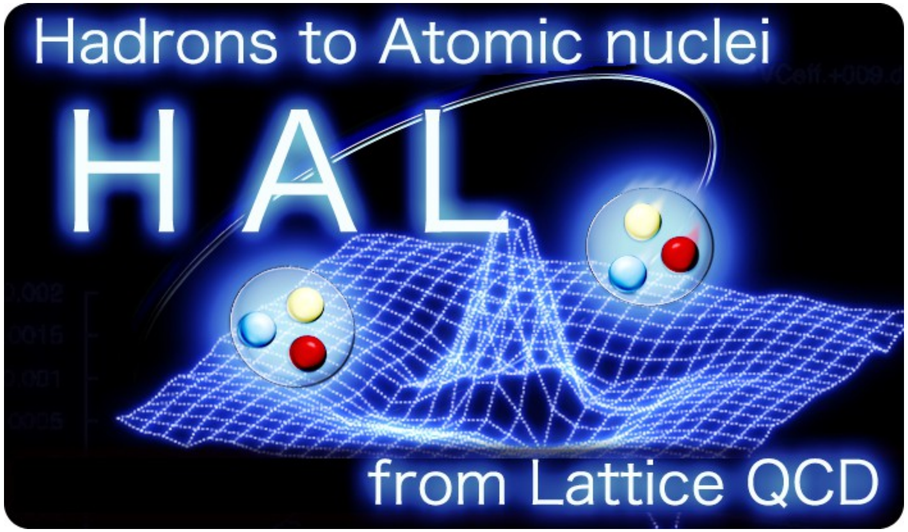
Study of H-dibaryon mass in Lattice QCD

Takashi Inoue   Nihon University

HAL QCD Collaboration

S. Aoki	Univ. Tsukuba
T. Doi	RIKEN
T. Hatsuda	RIKEN
Y. Ikeda	Tokyo Inst. Tech.
N. Ishii	Univ. Tsukuba
K. Murano	RIKEN
H. Nemura	Univ. Tsukuba
K. Sasaki	Univ. Tsukuba



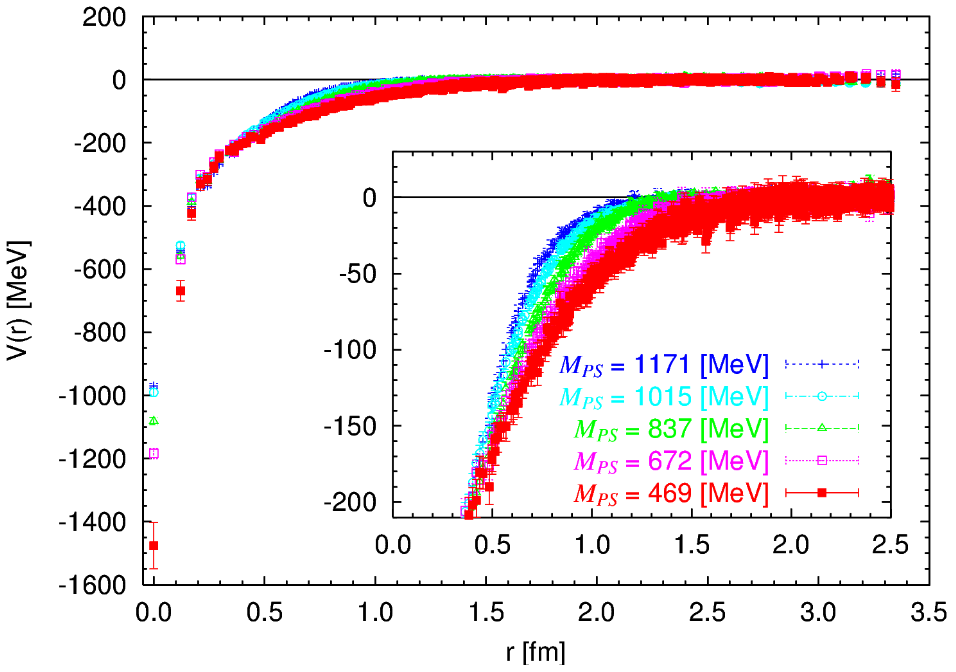
Lattice2012, June 27, 2012, Cairns

Introduction

- H-dibaryon: predicted compact 6-quark (B=2) state
  - R. L. Jaffe, Phys. Rev. Lett. 38 (1977) [MIT Bag model]
  - One of most famous candidates of exotic-hadron.
  - based on the observation of no Pauli exclusion due to 1_F nature, and a attractive contribution from gluon-exchange between quarks.
  - Does H exist in nature? $B_H > 7$ MeV is ruled out by discovery of $_{\Lambda\Lambda}$He. Possibility of a shallow bound state or a resonance still remains.
- Purpose: Search for H-dibayon in LQCD simulation.
  - We began with $SU(3)_F$ limit in order to avoid complicating calculation.
- At the Lattce2011, we reported …
  - If volume is large, it becomes difficult to separate energy eigenstates.
  - To overcome this problem, we developed a new technique.
  - With the new method, we found a bound state in 1_F BB channel.
  - A bound(stable) H was found in five $SU(3)_F$ world with different m_q.

@Lattice2011

K_uds	M_P.S. [MeV]	M_Bar [MeV]
0.13660	1170.9(7)	2274(2)
0.13710	1015.2(6)	2031(2)
0.13760	836.8(5)	1749(1)
0.13800	672.3(6)	1484(2)
0.13840	468.9(8)	1163(2)



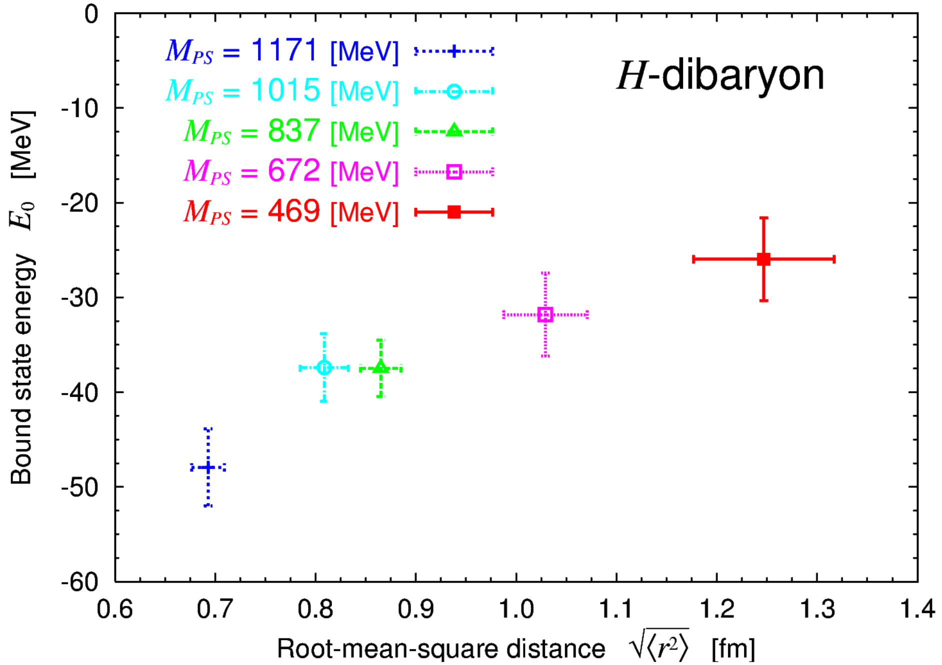
- Left: Table of the hopping parameter K and hadron masses M.
- Right: the extracted potential of the 1F BB channel.

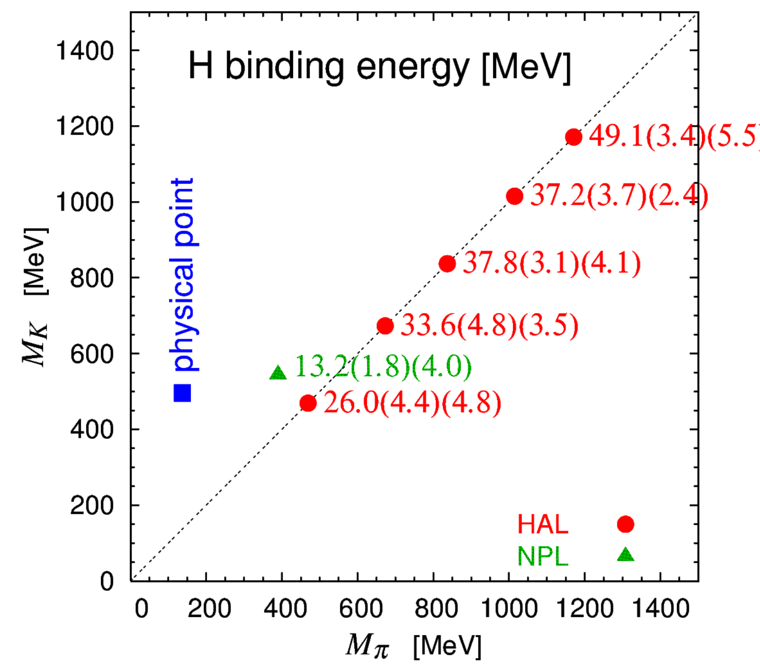
$$V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

HAL QCD
New technique
$\psi(\vec{r},t)$: NBS W.F.

- $V^{(1)}(r)$ become more attractive as quark mass decrease.

@Lattice2011

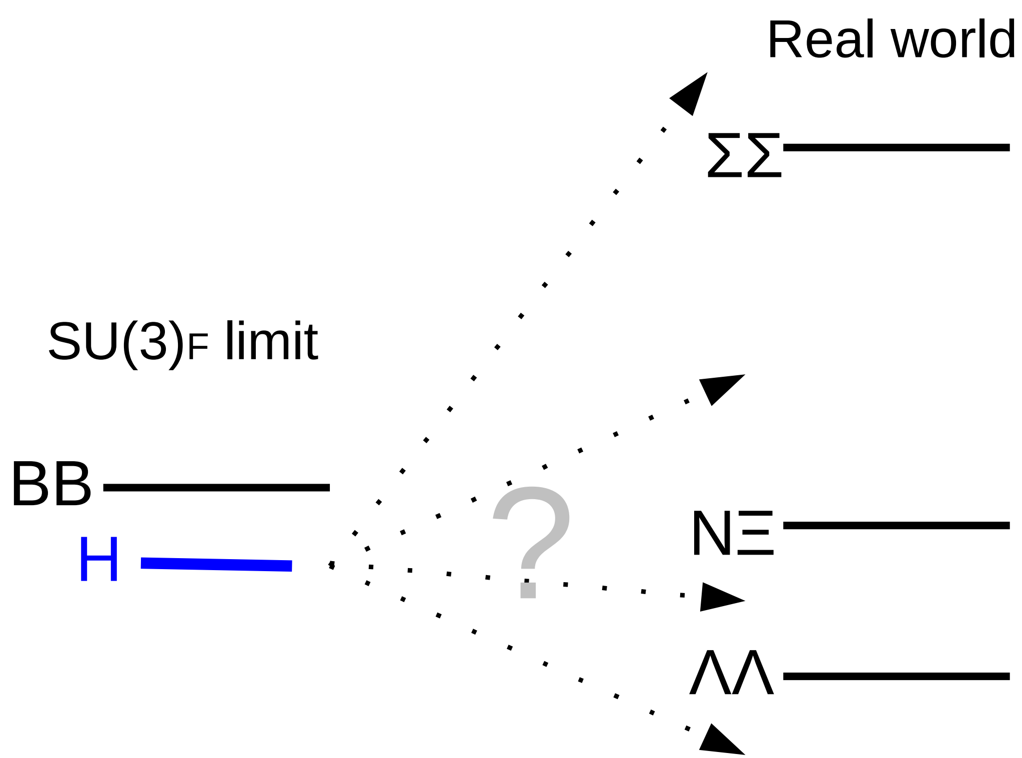




- Left: Observed bound state in 1_F BB channel.
  - Binding energy of H is 50 – 20 MeV depending on m_q.
- Right: Summary of H binding energy from FLQCD.
  - SR. Beane etal [NPLQCD colla.] Phys. Rev. Lett. 106, 162001 (2011)
  - Resulting H-dibaryon mass from the two groups looks consistent.

Introduction

- In order to estimate H-dibaryon in the real world, we study effect of $SU(3)_F$ breaking on H phenomenologically.



- What is expected? How can we approach in unbound case?

Outline

1. Introduction
2. Basis to describe BB interaction
3. Coupled $\Lambda\Lambda$ – $N\Xi$ – $\Sigma\Sigma$ $^1S_0$ scattering w/ $SU(3)_F$ breaking
4. Summary and Plan

Basis to describe BB interaction

- In SU(3)$_F$ limit, flavor irre. reps. give convenient basis.

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

- In flavor SU(3) broken world, e.g. the physical one, the baryon-basis are used instead of the flavor-basis.

$$\text{e.g.} \quad NN\,,\ \Lambda N\,,\ \Lambda\Lambda \ \text{ and so on}$$

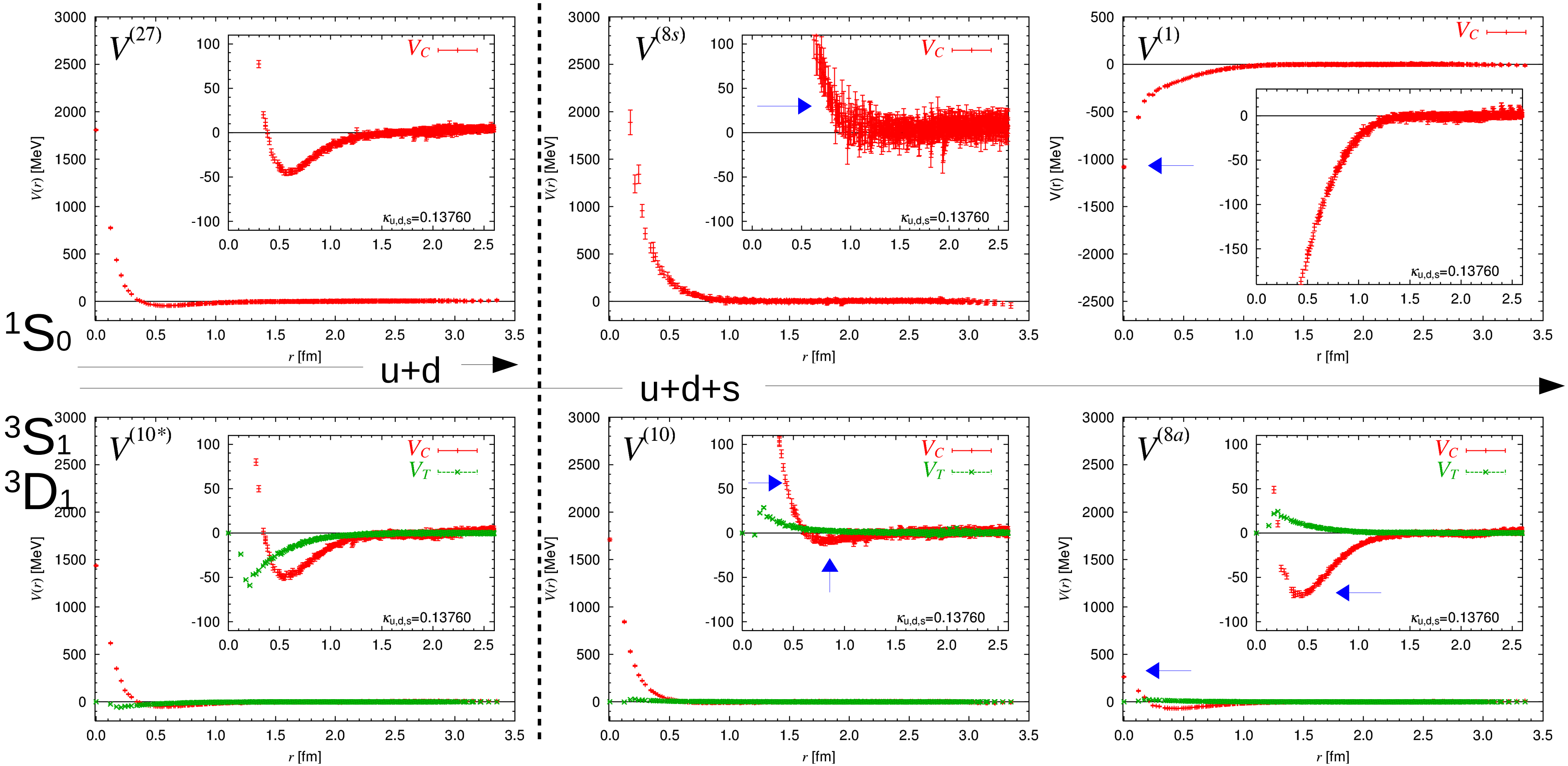
- In the SU(3)$_F$ limit, the baryon-base potential $V_{ij}(r)$ are given by a unitary rotation of the potential $V^{(a)}(r)$.

e.g. S=−2, I=0 sector

coupled channel

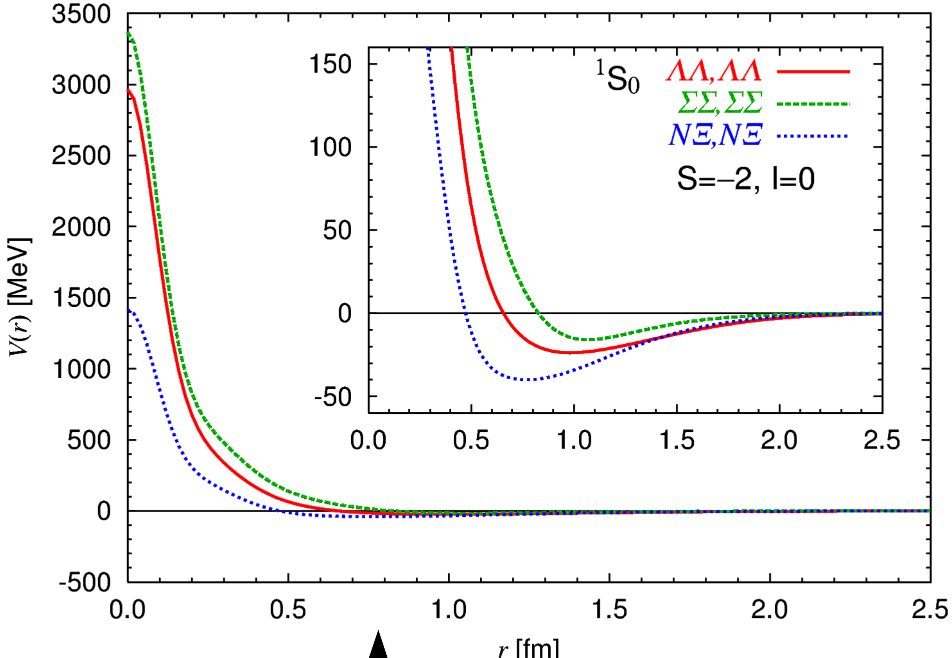
$$\begin{pmatrix} \langle \Lambda\Lambda | \\ \langle \Sigma\Sigma | \\ \langle \Xi N | \end{pmatrix} = U \begin{pmatrix} \langle 27 | \\ \langle 8 | \\ \langle 1 | \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t = \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$

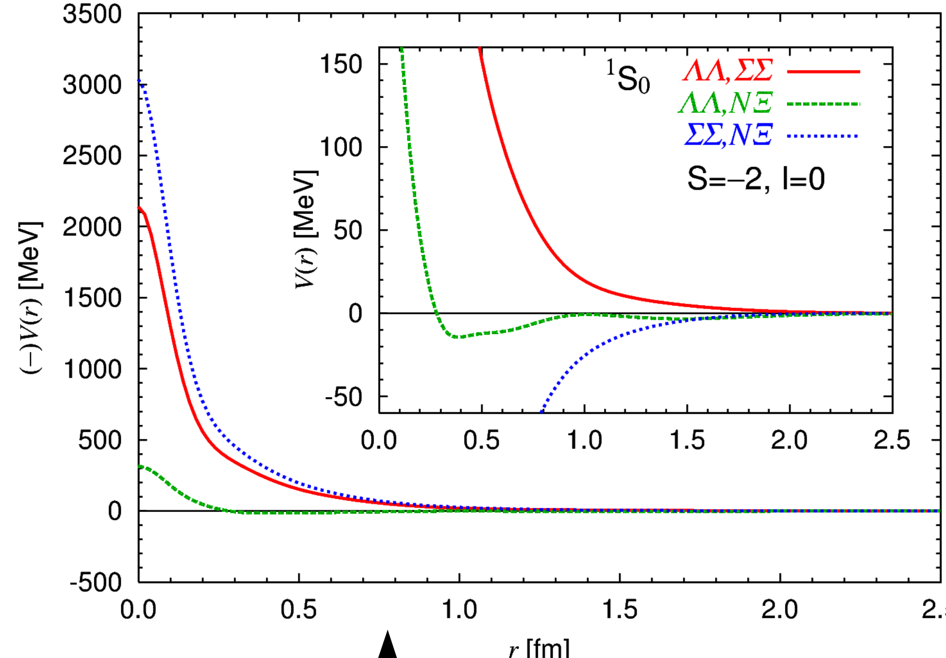
BB int. in flavor basis



- At the SU(3)$_F$ limit with m$_q$ corresponding to M$_{PS}$ = 837 [MeV].
- QM is true at small r. Especially, no repulsion in 1$_F$ channel.
- This V$^{(1)}$ supports a bound H-dibaryon in the SU(3)$_F$ limit.

BB int. in S=−2, I=0 sector





- Left: Diagonal pot. Right: Off-diagonal pot.
- In baryon basis, all three diagonal Interaction has repulsive core. NΞ channel is most attractive although it has not much meaning.
- Channel coupling interactions are comparable to diagonal ones, except for small ΛΛ – NΞ coupling (small sign change is artifact).

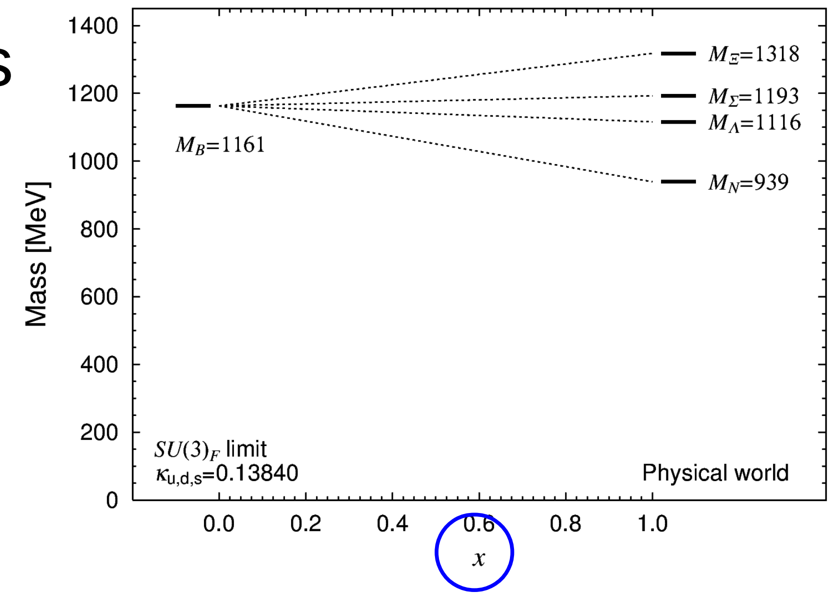
$\Lambda\Lambda - N\Xi - \Sigma\Sigma$ $^1S_0$ scattering

- To see $J^P=0^+$ states in S=−2, I=0 sector, we study this scatt.

$$T^{\alpha\beta} = V^{\alpha\beta} + \sum_{\gamma} V^{\alpha\gamma} G^{(0)}_{\gamma} T^{\gamma\beta}, \quad G^{(0)}_{\gamma} = \frac{1}{E - H^{(0)}_{\gamma} + i\epsilon}, \quad H^{(0)}_{\gamma} = \frac{p^2}{2\mu_{\gamma}} + M^{\gamma}_{1} + M^{\gamma}_{2}$$

- For baryon masses, we use values interpolated between SU(3)$_F$ limit one at K=0.13840 (M_{ps} = 469 MeV) and physical ones linearly.

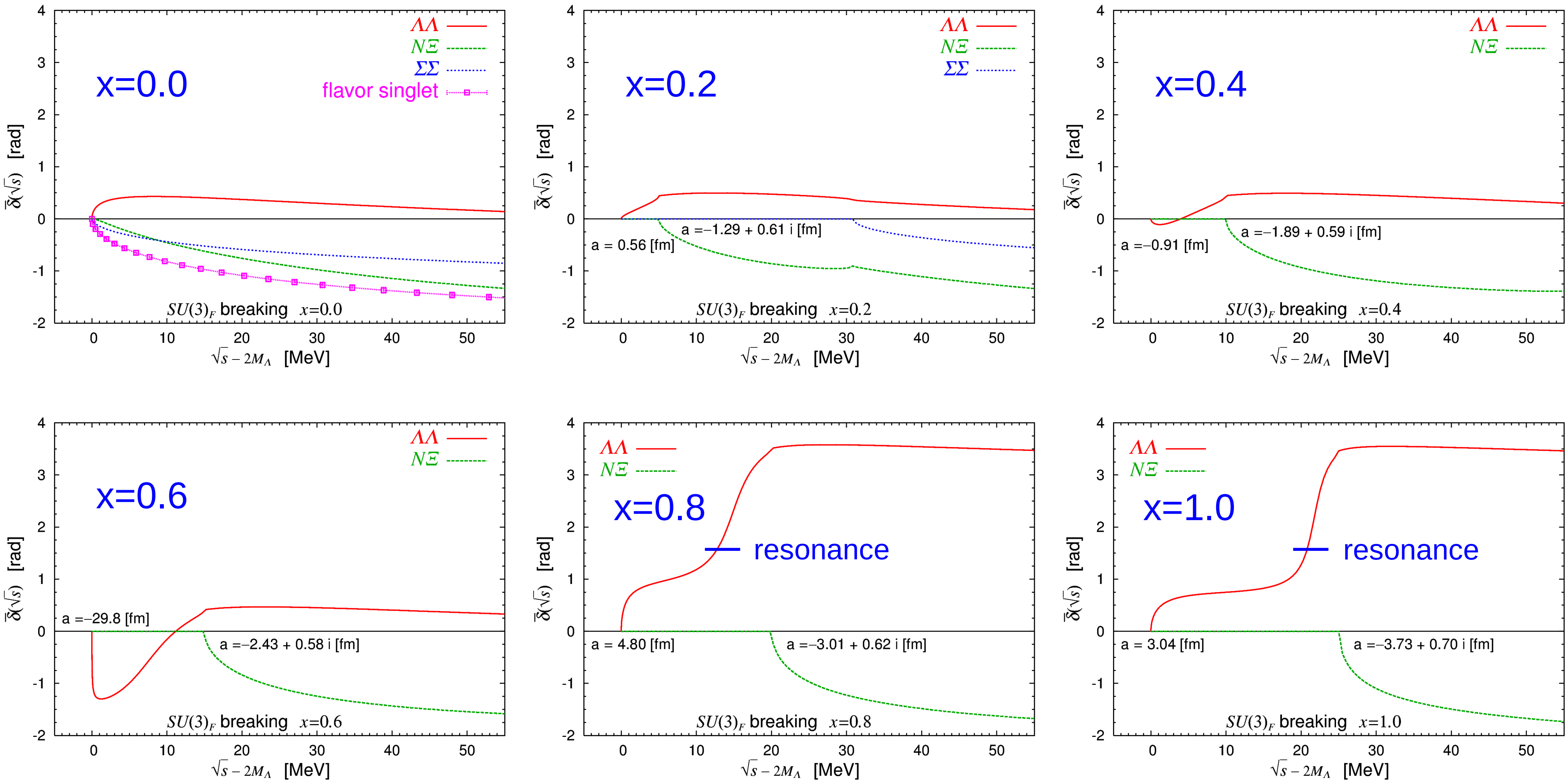
$$M_Y(x) = (1 - \underline{x})\, M_B^{SU(3)} + \underline{x}\, M_Y^{Phys}$$



- For $V^{\alpha\beta}$, we use ones given in the previous slide ie. at SU(3)$_F$
- This is just a trial study or demonstration for the moment !!
(based on the assumptions: 1.the mass of baryon has major effect,
 2. qualitative features of $V^{\alpha\beta}$ remain intact w/ SU(3)$_F$ breaking).

→ Thur. K. Sasaki

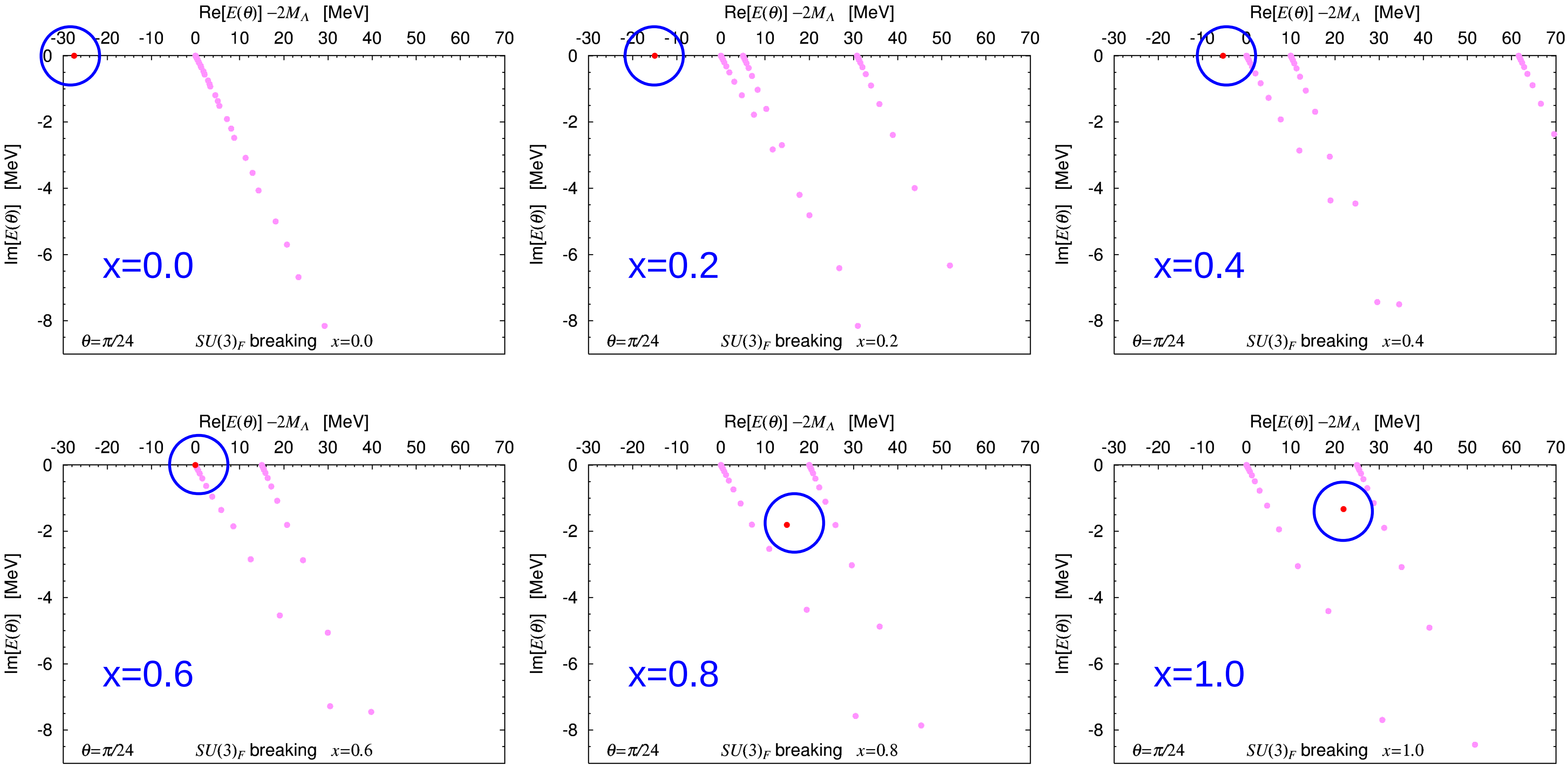
Phase-shifts



- Bar-phase-shifts in the Stapp parametrization: $S_{ii}^{l=0} = \eta_i e^{2i\bar{\delta}_i}$
- The ΛΛ phase-shift change drastically as the x increase, since
- H approaches the ΛΛ threshold from below and go though it.

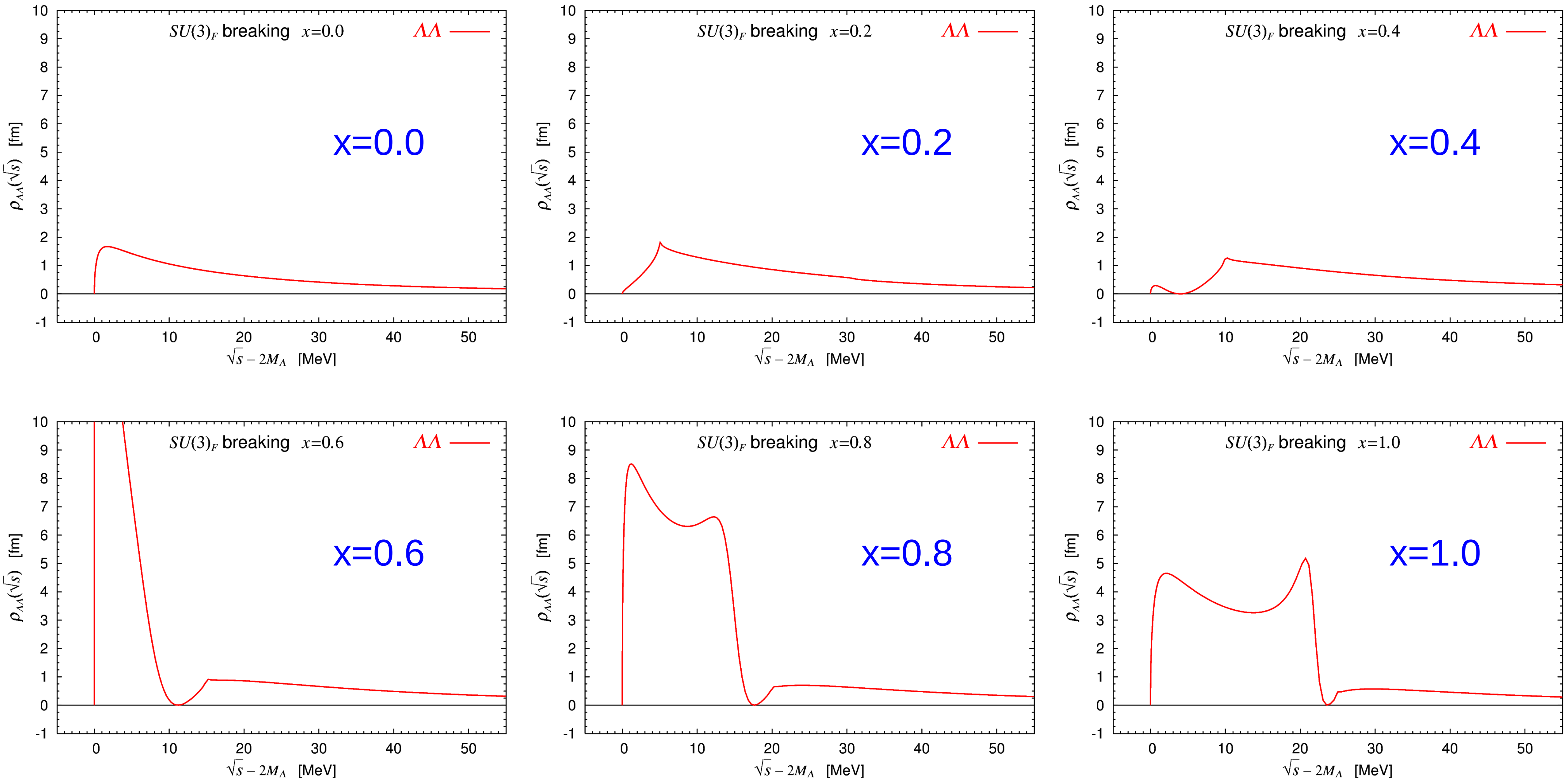
H-dibaryon in CSM

$$r \to r e^{i\theta} \qquad H(\theta)\,\Psi_\theta = E(\theta)\,\Psi_\theta$$



- Energy eigenvalues of the system in the Complex-Scaling-Method.
- H comes 3 MeV below the NΞ threshold at the empirical $SU(3)_F$ breaking in this phenomenological trial calculation.

ΛΛ invariant-mass spectrum



- Invariant-mass spectrum of ΛΛ calculated in S-wave dominance.

$$\rho_{\Lambda\Lambda}(\sqrt{s}) = |\, 1 - S_{\Lambda\Lambda}^{l=0} \,|^2 \,/\, k$$

- We may have a chance to find H in experiments counting two Λ.

Summary and Plan

- I've reviewed results we reported at Lattice2011.

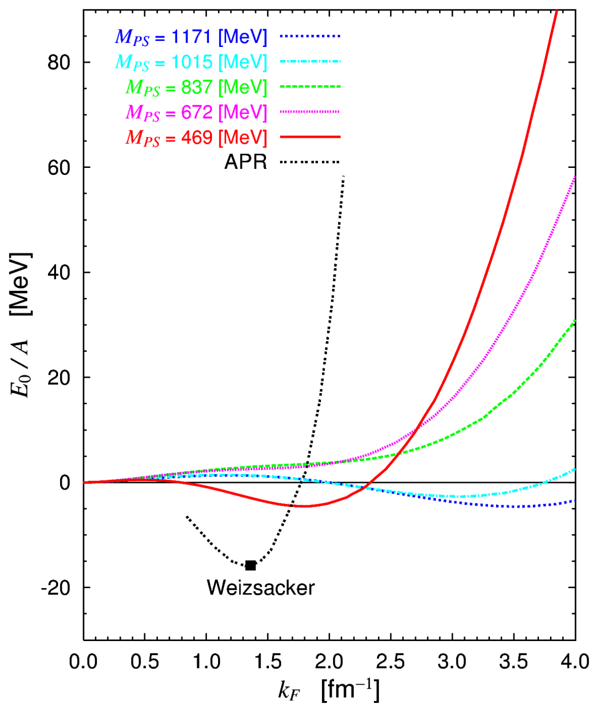
- We've estimated H in the real world w/ $SU(3)_F$ breaking.
  - in a bold phenomenological approach
  - H moves as the breaking increase and goes through ΛΛ, and comes to below NΞ at the physical breaking. (= resonance)
  - Because of the approximation, this result is less reliable. But it is unlikely that H is bound (below ΛΛ) in reality, I think.
  - We are able to study H in LQCD even in unbound case!

- Plan
  - Measurement for H at one more lighter quark $SU(3)_F$ point.
  - True $SU(3)_F$ breaking（2+1 flavor simulation & analysis).
  - High density baryon matter. → Thur. K. Sasaki

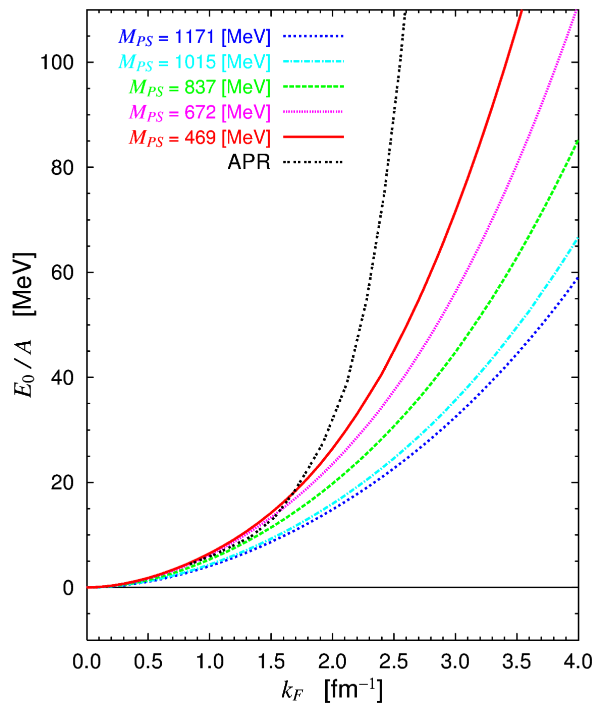
Backup slides

Matter EOS and Neutron Stars from LQCD

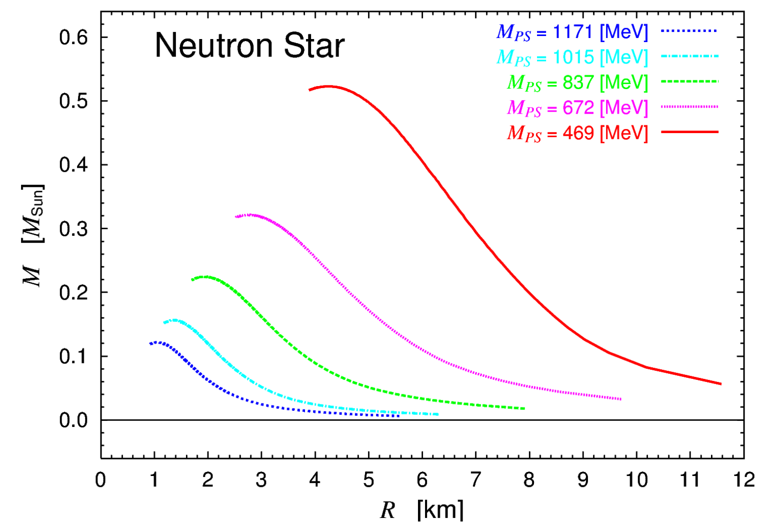
Symmetric nuclear



Pure neutron



Mass-radius curve



- Left: Nuclear matter EOS in the BHF with LQCD $V(r)$.
  - Include NN interaction in $^1S_0$, $^3S_1$, $^3D_1$ channels only.

- Right: Neutron Stars in LQCD with a hyperon free assumption.
  - Matter consists of n, p, e, μ in the chemical equilibrium.

Potential

S. Aoki, T. Hatsuda, N. Ishii, Prog. Theo. Phys. 123 89(2010)
N. Ishii etal. [HAL QCD coll.] in preparation

NBS wave function $\psi(\vec{r},t) = \phi_{Gr}(\vec{r})e^{-E_{Gr}t} + \phi_{1st}(\vec{r})e^{-E_{1st}t} \cdots$

DEFINE a "potential" through the "Schrödinger eq." for E-eigen-sates.

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{Gr}(\vec{r})e^{-E_{Gr}t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{Gr}(\vec{r}\,')e^{-E_{Gr}t} = E_{Gr}\phi_{Gr}(\vec{r})e^{-E_{Gr}t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\phi_{1st}(\vec{r})e^{-E_{1st}t} + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\phi_{1st}(\vec{r}\,')e^{-E_{1st}t} = E_{1st}\phi_{1st}(\vec{r})e^{-E_{1st}t}$$

Non-local but energy independent

By adding equations

$$\left[2M_B - \frac{\nabla^2}{2\mu}\right]\psi(\vec{r},t) + \int d^3\vec{r}\,' U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) = -\frac{\partial}{\partial t}\psi(\vec{r},t)$$

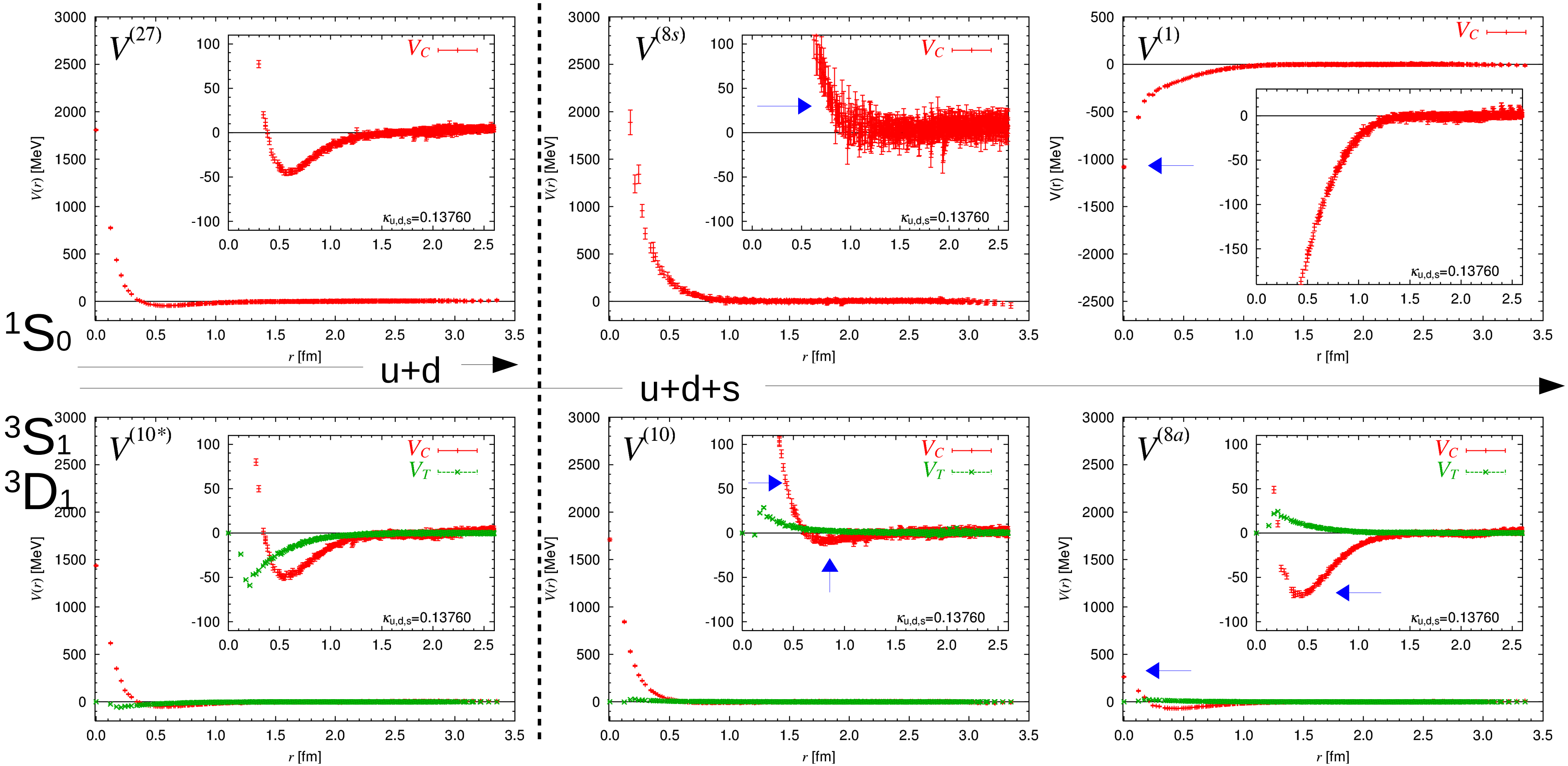
∇ expansion & truncation

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')[V(\vec{r}) + \cancel{\nabla + \nabla^2}\ldots]$$

Therefor, in the leading

$$\boxed{V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B}$$

BB int. in flavor basis



- At the SU(3)F limit corresponding to Mπ = MK = 837 [MeV].
- QM is true at small r. Especially, no repulsion in 1F channel.
- This indicate possibility of a bound H-dibaryon in the limit.

FAQ

1. Does your potential depend on the choice of source?

2. Does your potential depend on choice of operator at sink?

3. Does your potential $U(r,r')$ or $V(r)$ depend on energy?

FAQ

1. Does your potential depend on the choice of source?

➔ No. Some sources may enhance excited states in NBS w.f. However, the contamination is not problem in our new technique.

2. Does your potential depend on choice of operator at sink?

➔ Yes. It can be regarded as the “scheme” to define a potential. Note that a potential itself is not physical observable. We'll obtain unique result for observables irrespective to the choice, as long as the potential U(r,r') is deduced exactly.

3. Does your potential U(r,r') or V(r) depend on energy?

➔ By definition, U(r,r') is non-local but energy independent. While, determination and validity of its leading term V(r) obtained here, depend on energy because of the truncation. However, we know that the dependence in NN case is very small (thanks to our choice of sink operator = point) and negligible at least at $E_{lab.}$ = 0 – 90 MeV. We rely on this in this study. If we find some dependence, we'll determine the next leading term form it.

FAQ

4. Do you think energy dependence of $V^{(1)}(r)$ is also small?

5. Is the H a compact six-quark object or a tight BB bound state?

FAQ

4. Do you think energy dependence of $V^{(1)}(r)$ is also small?

➔ Yes. Because a large energy dependence means that

$$\left[2M_B - \frac{\nabla^2}{2\mu} + V_{\underline{Gr}}(\vec{r})\right]\phi_{Gr}(\vec{r})e^{-E_{Gr}t} = E_{Gr}\phi_{Gr}(\vec{r})e^{-E_{Gr}t}$$

$$\left[2M_B - \frac{\nabla^2}{2\mu} + V_{\underline{1\mathrm{st}}}(\vec{r})\right]\phi_{1\mathrm{st}}(\vec{r})e^{-E_{1\mathrm{st}}t} = E_{1\mathrm{st}}\phi_{1\mathrm{st}}(\vec{r})e^{-E_{1\mathrm{st}}t}$$

then $V(\vec{r}) \equiv \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$ would have a large t-dep.

5. Is the H a compact six-quark object or a tight BB bound state?

➔ Both. There is no distinct separation between two, because baryon is nothing but a 3-quark in QCD. Imagine a compact 6-quark object in $(0\mathrm{S})^6$ configuration. This configuration can be rewritten in a form of $(0\mathrm{S})^3 \times (0\mathrm{S})^3 \times \mathrm{Exp}(-a\, r^2)$ with relative coordinate r. This shows that a compact six-quark object can have a baryonic component, which we measure in the NBS w.f. We've established existence of a stable QCD eigenstate which couples to BB state. We do NOT insist that another “H” doesn't exist which cannot couple to BB.

FAQ

6. Do you insist such a deeply bound H exists in the real world?

7. What is the meaning of $\sqrt{\langle r^2 \rangle}$ of H?

FAQ

6. Do you insist such a deeply bound H exists in the real world?

- No. With SU(3)F breaking, three BB thresholds in S=−2,I=0 sector split as $E_{\Lambda\Lambda}^{\mathrm{Th}} < E_{N\Xi}^{\mathrm{Th}} < E_{\Sigma\Sigma}^{\mathrm{Th}}$. Therefore, we expect that the binding energy of H measured from $E_{\Lambda\Lambda}^{\mathrm{Th}}$ is much smaller than the present value, or even H is above $E_{\Lambda\Lambda}^{\mathrm{Th}}$ in the real world.

7. What is the meaning of $\sqrt{\langle r^2 \rangle}$ of H?

- It is a measure of spacial distribution of baryonic component in H. It corresponds to the "point matter root mean square distance" of deuteron (2 x 1.9 = 3.8 [fm]).