2+1 flavor QCD results of nuclear forces

Noriyoshi Ishii for HAL QCD Coll.

Background

- For many years,
we have studied nuclear forces and inter-baryon potentials
by using Nambu-Bethe-Salpeter (NBS) wave functions
based on the HAL QCD method.

- Last year,
we have developed an efficient method to obtain the HAL QCD potentials,
which does not require the ground state saturation of NBS wave functions.
("Time-dependent" Schrodinger-like equation)

- By using this new method,
we reanalyze 2+1 flavor lattice QCD results of NN potentials
with increasing statistics
and discuss the behaviors of NN phase shifts.

Lattice determination of nuclear potentials (HAL QCD method)

◆ Nambu-Bethe-Salpeter (NBS) wave function

$$\langle 0 | T[N(x)N(y)] | N(\vec{k})N(-\vec{k}), \mathrm{in}\rangle$$

a. It contains the information of S-matrix

$$S \equiv \langle N(\vec{p})N(-\vec{p}), \mathrm{out} | N(\vec{k})N(-\vec{k}), \mathrm{in}\rangle$$
$$= \mathrm{disc.} + i^2 \int d^4x_1 d^4x_2 e^{ip_1x_1}(\Box_1 + m^2) e^{ip_2x_2}(\Box_2 + m^2)$$
$$\times \langle 0 | T[N(x)N(y)] | N(\vec{k})N(-\vec{k}), \mathrm{in}\rangle$$

b. On the **equal-time** plane ($x_0 = y_0$), exactly the same behavior as scattering wave functions of QM.

$$\psi_{\vec{k}}(\vec{x}-\vec{y}) \equiv Z_N^{-1} \langle 0 | T[N(\vec{x}, x_0 = +0) N(\vec{y}, y_0 = -0)] | N(\vec{k})N(-\vec{k}), \mathrm{in}\rangle$$
$$\simeq e^{i\delta(k)} \frac{\sin(kr + \delta(k))}{kr} + \cdots \qquad (\text{for } |\vec{x}-\vec{y}| \to \infty)$$

[C.-J.D.Lin et al., NPB619(2001)467.]

c. Energy-independent potential U(x, x’) is defined from Schrodinger equation

$$(k^2 / m_N - H_0)\psi_{\vec{k}}(\vec{x}) = \int d^3x' U(\vec{x}, \vec{x}') \psi_{\vec{k}}(\vec{x}')$$

The resulting potential is faithful to the scattering phase $\delta(k)$ because of (b)

"Time-dependent" method (an efficient way to obtain HAL QCD potentials)

◆ **Normalized NN correlator (R-correlator)**

$$\Delta W(\vec{k}) \equiv 2\sqrt{m_N^2 + \vec{k}^2} - 2m_N$$

$$R(t,\vec{x}) \equiv e^{2m_N \cdot t} \langle 0 | T[N(\vec{x},t)N(\vec{y},t) \cdot \overline{\mathcal{J}}_{NN}(t=0)] | 0 \rangle$$

$$= \sum_{\vec{k}} a_{\vec{k}} \exp\left(-t\Delta W(\vec{k})\right) \psi_{\vec{k}}(\vec{x})$$

t has to be sufficiently large to suppress inelastic contribution ($E > 2m_N + m_{pion}$).

◆ **"Time-dependent" Schrodinger-like equation (derivation)**

$$\Delta W(\vec{k}) = \frac{\vec{k}^2}{m_N} - \frac{\Delta W(\vec{k})^2}{4m_N}$$

$$-\frac{\partial}{\partial t} R(t,\vec{x}) = \sum_{\vec{k}} a_{\vec{k}} \Delta W(\vec{k}) \exp\left(-t\Delta W(\vec{k})\right) \psi_{\vec{k}}(\vec{x})$$

$$= \sum_{\vec{k}} a_{\vec{k}} \left(\frac{\vec{k}^2}{m_N} - \frac{\Delta W(\vec{k})^2}{4m_N} \right) \exp\left(-t\Delta W(\vec{k})\right) \psi_{\vec{k}}(\vec{x})$$

$$= \sum_{\vec{k}} a_{\vec{k}} \left(H_0 + U - \frac{1}{4m_N} \frac{\partial^2}{\partial t^2} \right) \exp\left(-t\Delta W(\vec{k})\right) \psi_{\vec{k}}(\vec{x})$$

HAL QCD potential U satisfies

$$\left(H_0 + U\right) \psi_{\vec{k}}(\vec{x}) = \frac{\vec{k}^2}{m_N} \psi_{\vec{k}}(\vec{x})$$

⇨

$$\left(\frac{1}{4m_N} \frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right) R(t,\vec{x}) = \int d^3x' \, U(\vec{x},\vec{x}') R(t,\vec{x}')$$

The "time-dependent" Schrodinger-like equation enables us to obtain the nuclear potential without requiring the ground state saturation..

[N.Ishii et al.,PLB712(2012)437.]

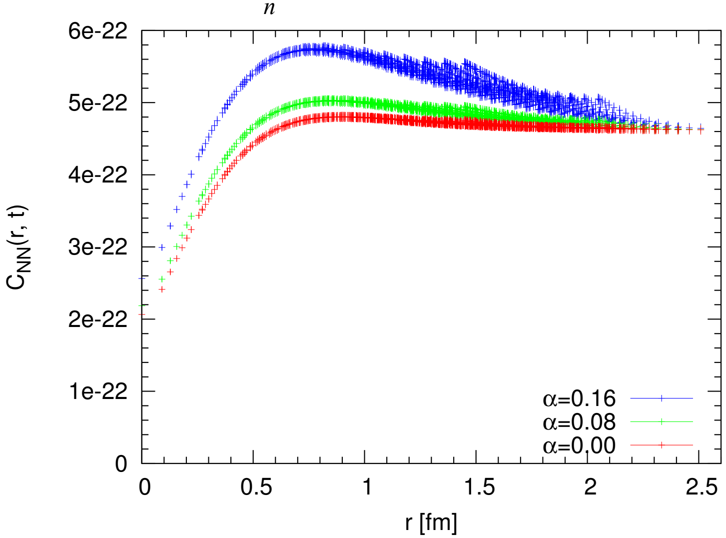
Ground state saturation is not needed. (an example)

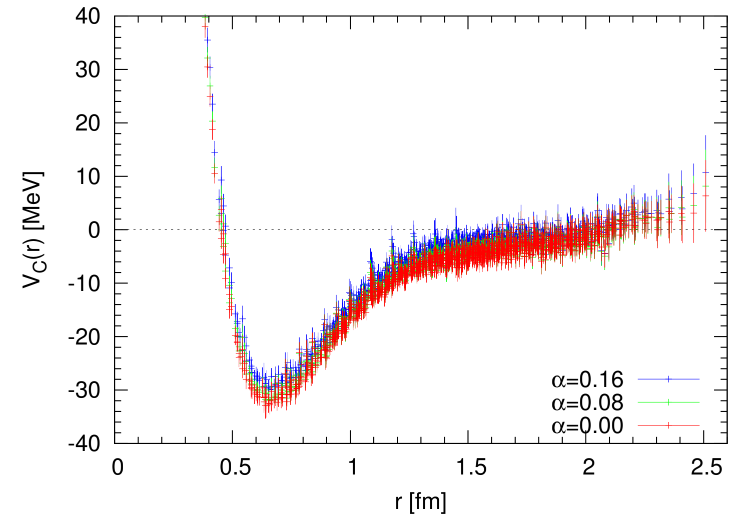
- Source function (with a single real parameter alpha)

$$f(x,y,z)=1+\alpha\left(\cos(2\pi x/L)+\cos(2\pi y/L)+\cos(2\pi z/L)\right)$$

- alpha is used change the mixtures of NBS wave function

$$C_{NN}(\vec{x}-\vec{y},t)\equiv\langle 0|T[N(\vec{x},t)N(\vec{y},t)\cdot\overline{\mathcal{J}}(t=0;\alpha)]|0\rangle$$
$$=\sum_n \psi_n(\vec{x}-\vec{y})\cdot a_n(\alpha)\cdot\exp(-E_n t)$$





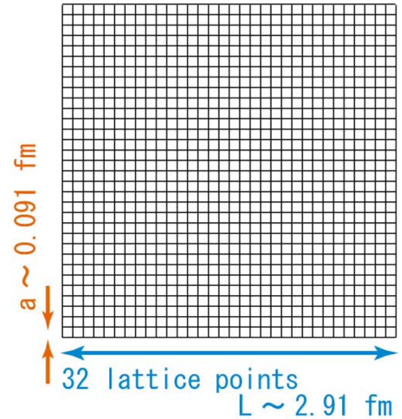
- "Time-dependent" Schrodinger-like equation leads to alpha-independent result.

$$\left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2}-\frac{\partial}{\partial t}-H_0\right)R(t,\vec{x})=\int d^3x'\,U(\vec{x},\vec{x}')R(t,\vec{x}')$$

2+1 flavor QCD results of nuclear forces by “time-depenent” method

2=1 flavor QCD results of nuclear forces

- lattice QCD setup
  - 2+1 flavor gauge configuration generated by PACS-CS Coll.
    - 32^3x64 lattice
    - Iwasage gauge action at beta=1.9
      → a=0.09 fm (L = 32a = 2.9 fm)
    - Nonperturbatively O(a) improved Wilson (clover) action
      with C_{SW} = 1.715
      - m_{pion} = 700 MeV
      - m_{pion} = 570 MeV
      - m_{pion} = 411 MeV
  - 4-point nucleon correlator
    for NBS wave functions and potentials
    - wall source
    - number of source points
      - m_{pion} = 700 MeV → 31 source points
      - m_{pion} = 570 MeV → 32 source points
      - m_{pion} = 411 MeV → 25 source points



PACS-CS

super computer T2K

ILDG

Japan Lattice Data Grid

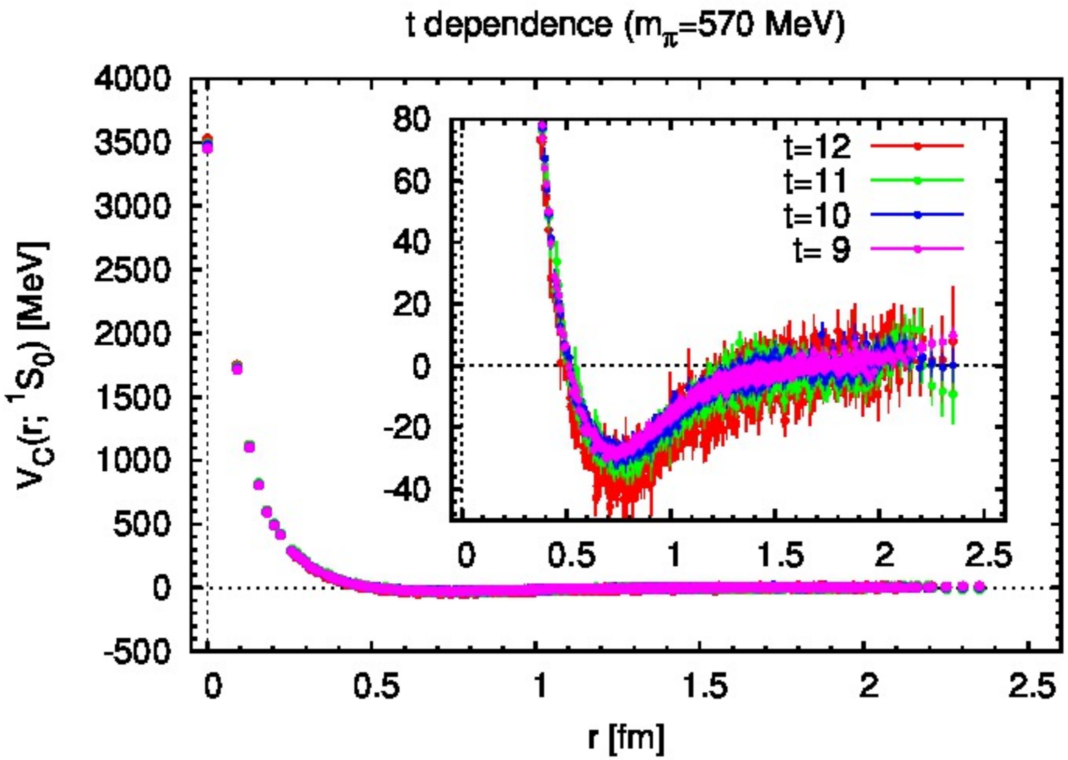
  - NN potentials are obtained at the leading order of derivative expansion:

$$U(\vec{x},\vec{x}') = V(\vec{x},\vec{\nabla})\delta(\vec{x}-\vec{x}')$$

$$V(\vec{x},\vec{\nabla}) \equiv V_C(\vec{x}) + V_T(\vec{x})S_{12} + O(\nabla)$$

Choise of t

$$V_C(\vec{r}) = -\frac{H_0 R(t,\vec{r})}{R(t,\vec{x})} - \frac{(\partial/\partial t) R(t,\vec{r})}{R(t,\vec{r})} + \frac{1}{4m_N}\frac{(\partial/\partial t)^2 R(t,\vec{r})}{R(t,\vec{r})}$$



Results at larger t lead to

- ❖ more reliable results
- ❖ larger statistical errors

➔

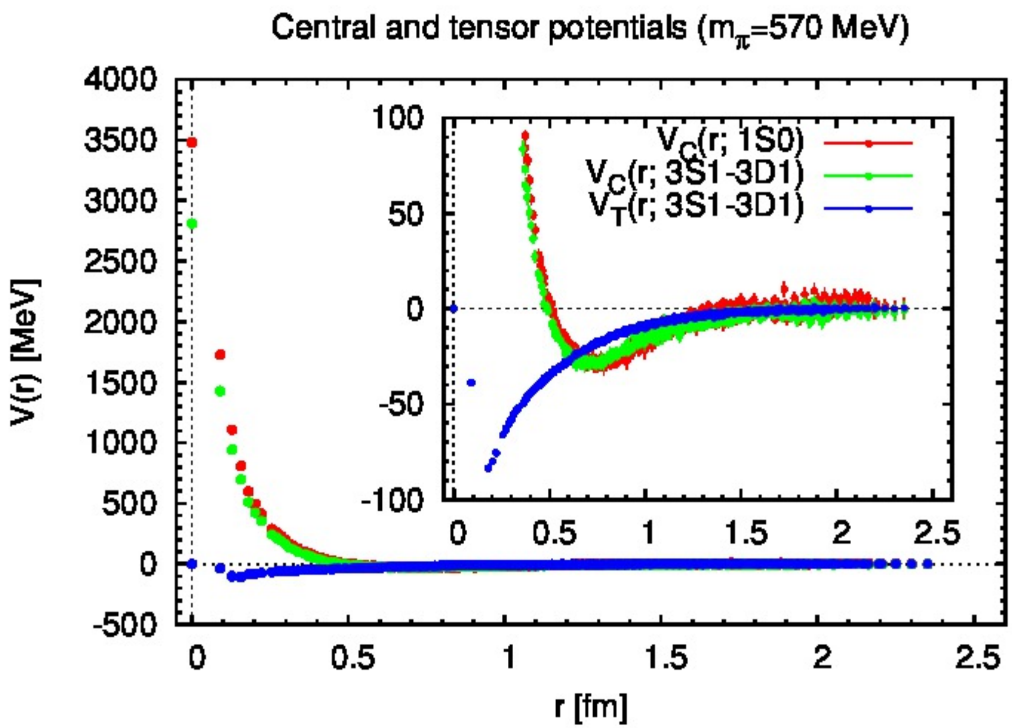
With this data, we choose t=10

Our choise of t in this talk:

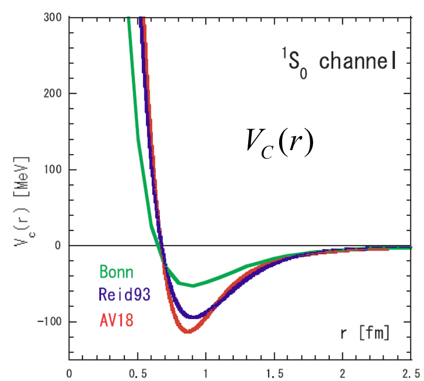
- ❖ m_{pion}=700: t = 10
- ❖ m_{pion}=570: t = 10
- ❖ m_{pion}=411: t = 8

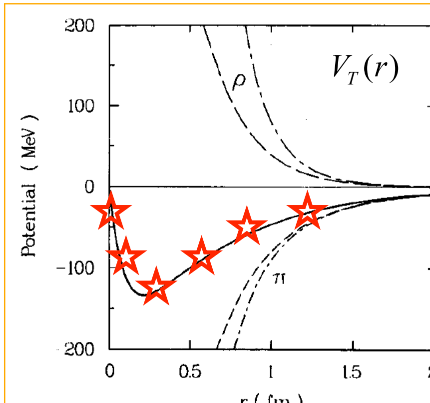
2+1 flavor QCD results of nuclear forces by “time-dependent” method

- Phenomenological properties of nuclear forces are reproduced



central force



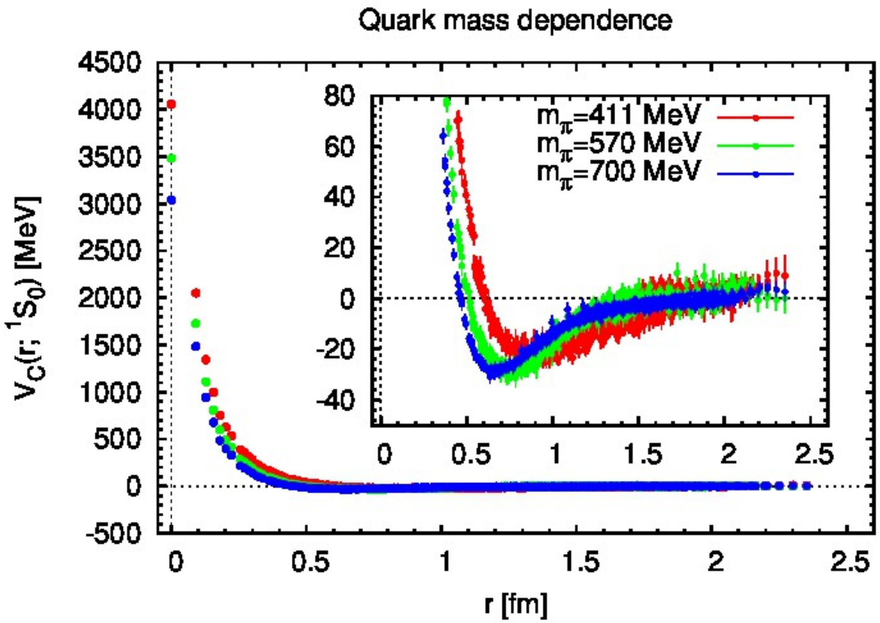


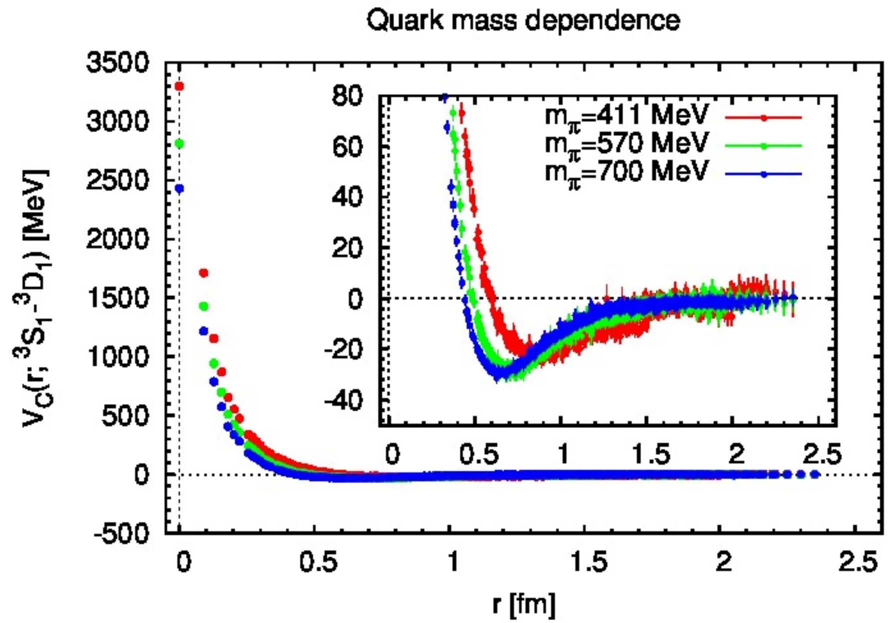
from R.Machleidt, Adv.Nucl.Phys.19

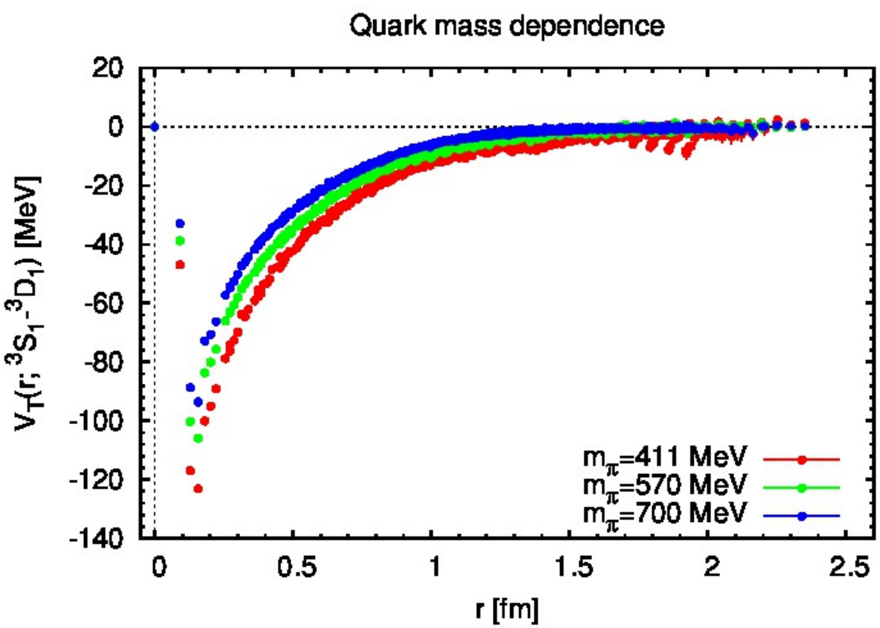
Fig. 3.7. The contributions from π and ρ (dashed) to the $T = 0$ tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

2+1 flavor QCD results of nuclear forces by "time-dependent" method

- quark mass dependence







With decreasing m_{pion},

- Repulsive core grows
- Attractive pocket grows
- Tensor force is enhanced

Fit of the potentials and Phase shift

Fit function

◆ **AV18-like fit function** (general form)

We do not use the constraints at the origin which are imposed on the fit parameters in the original AV18.

$$V_{NN}(r) \equiv v^{\pi}(r) + v^{R}(r)$$

$$v^{\pi}(r) \equiv f^2 \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2) \frac{m_\pi}{3} \left(Y(r;\, c) \cdot (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + T(r;\, c) \cdot S_{12} \right)$$

$$v_{ST}^{R}(r) \equiv v_{ST}^{c}(r) + v_{ST}^{t}(r) S_{12} + \cdots$$

$$v_{ST}^{i}(r) \equiv I_{TS}^{i} \cdot T^2(r;\, c) + \left(P_{TS}^{i} + (m_\pi r) Q_{TS}^{i} + (m_\pi r)^2 R_{TS}^{i} \right) W(r;\, r_0, a)$$

Values of m_π are fixed and are taken from PACS-CS Coll.,PRD79,034503('09)

$$Y(r;\, c) \equiv \frac{e^{-m_\pi r}}{m_\pi r} \left(1 - \exp(-c r^2) \right) \qquad \text{[Yukawa function]}$$

$$T(r;\, c) \equiv \left(1 + \frac{3}{m_\pi r} + \frac{3}{(m_\pi r)^2} \right) \frac{e^{-m_\pi r}}{m_\pi r} \left(1 - \exp(-c r^2) \right)^2 \qquad \text{[Tensor function]}$$

$$W(r;\, r_0, a) \equiv \left[1 + \exp\left(\frac{r - r_0}{a} \right) \right]^{-1} \qquad \text{[Woods-Saxon function]}$$

◆ ➔Simultaneous fit of two $V_C(r)$ and one $V_T(r)$ with **16 adjustable parameters**

- ☐ **Central potential (1S0)**

$$V_C(r;\, {}^1S_0) = -f^2 m_\pi Y(r;\, c) + I_{10}^{c} \cdot T^2(r;\, c) + \left(P_{10}^{c} + Q_{10}^{c} \cdot (m_\pi r) + R_{10}^{c} \cdot (m_\pi r)^2 \right) W(r;\, r_0, a)$$

- ☐ **Central potential (3S1-3D1)**

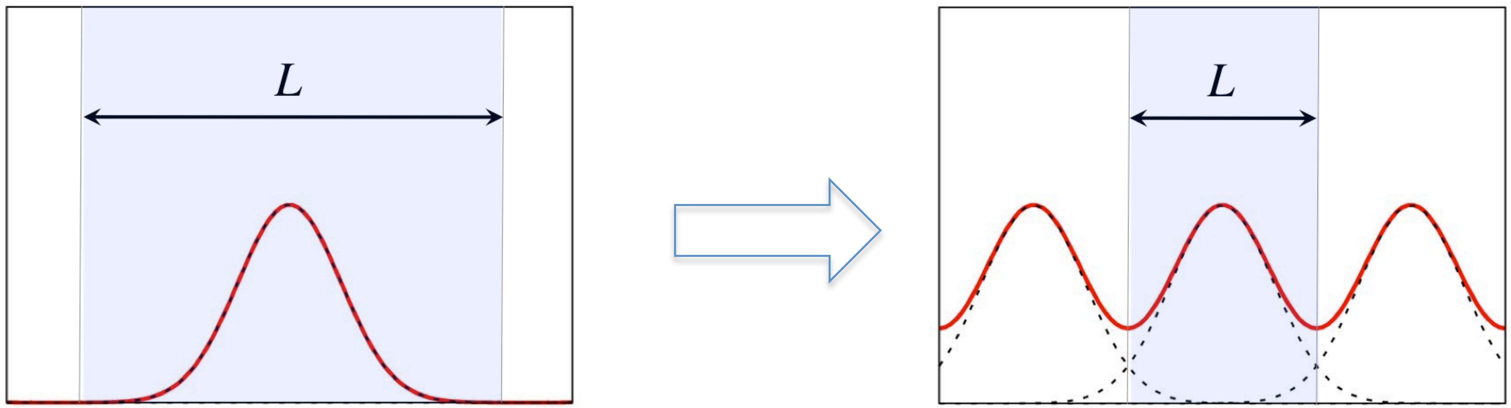
$$V_C(r;\, {}^3S_1 - {}^3D_1) = -f^2 m_\pi Y(r;\, c) + I_{01}^{c} \cdot T^2(r;\, c) + \left(P_{01}^{c} + Q_{01}^{c} \cdot (m_\pi r) + R_{01}^{c} \cdot (m_\pi r)^2 \right) W(r;\, r_0, a)$$

- ☐ **Tensor potential (3S1-3D1)**

$$V_T(r;\, {}^3S_1 - {}^3D_1) = -f^2 m_\pi T(r;\, c) + I_{01}^{t} \cdot T^2(r;\, c) + \left(P_{01}^{t} + Q_{01}^{t} \cdot (m_\pi r) + R_{01}^{t} \cdot (m_\pi r)^2 \right) W(r;\, r_0, a)$$

Fit function

- We attempt to take into account boundary effect

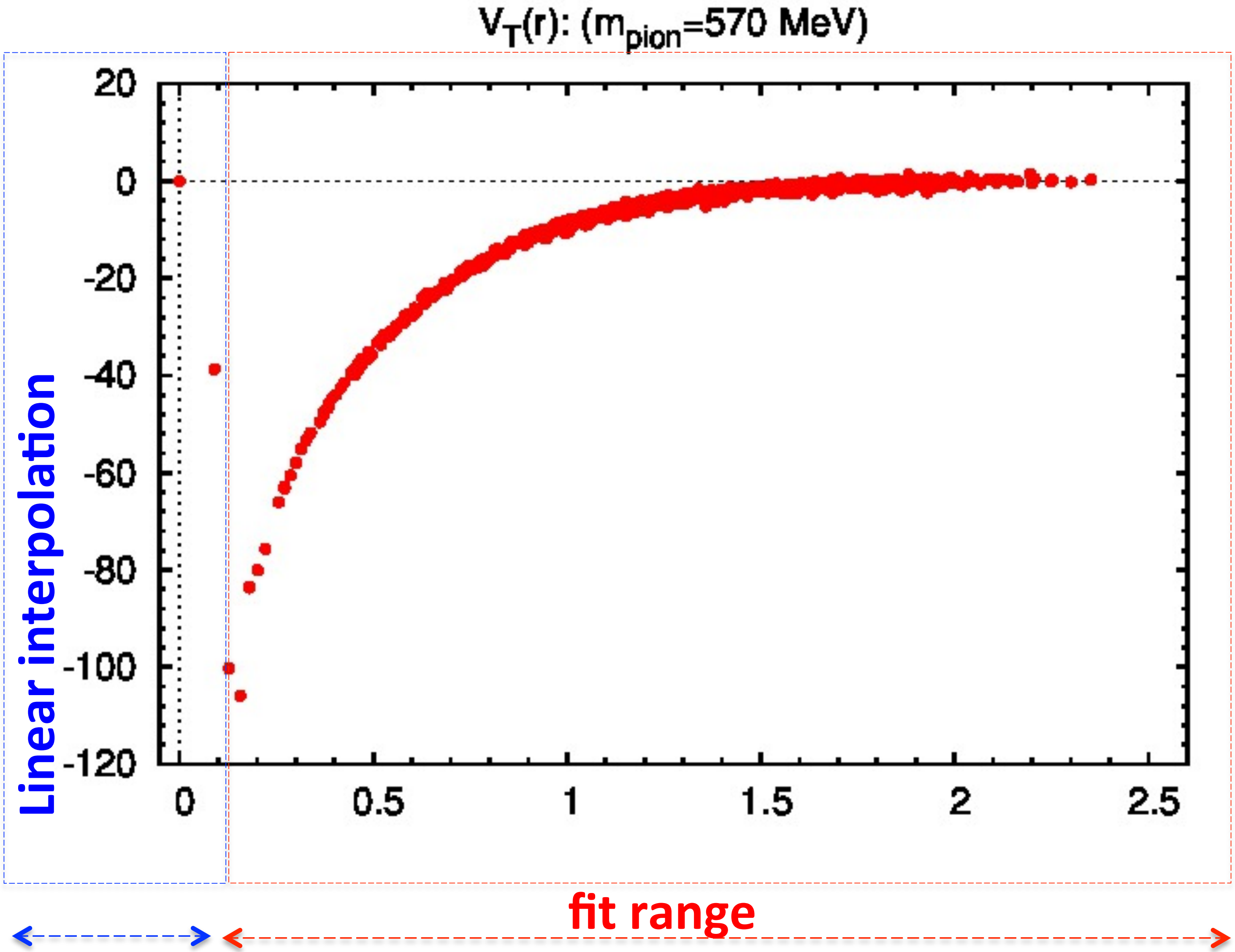


Receiving contributions from periodic images,
the original potential is modified as

$$V(\vec{r}) \Rightarrow \tilde{V}(\vec{r}) = \sum_{\vec{n}\in\mathbb{Z}^3} V(\vec{r}+L\vec{n})$$

Fitting region for the tensor potential

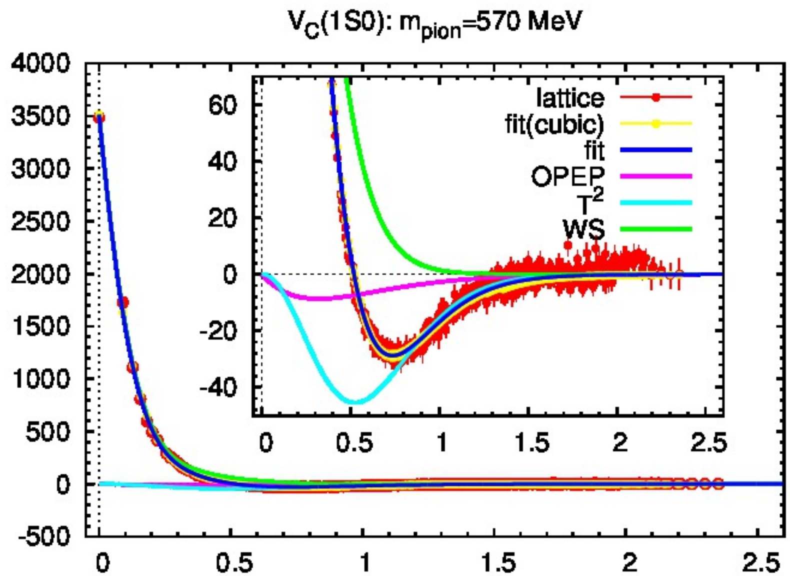
- Our tensor potential has a cusp around r = 0.12 fm, where a fit with a smooth function becomes difficult.

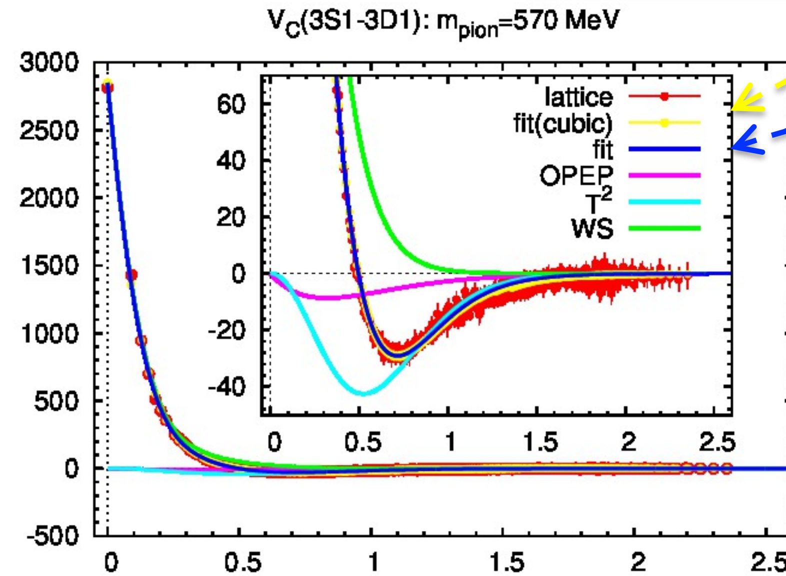


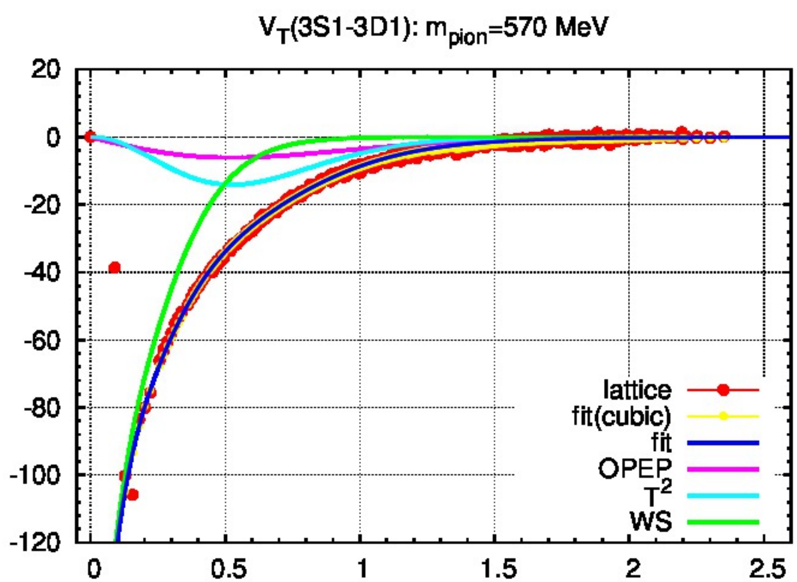
Fit (Results)

$$\tilde{V}(\vec{r}) = \sum_{\vec{n}\in\mathbb{Z}^3} V(\vec{r}+L\vec{n})$$

$V(\vec{r})$







$$V_{\rm C}(r;\ {}^1S_0) = -f^2 m_\pi Y(r;\ c) + I^c_{10}\cdot T^2(r;\ c) + \left(P^c_{10} + Q^c_{10}\cdot(m_\pi r) + R^c_{10}\cdot(m_\pi r)^2\right) W(r;\ r_0, a)$$

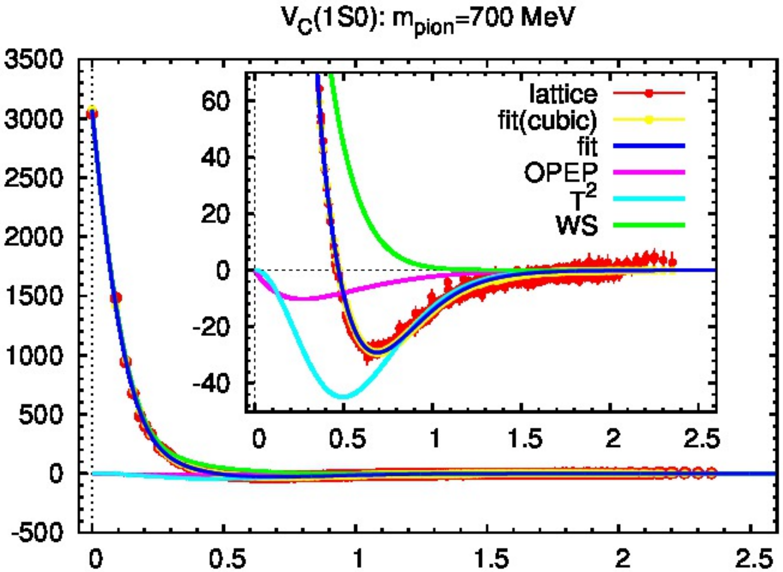
$$V_{\rm C}(r;\ {}^3S_1 - {}^3D_1) = -f^2 m_\pi Y(r;\ c) + I^c_{01}\cdot T^2(r;\ c) + \left(P^c_{01} + Q^c_{01}\cdot(m_\pi r) + R^c_{01}\cdot(m_\pi r)^2\right) W(r;\ r_0, a)$$

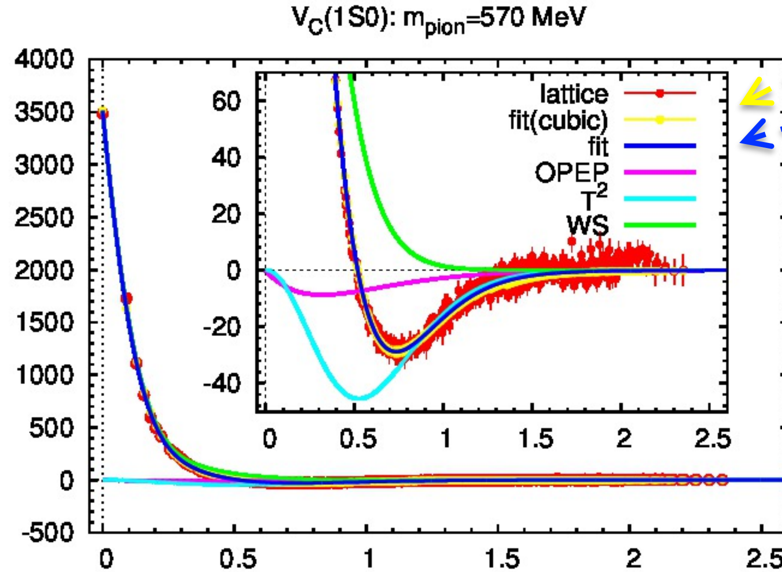
$$V_{\rm T}(r;\ {}^3S_1 - {}^3D_1) = -f^2 m_\pi T(r;\ c) + I^t_{01}\cdot T^2(r;\ c) + \left(P^t_{01} + Q^t_{01}\cdot(m_\pi r) + R^t_{01}\cdot(m_\pi r)^2\right) W(r;\ r_0, a)$$

These fit functions nicely parameterize the lattice data.

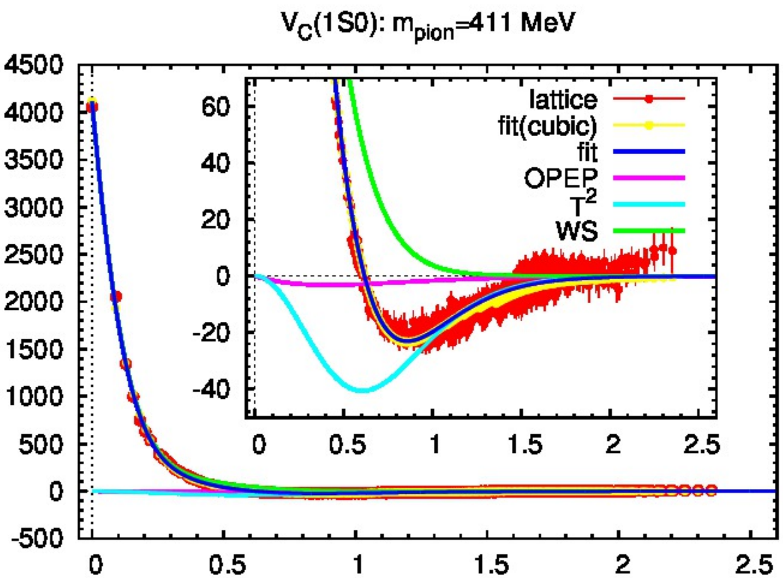
Fit(comment on the quark mass and the spatial volume)

$$\tilde{V}(\vec{r}) = \sum_{\vec{n}\in\mathbb{Z}^3} V(\vec{r}+L\vec{n})$$



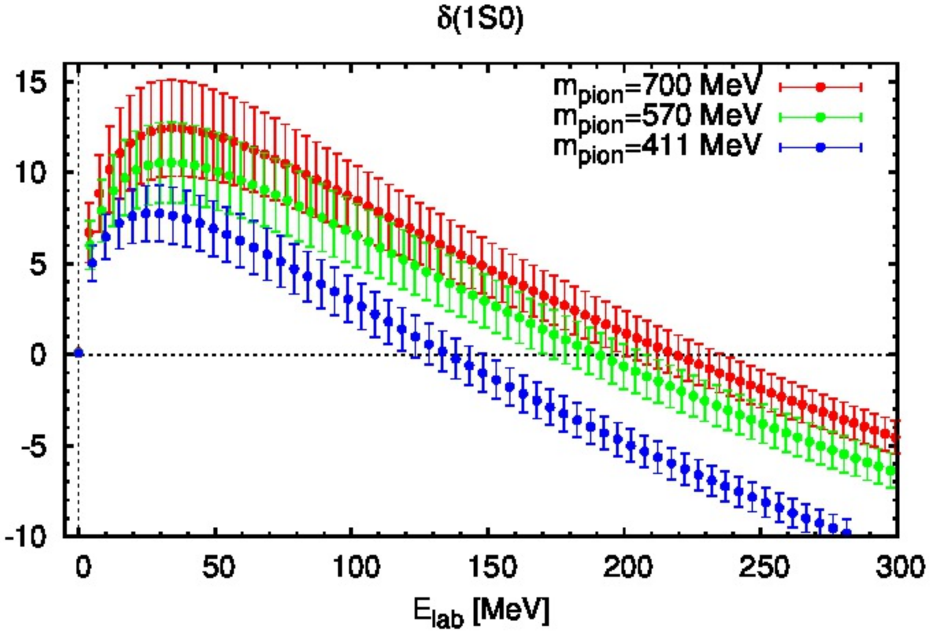


$V(\vec{r})$

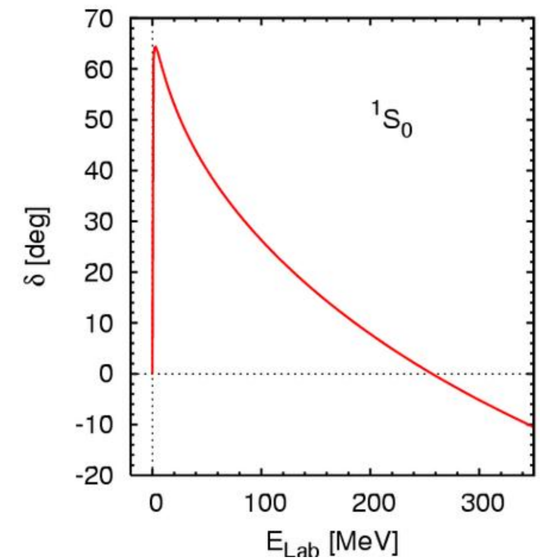


- The same fit functions work for other pion mass.
- Boundary effect becomes important for m_{pion} = 411 MeV. (See deviation between of blue from yellow)
- For calculation with m_{pion} < 411 MeV, Larger spatial volume (L > 3fm) should be used.

Scattering phase ($^1S_0$)



experiment

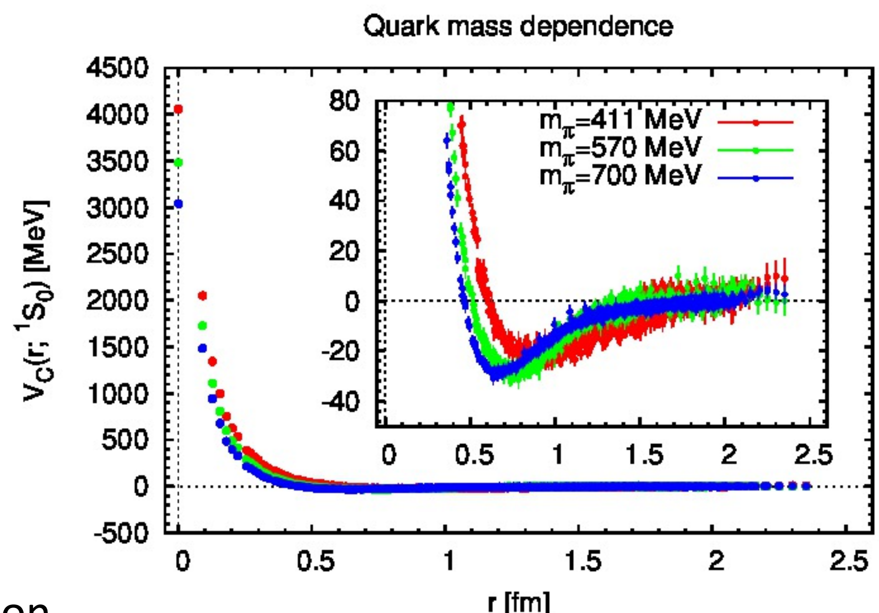


- ❖ Qualitatively reasonable behavior.
  But the strength is significantly weak.

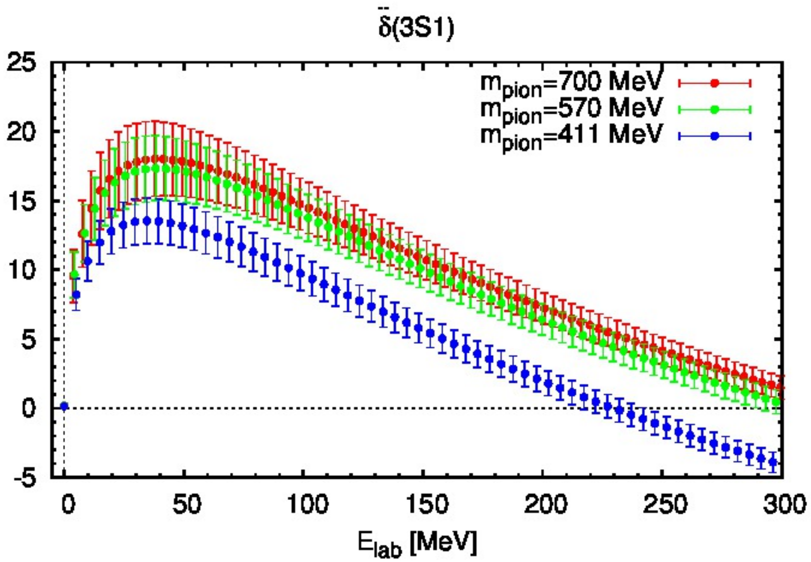
- ❖ Attraction shrinks gradually as m_{pion} decreases in this quark mass region m_{pion} = 411-700 MeV.

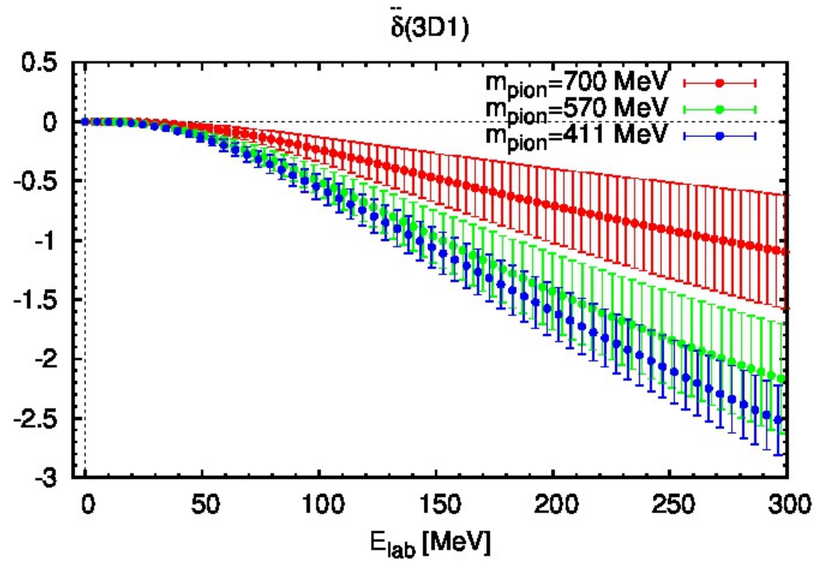
- ❖ The repulsive core grows more rapidly than the attraction grows.

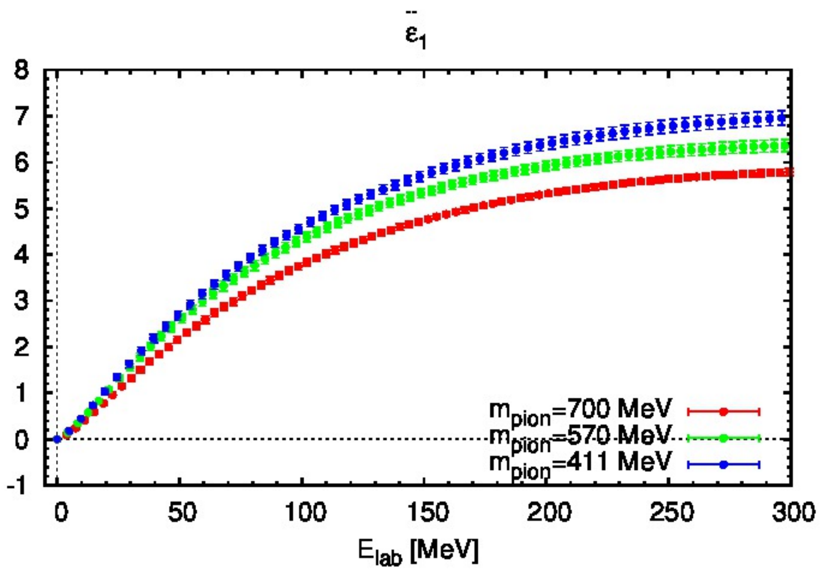
- ❖ It is important to go to smaller quark mass region.



Phase shifts and mixing parameter ($^3S_1$-$^3D_1$)







- Similar tendency as 1S0 is found.
- It is important to go to small quark mass region.

Stapp's convension is employed for the scattering phases and mixing parameter.

Summary

- We have applied the "time-dependent" Schrodinger-like equation to the 2+1 flavor QCD results of nuclear potentials for m_{pion} = 411, 570, 700 MeV.

$$\left(\frac{1}{4m_N}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} - H_0 \right) R(t,\vec{x}) = \int d^3x' U(\vec{x},\vec{x}') R(t,\vec{x}')$$

  The equation allows us to obtain the nuclear potentials without requiring the ground state saturation.

- The resulting potentials are parameterized by "AV18-like" fit functions, Smooth parameterization of the lattice data is obtained.

- By solving Schrodinger equation, we have obtained the scattering phases.
  - Behaviors are qualitatively reasonable. But the strength is not sufficient.
  - As m_{pion} decreases, the attraction shrinks.
    - Repulsive core grows more rapid than the attractive pocket grows.
    - It is important to go to much lighter quark mass region.

Backup slides

Existence of energy-independent interaction kernel

- We assume linear independence of NBS wave function below pion threshold.
  ➔ There is a dual basis

$$E \equiv 2\sqrt{m_N^2 + \vec{k}^2} < 2m_N + m_\pi$$

$$\int d^3r\, \widetilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r}) = (2\pi)^3\delta^3(\vec{k}' - \vec{k})$$

- We have

$$K_{\vec{k}}(\vec{r}) \equiv \left(\Delta + k^2\right)\psi_{\vec{k}}(\vec{r})$$

$$= \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r}) \int d^3r'\, \widetilde{\psi}_{\vec{k}'}(\vec{r})\psi_{\vec{k}}(\vec{r})$$

$$= \int d^3r' \left\{ \int \frac{d^3k}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\widetilde{\psi}_{\vec{k}'}(\vec{r}') \right\} \psi_{\vec{k}}(\vec{r}')$$

If we define

$$U(\vec{r},\vec{r}') \equiv \frac{1}{m_N} \int \frac{d^3k'}{(2\pi)^3} K_{\vec{k}'}(\vec{r})\widetilde{\psi}_{\vec{k}'}(\vec{r})$$

then we have

$$\frac{1}{m_N}\left(\Delta + k^2\right)\psi_{\vec{k}}(\vec{r}) = \int d^3r'\, U(\vec{r},\vec{r}')\psi_{\vec{k}}(\vec{r})$$

for $E \equiv 2\sqrt{m_N^2 + \vec{k}^2} < 2m_N + m_\pi$