Strange quark content of nucleon From Domain Wall Fermion

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Motivation

Strange quark content of the nucleon $\langle N|\bar{s}s|N\rangle$ is one of the quantities which traditionally has needed measurement of disconnected diagrams on the lattice, hence expensive/noisy. Recently, closely related quantities

$$\sigma_{\pi N} = m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle, \sigma_0 = m_{ud}\langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle$$

$$f_{T_s} = \frac{m_s\langle N|\bar{s}s|N\rangle}{m_N} = \frac{dm_N}{dm_s} \times \frac{m_s}{m_N}, \qquad y_N = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle}$$

has been getting attention, since it is crucial in Weakly Interacting Massive Particles(WIMP) search, via its interaction to Nucleon via Higgs.

Motivation(cont.)

- Despite the attention $\langle N|\bar{s}s|N\rangle$ has been getting, number of studies with chiral and, especially continuum ($a \to 0$) extrapolation is relatively limited.
- Good test of various techniques of noise reduction. Serves as a guide for future studies on more realistic ensembles with much noisier nucleons.
- Preliminary studies on the same ensembles presented 2 years ago, where existing propagators were not optimal for reweighting approach
    - for $a \sim 0.08$ ensemble, new contractions with smeared sink, reweighting factors on more configurations
    - for $a \sim 0.11$ ensemble, Increased measurements with box source

$\langle N|\bar{s}s|N\rangle$ from Lattice QCD

Different approaches to calculate $\langle N|\bar{s}s|N\rangle$:

- Direct measurement: measure 3-point function (ETMC, χQCD, $\cdots$)

$$\langle N|\bar{s}s|N\rangle = \lim_{0 \ll t' \ll t} \frac{\langle \bar{O}_N(0)\bar{s}s(t')O_N(t)\rangle}{\langle \bar{O}_N(0)O_N(t)\rangle}$$

- Use Feynman-Hellman theorem

$$\langle \bar{O}_N(0)O_N(t)\rangle \sim e^{-M_N t}, \qquad \langle N|\bar{s}s|N\rangle = \frac{dM_N}{dm_s}$$

  - First fitting M_N to a BChPT (Adelaide, BMW...)
  - Chain rule $\frac{dS}{dm_s} \sim \langle\bar{\psi}\psi\rangle(m_s)$ (MILC).
  - Numerical derivative, via reweighting

Group(arXiv)		a	m	$\langle N\|\bar{s}s\|N\rangle$	f_{T_s}	y_N
			$N_F = 2+1$			
Adelaide (0901.3310) (1205.5365)	ChPT	1	5		0.033(16)(4)(2) 0.022(6)(0)	
MILC (0905.2432) (1204.3866)	$\frac{dM}{dm}$	3 3		0.69(7)(9) 0.637(55)(74)	0.063(6)(9)	
BMW(1109.4265)	ChPT	3			$0.036(14)(^{+30}_{-25})$	$0.20(7)(^{+13}_{-17})$
QCDSF(1110.4971)	$\frac{dM}{dm}$ *	1			0.076(36)(63)	
Engelhardt (1011.6058)	3pt	1	1		0.041(12)	
χQCD(1204.0685)	3pt	1	1		0.048(15)	
JLQCD(1012.1907)	3pt	1	2	0.086(81)(107)	0.013(12)(16)	
			$N_F = 2+1+1$			
ETMC(1111.4857) (1111.5426) (1202.1480)	3pt	1	1		0.014(5)(1)	0.069(27) 0.066(11)(2) 0.082(16)(2)
MILC(HISQ)	$\frac{dM}{dm}$	4		0.44(8)(5)		

Reweighting of dynamical strange quark

Lattice spacing is a nontrivial function of β and one does not know it until it is measured on thermalized configurations.
In practice, ensembles with different strange quark masses are needed to interpolate (simple linear interpolation or SU(3) ChPT), or we have to reply on extrapolation.
Reweighting in strange quark has been shown to be effective for quantities such as f_π, m_π, B_K, which allows for elimination of systematic error from deviations of strange quark mass from physical value. RBC/UKQCD (arXiv:1011.0892)
Reweighting factors already exist for these ensembles. Only need nucleon 2pt to calculate $\langle N|\bar{s}s|N\rangle$.

Reweighting: Basics

$$w_i(m_s', m_s) = \det\left(\frac{D_2^\dagger D_2}{D_1^\dagger D_1}\right)^{1/2} = \det(\Omega)^{1/2}, \Omega = D_2^{-1} D_1 D_1^\dagger (D_2^\dagger)^{-1}$$

$$D_1 = D([U_i], m_l, m_s), D_2 = ([U_i], m_l, m_s')$$

$$w = \frac{\int e^{-\xi^\dagger \Omega^{1/2} \xi}}{\int e^{-\xi^\dagger \xi}} = \left\langle e^{-\xi^\dagger(\Omega^{1/2}-1)\xi} \right\rangle$$

$$w(m_s', m_s) = w(m_s' = m_n, m_{n-1}) \cdots w(m_2, m_1 = m_s)$$

Now observables for reweighted ensemble is calculated by

$$\langle O \rangle (m_s') = \frac{\Sigma_i O[U_i] w_i}{\Sigma_i w_i} \tag{1}$$

Measurement details

Calculate reweighted nucleon mass $M_N(a, m_l, m_s)$, fit to linear functions with a^2 coefficients.

$M_N(m_s, m_l, a) = c_0' + \langle N|\bar{s}s|N\rangle(m_l, a)m_s$

$\langle N|\bar{s}s|N\rangle(m_l, a) = c_0 + c_1 m_l(+c_2 a^2)$

(2+1) dynamical flavor DWF + Iwasaki gauge action generated by RBC/UKQCD (arXiv:1011.0892)

- 24^3 : $a^{-1} = 1.75(3)$ Gev, $m_{res} \sim 5$ Mev
  Box size=16, $am_s' = 0.03, 0.031, \cdots \underline{0.04}, 0.041, \cdots 0.05$
  EigCG used, Decrease cost by factor of ~ 3 for $am_l = 0.005$.

am_l	MD units	Propagator #
0.005	1420,1460$\cdots$8980	1520
0.010	1460,1500$\cdots$8540	1424
0.020	1900,1920$\cdots$3600	(660)

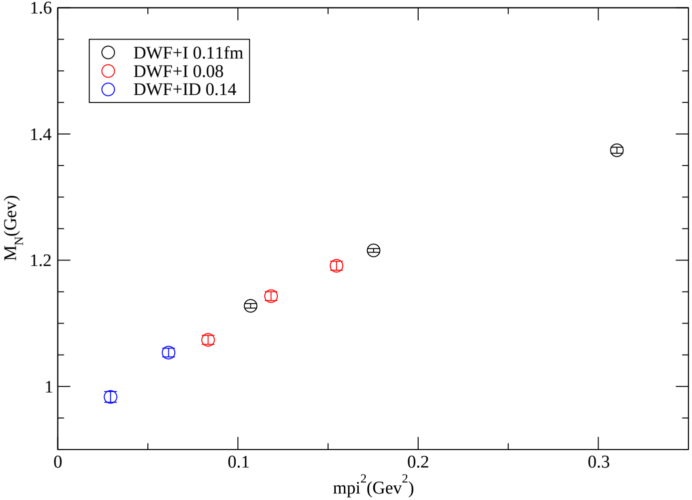
- 32^3 : $a^{-1} = 2.31(4)$Gev, $m_{res} \sim 1.5$Mev
  Gaussian source, $\langle r^2 \rangle^{1/2} \sim 6.0$ (generated by LHPC, arXiv:0907.4194), $am_s' = 0.025, 0.0255 \cdots \underline{0.03}$.

am_l	MD units	Propagator #
0.004	590,600$\cdots$6600	(1996)
0.006	544,552$\cdots$7600	(2600)
0.008	590,600$\cdots$6600	2064

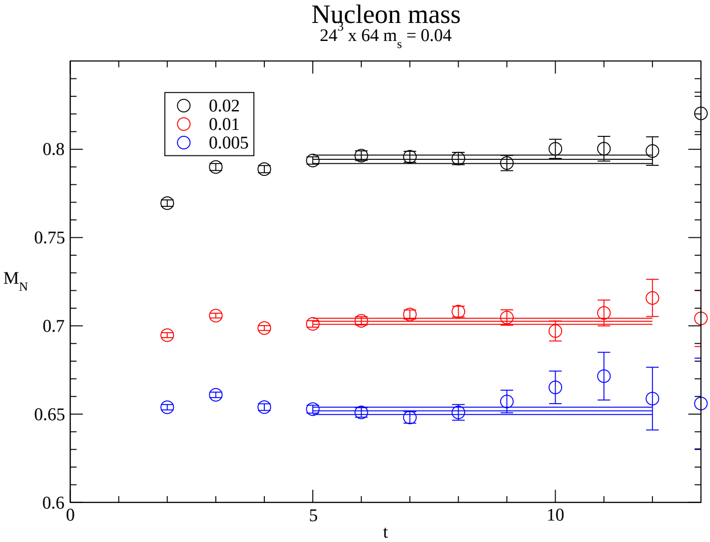
- DWF+ DSDR: $a^{-1} = 1.37(1)$ Gev, $m_{res} \sim 2.5$ Mev
  Gaussian smeared source, $am_s' = \underline{0.045}, 0.04525, \cdots 0.052$

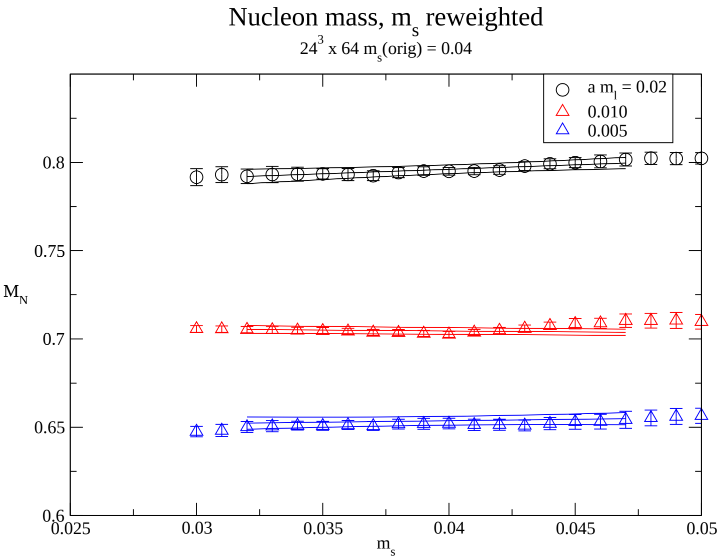
am_l	MD units	Propagator #
0.01	500,508$\cdots$2396	(764)
0.042	608,616$\cdots$1920	1320

m_N vs. m_π^2

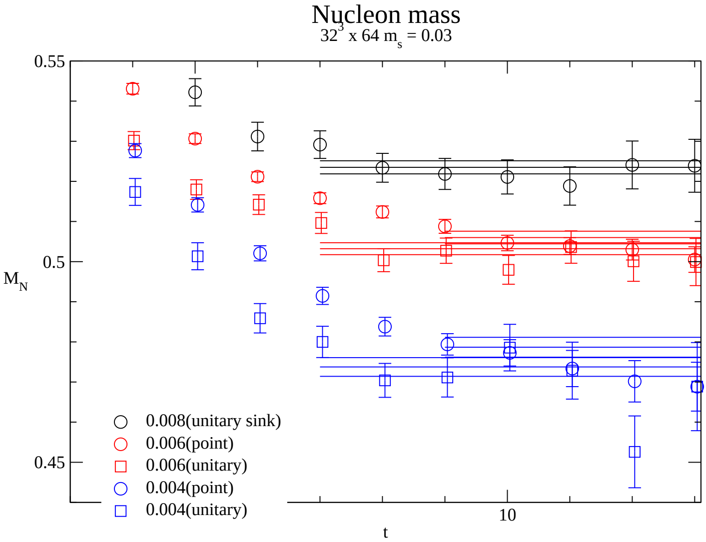

DWF+I 0.11fm
DWF+I 0.08
DWF+ID 0.14
1.6
1.4
1.2
1
0
0.1
0.2
0.3
M_N(Gev)
mpi^2(Gev^2)

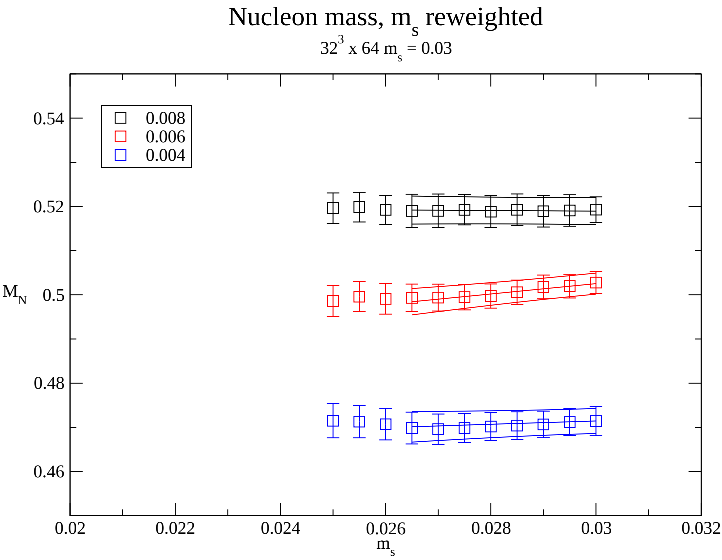
24^3 DWF nucleon effective mass

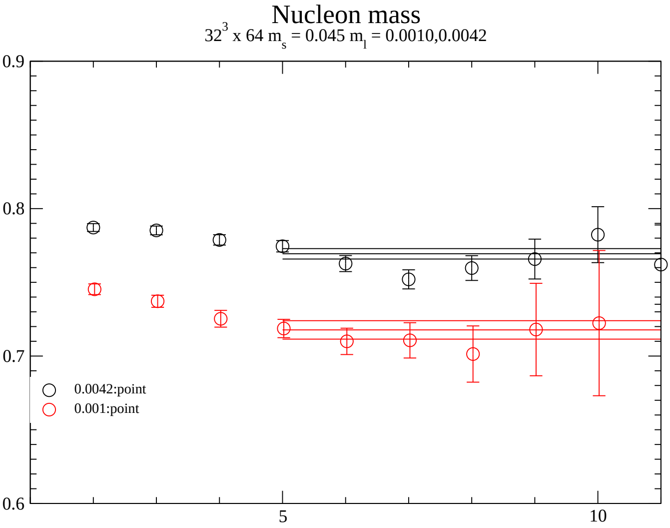


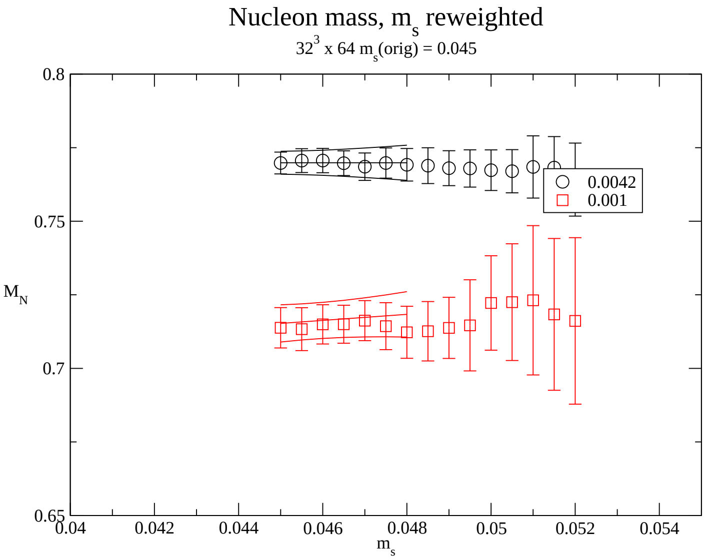

Nucleon mass, m_s reweighted
24^3 x 64 m_s(orig) = 0.04
a m_l = 0.02
0.010
0.005
M_N
0.8
0.75
0.7
0.65
0.6
0.025
0.03
0.035
0.04
0.045
0.05
m_s

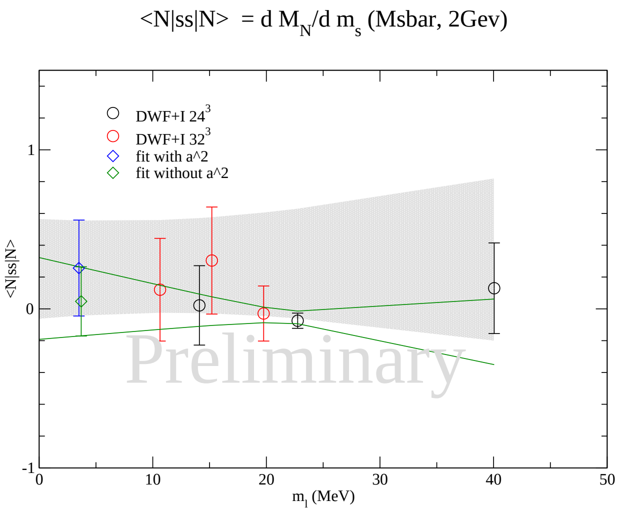
32^3 DWF nucleon effective mass



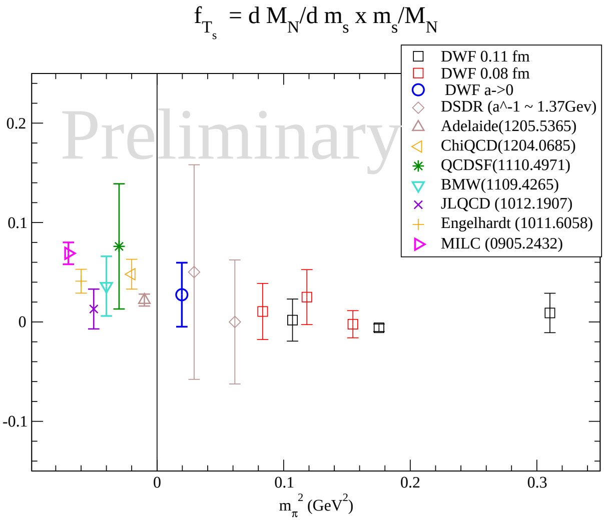

Nucleon mass, m_s reweighted
32^3 x 64 m_s = 0.03
0.008
0.006
0.004
M_N
0.54
0.52
0.5
0.48
0.46
0.02
0.022
0.024
0.026
0.028
0.03
0.032
m_s


Nucleon mass
32^3 x 64 m_s = 0.045 m_l = 0.0010,0.0042
0.9
0.8
0.7
0.6
5
10
0.0042:point
0.001:point


Nucleon mass, m s reweighted
32 3 x 64 m s (orig) = 0.045
0.8
0.75
0.7
0.65
M N
0.04
0.042
0.044
0.046
0.048
0.05
0.052
0.054
m s
0.0042
0.001



$$\langle N|\bar{s}s|N\rangle(\text{2Gev}, m_l = m_{phys}, \text{preliminary}) = 0.05(22)(\text{without } a^2)\ 0.25(32)\ (\text{with } a^2)$$



$$f_{T_s} \sim 0.026(32)$$

Summary & Outlook

Measurement of $M_N(m_s)$ by up to $\sim$ 20% from the dynamical m_s seems stable, allowing the extraction of dm_N/dm_s. Current preliminary continuum extrapolation is $\langle N|\bar{s}s|N\rangle$(2Gev)$\sim$ 0.25(32).
How can we improve?

- Statistical error: Smallness of the signal suggests more accurate measurement of M_N on each ensemble is crucial in improving the statistical error.

  Nucleon 2-point function is inherently noisy (S/N $\sim \exp[-(M_N - 3M_\pi)]$.

  We should calculate 2pt on multiple source positions per configuration. Various techniques are being developed to reduce the cost of multiple light quark inversions per configurations (Low Mode/All Mode Averaging(LMA,AMA), EigCG, Deflation, ...). It remains to be seen how effective and practical these techniques will continue to be as we approach the continuum limit (smaller a, larger V).

- Chiral extrapolation:
  Theory dependence significant. (How much can we trust BChPT?) Measurements at lighter pion mass would help greatly in controlling chiral extrapolation. Possible to combine these with DSDR lattices ($m_\pi \sim$ 180,250Mev) and/or new DWF+I ensembles near physical pion mass. $a \sim 0.11$ DWF+I started. $a \sim 0.08$ planned
- Continuum (a^2) extrapolation:
  Small statistical errors on more than 1 lattice spacing crucial for unconstrained fit. Despite difficulties from signal being small, would be very nice to be able to do contininuum extrapolation as rigorously as other quantities.

Given the ongoing efforts, maybe not too far away!

DWF with Dislocation Suppressing Determinant Ratio

Renfrew et. al., arXiv:0902.2587
Motivation: Dislocations which induce chiral symmetry breaking is probably the biggest hurdle for Ginsparg-Wilson fermions at larger lattice spacing. Critical slowing down at smaller lattice spacing makes it impractical to decrease a. $\rightarrow$ Introduce additional term to suppress dislocations at moderate lattice spacing.
Examples: QCD Thermodynamics, Nucleon matrix elements, Weak matrix elements ($K \rightarrow \pi\pi$)
Use a ratio of Dirac Operator with imaginary Wilson masses to control the suppression of eigenvalues near $-M_5$ while preserve larger eigenvalues.

$$\mathcal{W}(M_5, \epsilon_f, \epsilon_b) = \frac{\det[D_{\mathcal{W}}(-M_5 + \imath\epsilon_f\gamma^5)^\dagger D_{\mathcal{W}}(-M_5 + \imath\epsilon_f\gamma^5)]}{\det[D_{\mathcal{W}}(-M_5 + \imath\epsilon_b\gamma^5)^\dagger D_{\mathcal{W}}(-M_5 + \imath\epsilon_b\gamma^5)]}$$
$$= \frac{\det[D_{\mathcal{W}}(-M_5)^\dagger D_{\mathcal{W}}(-M_5)] + \epsilon_f^2}{\det[D_{\mathcal{W}}(-M_5)^\dagger D_{\mathcal{W}}(-M_5)] + \epsilon_b^2} = \prod_i \frac{\lambda_i^2 + \epsilon_f^2}{\lambda_i^2 + \epsilon_b^2}$$
$$\sim 1 \quad \text{for} \lambda_i \gg \epsilon_b, \epsilon_f, \qquad \sim \epsilon_f^2/\epsilon_b^2 \text{for} \lambda_i \ll \epsilon_f.$$