Strong coupling analysis of Aoki phase in Staggered-Wilson fermions

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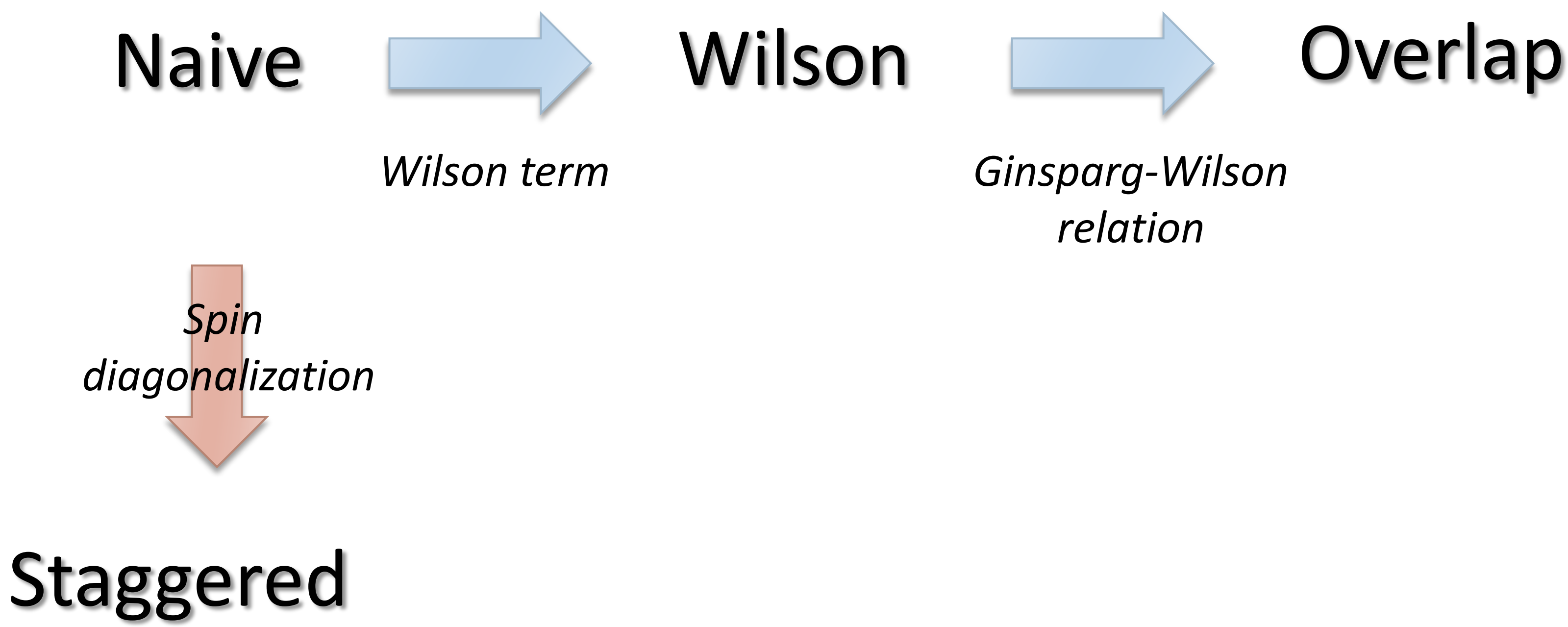
[arXiv:1205.6545]

Contents

- **Staggered-Wilson fermion**
- **Aoki phase (Purpose)**
- **Strong coupling study for Aoki phase in Staggered-Wilson fermion**
- **Summary & Future works**

Staggered-Wilson Fermion

- Lattice fermion → Doubling problem



Staggered-Wilson Fermion

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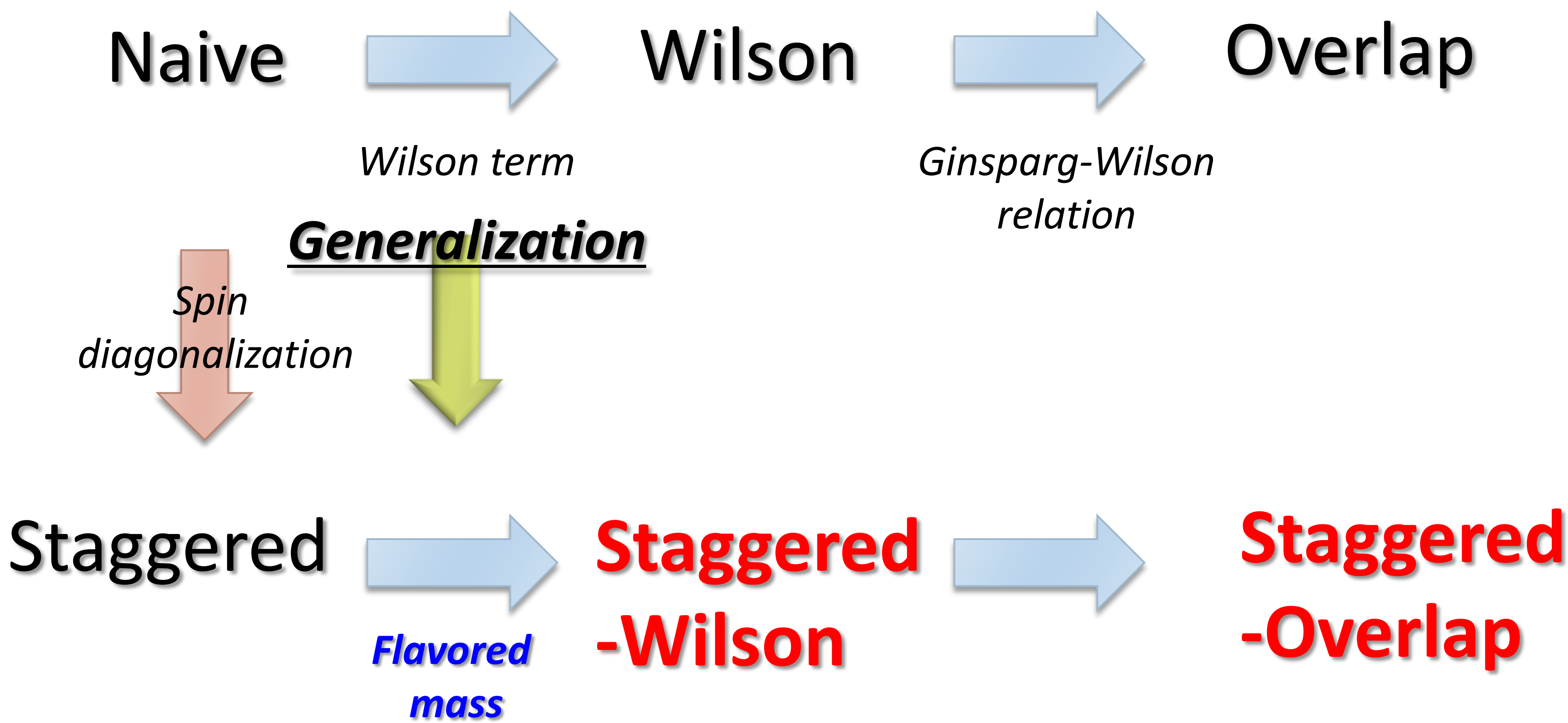

Adams (2011), Hoelbling (2011).
Creutz, Kimura, Misumi (2010)

D. H. Adams (2010)

Flavors of Staggered-Wilson Fermion

- Staggered-Wilson Fermion = Staggered + *Flavored Mass*
  - Adams type : *2* flavor (Doublers : 4 → 2) *Adams (2010,2011)*
  - Hoelbling type : *1* flavor (Doublers : 4 → 1) *Hoelbling (2011)*

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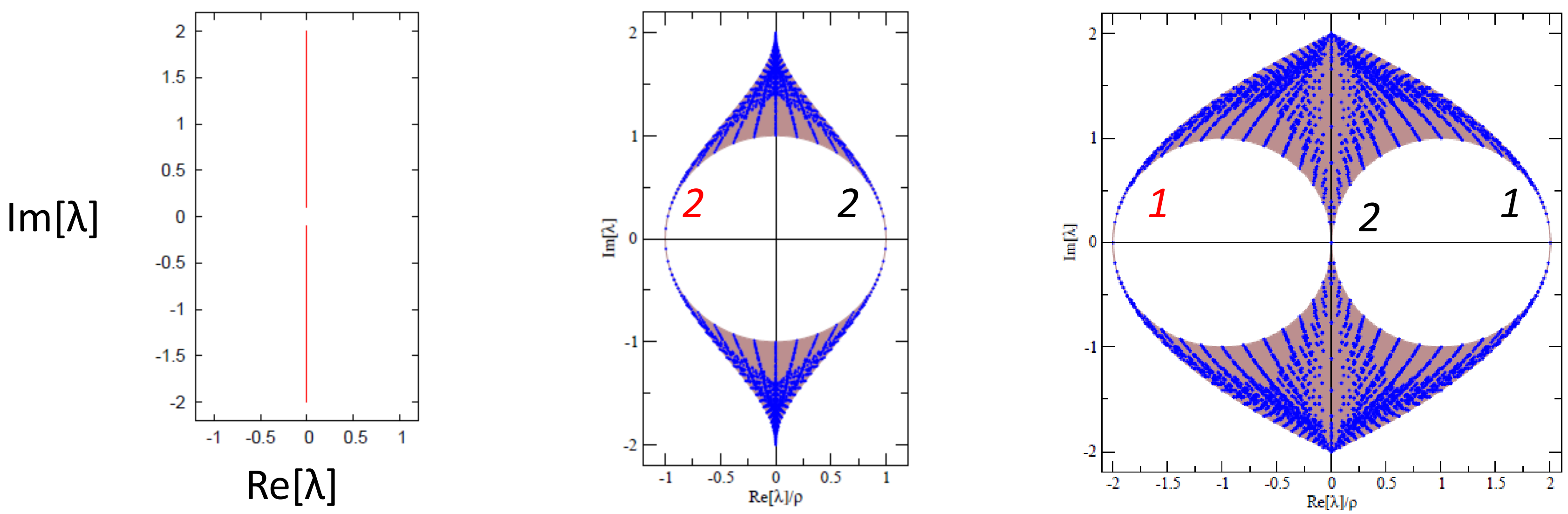
Dirac Spectrum (Free) *de Forcrand, Kurkela, Panero (2012)*

Staggered *Adams* *Hoelbling*



Properties of Staggered-Wilson Fermion

- Advantages
  - 1 or 2 Flavors, Anomaly (vs *Staggered Fermion*)
  - Lower Numerical Cost (vs *Wilson Fermion*) *de Forcrand, Kurkela, Panero (2011,2012)*
- Disadvantages
  - Lower Discrete Symmetries (Misumi's Plenary Talk, Monday)

Flavored Mass Terms

- Generalized Wilson terms *Golterman, Smit (1984)*
- Adams type (2 Flavor)
  - *4*-hopping

$$M_A = \sharp \sum_{sym} \epsilon \eta_5 \left(U_{\mu,x} U_{\nu,x+\hat{\mu}} U_{\rho,x+\hat{\mu}+\hat{\nu}} U_{\sigma,x+\hat{\mu}+\hat{\nu}+\hat{\rho}} + \cdots \right)$$

$$\sim 1 \otimes \xi_5$$

In spin-flavor representation

ξ_5 : γ_5 in flavor space

- Hoelbling type (1 Flavor)
  - *2*-hopping

$$M_H = \sharp \sum_{\mu \neq \nu} \eta_{\mu\nu} \left(U_{\mu,x} U_{\nu,x+\hat{\mu}} + \cdots \right)$$

$$\sim 1 \otimes \sigma_{\mu\nu}$$

$\eta_{\mu\nu}, \eta_5, \epsilon$: sign factor

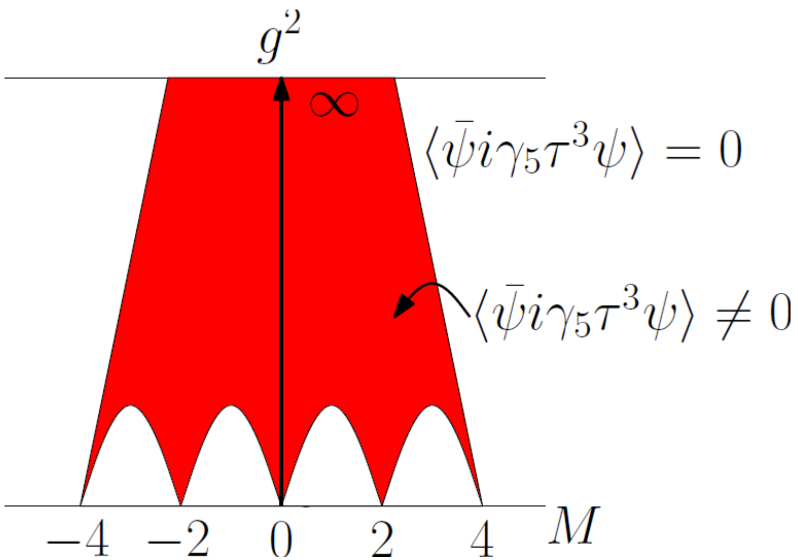
Contents

Phase Structure in Wilson Fermion

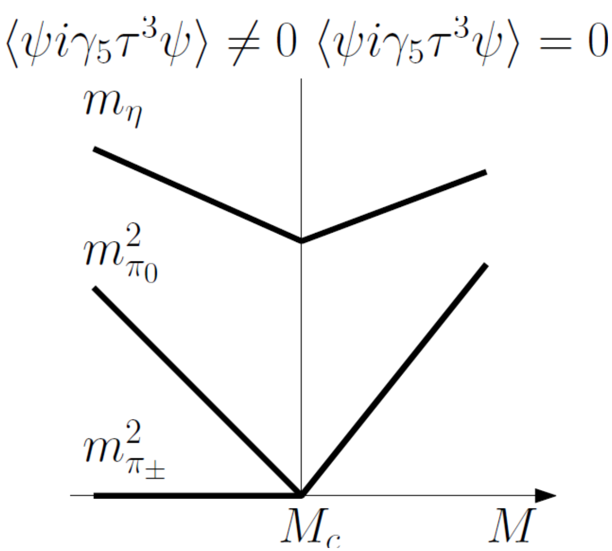
- Aoki Phase
  - Parity-flavor symmetry is spontaneously broken in some quark mass region. *Aoki(1984)*
- Quark mass tuning ➔ Chiral limit

Wilson Fermion (2 Flavor)

Phase structure



Pion Mass



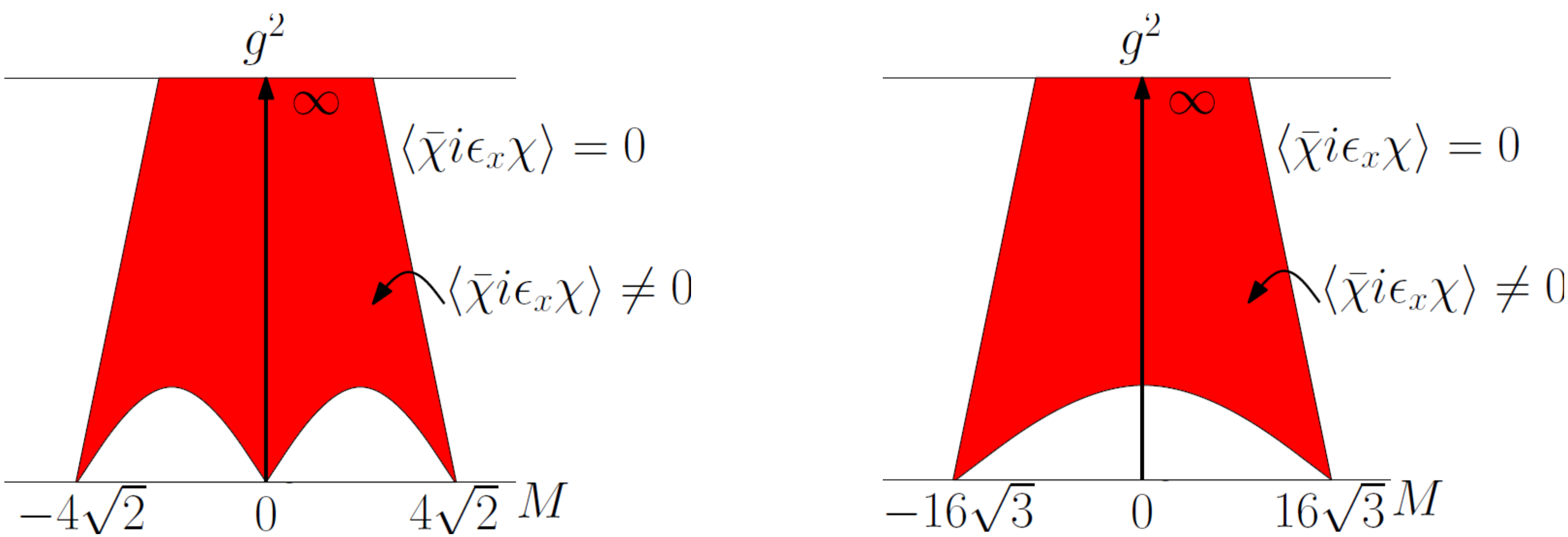
Aoki Phase in Staggered-Wilson Fermions?

- Flavored Mass : Generalized Wilson terms *Creutz, Kimura, Misumi (2010)*
- Previous study : Lattice Gross-Neveu model with flavored mass

  Creutz, Kimura, Misumi (2011)

Expected Phase Structure in Lattice QCD

Hoelbling type *Adams type*



Purpose

- We study Aoki phase of Staggered-Wilson fermion in strong coupling lattice QCD $\left(1/g^2 = 0\right)$.
  - Check the Chiral Limit

- Method
  - Hopping parameter expansion
  - Effective potential analysis

- Staggered-Wilson Fermion
  - Adams type , Hoelbling type

Contents

- Staggered-Wilson fermion
- Aoki phase (Purpose)
- **Strong coupling study for Aoki phase in Staggered-Wilson fermion**
  - Hopping parameter expansion
  - Effective potential
- Summary & Future works

Hopping Parameter Expansion (1 pt. Function)

- Diagrams (1 pt. function) ➔ $\langle \bar{\chi}_x \chi_x \rangle$ *Condensate*
  - Expansion for Hopping parameter *K* $K^{-1} = 2M$

e.g. Hoelbling type (1 Flavor)

$\langle \chi \bar{\chi} \rangle_x$

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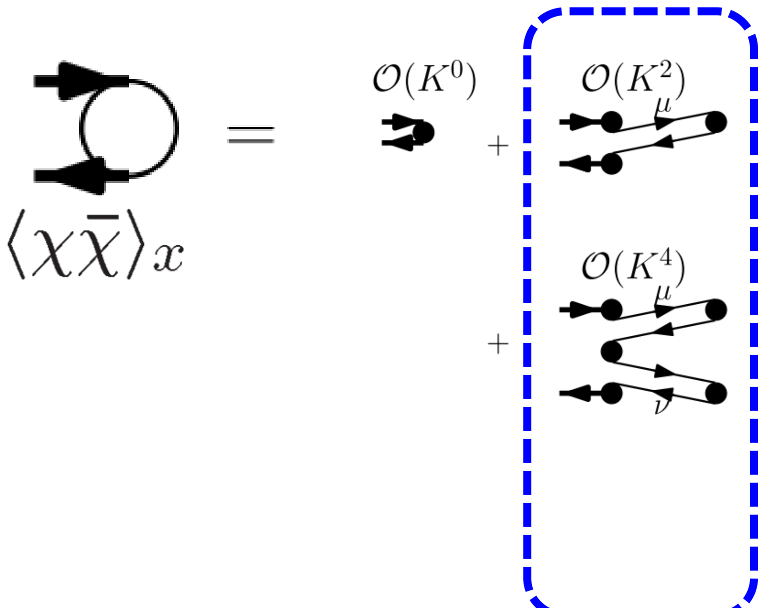

$+ \quad \mathcal{O}(K^5)$

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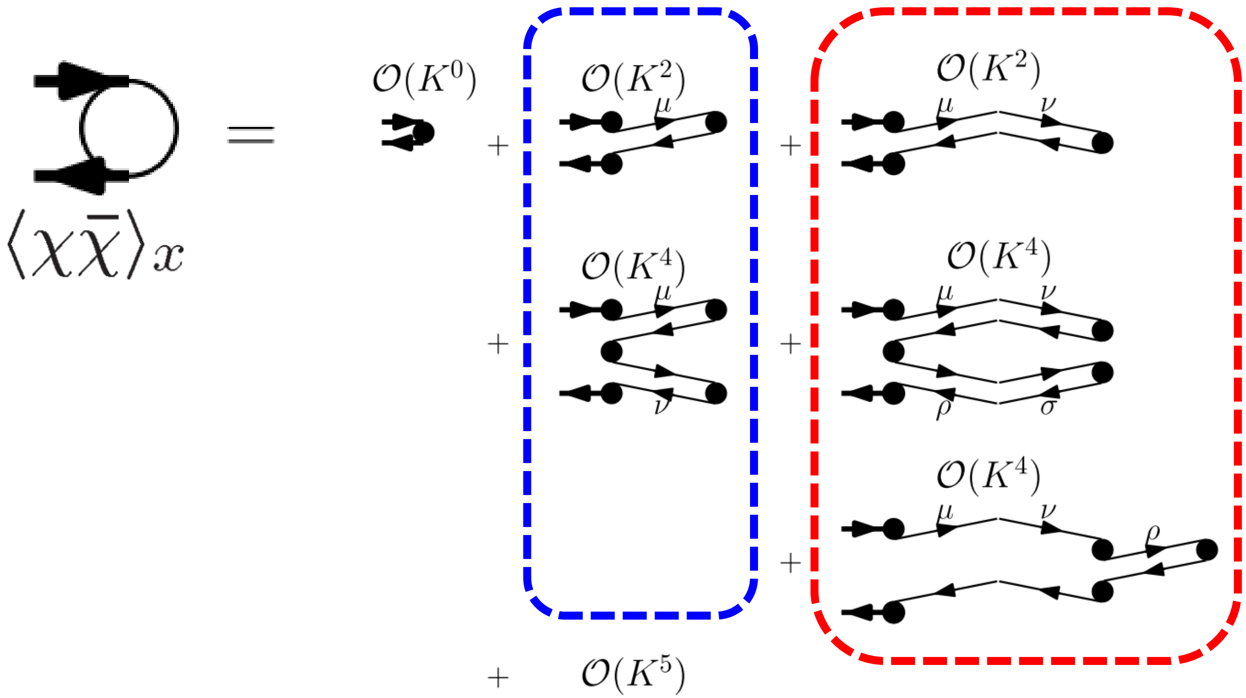
e.g. Hoelbling type (1 Flavor) *2 hopping (Flavored Mass)*



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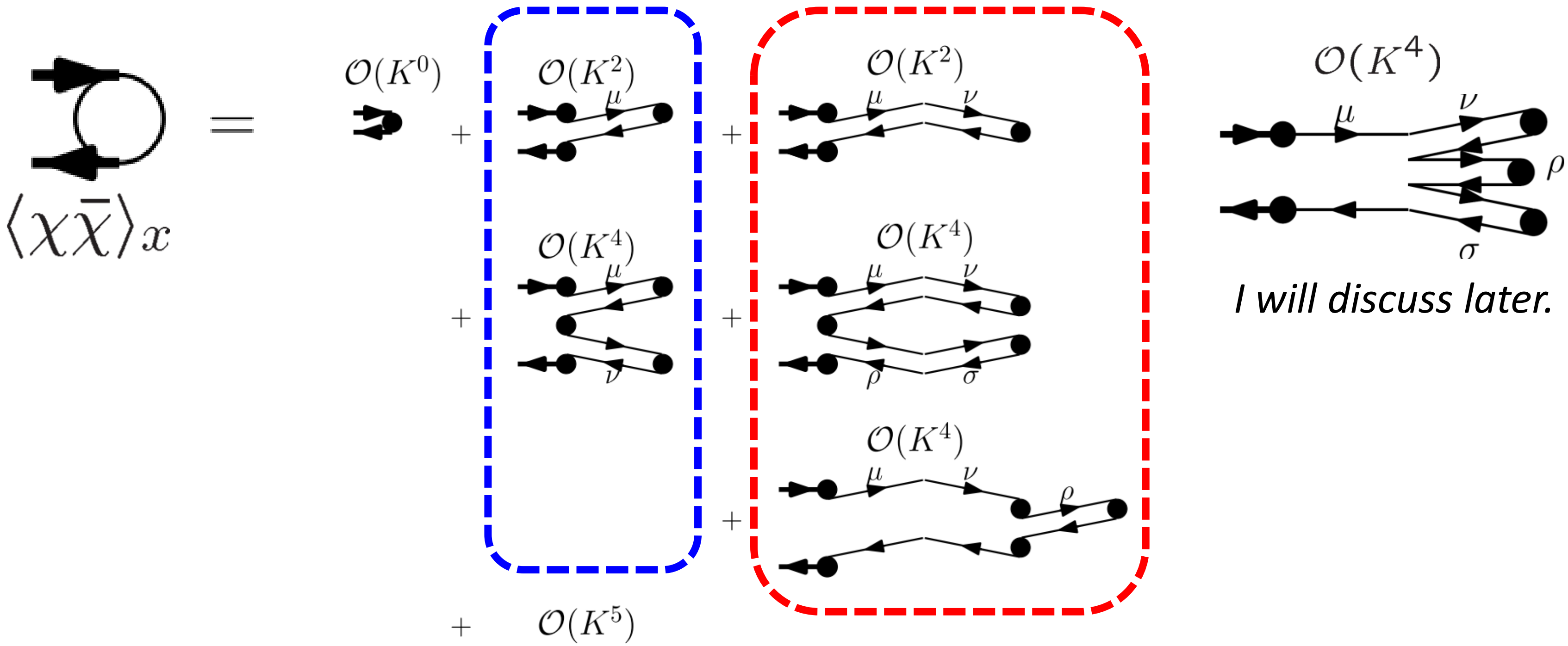
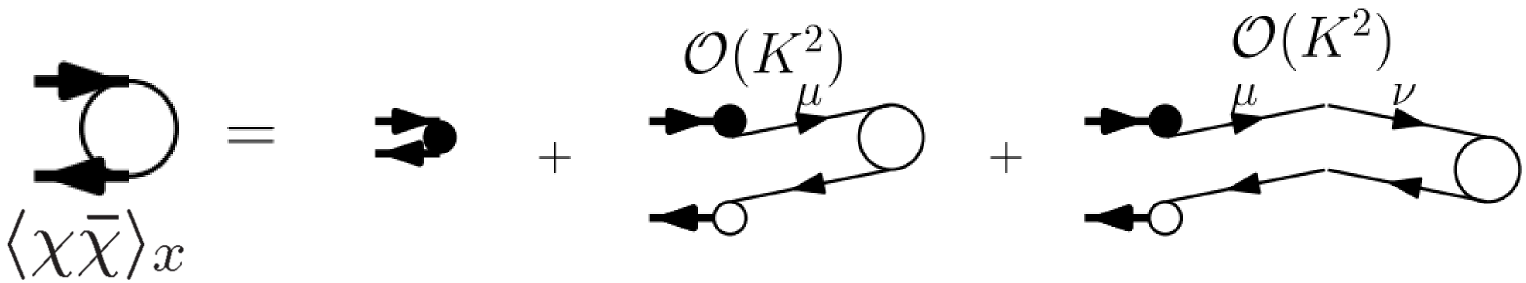


Hopping Parameter Expansion (1 pt. Function)

- Diagrams (1 pt. function) ➔ $\langle \bar{\chi}_x \chi_x \rangle$ *Condensate*
  - Summation for *K*. (K^2 , K^4 ,…)



Self-consistent equation

$$\langle \chi\bar{\chi} \rangle_x = -1 + 2K^2 \sum_{\mu} \langle \chi\bar{\chi} \rangle_x \langle \chi\bar{\chi} \rangle_{x+\hat{\mu}} - \frac{2}{3} K^2 \sum_{\mu \neq \nu} \langle \chi\bar{\chi} \rangle_x \langle \chi\bar{\chi} \rangle_{x+\hat{\mu}+\hat{\nu}}$$

Hopping Parameter Expansion (1 pt. Function)

- Mean-field treatment
  - Scalar $\langle \bar{\chi}\chi \rangle$
  - Pseudo-scalar $\langle \bar{\chi} i \epsilon_x \chi \rangle$

$$\epsilon_x \equiv (-1)^{x_1 + \cdots x_4} \sim \gamma_5 \otimes \xi_5$$

ξ_5 : γ_5 in flavor space

- Two solution
  - Parity symmetric

$$\frac{1}{N_c}\langle \bar{\chi}\chi \rangle = \frac{1}{2K}, \ \frac{1}{N_c}\langle \bar{\chi} i \epsilon_x \chi \rangle = 0$$

  - Parity broken

$$\frac{1}{N_c}\langle \bar{\chi}\chi \rangle = \frac{1}{8K}, \ \frac{1}{N_c}\langle \bar{\chi} i \epsilon_x \chi \rangle = \pm \frac{\sqrt{16K^2 - 1}}{8K}$$

For $|K| > \frac{1}{4}$*, Parity broken?*

Hopping Parameter Expansion (2 pt. Function)

- 2pt. Function $\langle \bar{\chi}_0^a i\epsilon_0 \chi_0^a \bar{\chi}_x^b i\epsilon_x \chi_x^b \rangle$ ➔ Pion mass

$$\cosh m_\pi = 1 + \frac{1 - 16K^2}{6K^2}$$

➡ For $|K| > \frac{1}{4}$, $m_\pi^2 < 0$?

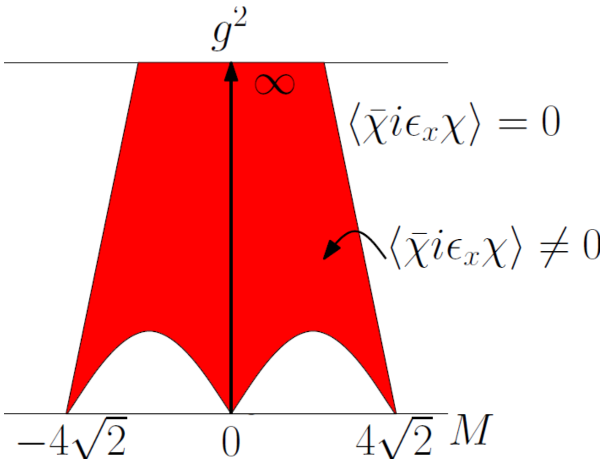
This result implies Aoki phase.

Effective Potential (Condensate)

- Effective Potential for Meson fields (large N_c)
  ➔We can identify the Parity-broken phase

$$V_{\text{eff}} = \frac{1}{V_4}\int \mathcal{D}[\chi,\bar{\chi},U]\exp\left[S_F\right]$$

	$\frac{1}{N_c}\langle\bar{\chi}\chi\rangle$	$\frac{1}{N_c}\langle\bar{\chi}i\epsilon_x\chi\rangle$
$M^2 > 4$	$\frac{1}{M}$	0
$M^2 < 4$	$\frac{M}{8-M^2}$	$\pm\frac{\sqrt{2(4-M^2)}}{8-M^2}$



For $M^2 < 4\left(|K| > \frac{1}{4}\right)$, $\langle\bar{\chi}i\epsilon_x\chi\rangle \neq 0$

Effective Potential (Pion Mass)

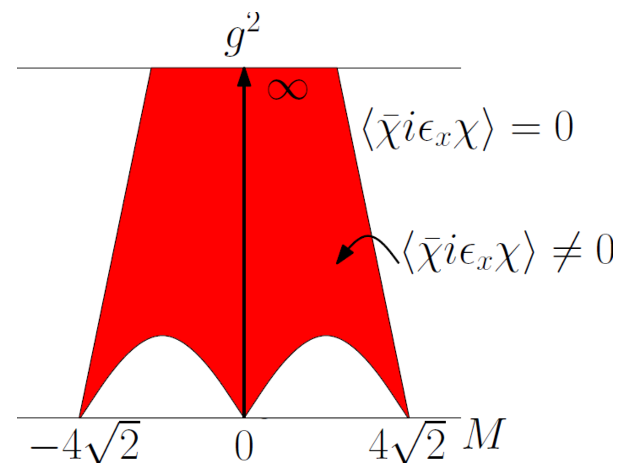
- Critical Mass (M_c) corresponds to the chiral limit.

$$m_\pi \leftarrow \frac{\partial^2 S_{\text{eff}}(M)}{\partial M_x \partial M_y}$$

M_x : Meson fields

$$\cosh m_\pi = 1 + \frac{2M^2 - 8}{3} \text{ for } M^2 > 4$$

$$\Rightarrow \quad m_\pi^2 = 0 \text{ for } M^2 = 4$$



Discussion on 2 Flavor Case

- Hoelbling (2 taste)
- Possibility
  - (1)Parity symmetry breaking

$$\langle \bar{\chi} i\epsilon_x \chi \rangle \neq 0, \langle \bar{\chi} i\epsilon_x \tau^3 \chi \rangle = 0$$

  - (2)Parity-flavor symmetry breaking

$$\langle \bar{\chi} i\epsilon_x \chi \rangle = 0, \langle \bar{\chi} i\epsilon_x \tau^3 \chi \rangle \neq 0$$

- Parity-flavor symmetry breaking
  - Vafa-Witten's theorem ➔ $\langle \bar{\chi} i\epsilon_x \chi \rangle = 0$

Vafa, Witten (1984)

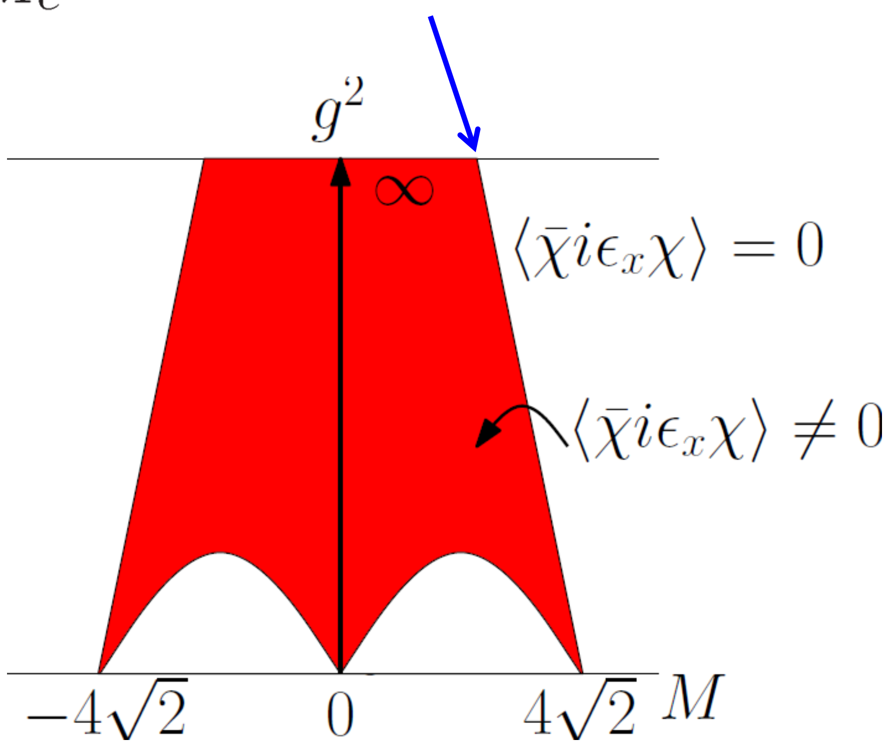
Brief Summary

- Hoelbling type (1 flavor)

$$\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0 \text{ for } |M| < M_c$$
$$m_\pi^2 = 0 \text{ for } |M| = M_c$$

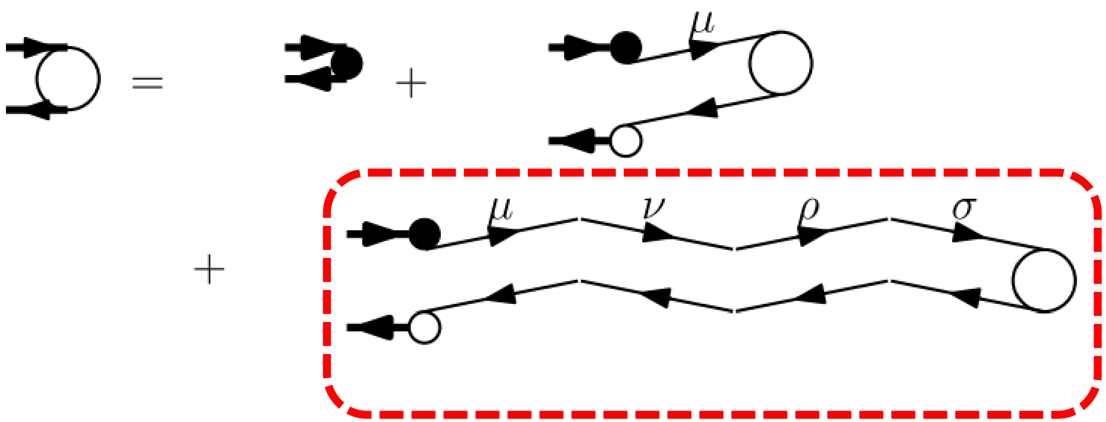
- We can perform the lattice simulation with staggered -Wilson fermions by tuning mass parameter.

Chiral Limit



Analysis in Adams type

- Similar way as Hoelbling type
  - e.g. Hopping Parameter Expansion (1 pt. function)

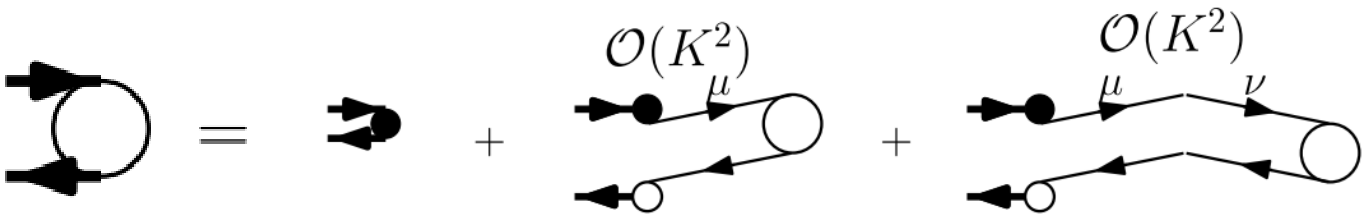


4 hopping (Flavored Mass)

$$\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0 \text{ for } |M| < M_c$$
$$m_\pi^2 = 0 \text{ for } |M| = M_c$$

Higher Order (Preliminary)

- This analysis is exact up to $O(K^3)$ *e.g. Hoelbling type (1 pt. Function)*



$\langle\chi\bar{\chi}\rangle_x$

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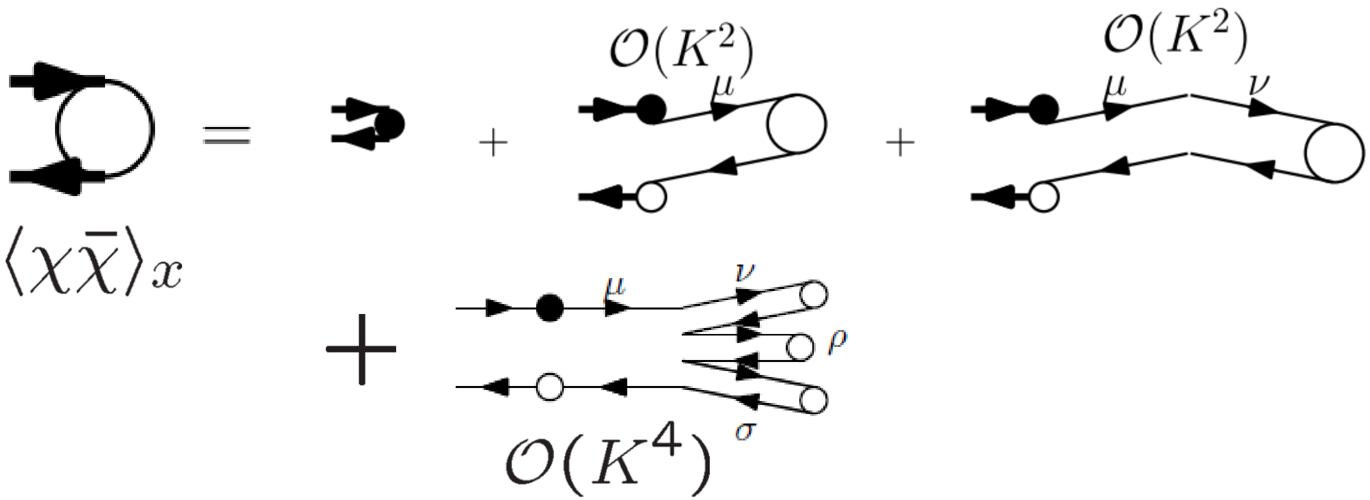

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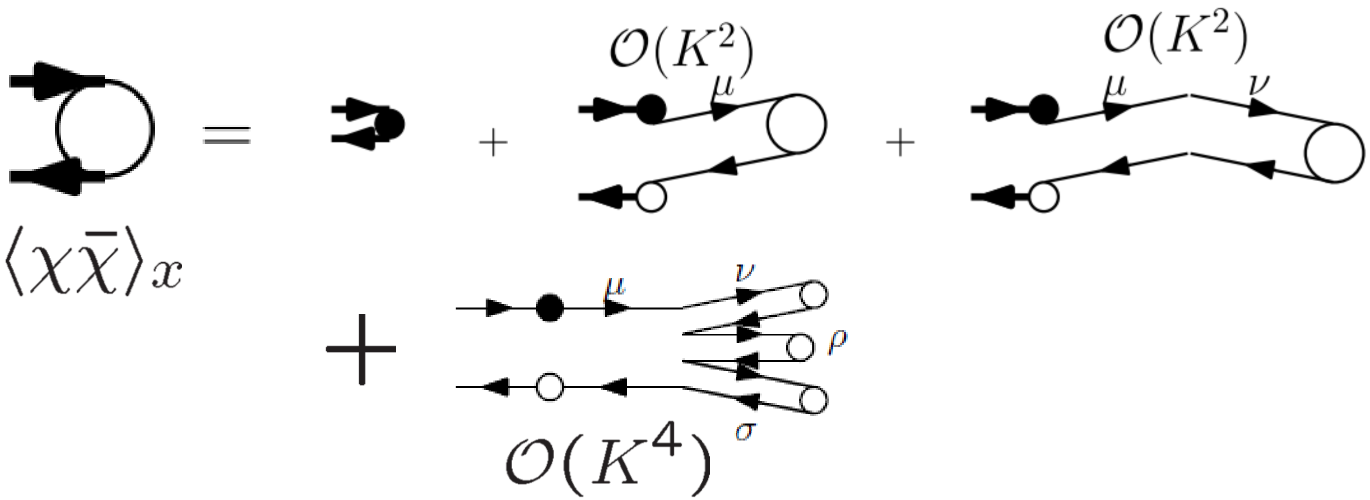
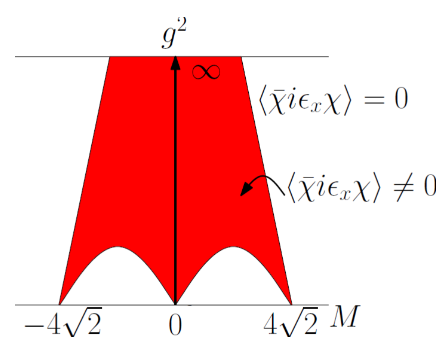


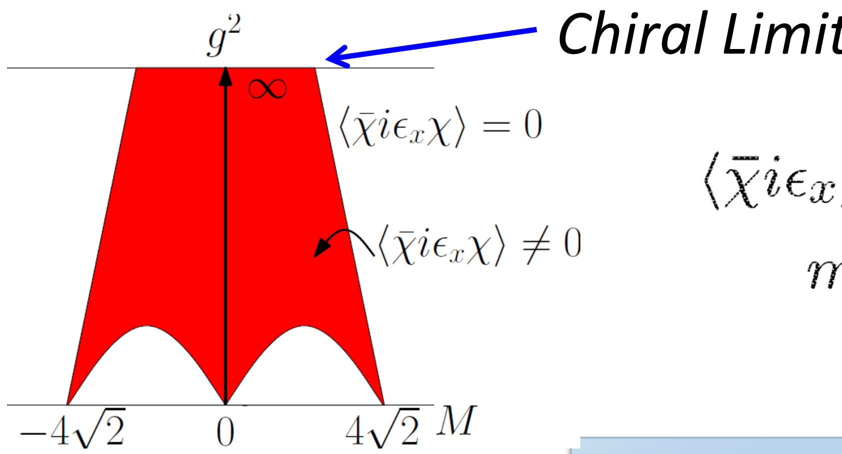
- The region of Aoki Phase is enlarged, but the symmetric phase exist.

$$K_c \searrow \quad (M_c \nearrow)$$



Summary & Future Works

- We have studied Aoki phase in Staggerd-Wilson fermion in strong coupling QCD.



$$\langle \bar{\chi} i \epsilon_x \chi \rangle \neq 0 \text{ for } |M| < M_c$$
$$m_\pi^2 = 0 \text{ for } |M| = M_c$$

- We can perform the lattice simulation with staggered-Wilson fermions by tuning mass parameter.
- Future Works
  - Evaluation for higher-order terms