smeared).
- 24^3x128, $\xi \approx 3.5$, $a_s \approx 0.12$ fm., $a_t \approx 0.035$ fm.
- Two light quark masses $a_t m_l = -0.0840, -0.0860$ correspond to $m_\pi \approx 220, 380$ MeV.
- $a_t m_s = -0.0743$, $a_t m_{crit} \approx -0.087$, $m_K \sim 10\%$ heavy
- Measurements on 416 configurations at $a_t m_l = -0.0840$, 580 configurations at $a_t m_l = -0.0860$

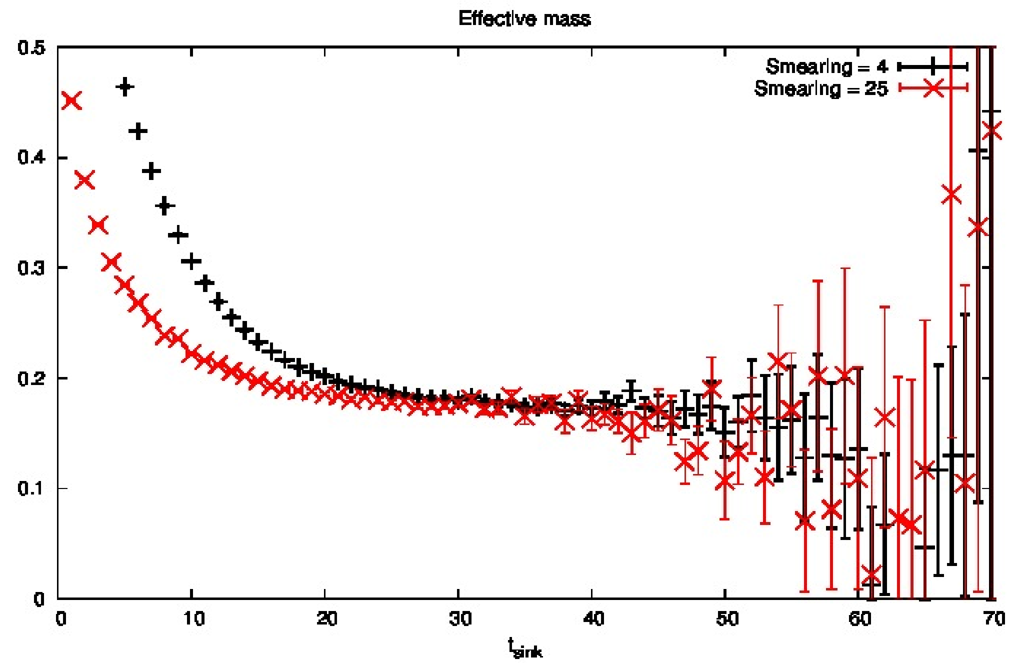
Two-point functions

- Nucleon two-point functions
- 32 sources/configuration, every 4 time slices
- 3 source/3 sink smear
- 4, 16, 25 APE smearing steps with nearest-neighbor weights = 1/7

$$
\begin{aligned}
G^{(2)}(t_{sink}) \quad = \quad & (1+\gamma_4)^{\alpha\beta} \sum_{\vec{x}} \left\langle N^{\beta}(\vec{x}, t_{sink}) \overline{N}^{\alpha}(x_0, t_{src}=0) \right\rangle \\
& +(1-\gamma_4)^{\alpha\beta} \sum_{\vec{x}} \left\langle N^{\beta}(\vec{x}, N_t - t_{sink}) \overline{N}^{\alpha}(x_0, t_{src}=0) \right\rangle
\end{aligned}
$$

Effective Masses

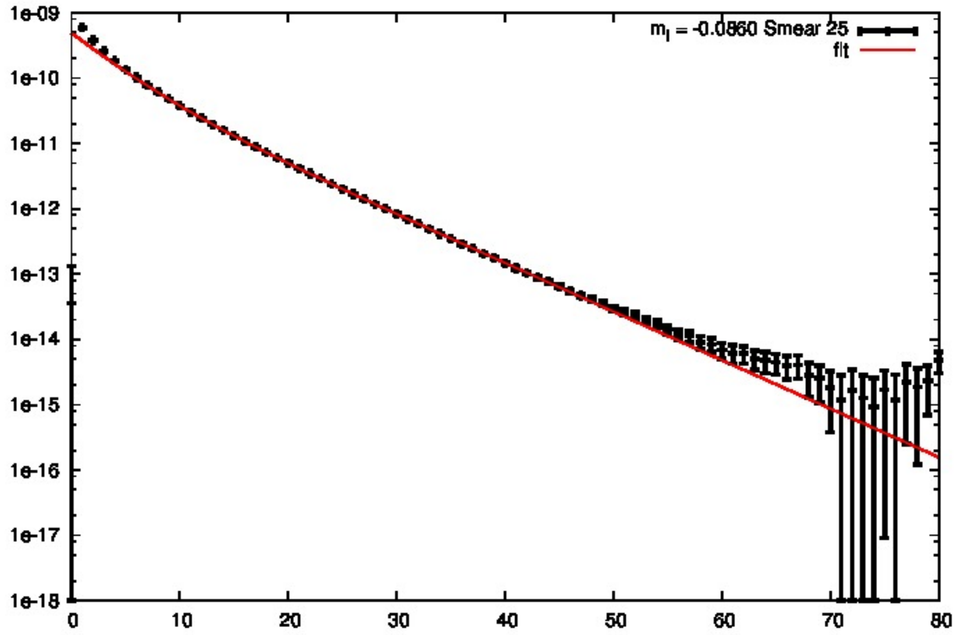


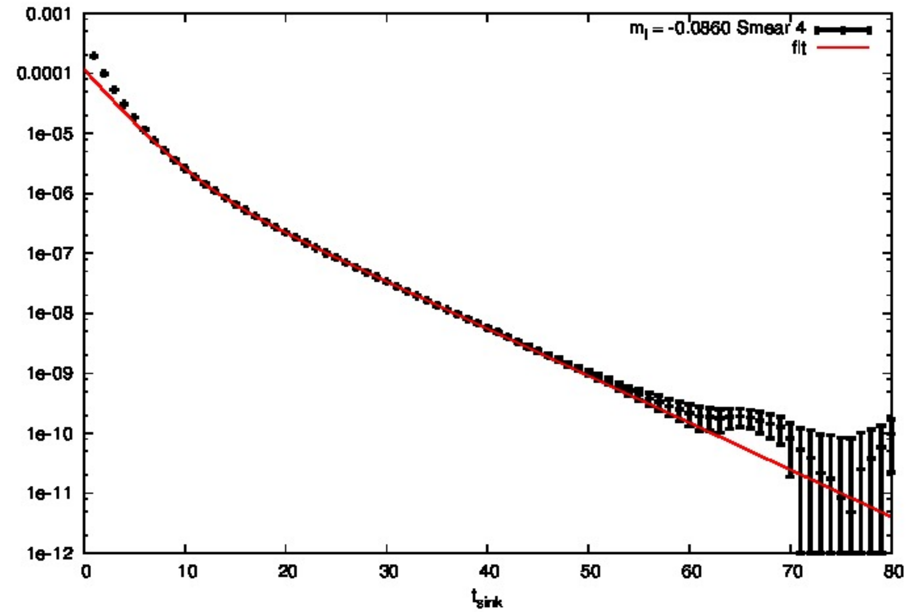


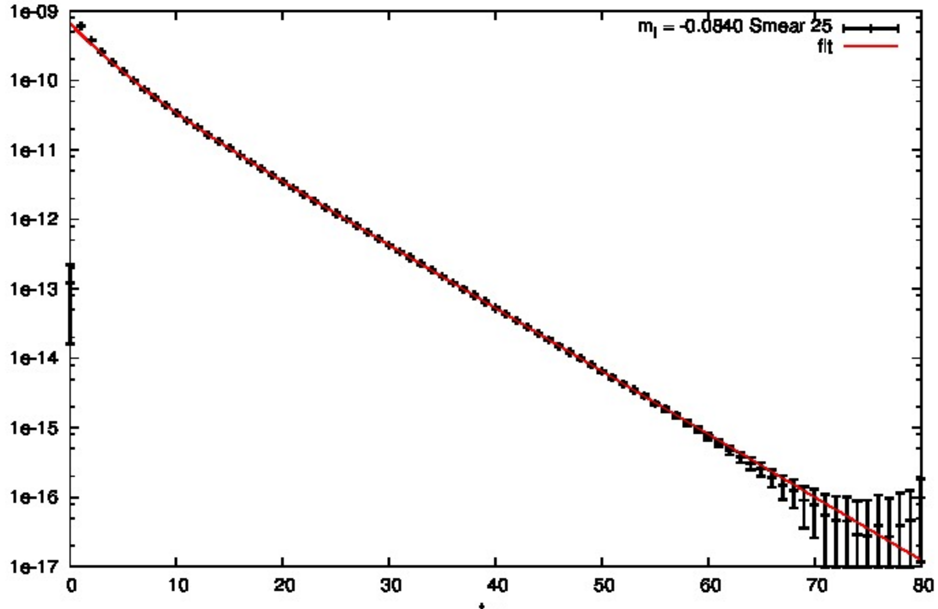
- Two-point function is fairly well-determined.
- Smearing – reduce contribution from excited states at the cost of poorer statistical error.
- Fit to double exponential:

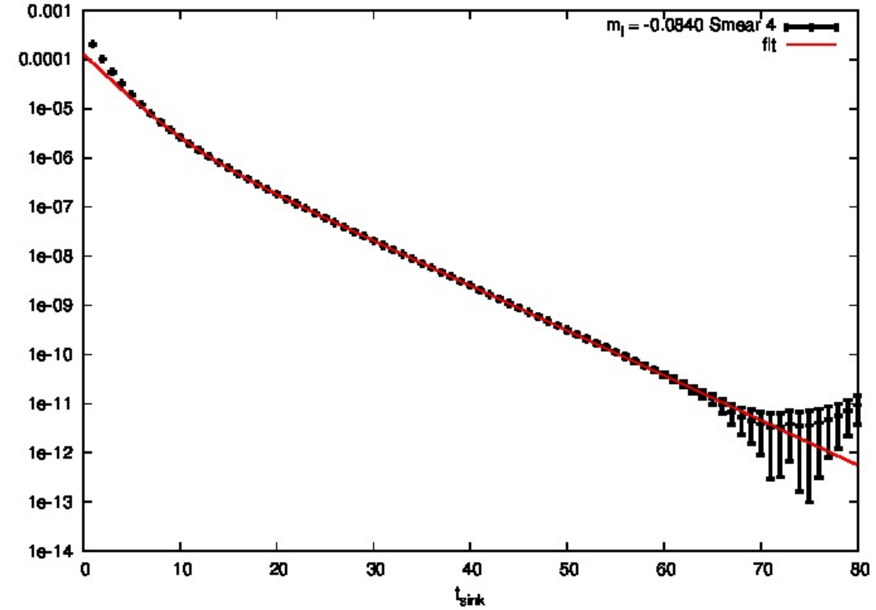
$$G^{(2)}(t_{sink}, t_{src}) = c_1 \exp(m_N(t_{sink} - t_{src}) + c_2 \exp(m_{ex}(t_{sink} - t_{src})$$

Excited state fits

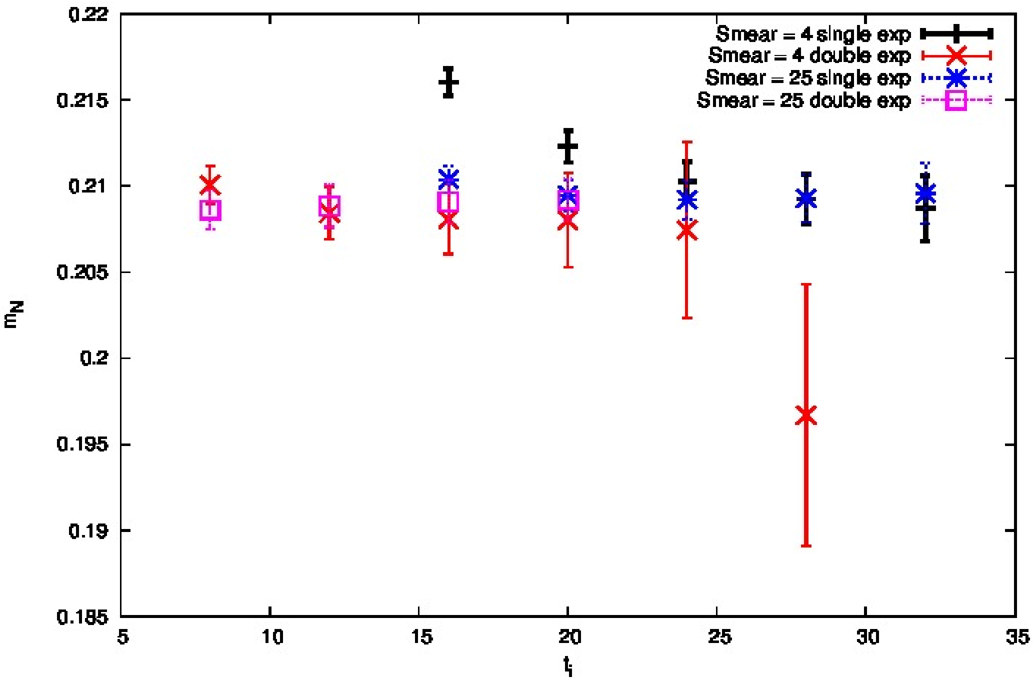


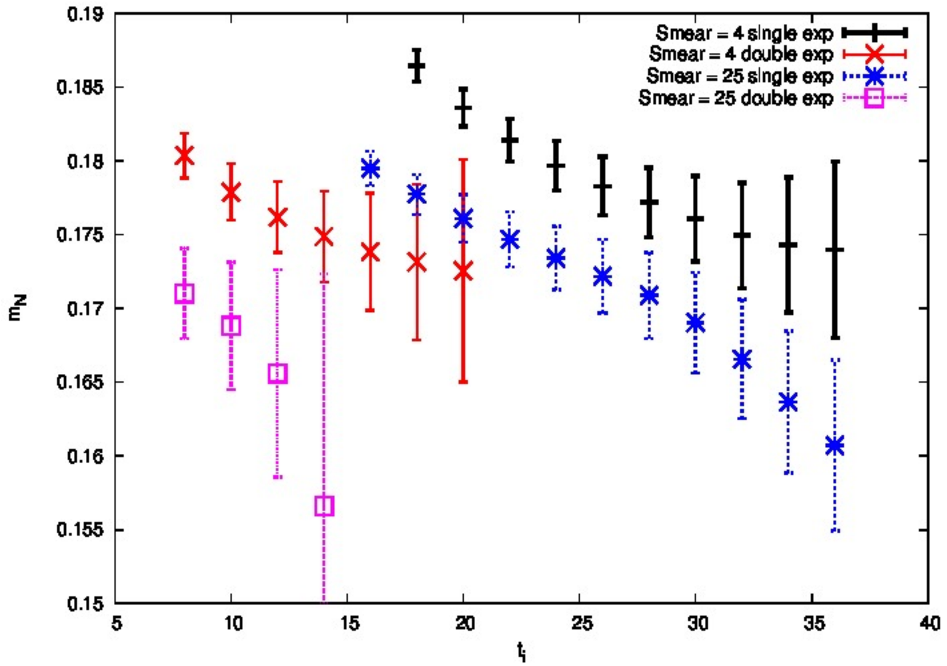






Two point function results

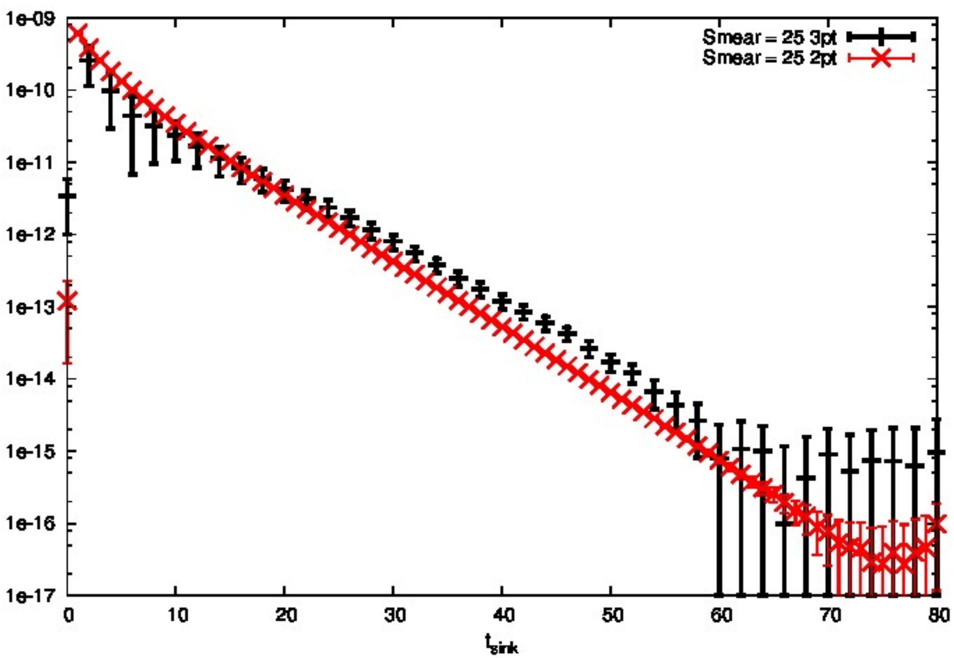


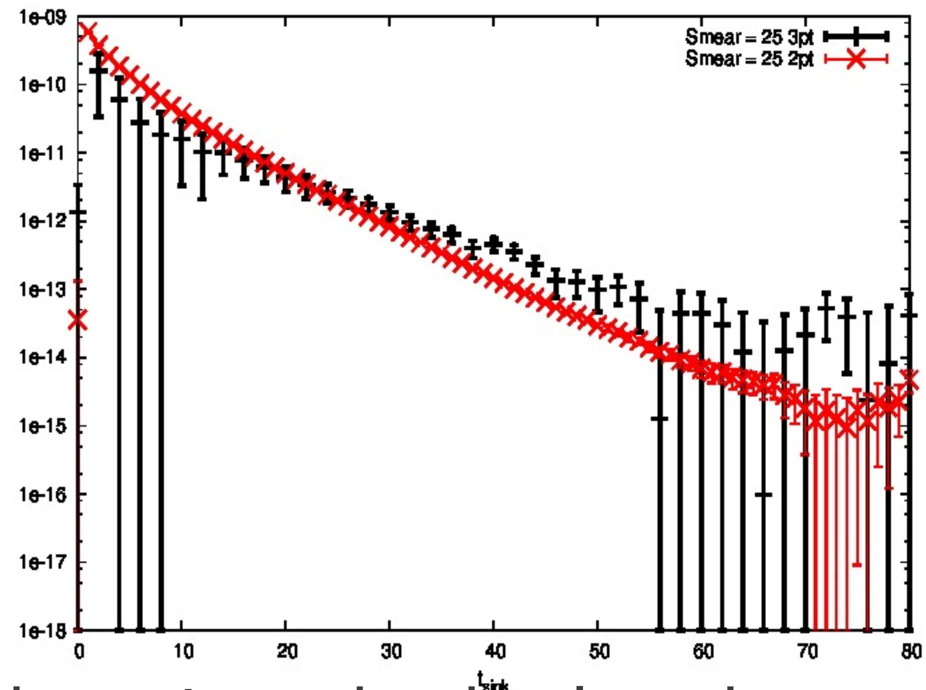


- Double exponential fit gives robust values for m_N (and also c_1) for $a_t m_l$ = -0.0840: $a_t m_N$ = 0.209(1)
- Excited state contamination is greater at $a_t m_l$ = -0.0860, 25 smearing steps gives unstable fits.
- $a_t m_N$ = 0.17(1)

Three point function

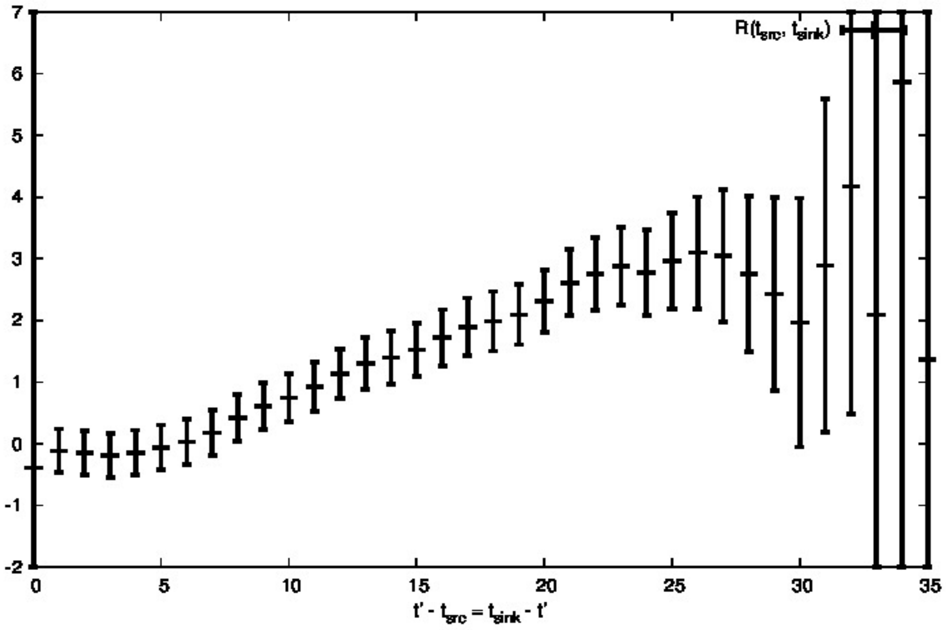
$$G_S^{(3)}(t_{sink}, t') = (1+\gamma_4)^{\alpha\beta} \sum_{\vec{x},\vec{x}'} \left\langle N^{\beta}(\vec{x}, t_{sink}) \left[\overline{s}s(\vec{x}', t') - \langle \overline{s}s(\vec{x}', t')\rangle\right] \overline{N}^{\alpha}(x_0, t_{src}=0) \right\rangle$$

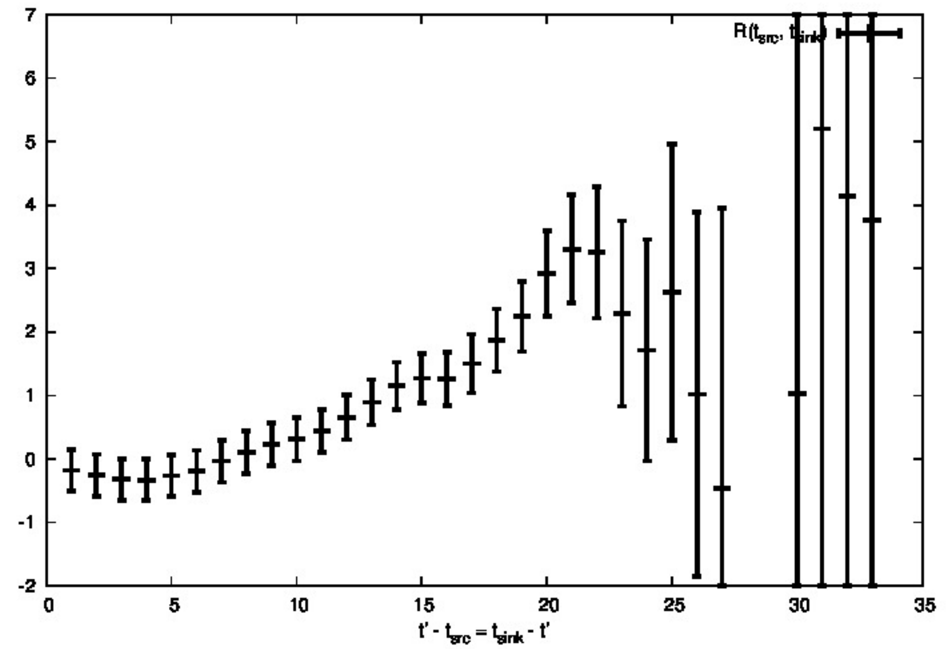




- 1D slice of three-point function – Insertion is placed symmetricallyt' = ($t_{sink\,–}$ t_{src})/2.
- Matrix element given by large time behavior of ratio.
- Excited state contamination is evident when insertion distance is small.

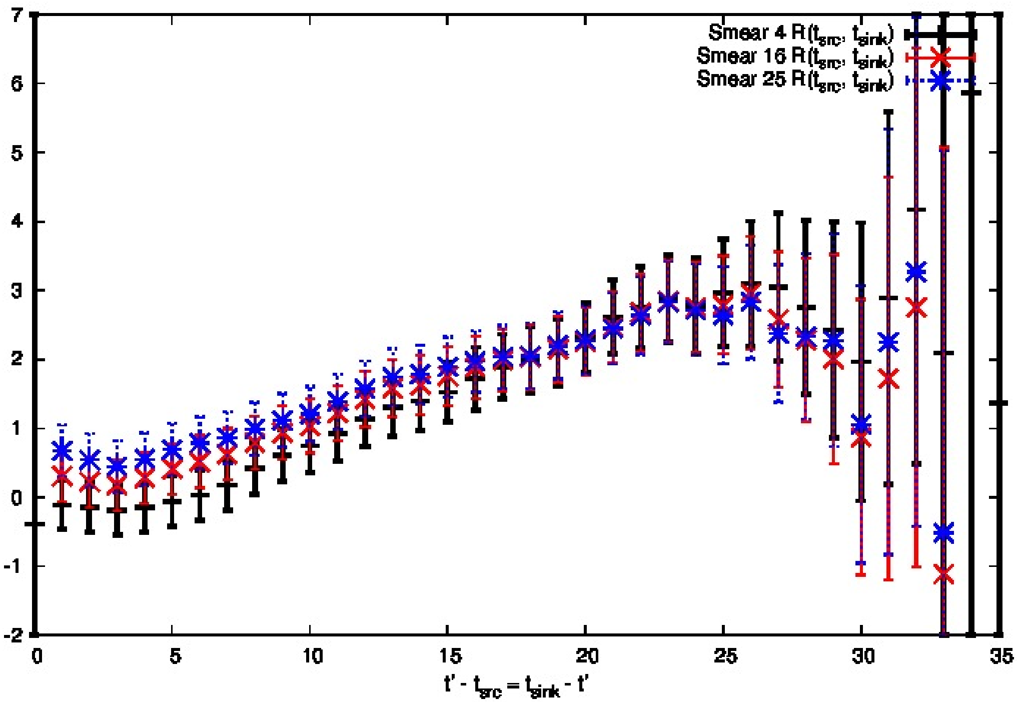
Ratios

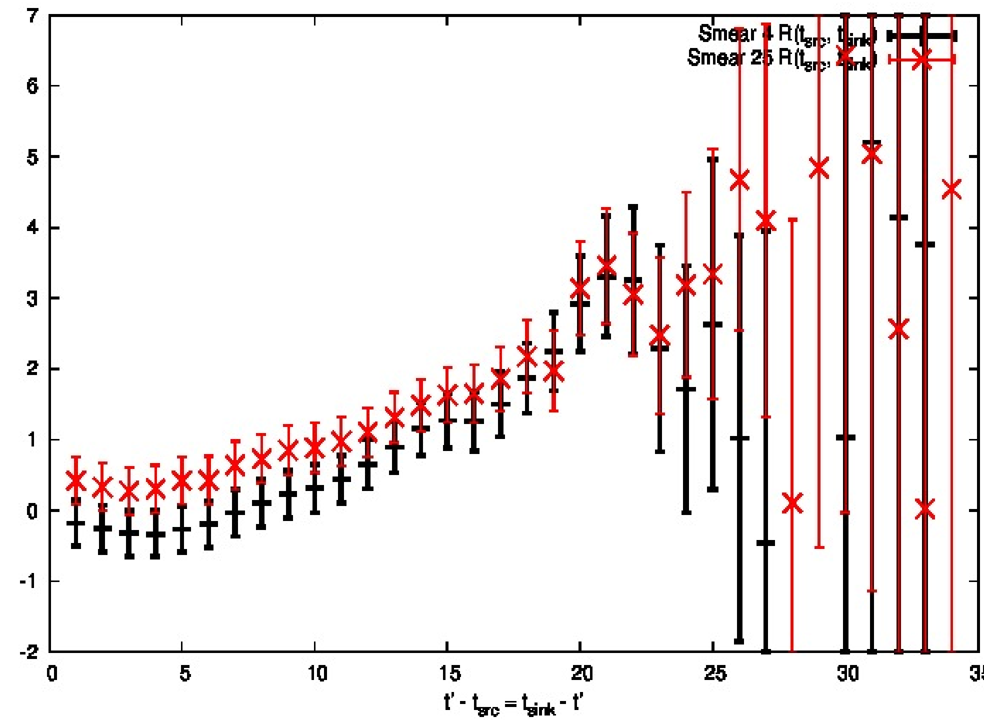




- Expect R(t_{src}, t_{sink}) to plateau for sufficiently large t_{sink} – t_{src}
- Excited state contributions present, as in two-point function.
- Plateau not reached until t_{sink} – t_{src} ~ 20

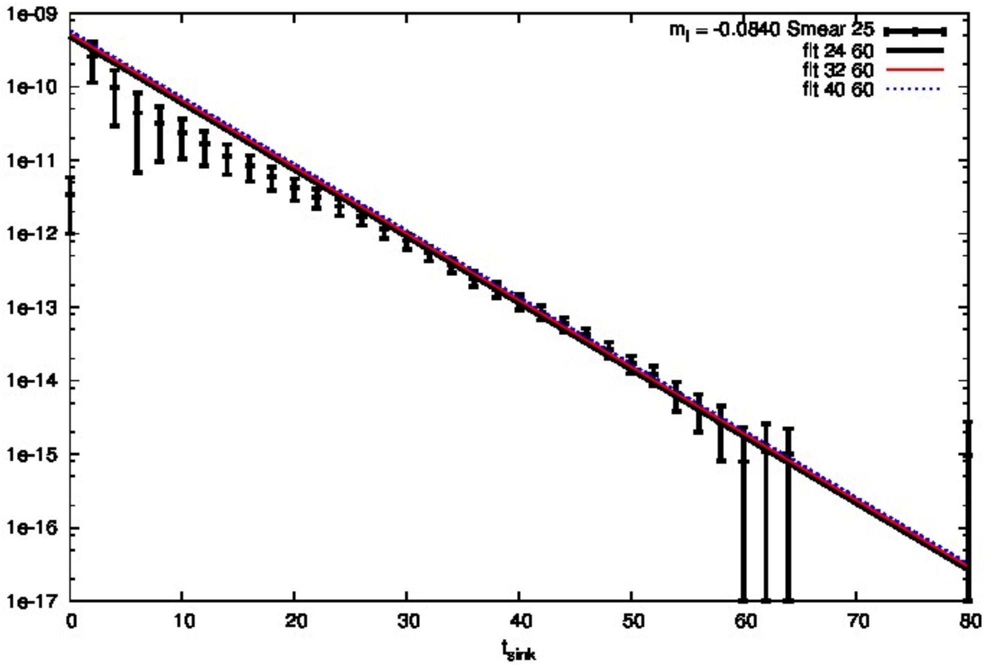
Ratios

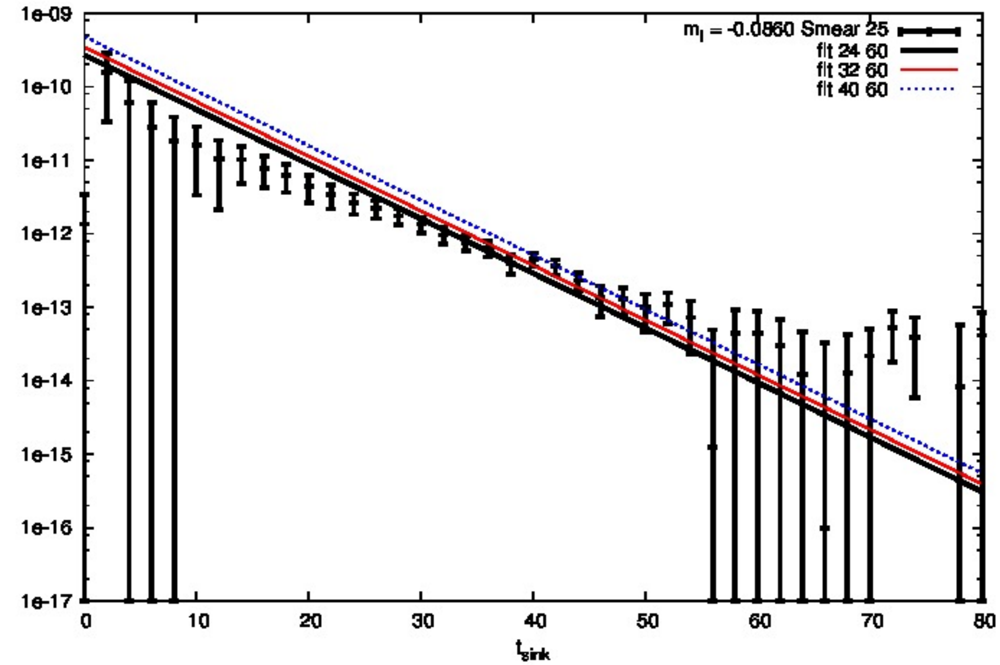




- Different source/sink smearing gives consistent results.
- Slightly less excited state contribution at smaller time separations with more smearing.

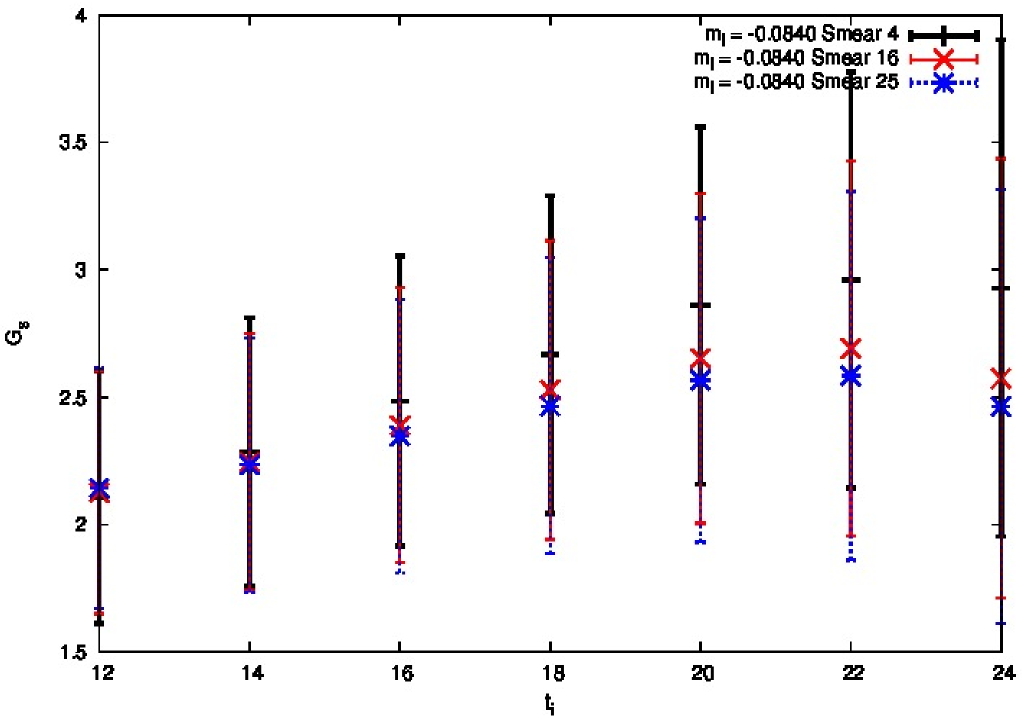
Fits

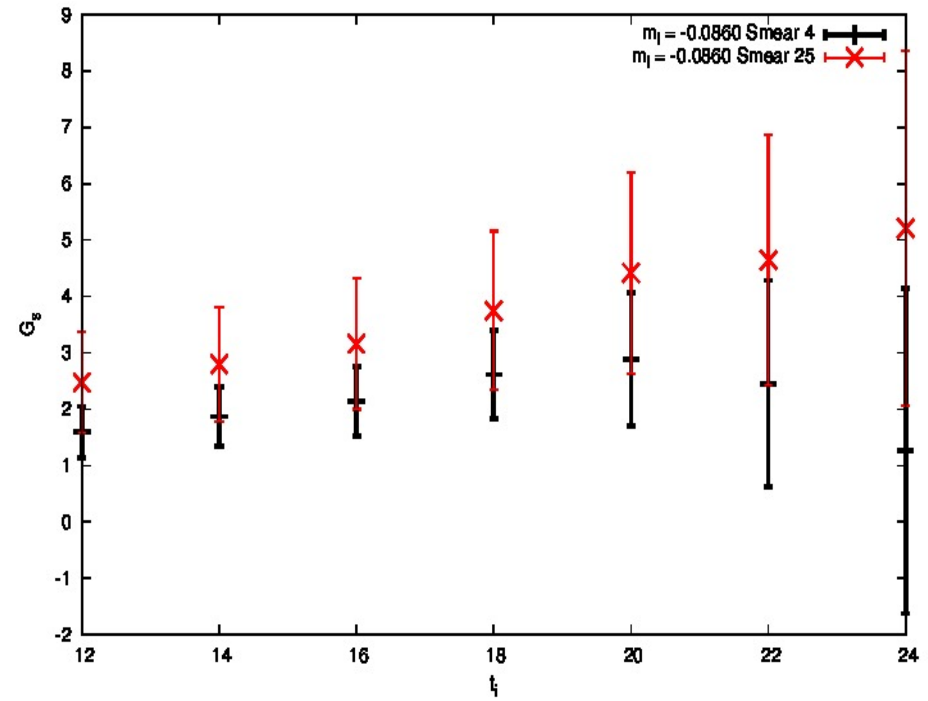




- Fit three-point function to some window $t_{min} \leq (t' - t_{src}) \leq t_{max}$
- Excited states evidently give negative contribution to three-point function.
- Cancellation of ground state terms at small separations – difficult to get robust fit to excited contribution
- Fix m_N to value obtain from two-point functions.

Fits





t_{min} = 20, t_{max} = 30,
Smearing = 4, G_s = 2.9(7)
Smearing = 16, G_s = 2.7(6)
Smearing = 25, G_s = 2.6(6)

t_{min} = 20, t_{max} = 30,
Smearing = 4, G_s = 4.4(1.8)
Smearing = 25, G_s = 2.9(1.2)

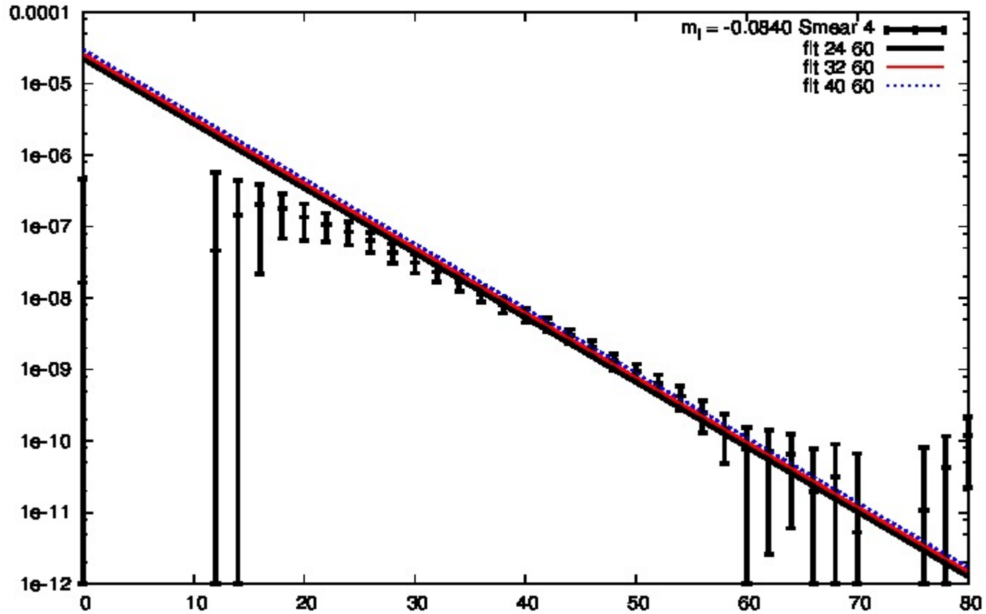
Results

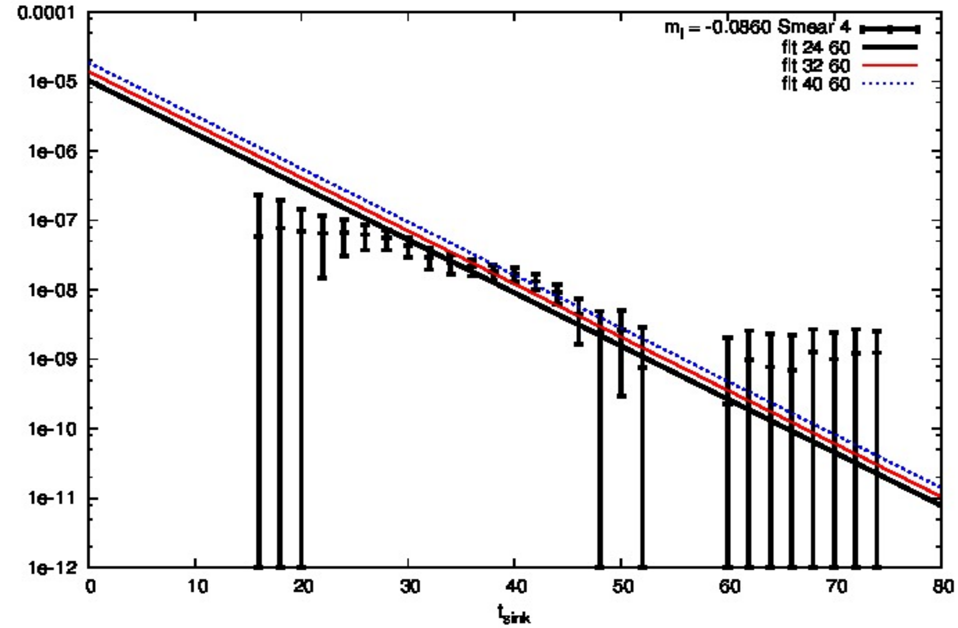
- Naively, $a_t(m_s - m_{crit}) \approx 0.013$, $a_t m_N \approx 0.2$, $\langle N|\bar{s}s|N\rangle \approx 3$
- $(m_s - m_{crit}) \langle N|\bar{s}s|N\rangle \approx 200$ MeV
- $f_{Ts} \approx 0.2$ – larger than other lattice calculations, $f_{Ts} \leq 0.1$
- Not yet renormalized!
- Lack of chiral symmetry in Wilson formulation – mixing between light and strange scalar matrix elements.
- Plans to carry out renormalization procedure.

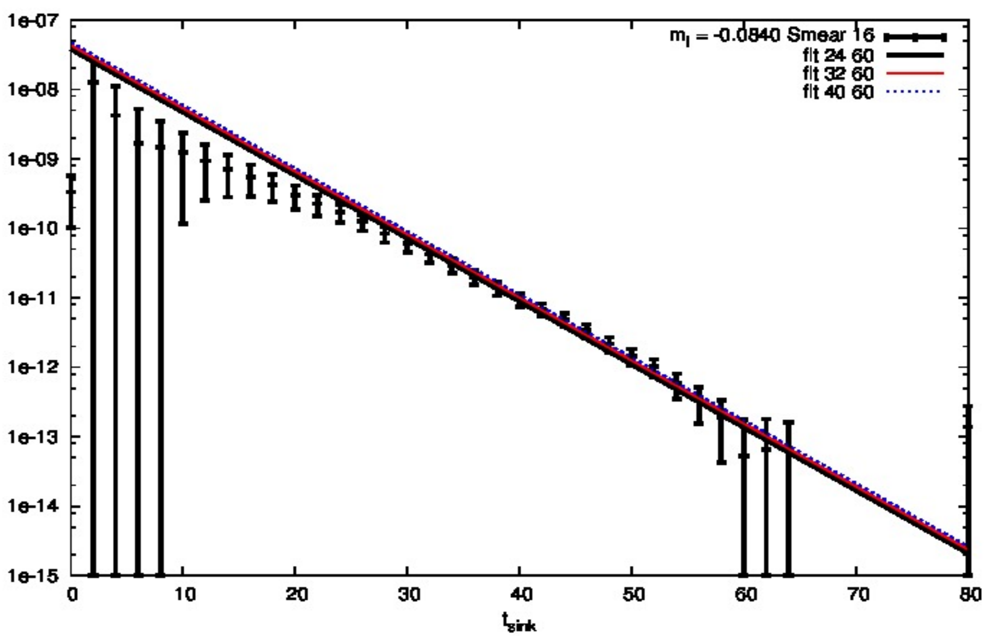
Summary

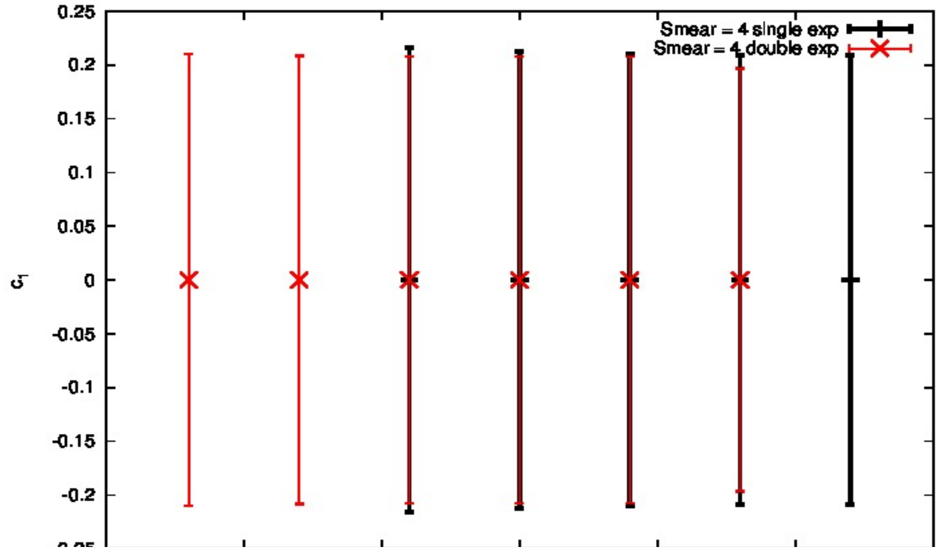
- Calculation of strange scalar form factor.
- ***Preliminary*** value $\langle N|\bar{s}s|N\rangle \sim 3$, 30% statistical error.
- Need to carry out calculation of renormalization coefficients to correctly calculate mixing.
- Multiple source/sink smearing combinations. Apply variational methods to reduce excited state contamination.
- Can perform additional measurments of nucleon correlator on existing lattices.
- Calculate axial, electric, magnetic form factors, also momentum dependence.

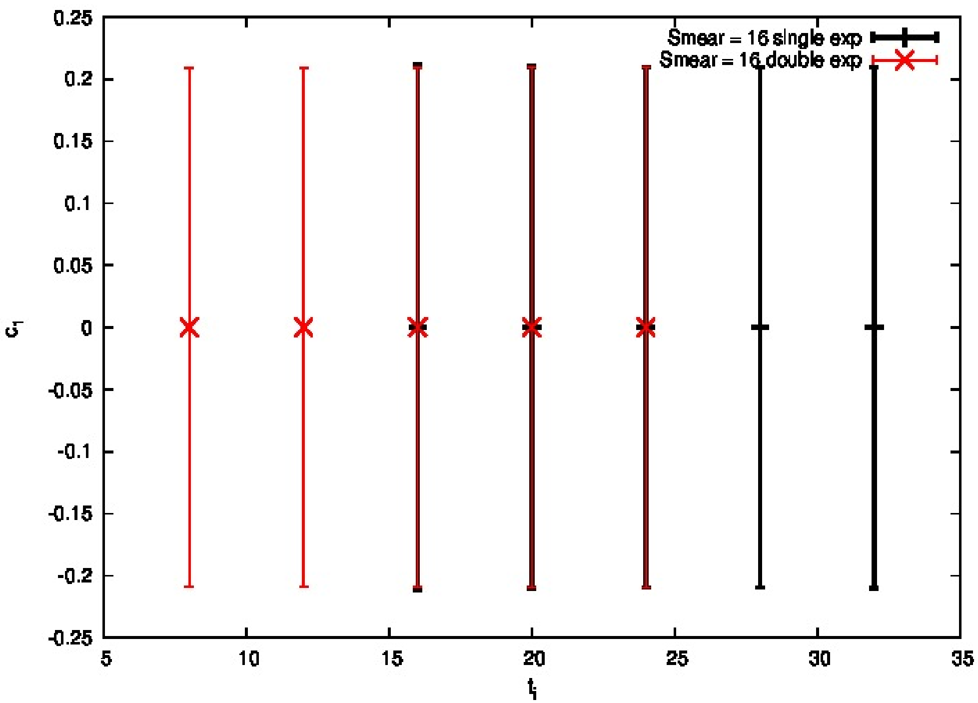
Fits

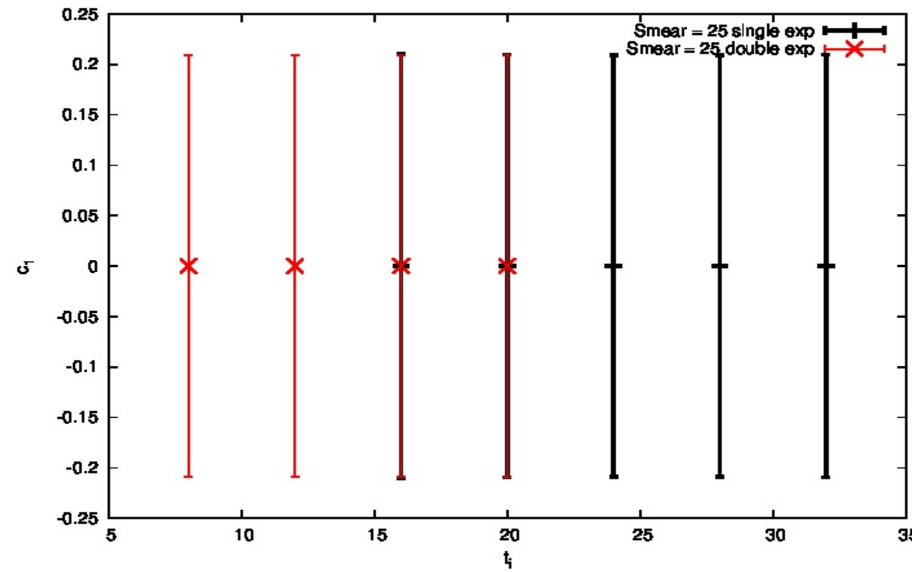

m_l = -0.0840 Smear 4
fit 24 60
fit 32 60
fit 40 60

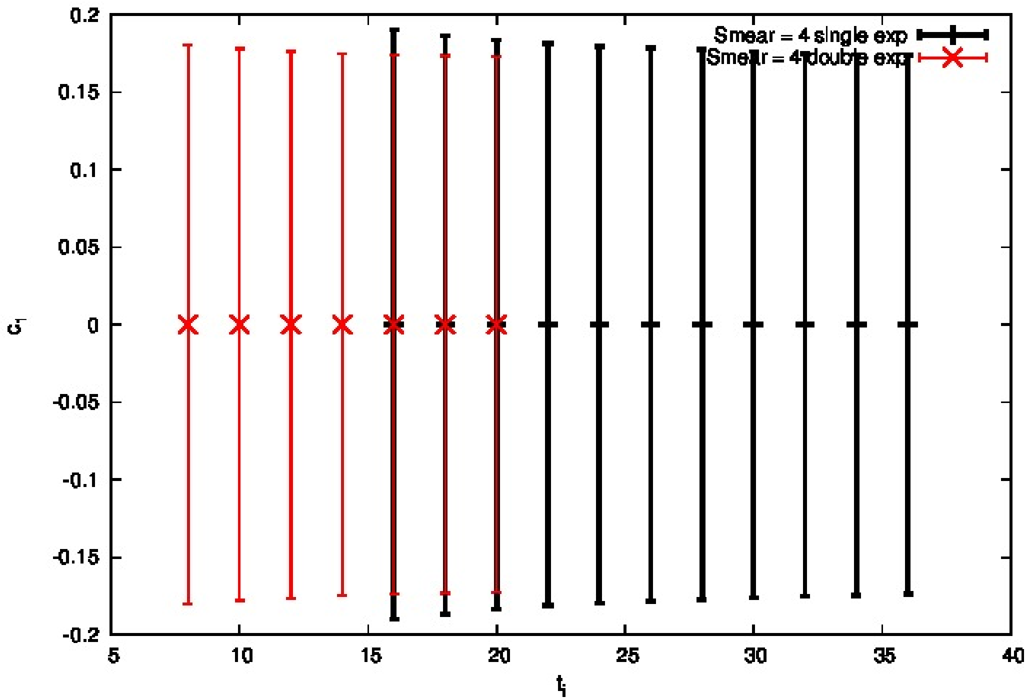

m_l = -0.0860 Smear 4
fit 24 60
fit 32 60
fit 40 60
t_sink

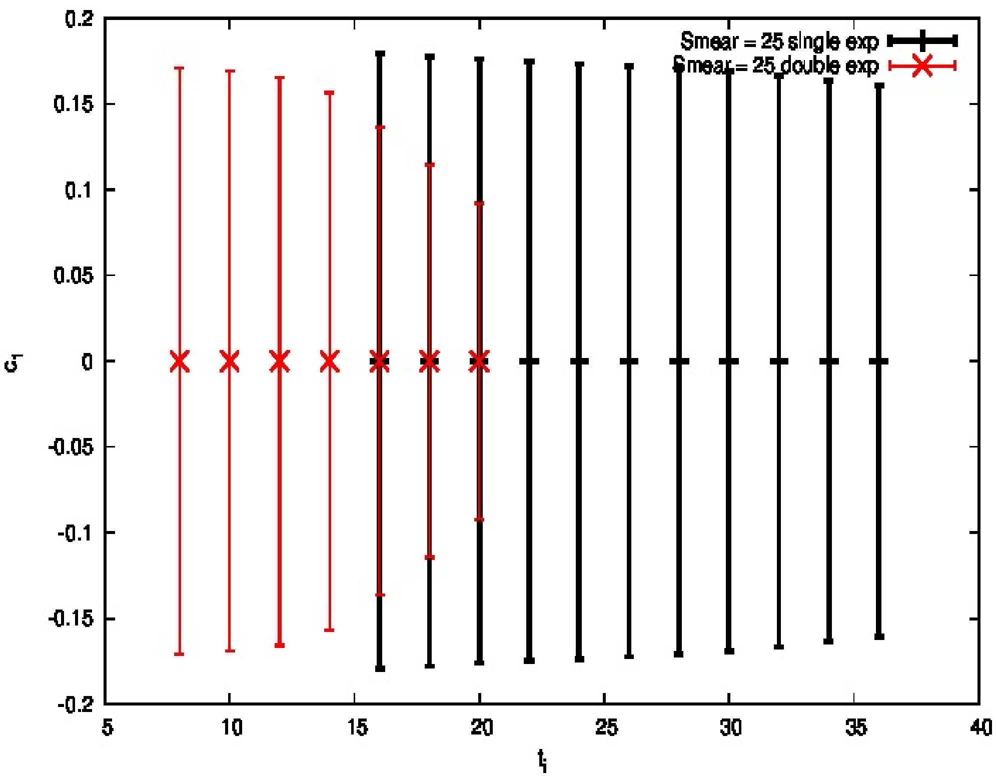

m_l = -0.0840 Smear 16
fit 24 60
fit 32 60
fit 40 60
t_sink


Smear = 4 single exp
Smear = 4 double exp
c_i


Smear = 16 single exp
Smear = 16 double exp
c_i
t_i


Smear = 25 single exp
Smear = 25 double exp
c_i
t_i


Smear = 4 single exp
Smear = 4 double exp
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
5
10
15
20
25
30
35
40
c_i
t_i


Smear = 25 single exp
Smear = 25 double exp
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2
5
10
15
20
25
30
35
40
c_i
t_i