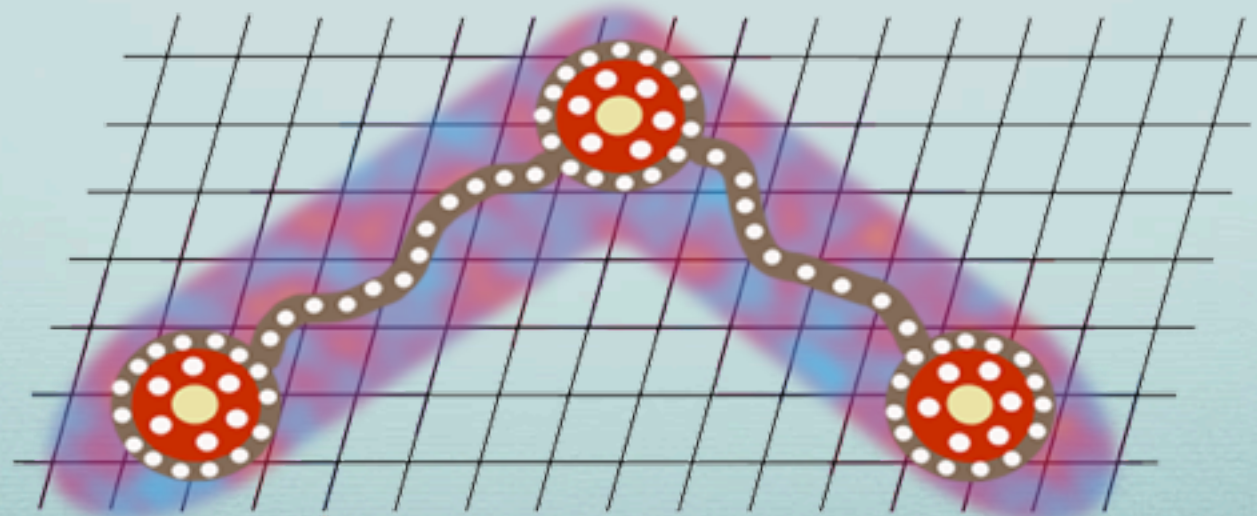




MULTI-BARYON SYSTEMS

Kostas Orginos
Will Detmold

College of William & Mary -- JLab

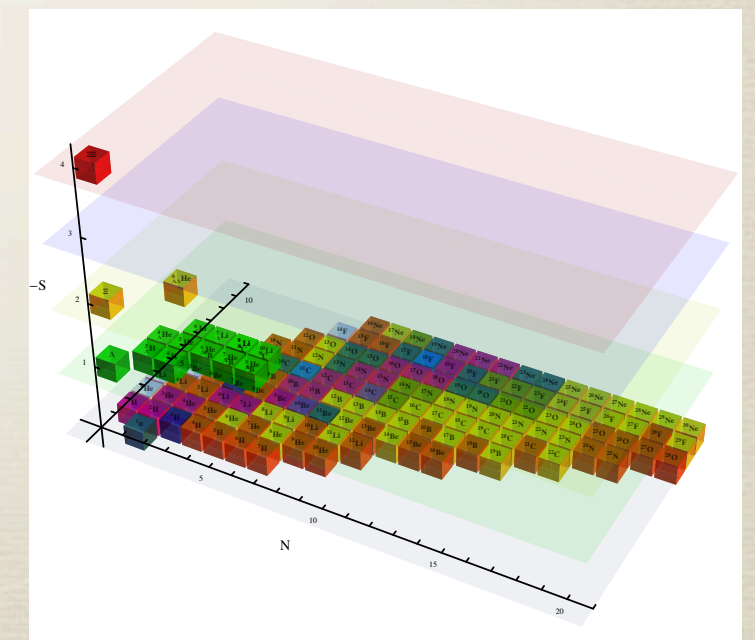


Lattice 2012
Cairns, Australia

Hadron Interactions

Goals:

- * Challenge: Compute properties of nuclei from QCD
- * Spectrum and structure
- * Confirm well known experimental observations for two nucleon systems
- * Explore the largely unknown territory of hyper-nuclear physics
- * Provide input for the equation of state for nuclear matter in neutron stars
- * Provide input for understanding the properties of multi-baryon systems



More than two body

- * Construct interpolating fields
- * Find efficient ways to perform the Wick contractions
- * Address the signal to noise problem
- * Extract the energy spectrum
- * Interpret the energy spectrum

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Interpolating fields

Most general multi-baryon interpolating field

$$\bar{\mathcal{N}}^h = \sum_{\mathbf{a}} w_h^{a_1, a_2 \cdots a_{n_q}} \bar{q}(a_1) \bar{q}(a_2) \cdots \bar{q}(a_{n_q})$$

The indices $\mathcal{a}$ are composite including space, spin, color and flavor

- * The goal is to calculate the tensors $\mathcal{w}$
- * The tensors $\mathcal{w}$ are completely antisymmetric
- * Number of terms in the sum are

$$\frac{N!}{(N - n_q)!}$$

Imposing the anti-symmetry:

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1, a_2 \cdots a_{n_q}), k} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, \cdots, i_{n_q}} \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \cdots \bar{q}(a_{i_{n_q}})$$

Reduced weights

Totally anti-symmetric tensor

$$\epsilon^{1,2,3,4,\cdots,n_q} = 1$$

* Total number of reduced weights:

$$\frac{N!}{n_q!(N - n_q)!}$$

Hadronic Interpolating field

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{M_w} \tilde{W}_h^{(b_1, b_2 \cdots b_A)} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, \cdots, i_A} \bar{B}(b_{i_1}) \bar{B}(b_{i_2}) \cdots \bar{B}(b_{i_A})$$

hadronic reduced weights

baryon composite interpolating field

$$\bar{B}(b) = \sum_{k=1}^{N_{B(b)}} \tilde{w}_b^{(a_1, a_2, a_3), k} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, i_3} \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \bar{q}(a_{i_3})$$

Basak et.al. PhysRevD.72.074501 (2005)

Calculation of weights

- * Compute the hadronic weights
- * Replace baryons by quark interpolating fields
- * Perform Grassmann reductions
- * Read off the reduced weights for the quark interpolating fields
- * Computations done in: **algebra (C++)**

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{M_w} \tilde{W}_h^{(b_1, b_2 \cdots b_A)} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, \cdots, i_A} \bar{B}(b_{i_1}) \bar{B}(b_{i_2}) \cdots \bar{B}(b_{i_A})$$



$$\bar{\mathcal{N}}^h = \sum_{k=1}^{N_w} \tilde{w}_h^{(a_1, a_2 \cdots a_{n_q}), k} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, \cdots, i_{n_q}} \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \cdots \bar{q}(a_{i_{n_q}})$$

Interpolating fields

NPLQCD arXiv:1206.5219

Label	A	s	I	J^π	Local SU(3) irreps	int. field size
N	1	0	1/2	$1/2^+$	8	9
Λ	1	-1	0	$1/2^+$	8	12
Σ	1	-1	1	$1/2^+$	8	9
Ξ	1	-2	1/2	$1/2^+$	8	9
d	2	0	0	1^+	$\overline{10}$	21
nn	2	0	1	0^+	27	21
$n\Lambda$	2	-1	1/2	0^+	27	96
$n\Lambda$	2	-1	1/2	1^+	$8_A, \overline{10}$	48, 75
$n\Sigma$	2	-1	3/2	0^+	27	42
$n\Sigma$	2	-1	3/2	1^+	10	27
$n\Xi$	2	-2	0	1^+	8_A	96
$n\Xi$	2	-2	1	1^+	$8_A, 10, \overline{10}$	52, 66, 75
H	2	-2	0	0^+	1, 27	90, 132
${}^3\text{H}, {}^3\text{He}$	3	0	1/2	$1/2^+$	$\overline{35}$	9
${}^3_\Lambda\text{H}(1/2^+)$	3	-1	0	$1/2^+$	$\overline{35}$	66
${}^3_\Lambda\text{H}(3/2^+)$	3	-1	0	$3/2^+$	$\overline{10}$	30
${}^3_\Lambda\text{He}, {}^3_\Lambda\tilde{\text{H}}, nn\Lambda$	3	-1	1	$1/2^+$	27, $\overline{35}$	30, 45
${}^3_\Sigma\text{He}$	3	-1	1	$3/2^+$	27	21
${}^4\text{He}$	4	0	0	0^+	$\overline{28}$	1
${}^4_\Lambda\text{He}, {}^4_\Lambda\text{H}$	4	-1	1/2	0^+	$\overline{28}$	6
${}^4_{\Lambda\Lambda}\text{He}$	4	-2	1	0^+	27, $\overline{28}$	15, 18
$\Lambda\Xi^0 pnn$	5	-3	0	$3/2^+$	$\overline{10} + \dots$	1

Interpolating fields

NPLQCD arXiv:1206.5219

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${}^4_{\Lambda\Lambda}\text{He}$	4	-2	1	0^+	27, $\overline{28}$	15, 18
$\Lambda\Xi^0 pnn$	5	-3	0	$3/2^+$	$\overline{10} + \dots$	1

Hadronic interpolating field

$$\bar{\mathcal{N}}^h = \sum_{k=1}^{M_w} \tilde{W}_h^{(b_1, b_2 \dots b_A)} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, \dots, i_A} \bar{B}(b_{i_1}) \bar{B}(b_{i_2}) \dots \bar{B}(b_{i_A})$$

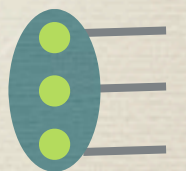
Quark interpolating field

$$\bar{\mathcal{N}}^h = \sum_{\mathbf{a}} w_h^{a_1, a_2 \dots a_{n_q}} \bar{q}(a_1) \bar{q}(a_2) \dots \bar{q}(a_{n_q})$$

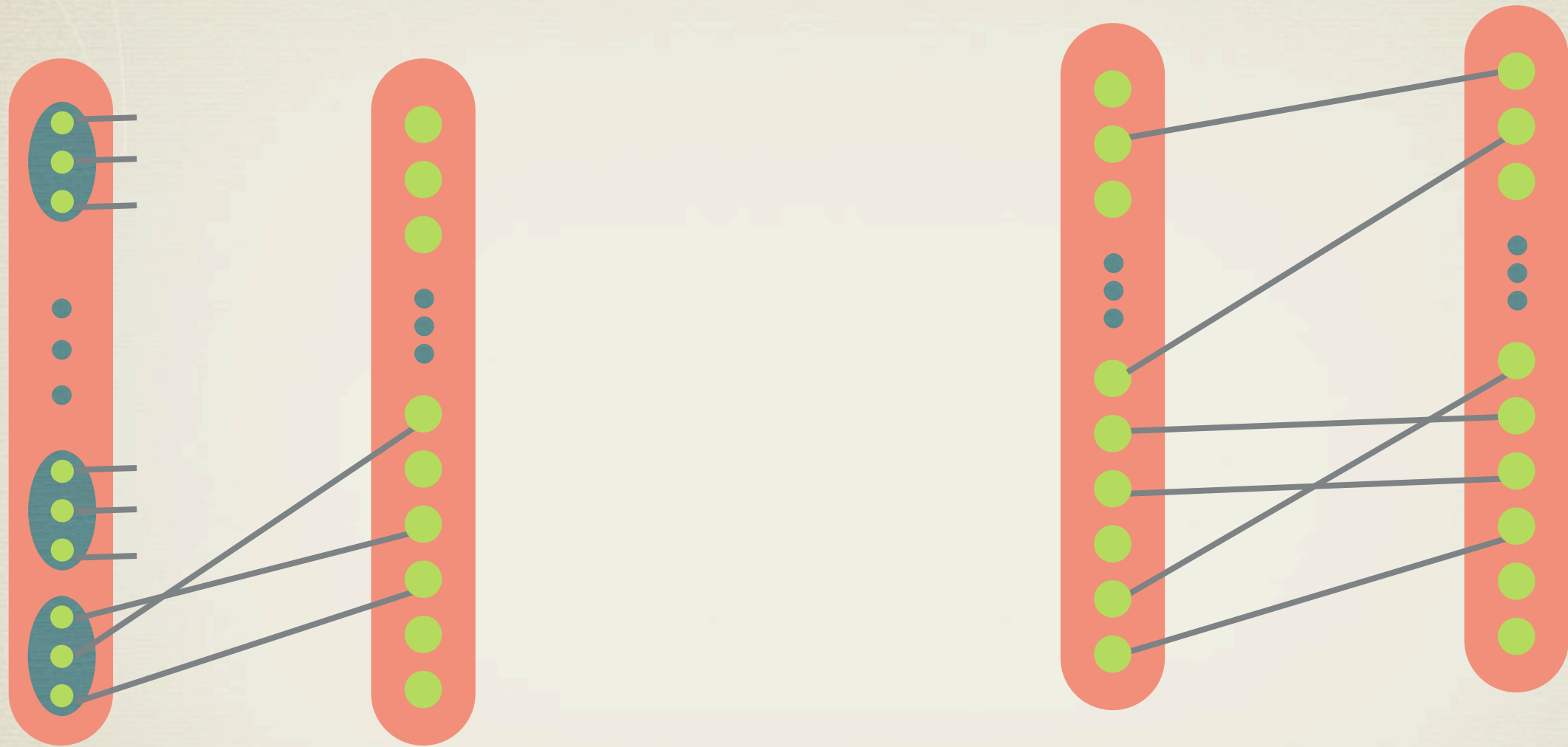
Baryon Block

$$\mathcal{B}^{a_1, a_2, a_3}(b, \mathbf{p}; x_0) = \sum_{\mathbf{x}} e^{i\mathbf{p} \cdot \mathbf{x}} \sum_{k=1}^{N_B(b)} \tilde{w}_b^{(c_1, c_2, c_3), k} \sum_{\mathbf{i}} \epsilon^{i_1, i_2, i_3} S(c_{i_1}, \mathbf{x}; a_1, x_0) S(c_{i_2}, \mathbf{x}; a_2, x_0) S(c_{i_3}, \mathbf{x}; a_3, x_0)$$

Quark propagator

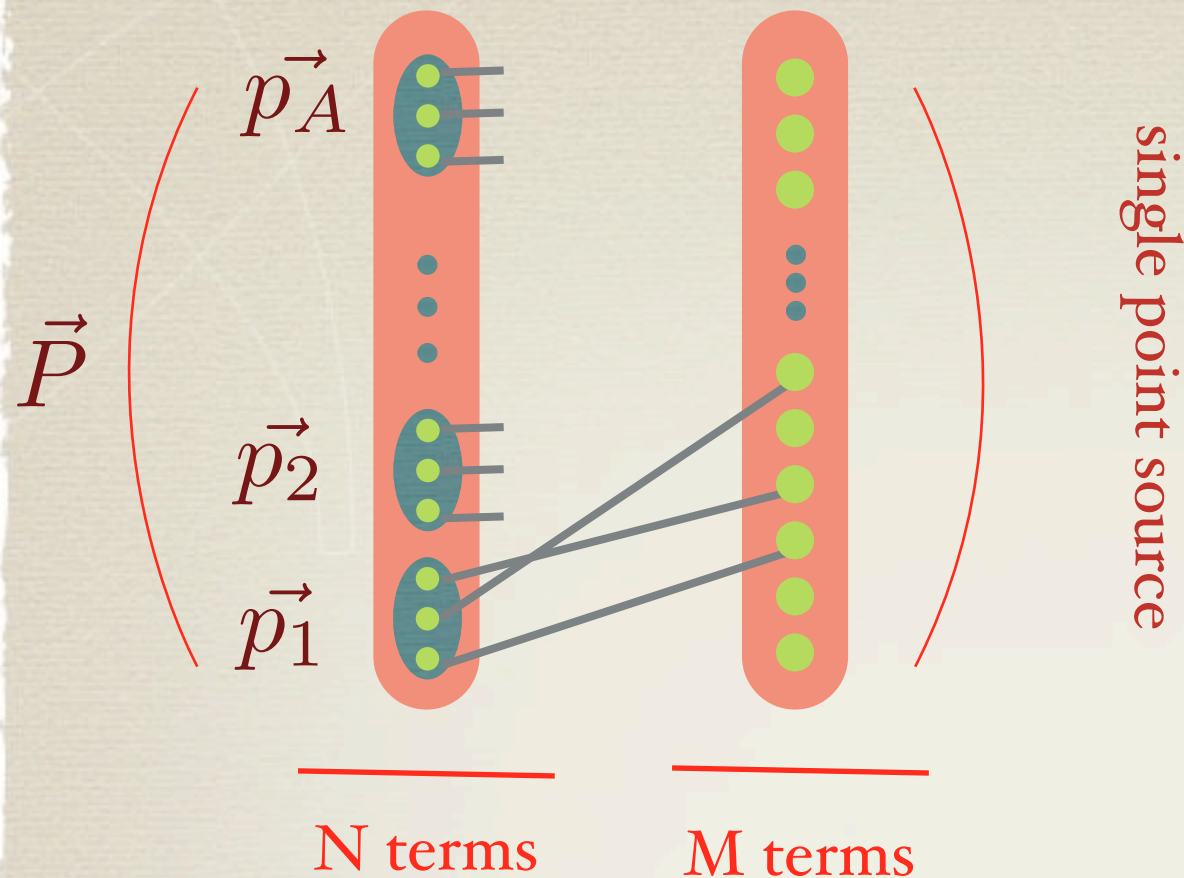


Contraction methods



- * quark to hadronic interpolating fields
- * quark to quark interpolating fields

Quarks to Hadrons



Cost: $M \cdot N \frac{n_u!n_d!n_s!}{2^{(A-n_{\Sigma^0}-n_{\Lambda})}}$

- * Loop over all source and sink terms
- * Connect each baryon in all possible ways to the source quarks (selecting the indices from the block)
- * ^4He cost: 0.8 s per time slice on a single core of a Dual Core AMD Opteron 285 processor

Quarks to Quarks

$$\begin{aligned}
 [\mathcal{N}_1^h(t) \bar{\mathcal{N}}_2^h(0)] &= \int \mathcal{D}q \mathcal{D}\bar{q} e^{-\mathcal{S}_{QCD}} \sum_{k'=1}^{N'_w} \sum_{k=1}^{N_w} \tilde{w}_h^{(a'_1, a'_2 \dots a'_{n_q}), k'} \tilde{w}_h^{(a_1, a_2 \dots a_{n_q}), k} \times \\
 &\times \sum_{\mathbf{j}} \sum_{\mathbf{i}} \epsilon^{j_1, j_2, \dots, j_{n_q}} \epsilon^{i_1, i_2, \dots, i_{n_q}} q(a'_{j_{n_q}}) \dots q(a'_{j_2}) q(a'_{j_1}) \times \bar{q}(a_{i_1}) \bar{q}(a_{i_2}) \dots \bar{q}(a_{i_{n_q}}) \\
 &= e^{-\mathcal{S}_{eff}} \sum_{k'=1}^{N'_w} \sum_{k=1}^{N_w} \tilde{w}_h^{(a'_1, a'_2 \dots a'_{n_q}), k'} \tilde{w}_h^{(a_1, a_2 \dots a_{n_q}), k} \times \\
 &\times \sum_{\mathbf{j}} \sum_{\mathbf{i}} \epsilon^{j_1, j_2, \dots, j_{n_q}} \epsilon^{i_1, i_2, \dots, i_{n_q}} S(a'_{j_1}; a_{i_1}) S(a'_{j_2}; a_{i_2}) \dots S(a'_{j_{n_q}}; a_{i_{n_q}})
 \end{aligned}$$

Define the matrix:

$$G(j, i)^{(a'_1, a'_2 \dots a'_{n_q}); (a_1, a_2 \dots a_{n_q})} = S(a'_j; a_i)$$

Quarks to Quarks

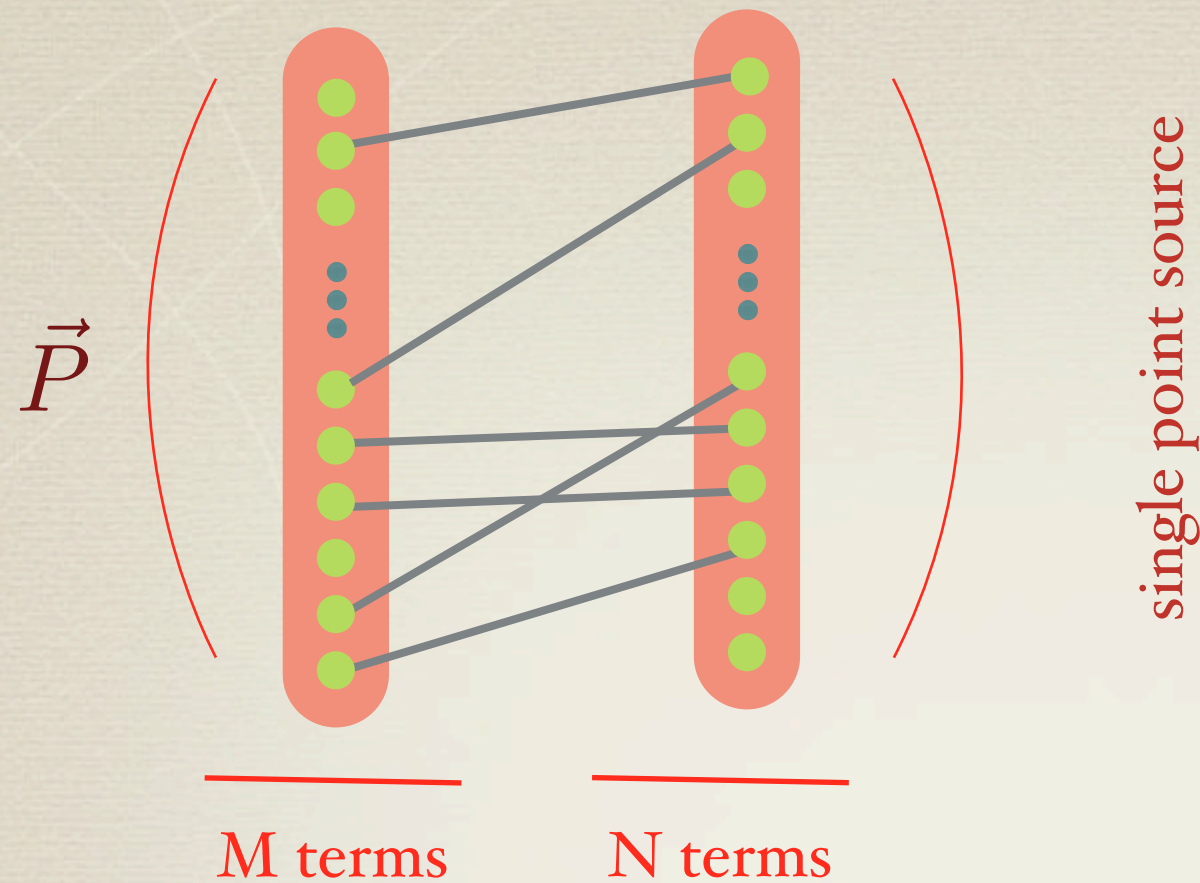
The matrix:

$$G(j, i)^{(a'_1, a'_2 \cdots a'_{n_q}); (a_1, a_2 \cdots a_{n_q})} = S(a'_j; a_i)$$

The Correlation function:

$$[\mathcal{N}_1^h(t) \bar{\mathcal{N}}_2^h(0)] = \sum_{k'=1}^{N'_w} \sum_{k=1}^{N_w} \tilde{w}_h^{(a'_1, a'_2 \cdots a'_{n_q}), k'} \tilde{w}_h^{(a_1, a_2 \cdots a_{n_q}), k} \times \left| G^{(a'_1, a'_2 \cdots a'_{n_q}); (a_1, a_2 \cdots a_{n_q})} \right|$$

Total momentum projection is implicit in the above



Quarks to Quarks

Naive Cost: $n_u!n_d!n_s! \times NM$

Actual Cost: $n_u^3n_d^3n_s^3 \times MN$

- * Loop over all source and sink terms
- * Compute the determinant for each flavor
- * Cost is polynomial in quark number

Lattice Setup

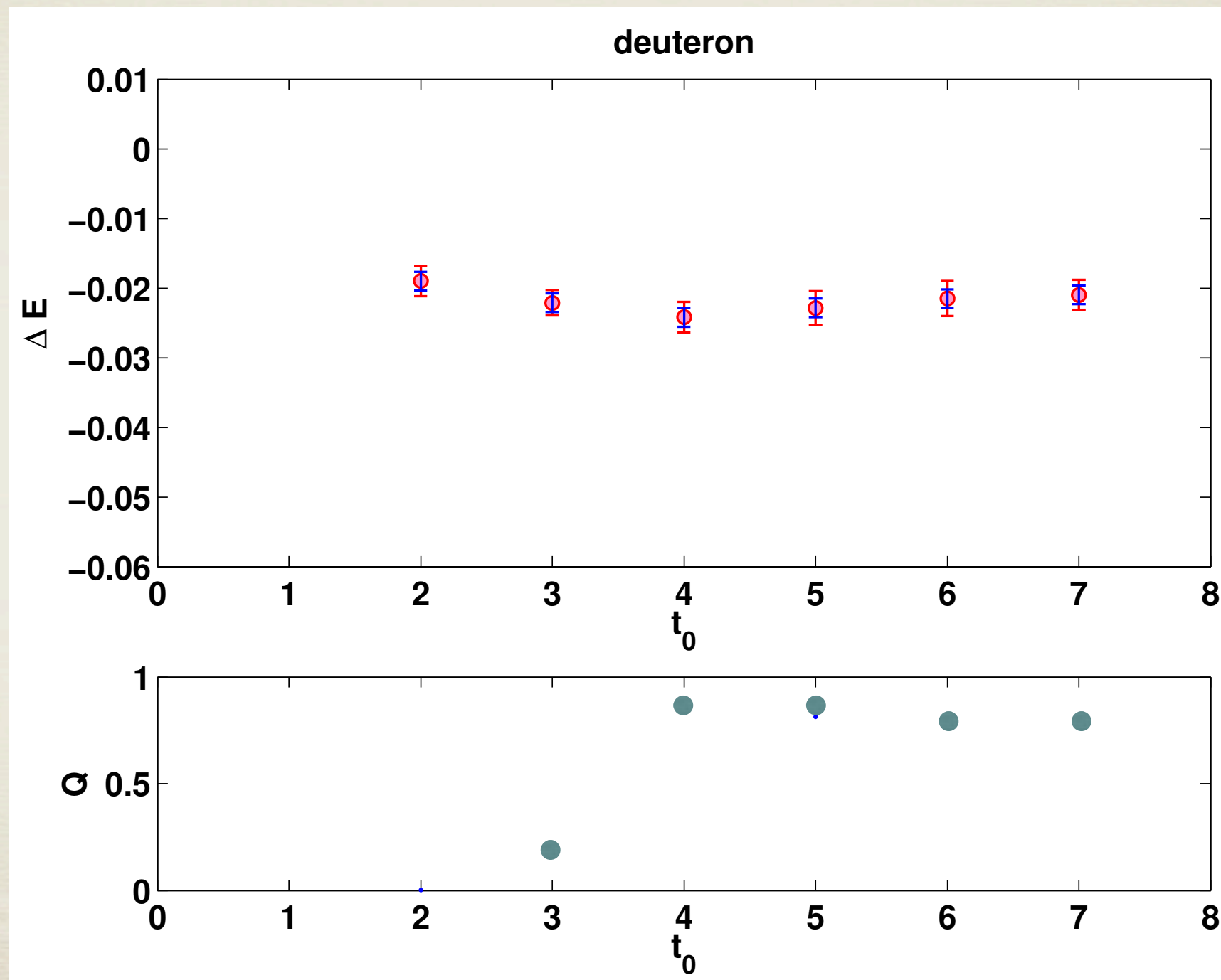
- * Isotropic Clover Wilson with LW gauge action
 - * Stout smeared (1-level)
 - * Tadpole improved
- * **SU(3)** symmetric point
 - * Defined using m_π/m_Ω
- * Lattice spacing **0.145 fm**
 - * Set using Y spectroscopy
- * Large volumes
 - * $24^3 \times 48$ $32^3 \times 48$ $48^3 \times 64$
 - * **3.5 fm** **4.5 fm** **7.0 fm**

NPLQCD arXiv:1206.5219

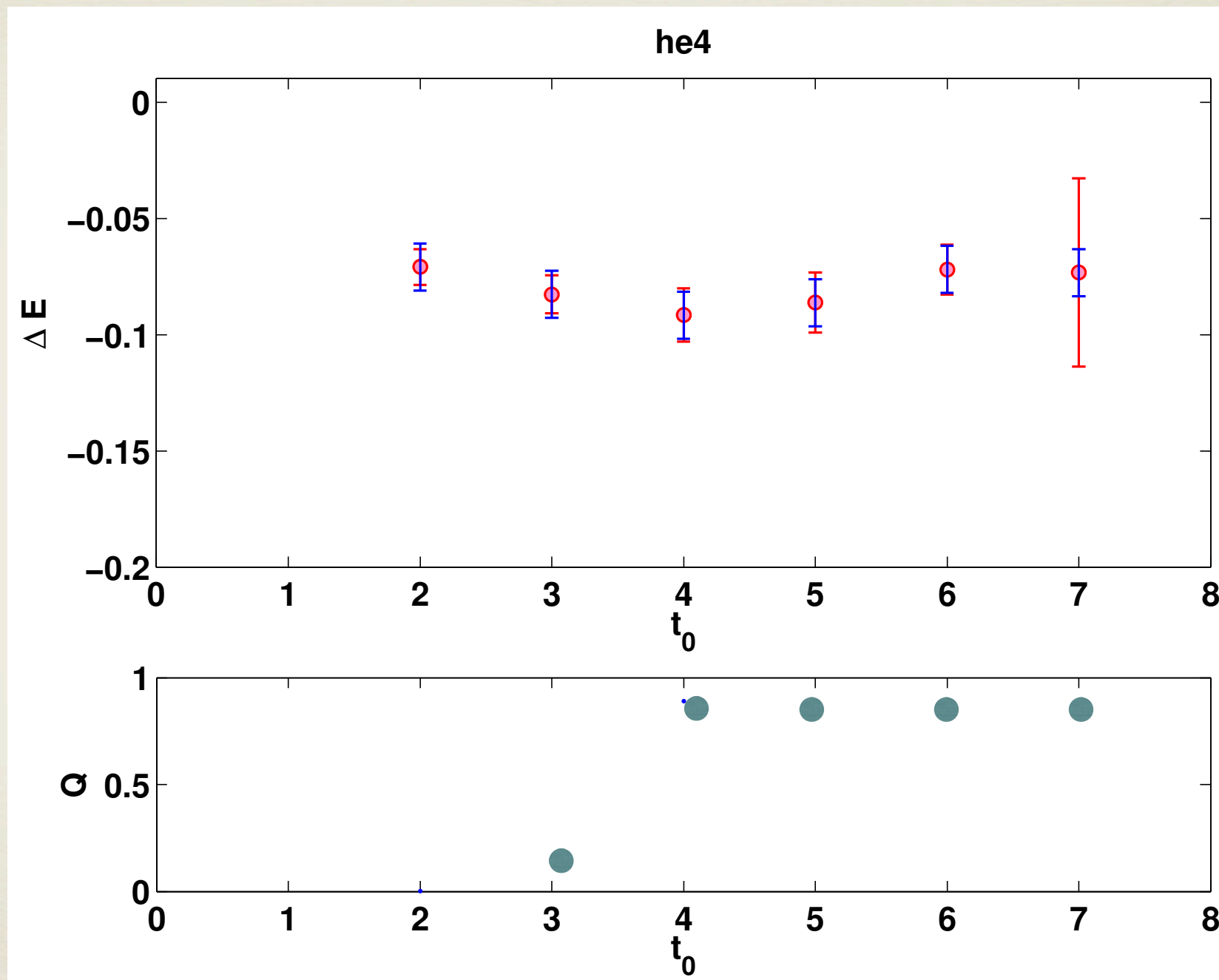
Using Quark to Hadron contractions

- * Calculate the spectrum of nuclei and hyper-hypernuclei
 - * Work with $A < 6$
- * Use single point source for quark propagators
- * In certain cases introduce nontrivial spatial wave function using a plane wave basis ($p^2 < 5$)
- * Use boosted states interpolating fields to check for finite volume effects
- * High statistics: 4K, 3K, 1.2K lattices with multiple correlators per lattice
- * Cost of contractions negligible

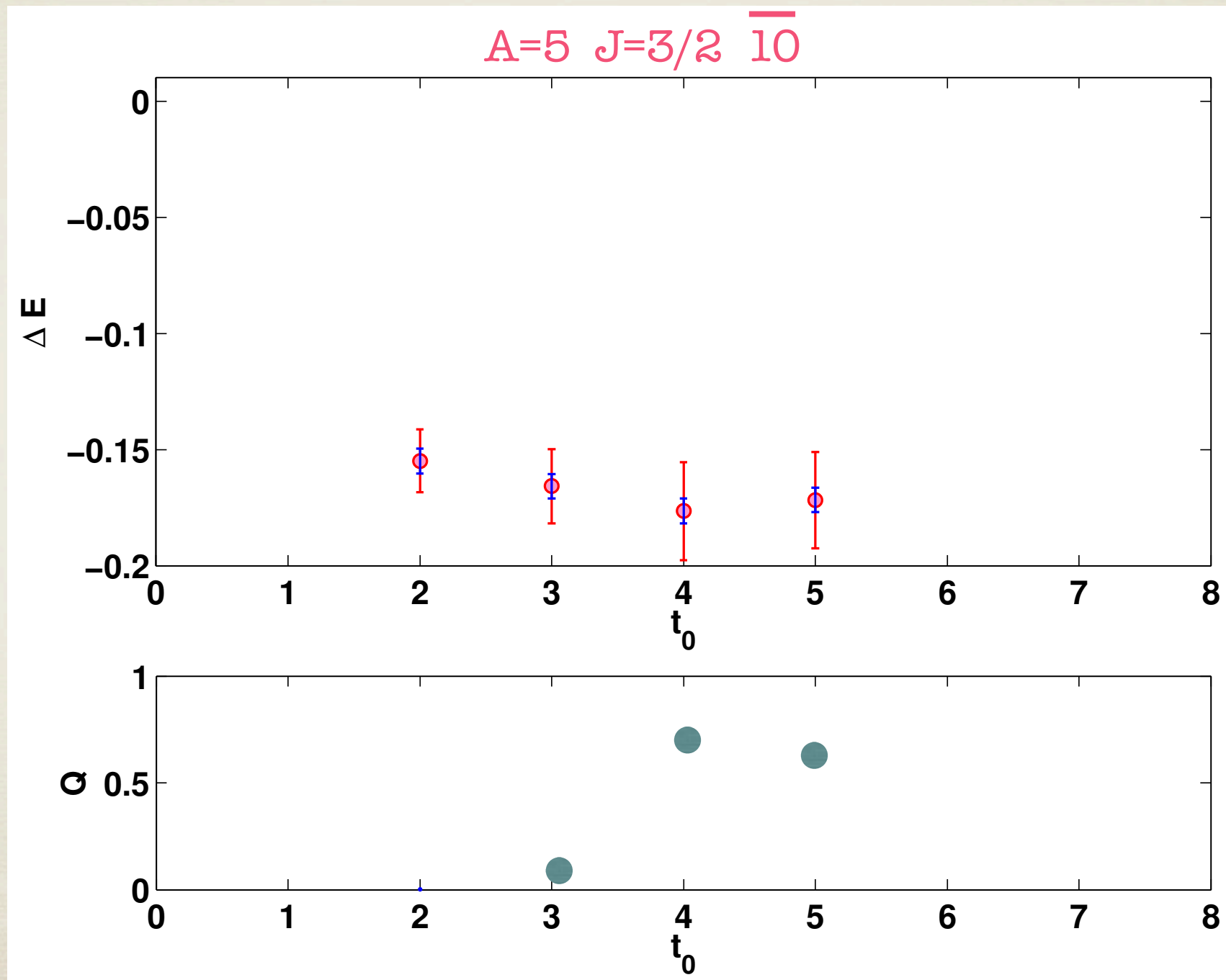
Fits



Fits

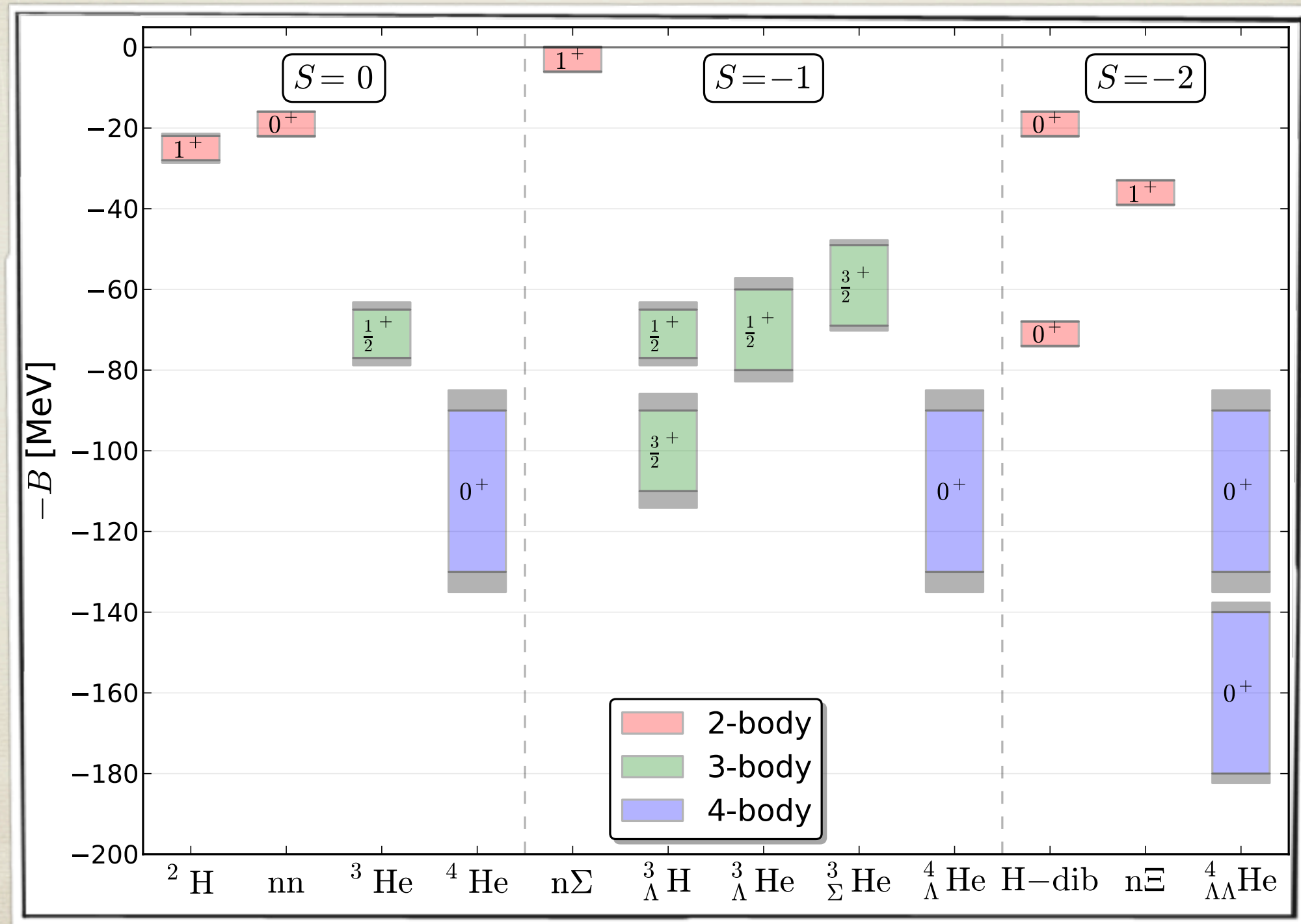


Fits



Nuclear spectrum

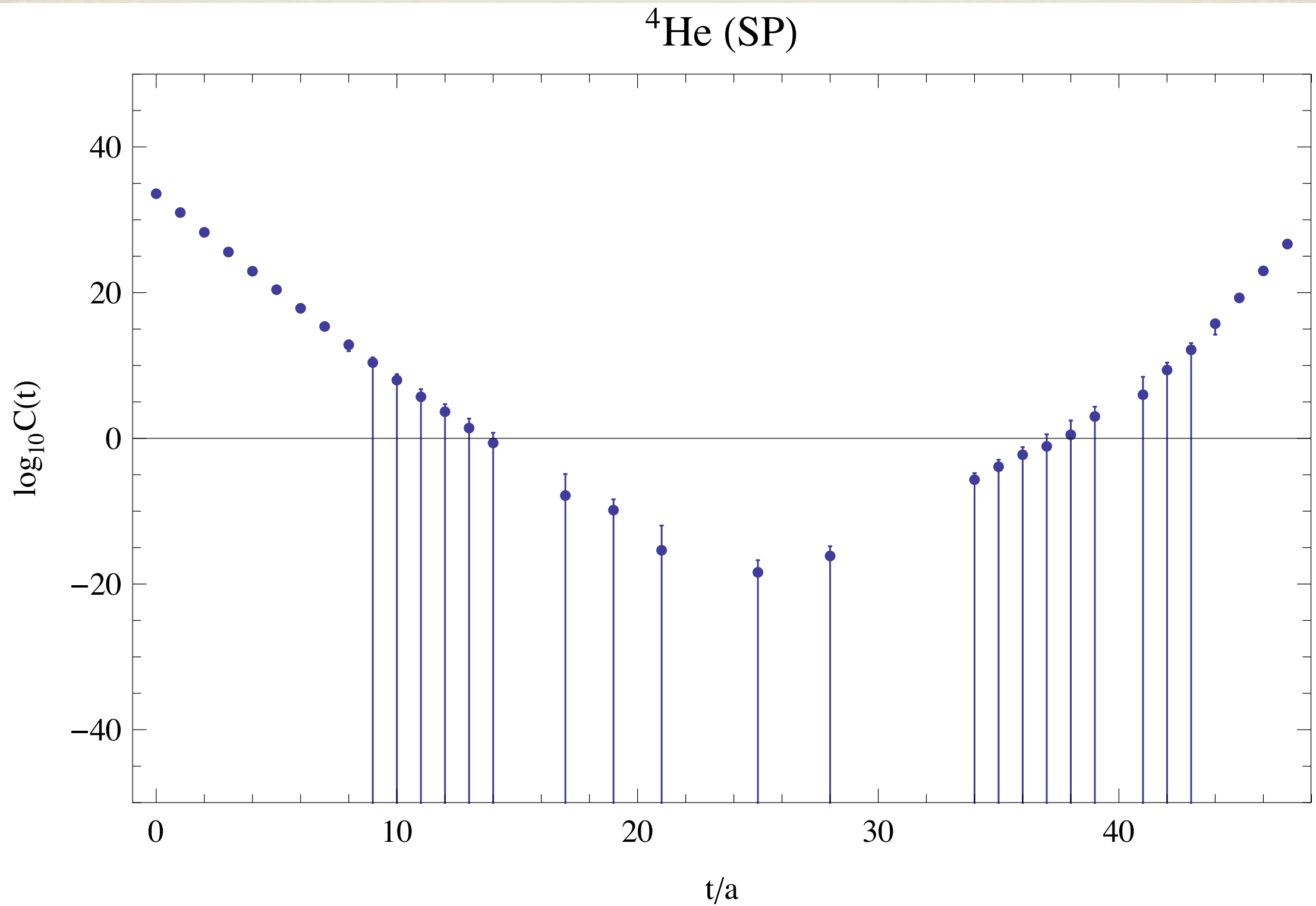
NPLQCD



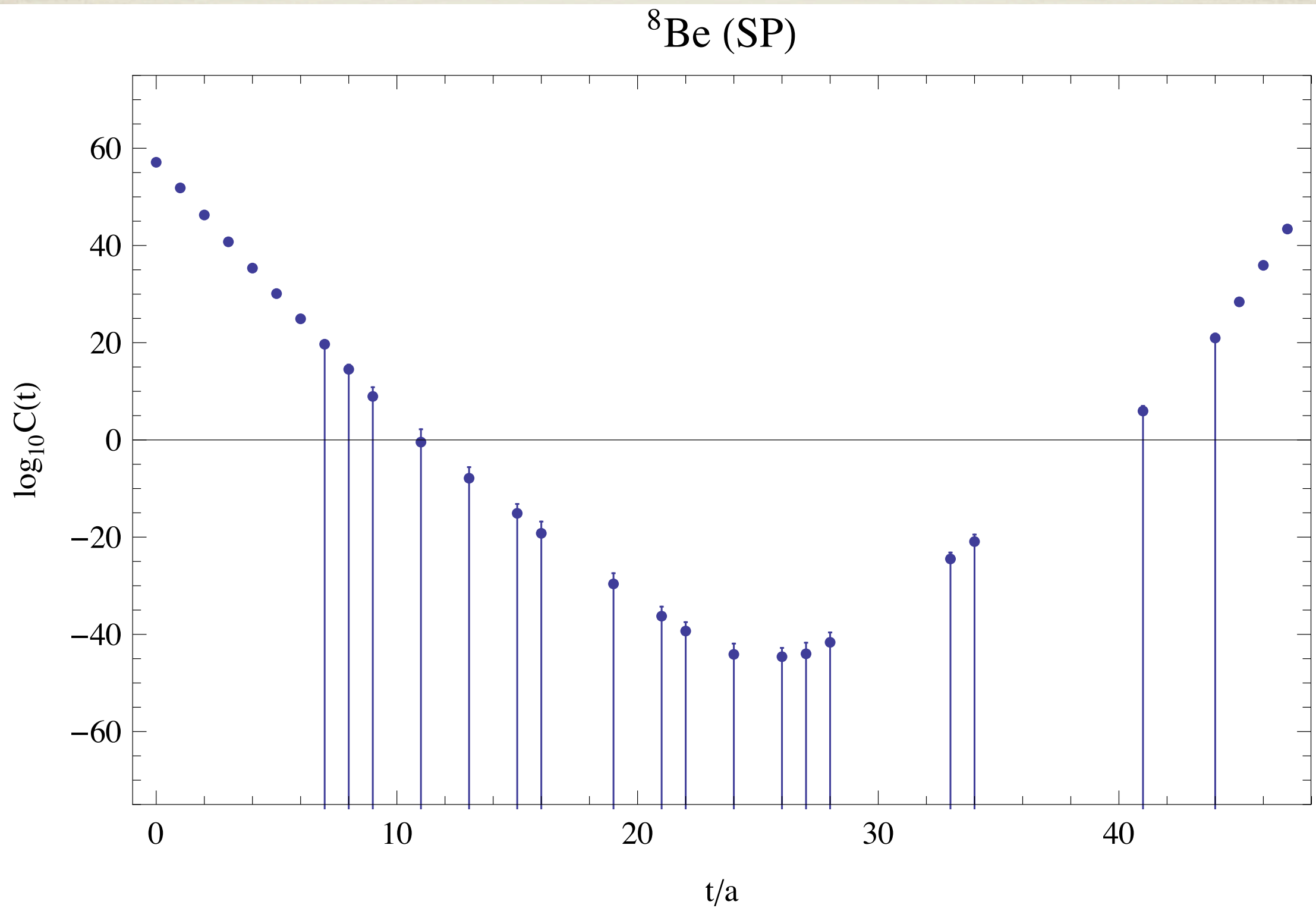
Using Quark to Quark contractions

- * Push to large quark numbers
- * Need to use more than one point for spacial wave function
- * Pauli exclusion principle ...
- * Low statistics this was just a feasibility demonstration
- * Only one volume: 32^3
- * No attempt to extract physics

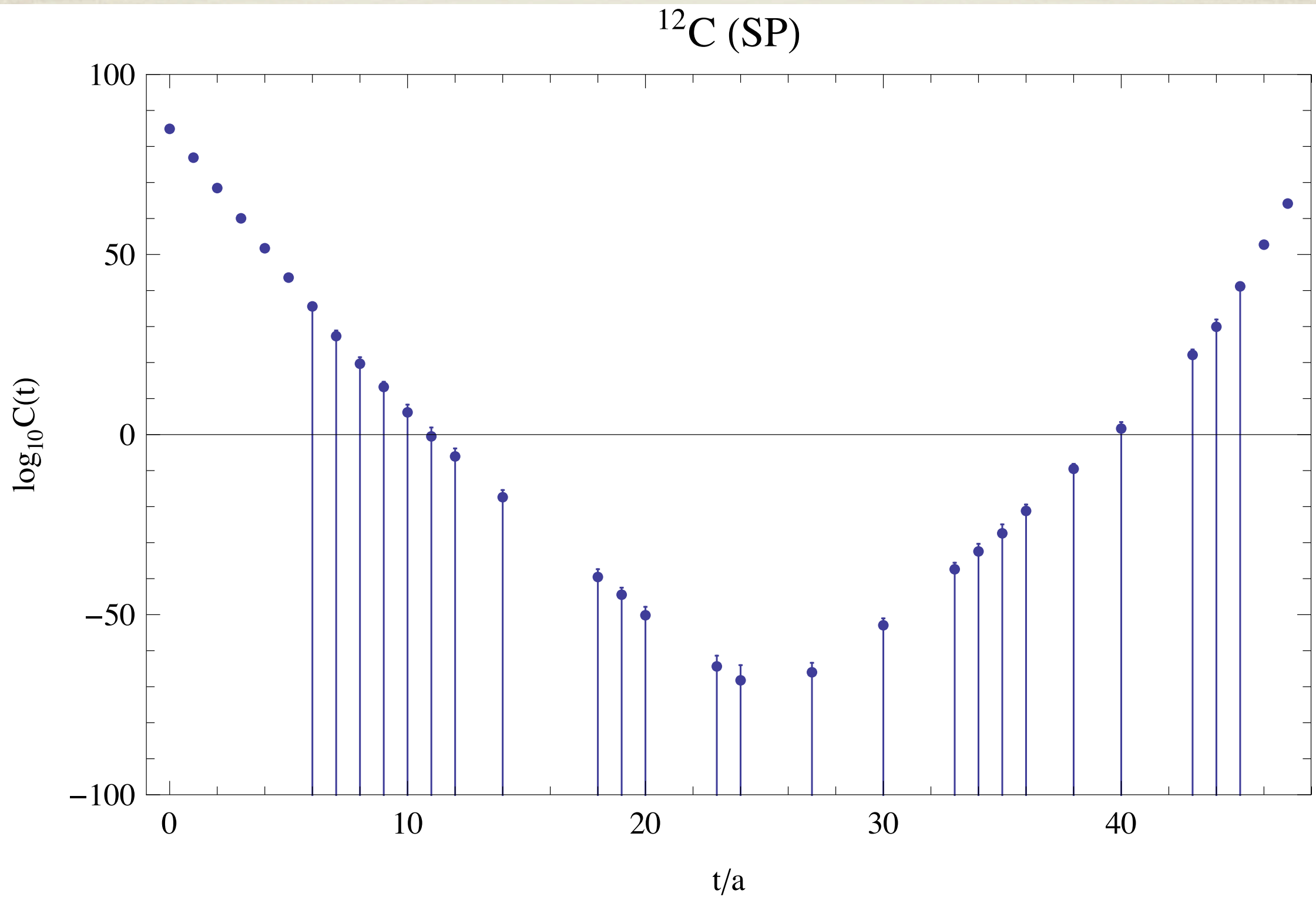
Correlators



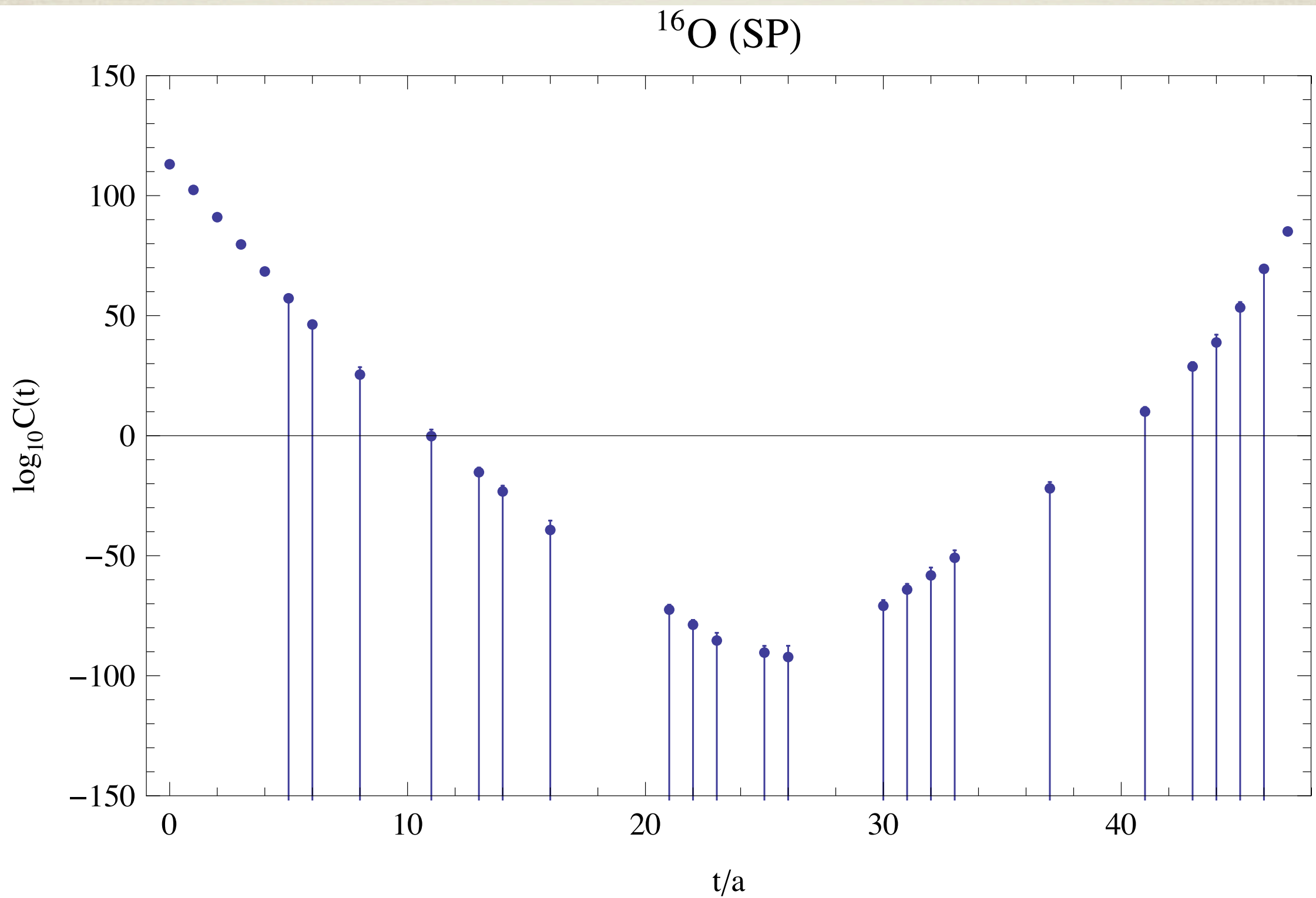
Correlators



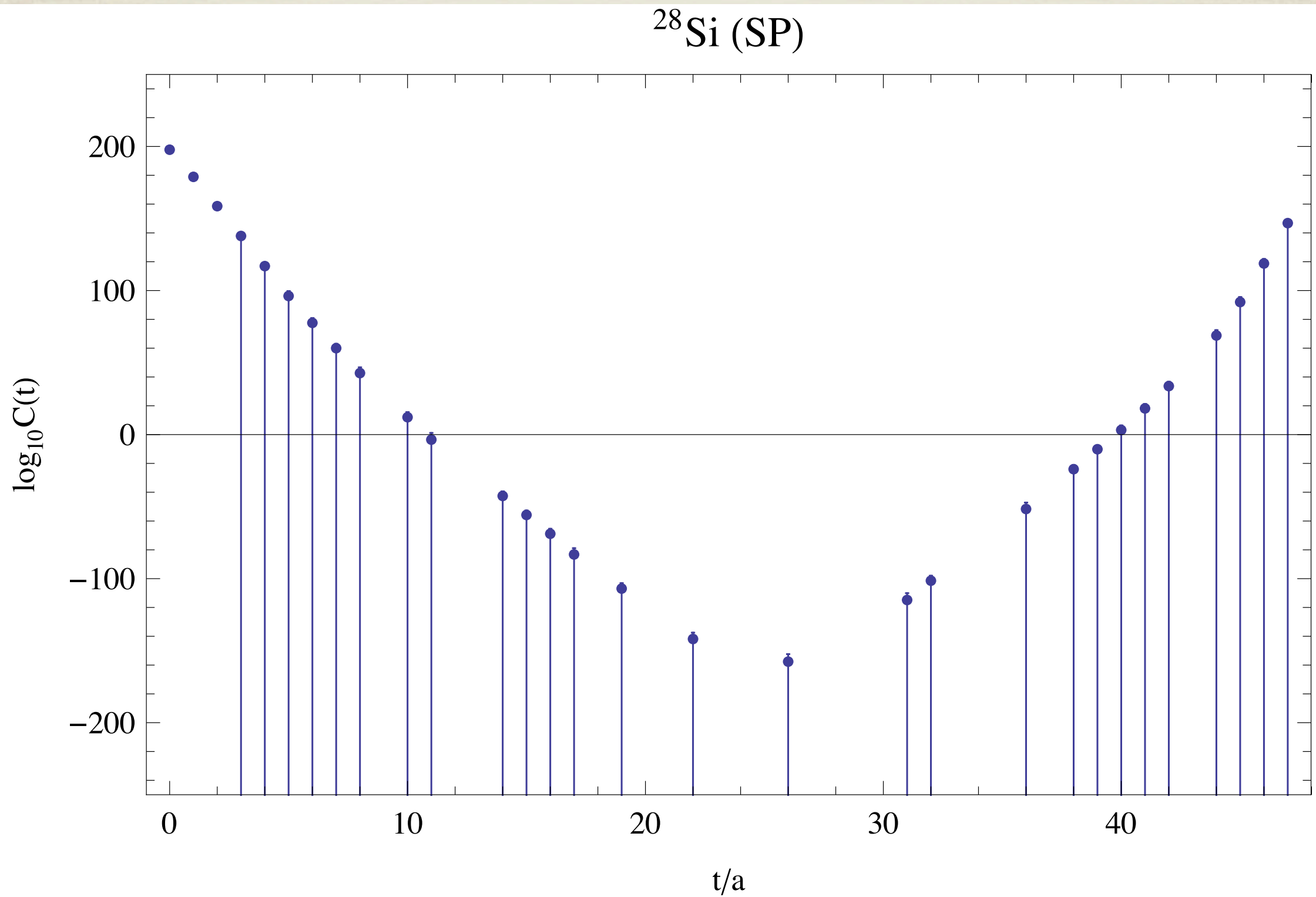
Correlators



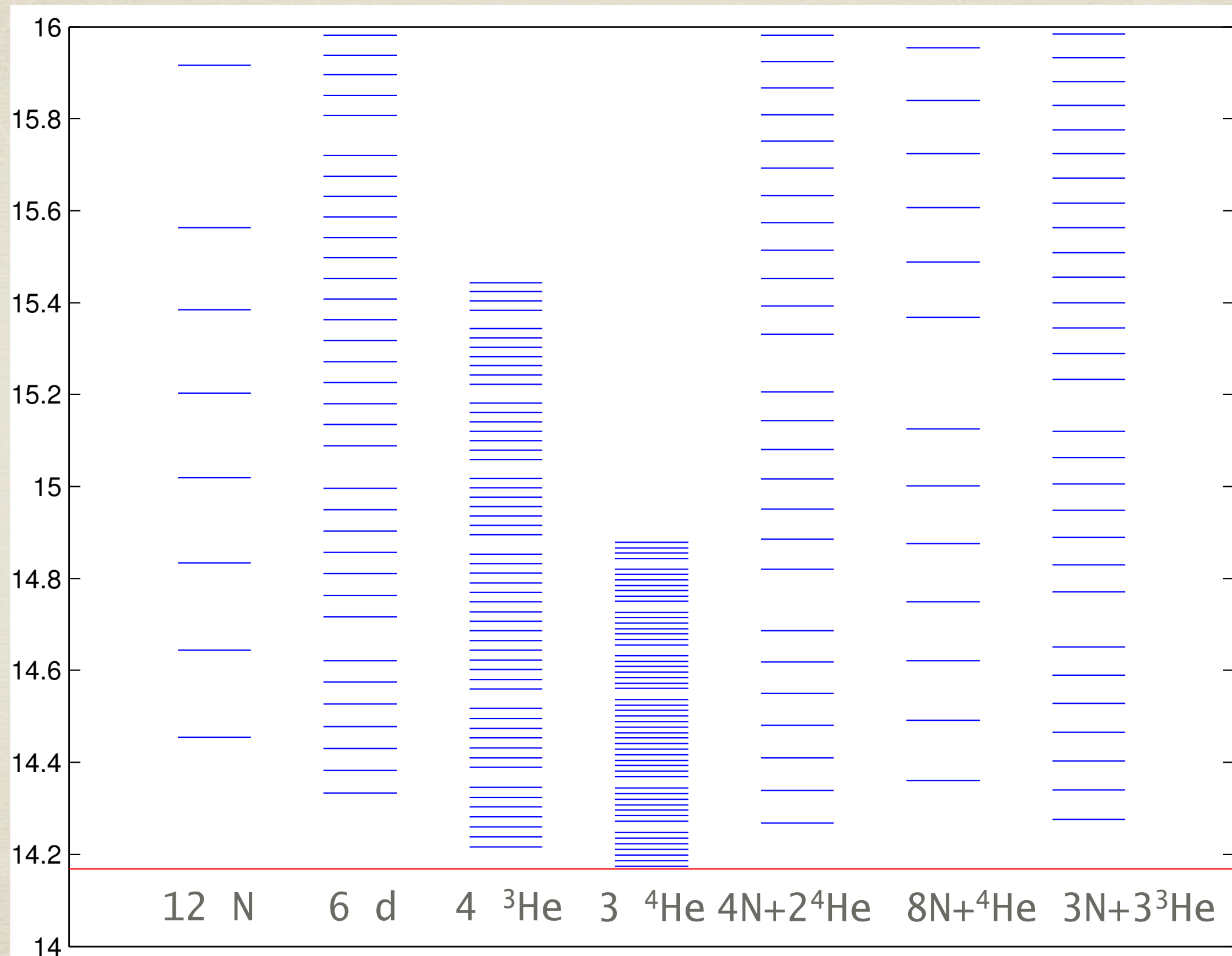
Correlators



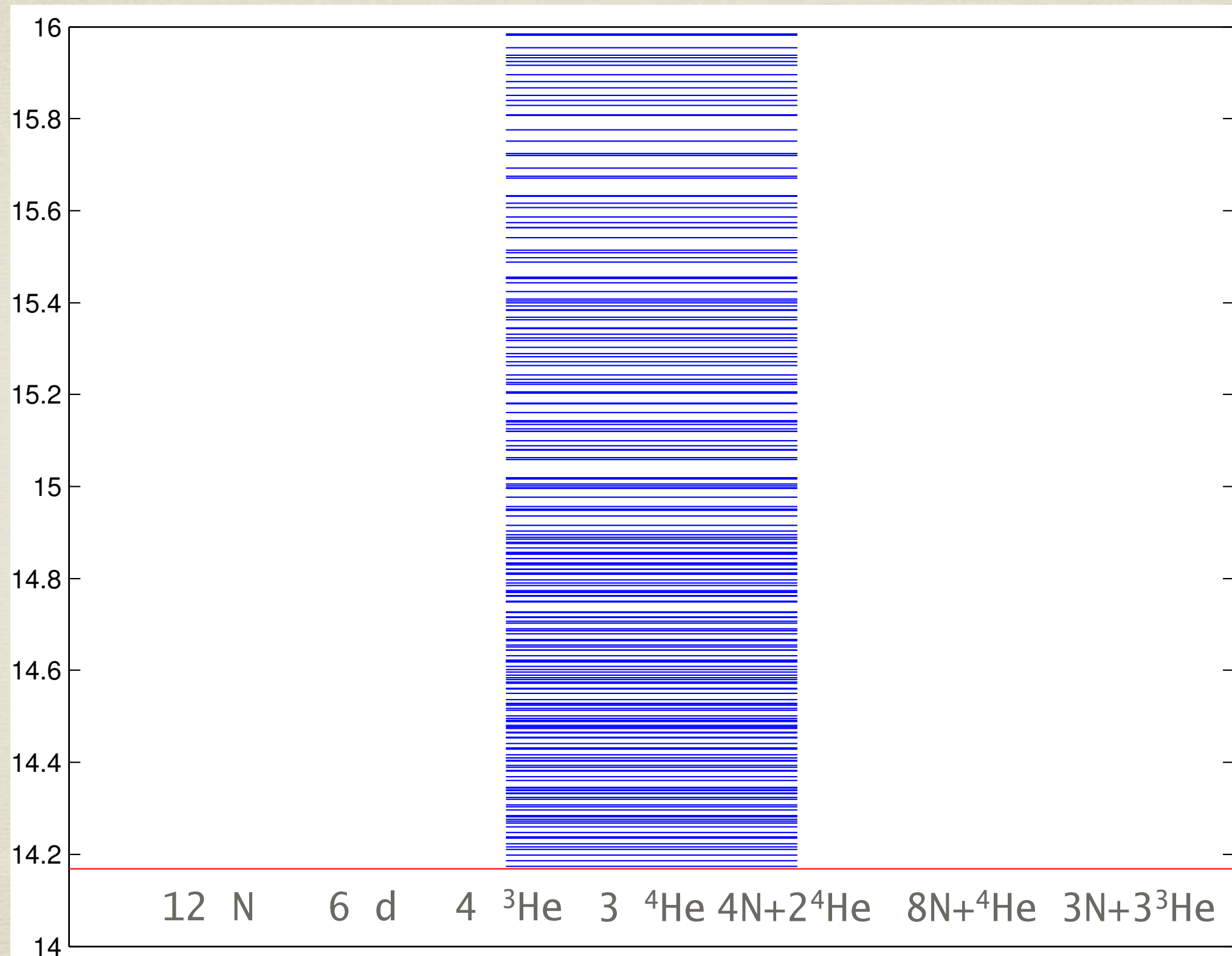
Correlators



Expected Carbon spectrum in the 32^3 box



Expected Carbon spectrum in the 32^3 box



Conclusions

- * We have a systematic way of constructing all possible interpolating fields
- * NPLQCD: Presented results for nuclei with $A < 5$ and $S > -3$
NPLQCD arXiv:1206.5219
- * Special care needs to be given to the selection of interpolating fields
 - * Minimize number of terms in the interpolating field and optimize the signal
- * We have an algorithm for quark contractions in polynomial time