

Strong coupling analysis of Aoki phase in Staggered-Wilson fermions

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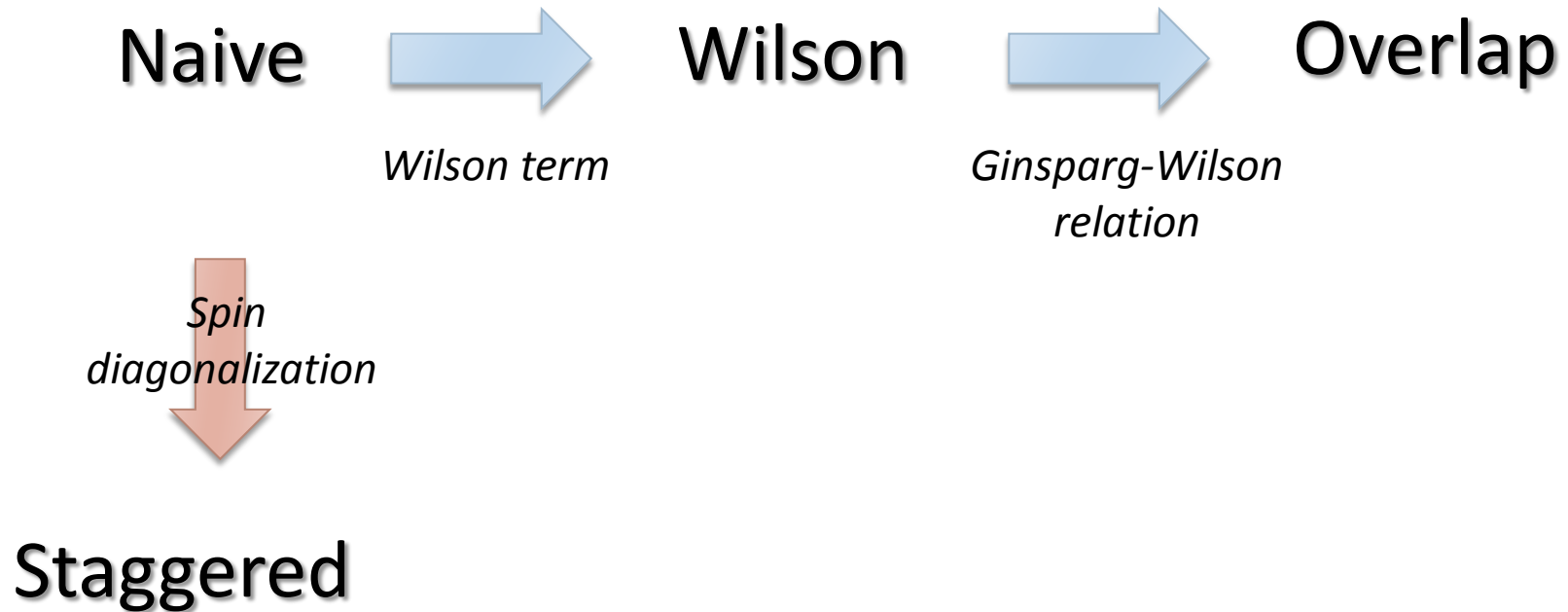
[arXiv:1205.6545]

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- ▶ **Staggered-Wilson fermion**
- ▶ **Aoki phase (Purpose)**
- ▶ **Strong coupling study for Aoki phase in Staggered-Wilson fermion**
- ▶ **Summary & Future works**

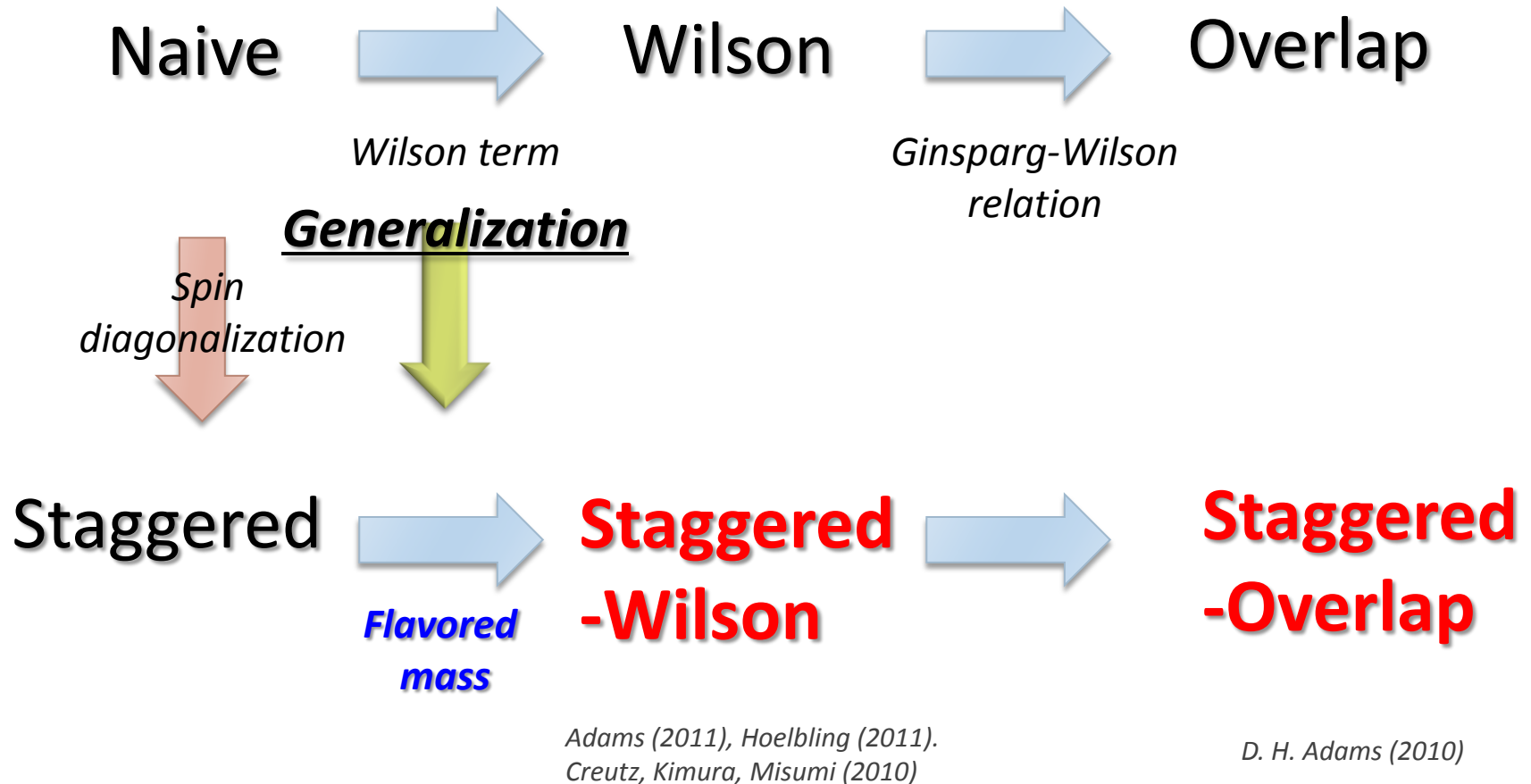
Staggered-Wilson Fermion

- ▶ Lattice fermion → Doubling problem



Staggered-Wilson Fermion

- ▶ Lattice fermion → Doubling problem



Flavors of Staggered-Wilson Fermion

- ▶ Staggered-Wilson Fermion = Staggered + *Flavored Mass*
 - ▶ Adams type : *2* flavor (Doubblers : 4 $\rightarrow$ 2) *Adams (2010,2011)*
 - ▶ Hoelbling type : *1* flavor (Doubblers : 4 $\rightarrow$ 1) *Hoelbling (2011)*

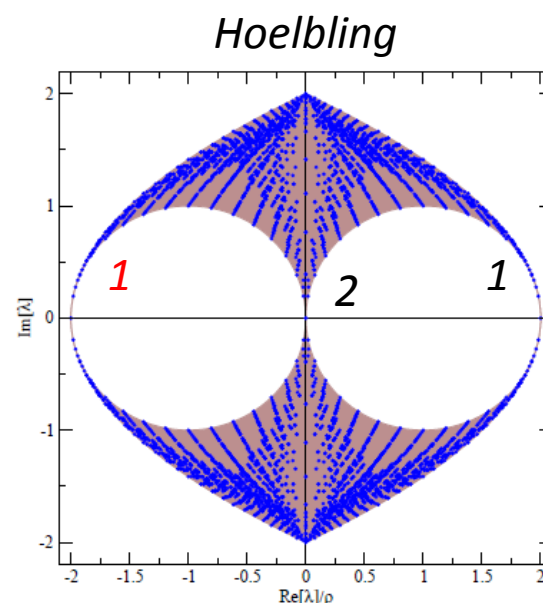
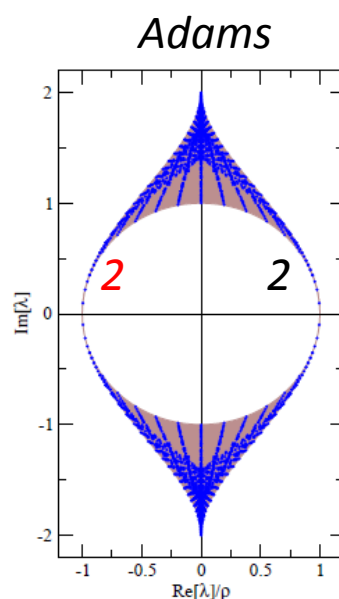
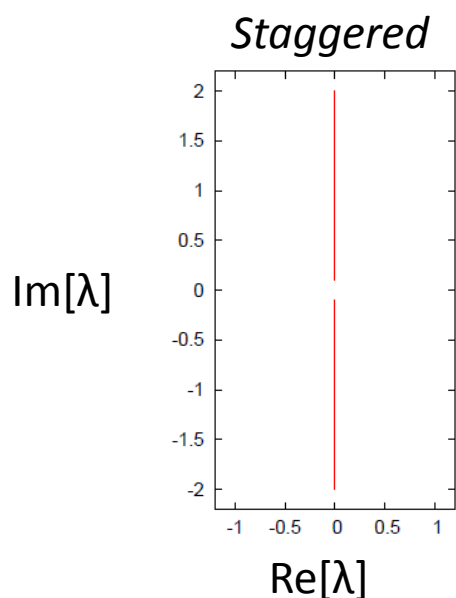
Flavors of Staggered-Wilson Fermion

► Staggered-Wilson Fermion = Staggered + *Flavored Mass*

► Adams type : **2** flavor (Doublers : 4 $\rightarrow$ 2) *Adams (2010,2011)*

► Hoelbling type : **1** flavor (Doublers : 4 $\rightarrow$ 1) *Hoelbling (2011)*

Dirac Spectrum (Free) de Forcrand, Kurkela, Panero (2012)



Properties of Staggered-Wilson Fermion

► Advantages

- 1 or 2 Flavors, Anomaly (vs *Staggered Fermion*)
- Lower Numerical Cost (vs *Wilson Fermion*) *de Forcrand, Kurkela, Panero (2011,2012)*

► Disadvantages

- Lower Discrete Symmetries (Misumi's Plenary Talk, Monday)

Flavored Mass Terms

- ▶ Generalized Wilson terms

Golterman, Smit (1984)

- ▶ Adams type (2 Flavor)

- ▶ 4-hopping

$$M_A = \# \sum_{sym} \epsilon \eta_5 \left(U_{\mu,x} U_{\nu,x+\hat{\mu}} U_{\rho,x+\hat{\mu}+\hat{\nu}} U_{\sigma,x+\hat{\mu}+\hat{\nu}+\hat{\rho}} + \cdots \right)$$

$$\sim 1 \otimes \xi_5$$

In spin-flavor representation

ξ_5 : γ_5 in flavor space

- ▶ Hoelbling type (1 Flavor)

- ▶ 2-hopping

$$M_H = \# \sum_{\mu \neq \nu} \eta_{\mu\nu} \left(U_{\mu,x} U_{\nu,x+\hat{\mu}} + \cdots \right)$$

$$\sim 1 \otimes \sigma_{\mu\nu}$$

$\eta_{\mu\nu}, \eta_5, \epsilon$: sign factor

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- ▶ Staggered-Wilson fermion
- ▶ **Aoki phase (Purpose)**
 - ▶ In Wilson Fermion
 - ▶ In Staggered-Wilson Fermion
- ▶ Strong coupling study for Aoki phase in Staggered-Wilson fermion
- ▶ Summary & Future works

Phase Structure in Wilson Fermion

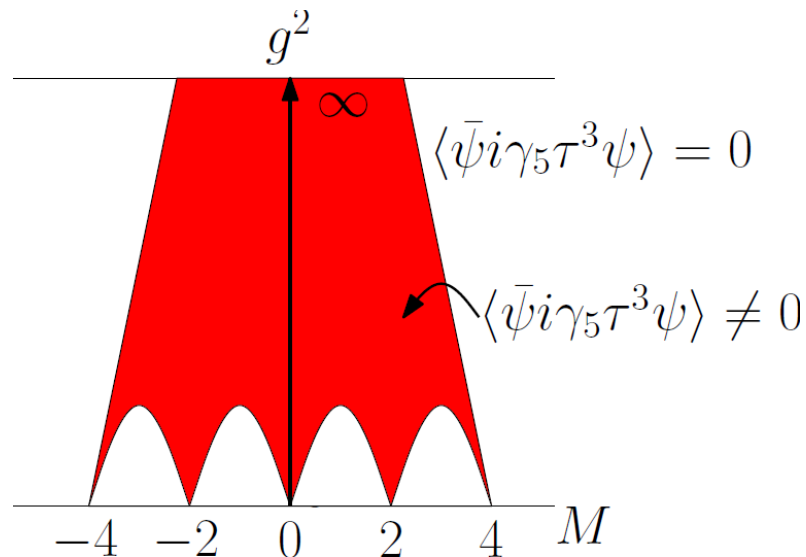
► Aoki Phase

- **Parity-flavor symmetry is spontaneously broken** in some quark mass region. Aoki(1984)

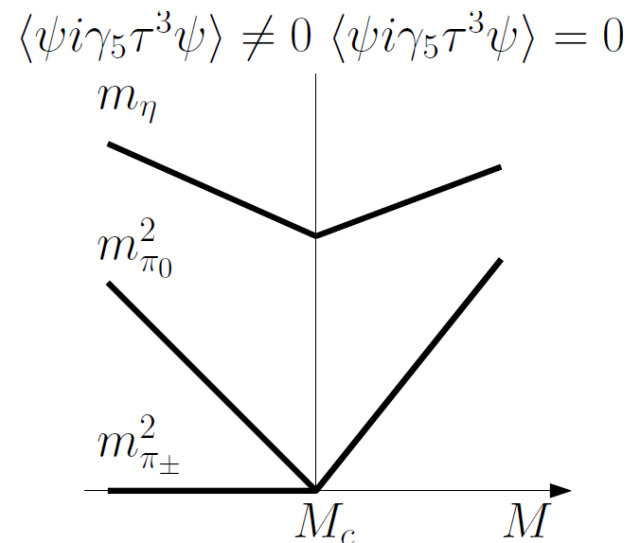
► Quark mass tuning → Chiral limit

Wilson Fermion (2 Flavor)

Phase structure



Pion Mass

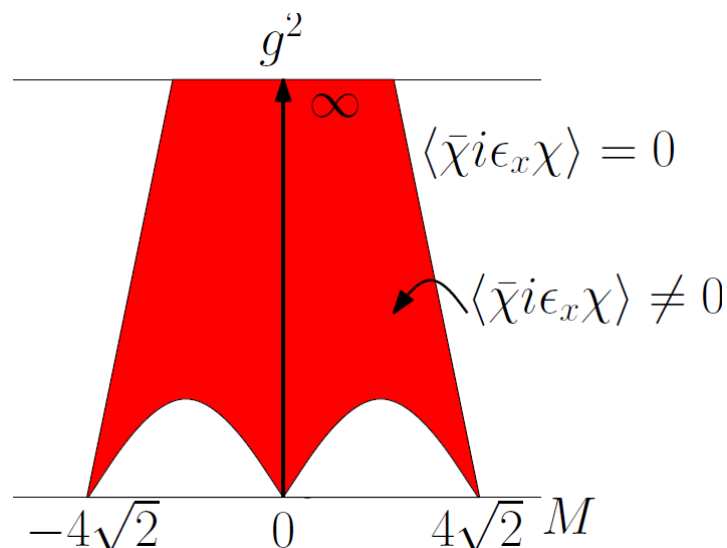


Aoki Phase in Staggered-Wilson Fermions?

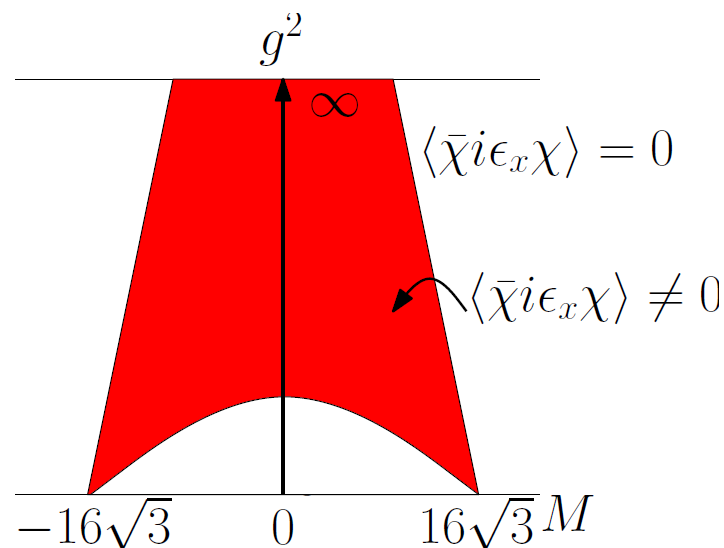
- ▶ Flavored Mass : Generalized Wilson terms *Creutz, Kimura, Misumi (2010)*
- ▶ Previous study : Lattice Gross-Neveu model with flavored mass
Creutz, Kimura, Misumi (2011)

Expected Phase Structure in Lattice QCD

Hoelbling type



Adams type



Purpose

- ▶ We study Aoki phase of Staggered-Wilson fermion in strong coupling lattice QCD ($1/g^2 = 0$).
 - ▶ Check the Chiral Limit
- ▶ Method
 - ▶ Hopping parameter expansion
 - ▶ Effective potential analysis
- ▶ Staggered-Wilson Fermion
 - ▶ Adams type , Hoelbling type

Contents

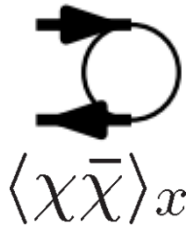
- ▶ Staggered-Wilson fermion
- ▶ Aoki phase (Purpose)
- ▶ **Strong coupling study for Aoki phase in Staggered-Wilson fermion**
 - ▶ Hopping parameter expansion
 - ▶ Effective potential
- ▶ Summary & Future works

Hopping Parameter Expansion (1 pt. Function)

► Diagrams (1 pt. function) $\rightarrow \langle \bar{\chi}_x \chi_x \rangle$ *Condensate*

► Expansion for Hopping parameter K $K^{-1} = 2M$

e.g. Hoelbling type (1 Flavor)



Hopping Parameter Expansion (1 pt. Function)

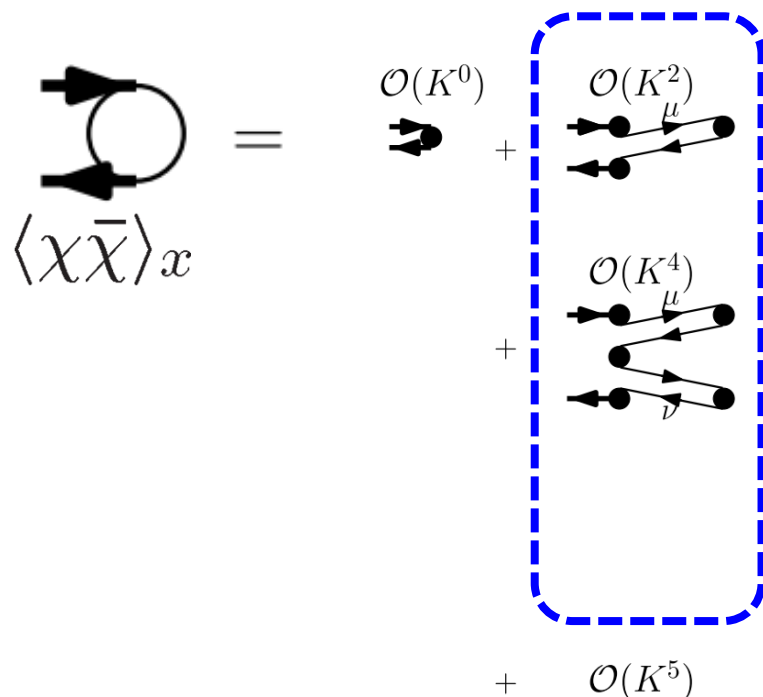
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► Expansion for Hopping parameter K

$$K^{-1} = 2M$$

► Link variables appear as a pair of $(U_\mu, U_\mu^\dagger)$.

e.g. Hoelbling type (1 Flavor)



Hopping Parameter Expansion (1 pt. Function)

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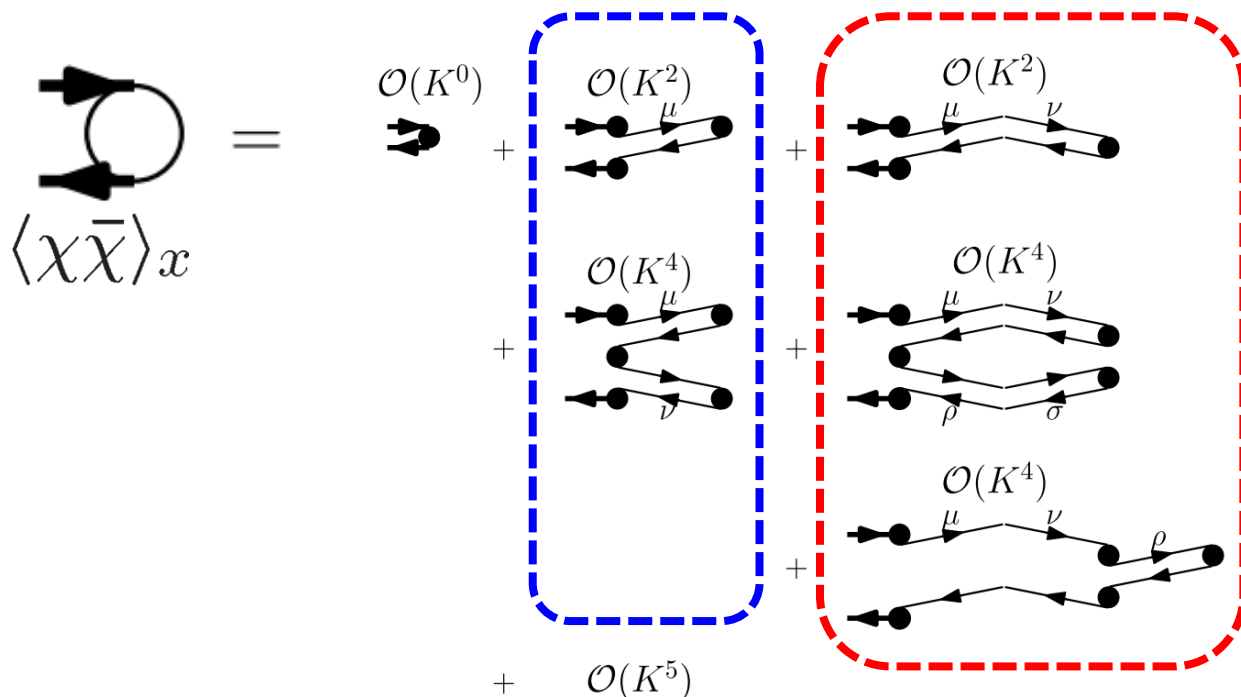
► Expansion for Hopping parameter K

$$K^{-1} = 2M$$

► Link variables appear as a pair of $(U_\mu, U_\mu^\dagger)$, $(U_\mu U_\nu, U_\mu^\dagger U_\nu^\dagger)$, $(U_\mu^\dagger U_\nu, U_\mu U_\nu^\dagger)$

e.g. Hoelbling type (1 Flavor)

2 hopping (Flavored Mass)



Hopping Parameter Expansion (1 pt. Function)

► Diagrams (1 pt. function) $\rightarrow \langle \bar{\chi}_x \chi_x \rangle$ *Condensate*

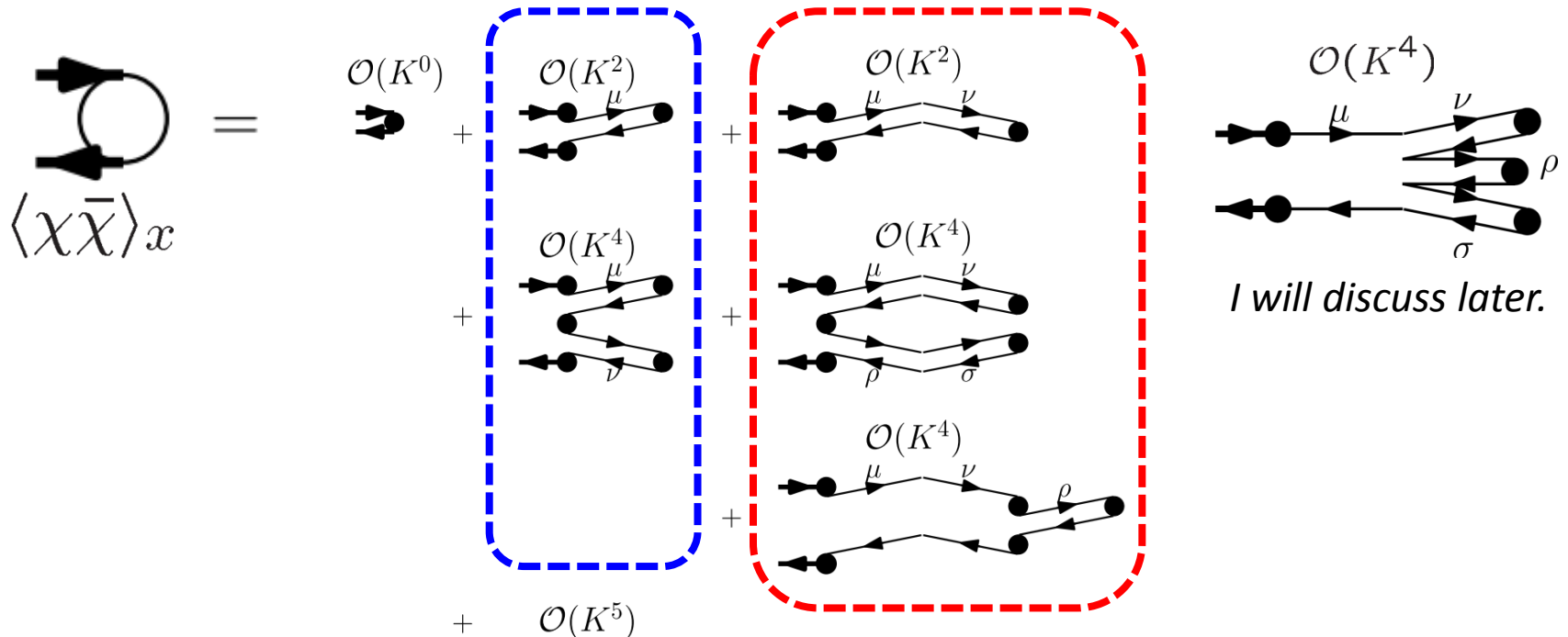
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e.g. Hoelbling type (1 Flavor)

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Hopping Parameter Expansion (1 pt. Function)

- ▶ Diagrams (1 pt. function) $\rightarrow \langle \bar{\chi}_x \chi_x \rangle$ *Condensate*
 - ▶ Summation for K . ($K^2, K^4, \dots$)

The diagram shows the expansion of the 1-point function $\langle \chi \bar{\chi} \rangle_x$ as a sum of diagrams. On the left, a circle with two arrows (one entering, one exiting) is labeled $\langle \chi \bar{\chi} \rangle_x$. This is equal to a sum of three terms: 1) a tadpole diagram (a circle with one external line), 2) a diagram labeled $\mathcal{O}(K^2)$ showing a tadpole with an internal loop and an external line labeled μ , and 3) a diagram labeled $\mathcal{O}(K^2)$ showing a tadpole with two internal lines labeled μ and ν .

Self-consistent equation

$$\langle \chi \bar{\chi} \rangle_x = -1 + 2K^2 \sum_{\mu} \langle \chi \bar{\chi} \rangle_x \langle \chi \bar{\chi} \rangle_{x+\hat{\mu}} - \frac{2}{3}K^2 \sum_{\mu \neq \nu} \langle \chi \bar{\chi} \rangle_x \langle \chi \bar{\chi} \rangle_{x+\hat{\mu}+\hat{\nu}}$$

Hopping Parameter Expansion (1 pt. Function)

- ▶ Mean-field treatment

- ▶ Scalar $\langle \bar{\chi} \chi \rangle$
- ▶ Pseudo-scalar $\langle \bar{\chi} i \epsilon_x \chi \rangle$

$$\epsilon_x \equiv (-1)^{x_1 + \dots + x_4} \sim \gamma_5 \otimes \xi_5$$

ξ_5 : γ_5 in flavor space

- ▶ Two solution

- ▶ Parity symmetric

$$\frac{1}{N_c} \langle \bar{\chi} \chi \rangle = \frac{1}{2K}, \quad \frac{1}{N_c} \langle \bar{\chi} i \epsilon_x \chi \rangle = 0$$

- ▶ Parity broken

$$\frac{1}{N_c} \langle \bar{\chi} \chi \rangle = \frac{1}{8K}, \quad \frac{1}{N_c} \langle \bar{\chi} i \epsilon_x \chi \rangle = \pm \frac{\sqrt{16K^2 - 1}}{8K}$$

 *For $|K| > \frac{1}{4}$, Parity broken?*

Hopping Parameter Expansion (2 pt. Function)

- 2pt. Function $\langle \bar{\chi}_0^a i\epsilon_0 \chi_0^a \bar{\chi}_x^b i\epsilon_x \chi_x^b \rangle \rightarrow$ Pion mass

$$\cosh m_\pi = 1 + \frac{1 - 16K^2}{6K^2}$$

 For $|K| > \frac{1}{4}$, $m_\pi^2 < 0$?

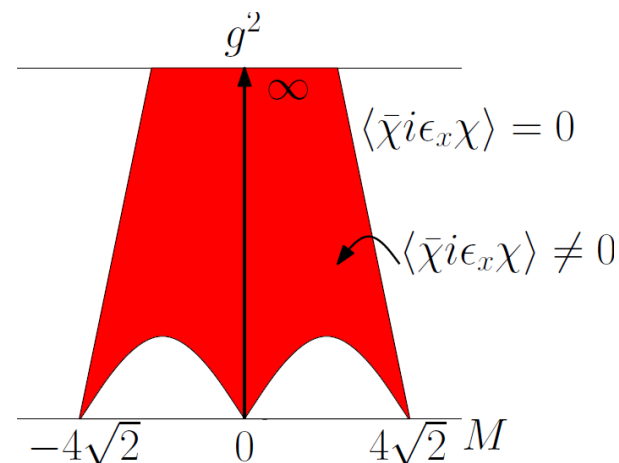
This result implies Aoki phase.

Effective Potential (Condensate)

- ▶ Effective Potential for Meson fields (large N_c)
- ➔ We can identify the Parity-broken phase

$$V_{\text{eff}} = \frac{1}{V_4} \int \mathcal{D}[\chi, \bar{\chi}, U] \exp [S_F]$$

	$\frac{1}{N_c} \langle \bar{\chi} \chi \rangle$	$\frac{1}{N_c} \langle \bar{\chi} i \epsilon_x \chi \rangle$
$M^2 > 4$	$\frac{1}{M}$	0
$M^2 < 4$	$\frac{M}{8 - M^2}$	$\pm \frac{\sqrt{2(4 - M^2)}}{8 - M^2}$



➔ For $M^2 < 4 \left(|K| > \frac{1}{4} \right)$, $\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0$

Effective Potential (Pion Mass)

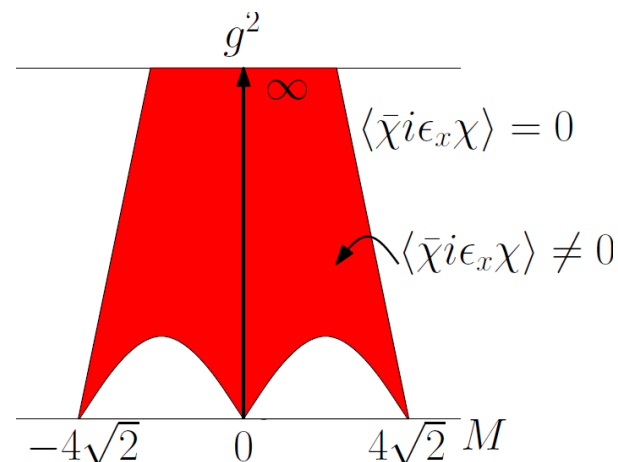
- ▶ Critical Mass (M_c) corresponds to the chiral limit.

$$m_\pi \leftarrow \frac{\partial^2 S_{\text{eff}}(M)}{\partial M_x \partial M_y}$$

M_x : Meson fields

$$\cosh m_\pi = 1 + \frac{2M^2 - 8}{3} \text{ for } M^2 > 4$$

➡ $m_\pi^2 = 0 \text{ for } M^2 = 4$



Discussion on 2 Flavor Case

- ▶ Hoelbling (2 taste)

- ▶ Possibility

- ▶ (1) Parity symmetry breaking

$$\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0, \langle \bar{\chi} i \epsilon_x \tau^3 \chi \rangle = 0$$

- ▶ (2) Parity-flavor symmetry breaking

$$\langle \bar{\chi} i \epsilon_x \chi \rangle = 0, \langle \bar{\chi} i \epsilon_x \tau^3 \chi \rangle \neq 0$$

- ▶ Parity-flavor symmetry breaking

- ▶ Vafa-Witten's theorem $\Rightarrow \langle \bar{\chi} i \epsilon_x \chi \rangle = 0$

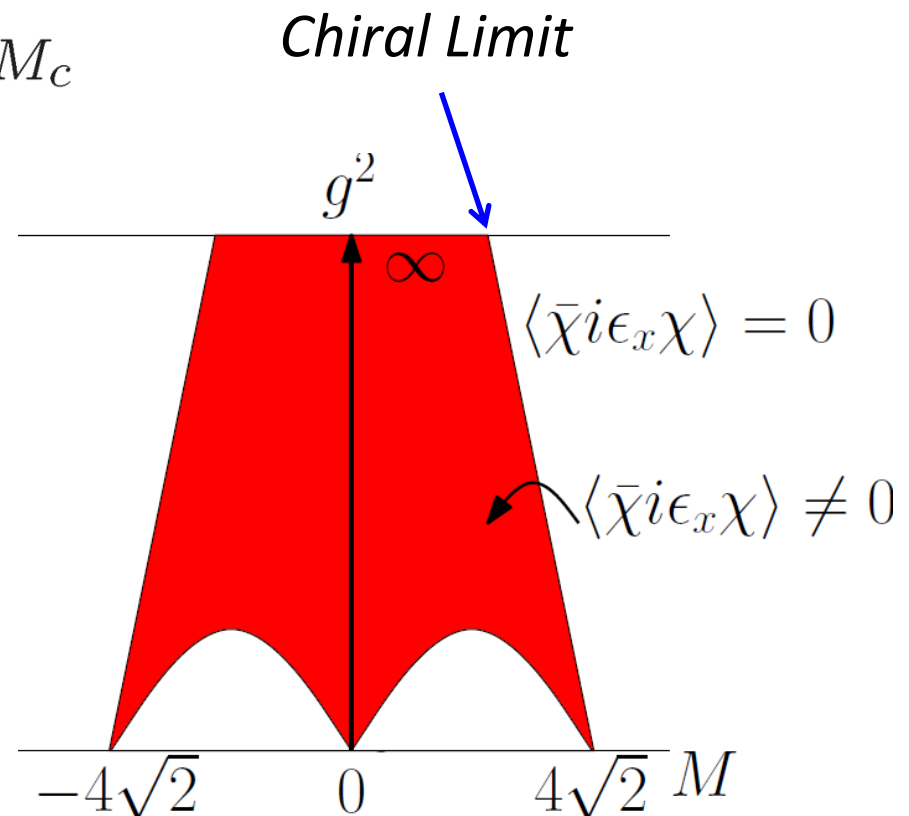
Vafa, Witten (1984)

Brief Summary

► Hoelbling type (1 flavor)

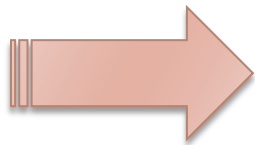
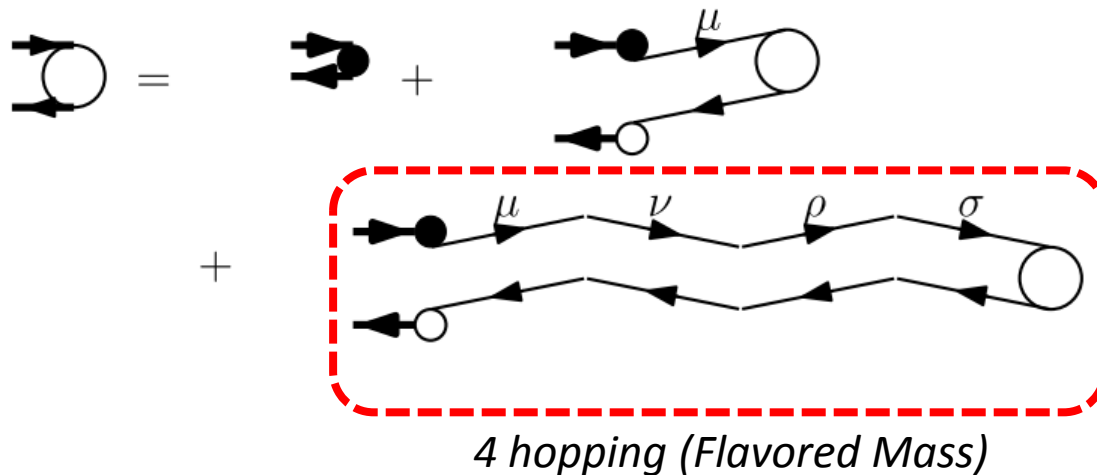
$$\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0 \text{ for } |M| < M_c$$
$$m_\pi^2 = 0 \text{ for } |M| = M_c$$

- We can perform the lattice simulation with staggered-Wilson fermions by tuning mass parameter.



Analysis in Adams type

- ▶ Similar way as Hoelbling type
 - ▶ e.g. Hopping Parameter Expansion (1 pt. function)



$$\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0 \text{ for } |M| < M_c$$

$$m_\pi^2 = 0 \text{ for } |M| = M_c$$

Higher Order (Preliminary)

- This analysis is exact up to $O(K^3)$ *e.g. Hoelbling type (1 pt. Function)*

The diagram shows the expansion of the two-point function $\langle \chi \bar{\chi} \rangle_x$ into three terms:

- First term:** A tadpole diagram with two external fermion lines (represented by arrows) meeting at a single vertex, which is connected to a loop.
- Second term:** A diagram labeled $O(K^2)$. It features a fermion line with a self-energy insertion (a loop) and an external fermion line.
- Third term:** A diagram labeled $O(K^2)$. It shows a fermion line with a self-energy insertion (a loop) and an external fermion line, with additional indices μ and ν associated with the loop.

The expansion is written as:

$$\langle \chi \bar{\chi} \rangle_x = \text{tadpole} + O(K^2) \text{ (self-energy)} + O(K^2) \text{ (self-energy with indices)}$$

Higher Order (Preliminary)

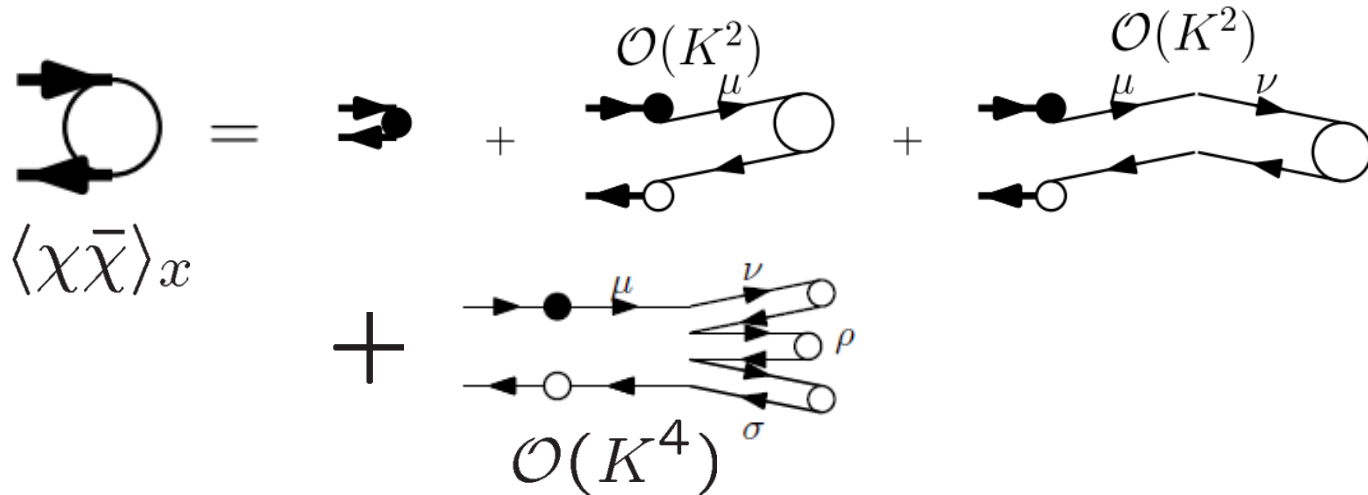
- This analysis is exact up to $\mathcal{O}(K^3)$ *e.g. Hoelbling type (1 pt. Function)*

The diagram shows the expansion of the two-point function $\langle \chi \bar{\chi} \rangle_x$ into a sum of terms:

- The first term is a tadpole diagram with two external legs meeting at a vertex, labeled $\langle \chi \bar{\chi} \rangle_x$.
- The second term is a tadpole diagram with a self-energy loop on the internal line, labeled $\mathcal{O}(K^2)$.
- The third term is a tadpole diagram with a self-energy loop on the internal line, labeled $\mathcal{O}(K^2)$.
- The fourth term is a tadpole diagram with a self-energy loop on the internal line, labeled $\mathcal{O}(K^4)$.

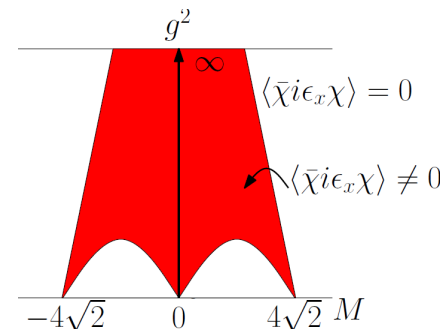
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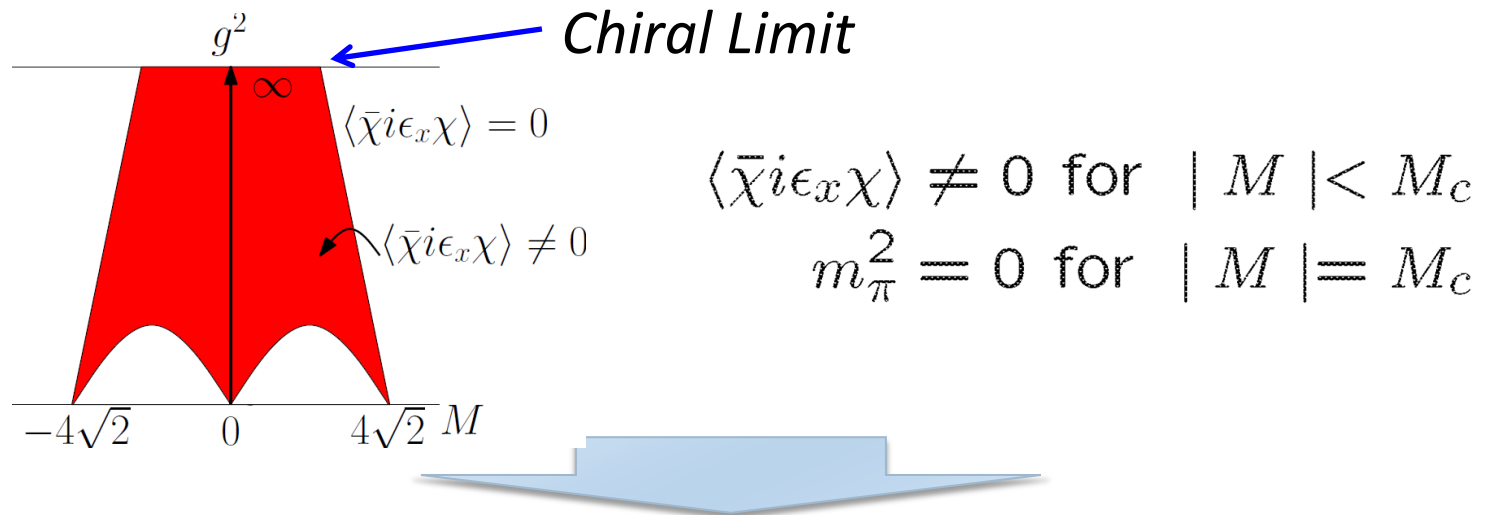
- ▶ The region of Aoki Phase is enlarged, but the symmetric phase exist.

$$K_c \searrow \quad (M_c \nearrow)$$



Summary & Future Works

- ▶ We have studied Aoki phase in Staggered-Wilson fermion in strong coupling QCD.



- ▶ We can perform the lattice simulation with staggered-Wilson fermions by tuning mass parameter.
- ▶ Future Works
 - ▶ Evaluation for higher-order terms