Low lying hadron spectrum and chiral condensate with two flavors of naive Wilson fermions

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- Wilson fermions break chiral symmetry explicitly on lattice. Naive Wilson fermions have O(a) cut-off effects.

- Emergence of chiral regime from Lattice QCD with naive Wilson fermions has had a questionable past, showing large scaling violations in hadronic observables, lack of suppression of topological susceptibility with decreasing quark masses etc, although most of the studies were done a decade or more ago (mostly quenched) and were done at large pion masses, large lattice spacings and small volumes.

- Simulations with Wilson fermions were also unreliable and difficult because of the so-called exceptional configurations which result from the lack of chiral symmetry.

- However, simulations of lattice QCD with Wilson fermions with smaller quark masses, smaller lattice spacings and larger volumes are now possible with improved algorithms, e.g., DDHMC (Luescher 2004).

- It has also been shown that simulations are safe from exceptional configurations if the volume is large enough

So the questions are:

- Whether the chiral regime of QCD can emerge easily from lattice QCD with naive Wilson fermions? Obviously this also implies that cut-offs effects then have to be under control.

- At what sort of lattice scale the above can happen, if at all?

To investigate these issues, we have initiated a fairly detailed study of lattice QCD with 2 degenerate flavors of naive Wilson fermions and with plaquette gauge action.

This is an on-going investigation, and in this talk I will present data which do not all have the same degree of finesse, some actually will be quite preliminary.

The first result was presented last year (LATTICE 2011):

- topological susceptibility with naive Wilson fermions was shown for the first time to be unambiguously suppressed by going to lower quark masses, consistent with expectations from the chiral Ward identity and chiral perturbation theory (PLB 2012).

- results of topological charge density correlators have been presented this year (*parallel talk by S. Mondal, 27 June Lattice 2012, chiral symmetry session*)

We have, so far, done simulations at 2 gauge couplings, 3 lattice volumes, and a number of quark masses.

The set of data points discussed in this talk are as follows:

$\beta = 5.6, \;\; 16^3 32, \;\; 24^3 48, \;\; 32^3 64, \;\; a = 0.069 \; fm$
pion masses: from 790 MeV down to 315 MeV

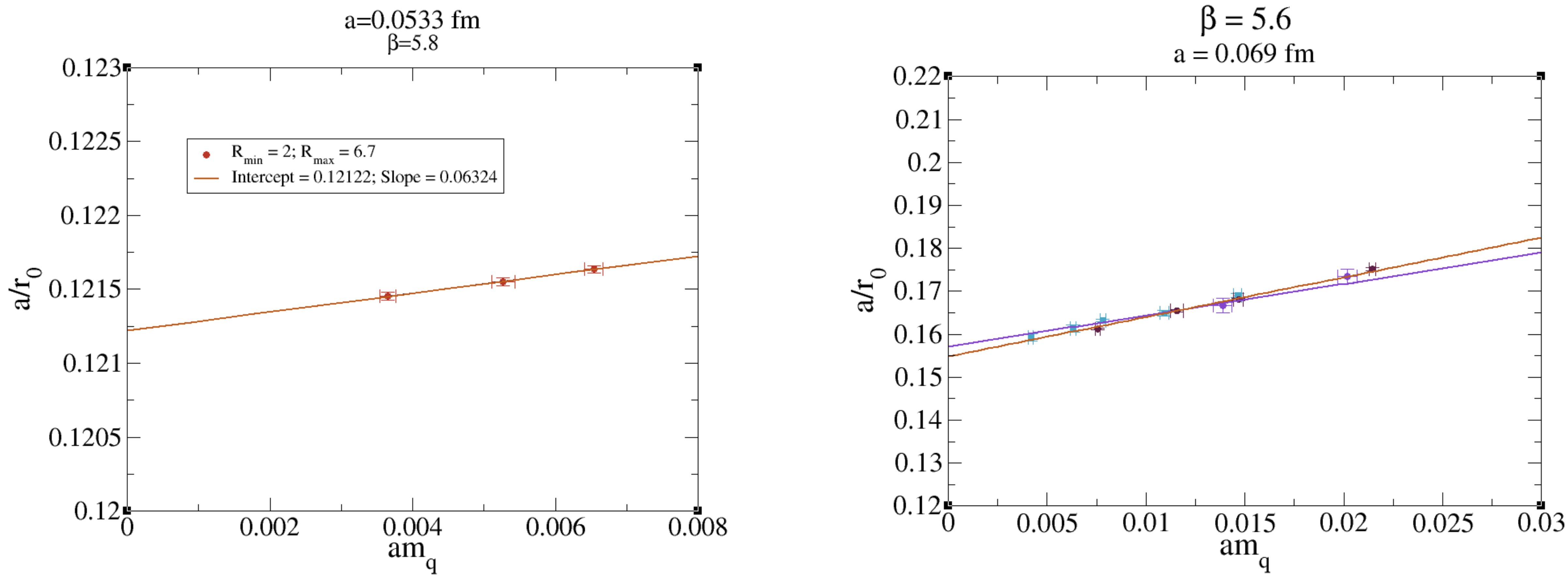
$\beta = 5.8, \;\; 32^3 64, \;\; a = 0.053 \; fm$
pion masses: from 600 MeV to 275 MeV

A reasonably detailed study of autocorrelations of several operators have already been carried out (*parallel talk: Santanu Mondal, 25 June, Machines & algorithms session*)

Some of the data points are a bit preliminary.

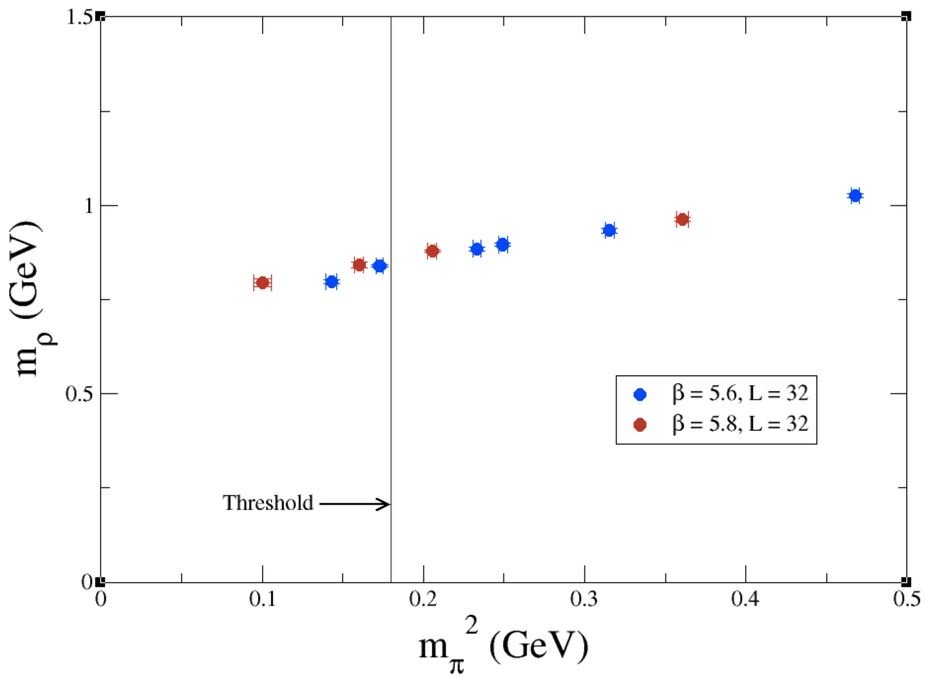
Other data points (especially lower mass and larger volumes) are either planned or are being simulated currently.

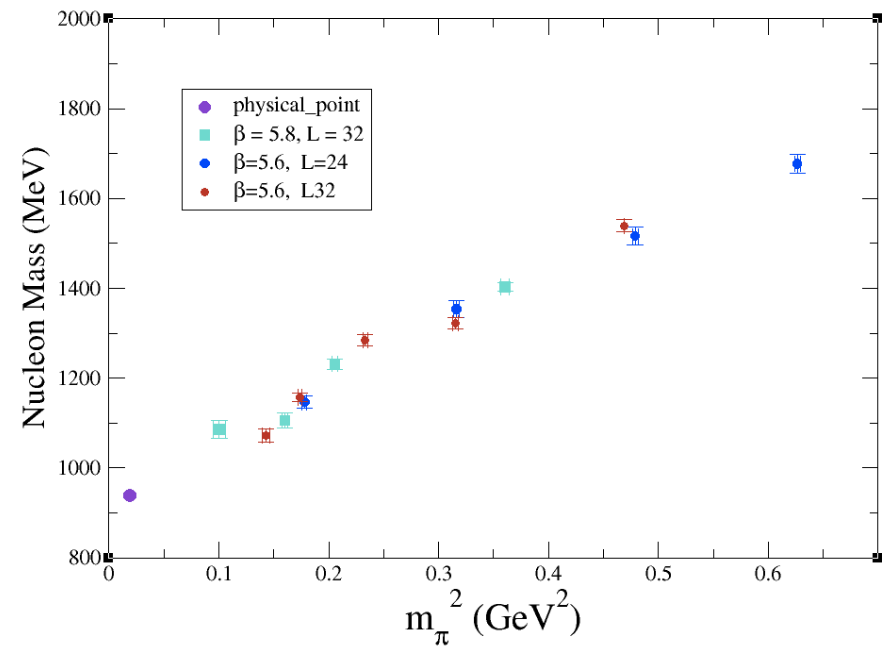
Quick look at the scale and r_0

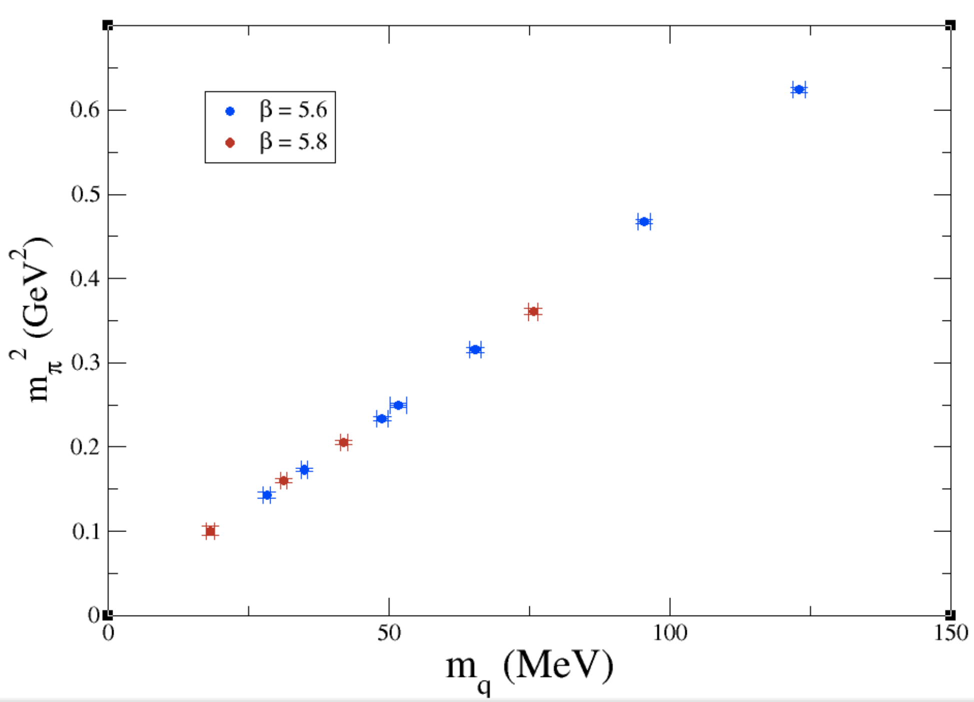


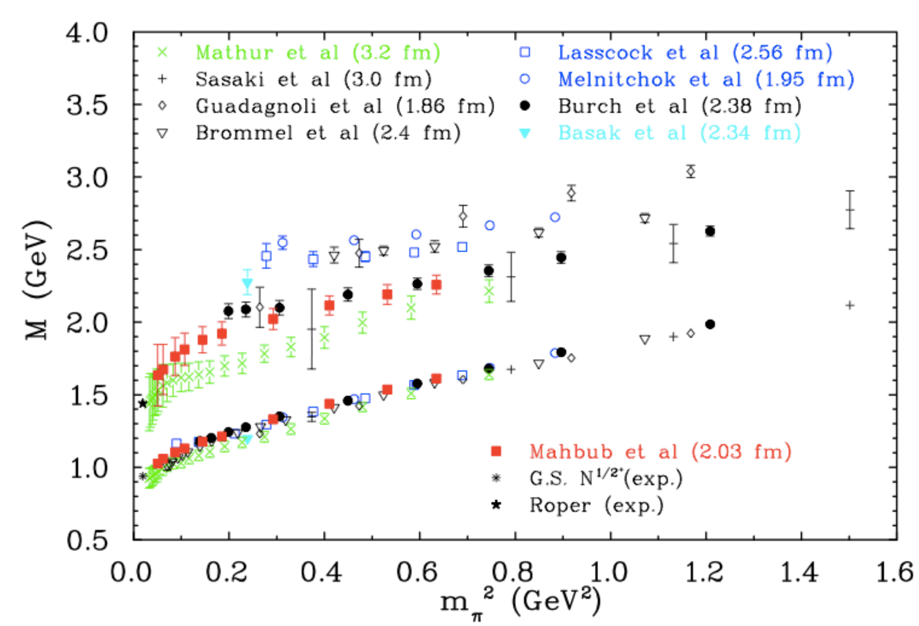
r_0 is determined from pion-nucleon mass ratio plotted vs pion mass squared (in units of r_0^2): $r_0 = 0.44\ fm$

No significant scaling violation observed



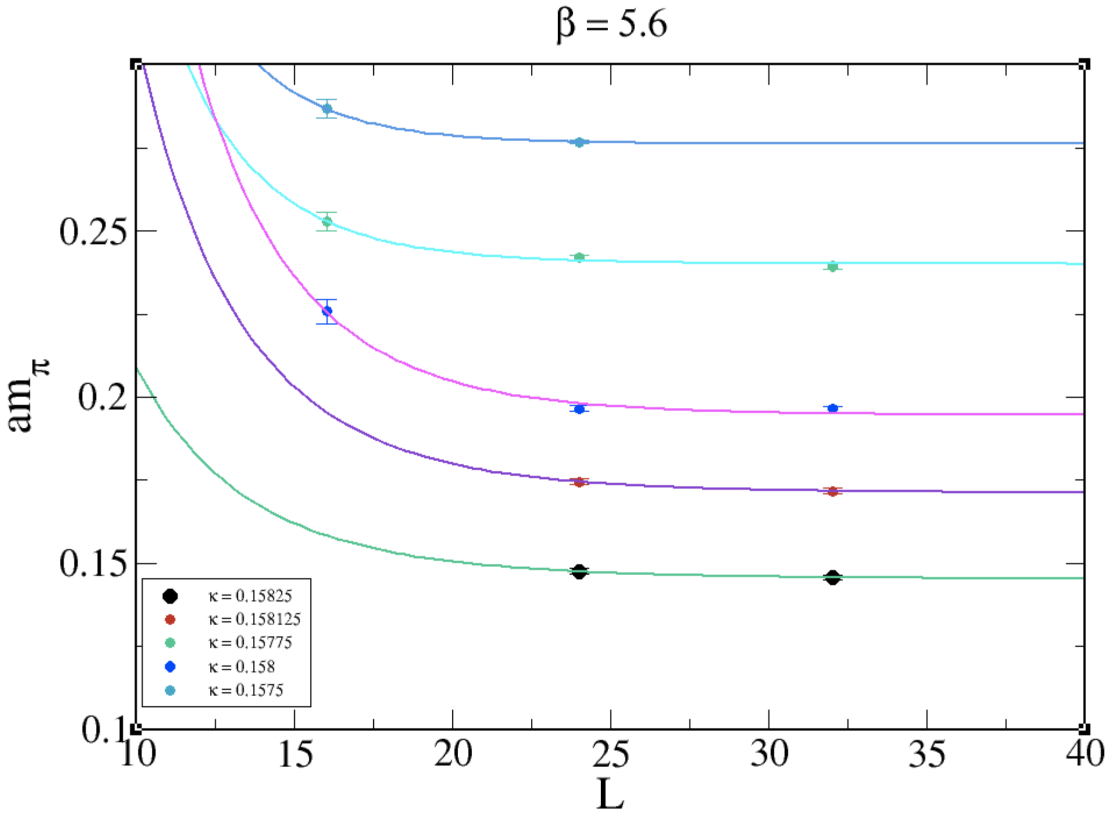


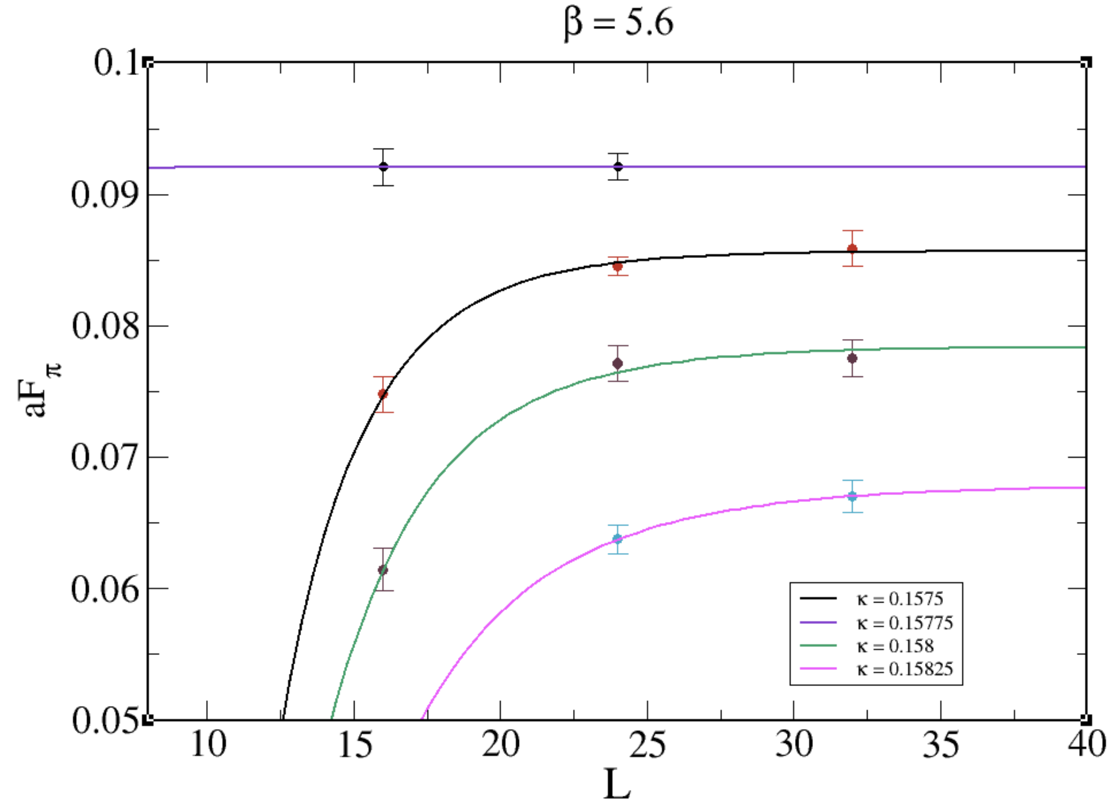




Mahbub et.al. PoS (2009), global plot for comparison

Finite Size Extrapolations





The Luescher's asymptotic formula fits the data reasonably well.
However, the coeffs. are significantly off, if the 16^3 data are included.

$$m_\pi(L) = m_\pi + cL^{-3/2} e^{-m_\pi L}, \qquad c = \frac{3}{4(2\pi)^{3/2}} \frac{m_\pi^{3/2}}{f_\pi^2}$$

Finite Volume Extrapolations ....

The 1-loop finite volume ChiPT expressions (Gasser & Leutwyler 1987) were used:

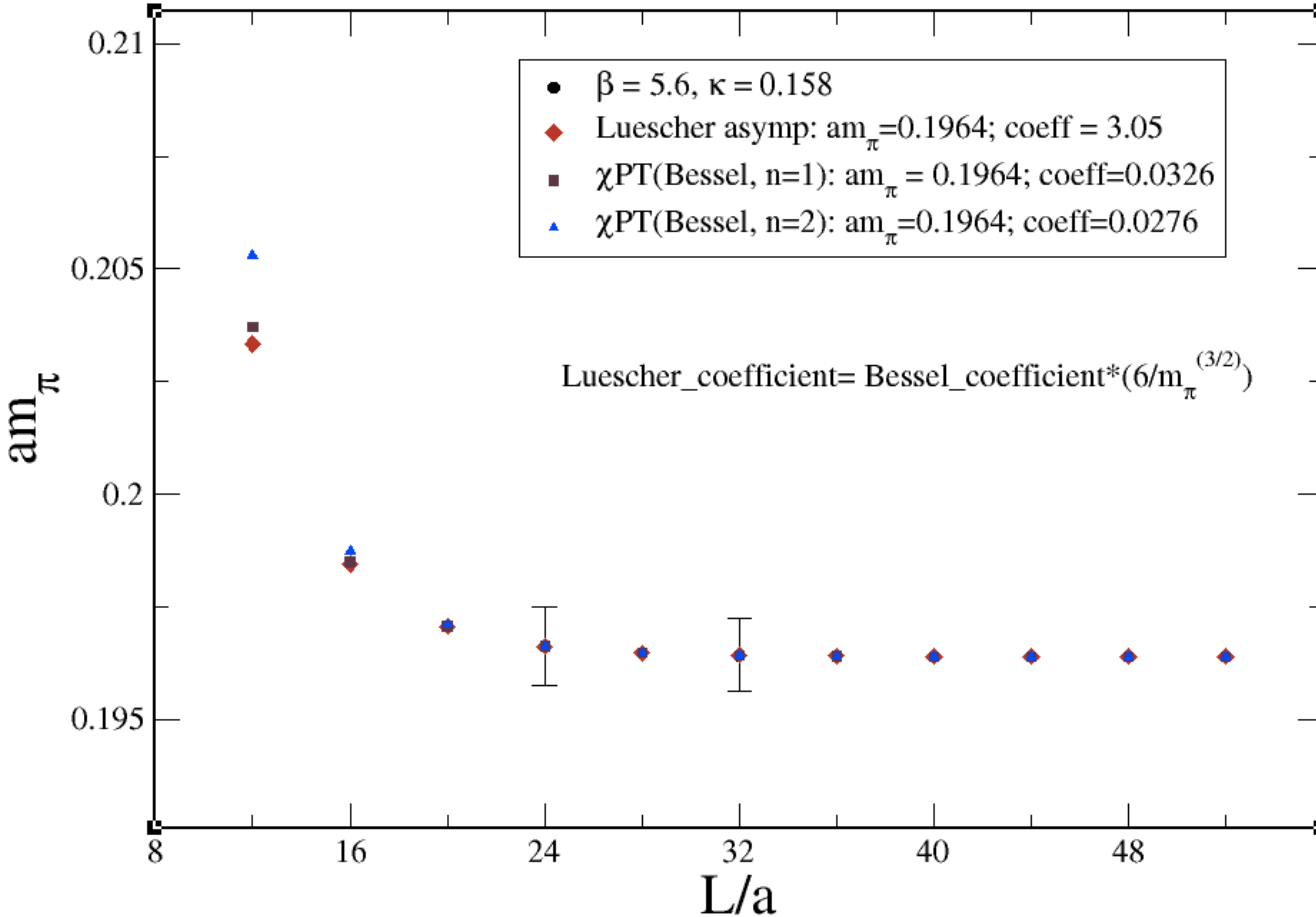
$$m_\pi(L) = m_\pi \left[1 + \frac{1}{4}\frac{m_\pi^2}{(4\pi f_\pi)^2}\tilde{g}_1(m_\pi L)\right]$$

$$f_\pi(L) = f_\pi \left[1 - \frac{m_\pi^2}{(4\pi f_\pi)^2}\tilde{g}_1(m_\pi L)\right]$$

$$\tilde{g}_1(m_\pi L) = \sum_{n=1}^{\infty} \frac{4m(n)}{\sqrt{n}\, m_\pi L} K_1(\sqrt{n}\, m_\pi L)$$

The summation was carried out till n = 2

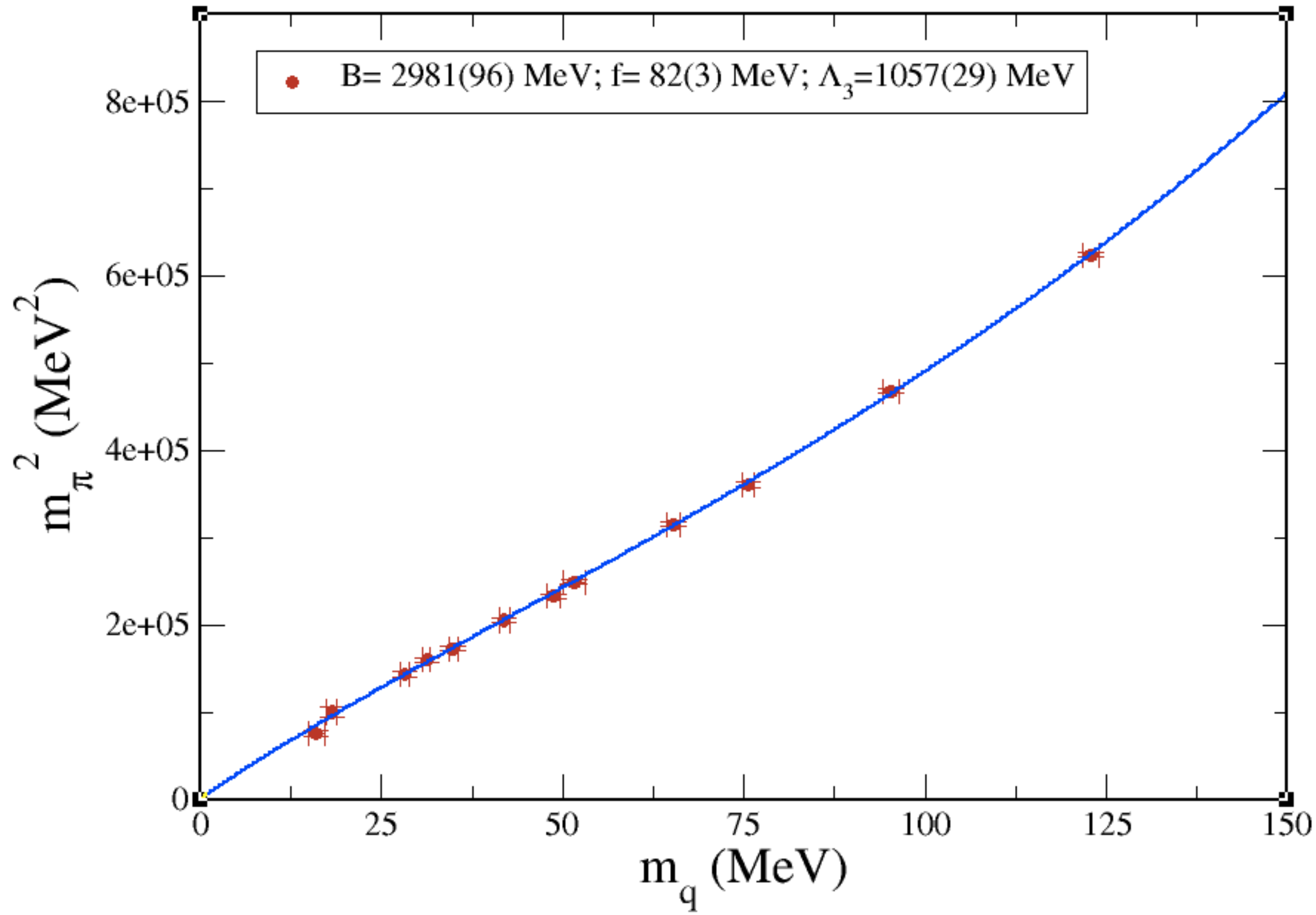
Only 24^3 and 32^3 data were used for the infinite volume extrapolation. These fits were most satisfactory when pion masses were relatively small and $m_\pi L > 4$



0.21
0.205
0.2
0.195
am_π
8
16
24
32
40
48
L/a
β = 5.6, κ = 0.158
Luescher asymp: am_π=0.1964; coeff = 3.05
χPT(Bessel, n=1): am_π = 0.1964; coeff=0.0326
χPT(Bessel, n=2): am_π=0.1964; coeff=0.0276
Luescher_coefficient= Bessel_coefficient*(6/$m_\pi^{(3/2)}$)

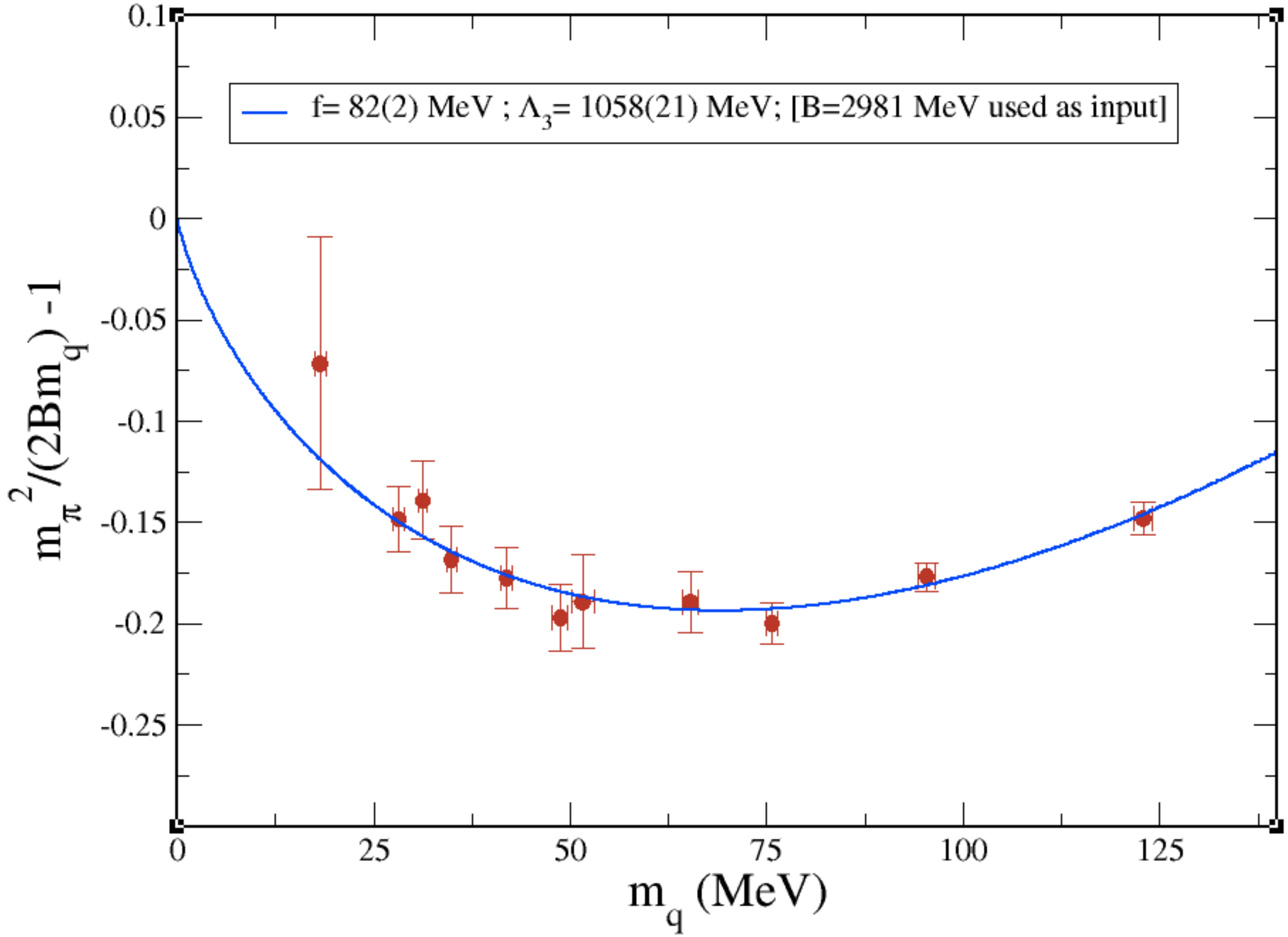
Pion Sector

NLO χPT fit to combined data of β = 5.6, 5.8



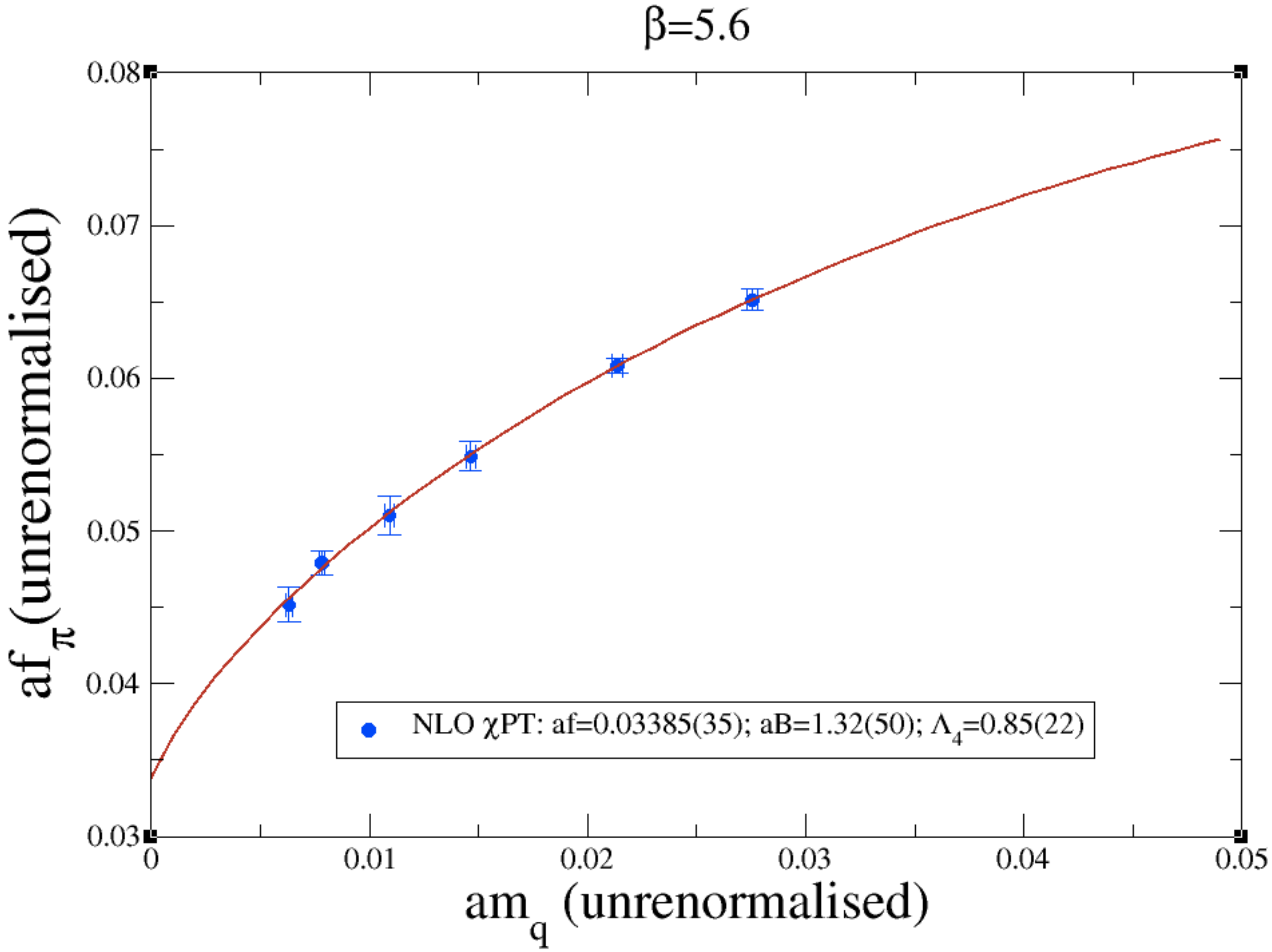
Leads to $\tilde{l}_3 = 4.04(5)$

Pion Sector ...



$$\Rightarrow \quad \Sigma^{1/3} \sim 272 \;\; \text{MeV}$$

Pion Sector ...



$$\Sigma^{1/3} \sim 241 MeV$$

(with renormalization constants put in from Becirevic et.al. 2005)

Chiral Condensate from Ward Identity

$$< \delta \mathcal{O} > = < \delta S \; \mathcal{O} >$$

Now consider axial transformation:

$$\delta\psi = i\alpha_A^a \frac{1}{2}\lambda_a \gamma_5 \; \psi$$
$$\delta\bar{\psi} = i\,\bar{\psi}\;\alpha_A^a \frac{1}{2}\lambda_a \gamma_5$$

and take the operator as:

$$\mathcal{O} \equiv P^b = \bar{\psi}\,\lambda^b \gamma_5\,\psi$$

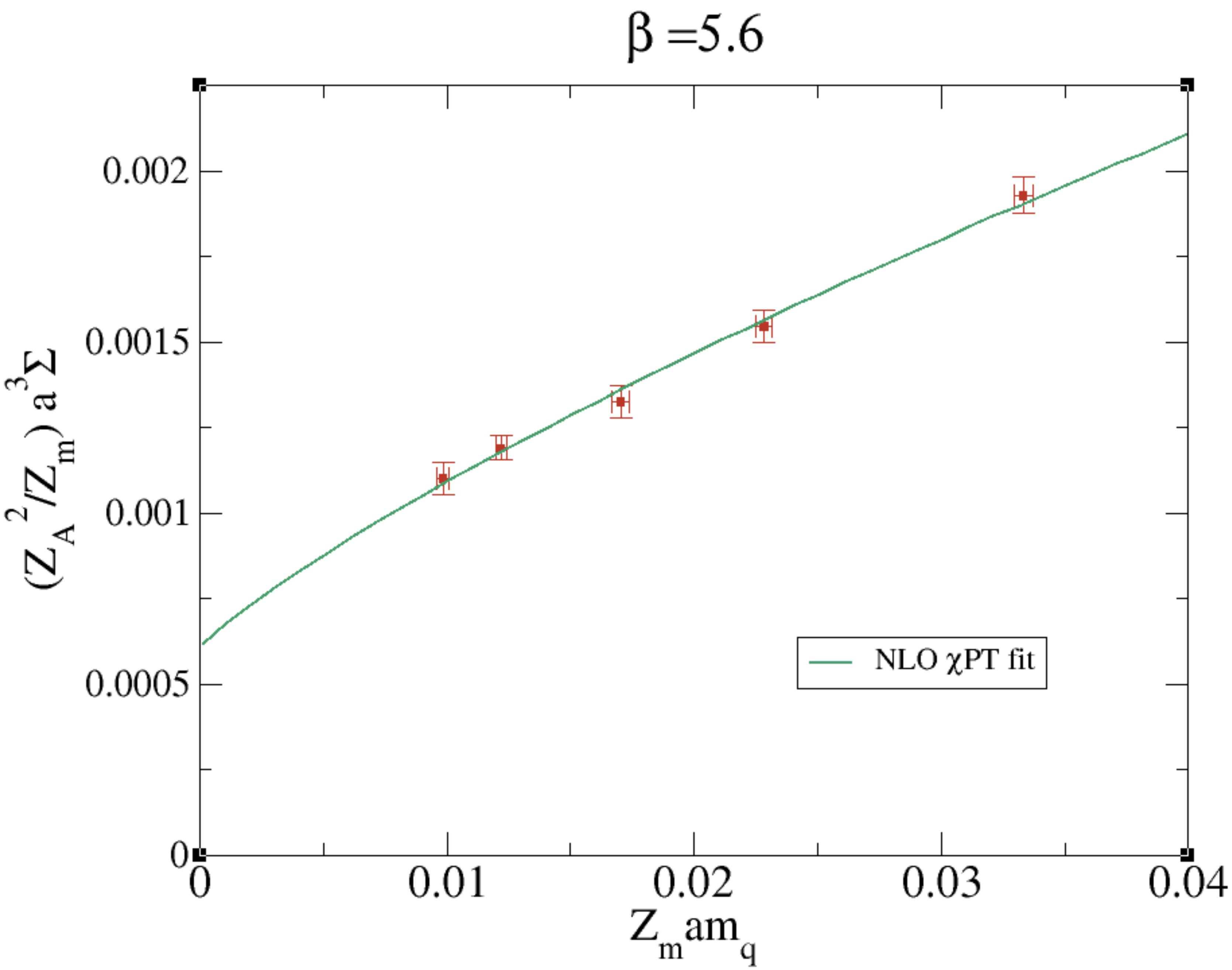
The Ward Identity is:

$$\partial_x^\mu < A_\mu^a(x) P^b(0) > - < \bar{\psi}(x)\{\frac{1}{2}\lambda^a, M\}\gamma_5\,\psi\,P^b(0) > = -\delta(x)\delta^{ab}\frac{1}{N_F} \; < \bar{\psi}\psi >$$

$$\Longrightarrow \qquad \frac{1}{N_F} < \bar{\psi}\psi > = m_q \int d^4x < P^a(x) P^b(0) >$$

$$\Longrightarrow \qquad < \bar{\psi}\psi > = \frac{2 m_q C_{PP}}{m_\pi} = C_{AP}$$

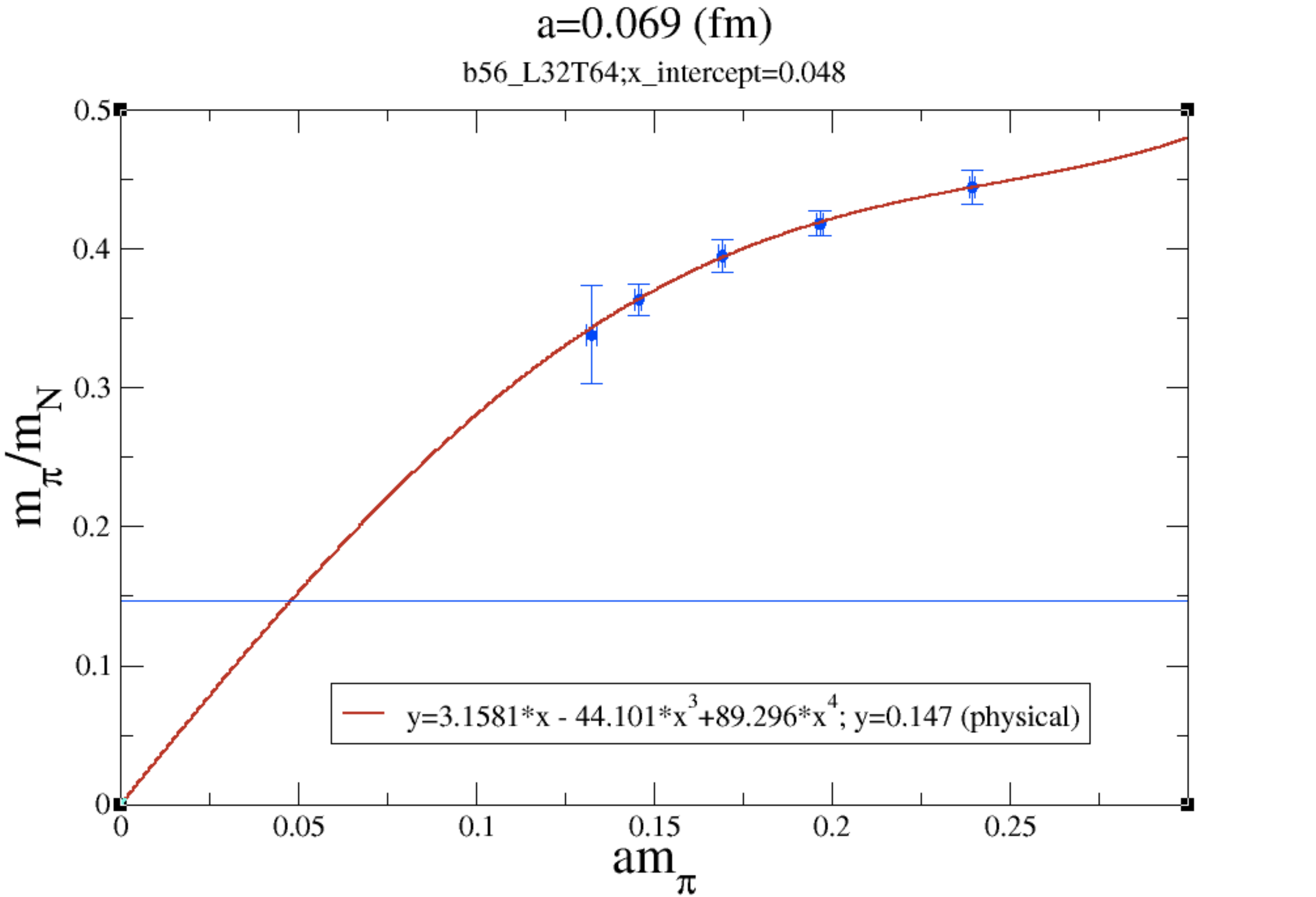
Condensate ...



$$\Sigma^{1/3} \sim 241 \ \text{MeV}$$

Conclusions

- Wilson lattice QCD looks very decent at scales 0.05 - 0.07 fm
- No significant scaling violations
- Chiral properties emerging already at 300 - 400 MeV pions
- Bigger volumes especially at the larger beta absolutely necessary to enable us to reach lower masses and infinite volume extrapolations
- Preliminary numbers (e.g. LECs and condensates) coming out are also encouraging and well within the ballpark region of estimates by other groups
- Need accurate renormalization constants, especially Z_A



a=0.069 (fm)
b56_L32T64;x_intercept=0.048
0.5
0.4
0.3
0.2
0.1
0
0
0.05
0.1
0.15
0.2
0.25
m_π/m_N
am_π
y=3.1581*x - 44.101*x^3+89.296*x^4; y=0.147 (physical)