Topological charge density correlator in Lattice QCD with two flavours of unimproved Wilson fermion

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Can unimproved Wilson fermion reproduce the chiral properties of continuum QCD?

Recently we have demonstrated the suppression of topological susceptibility with decreasing quark mass in the case of unimproved Wilson fermion.

Phys. Lett. B **707**, 228 (2012),
PoS LATTICE **2011**, 099 (2011).

Topological susceptibility (χ) is the volume integral of topological charge density correlator

$$\chi = \int d^4x \, \langle q(x) q(0) \rangle$$

where $q(x)$ is the topological charge density.

In this work we study the properties of the topological charge density correlator with unimproved Wilson fermion.

Topological charge density correlator $C(r)$:

$$C(r) = \langle q(x) q(0) \rangle, \quad r = |x|.$$

- Due to the pseudoscalar nature of $q(x)$ and reflection positivity of the Euclidean theory

$$C(r) = \langle q(x) q(0) \rangle \leq 0 \text{ for } x \neq 0.$$

- $C(r)$ has nonintegrable positive divergence at the origin and nonintegrable negative divergence close to the origin.
  E. Seiler, Phys. Lett. B **525**, 355 (2002),
  E. Seiler and I. O. Stamatescu, MPI-PAE/PTh 10/87.

Complications on Lattice:

- Lattice theory defined by a particular action may not be reflection positive $\rightarrow$ This is not a concern for Wilson fermion.
- The lattice operator for $q(x)$ may extend over several lattice spacings, and thus for sufficiently small x the continuum like behaviors are not expected. But, nevertheless these properties should emerge as lattice spacings become smaller and smaller.

- For topological charge density, we use the lattice approximation developed for $SU(2)$ by DeGrand, Hasenfratz and Kovacs (Nucl. Phys. **B505**, 417-441 (1997)), modified for $SU(3)$ by Hasenfratz and Nieter (Phys. Lett. **B439**, 366-372 (1998)).
- We used HYP smearing with optimized smearing coefficients $\alpha = 0.75$, $\alpha_2 = 0.6$ and $\alpha_3 = 0.3$ (A. Hasenfratz, F. Knechtli, Phys. Rev. **D64**, 034504 (2001)) to suppress the ultraviolet lattice artifacts.

Simulation Details

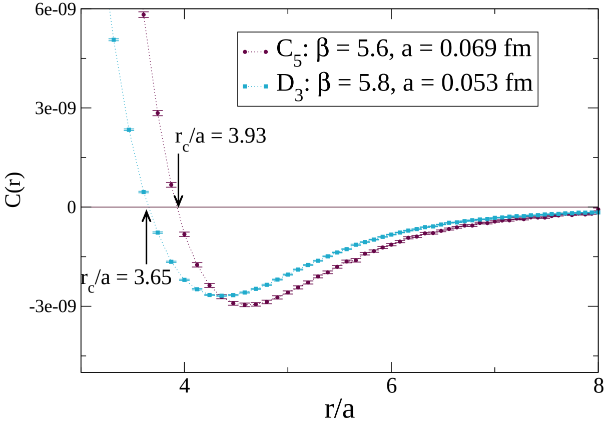
$\beta = 5.6$

tag	lattice	κ	block	N_2	N_{trj}	τ
A_{2b}	$16^3 \times 32$	0.158	8^4	10	6816	0.5
B_{1b}	$24^3 \times 48$	0.1575	$12^2 \times 6^2$	18	13128	0.5
B_{3a}	,,	0.158	$6^3 \times 8$	6	7200	0.5
B_{3b}	,,	0.158	$12^2 \times 6^2$	18	13646	0.5
B_{4a}	,,	0.158125	$6^3 \times 8$	8	9360	0.5
B_{4b}	,,	0.158125	$12^2 \times 6^2$	18	11328	0.5
B_{5a}	,,	0.15825	$6^3 \times 8$	8	6960	0.5
B_{5b}	,,	0.15825	$12^2 \times 6^2$	18	12820	0.5
C_2	$32^3 \times 64$	0.158	$8^3 \times 16$	8	7576	0.5
C_3	,,	0.15815	$8^3 \times 16$	8	9556	0.5
C_5	,,	0.1583	$8^3 \times 16$	8	11200	0.5

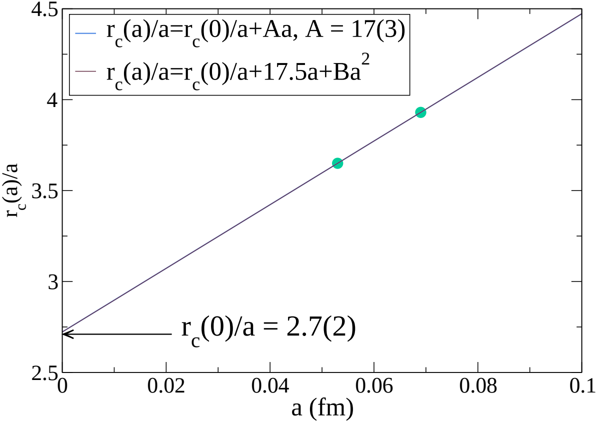
$\beta = 5.8$

tag	lattice	κ	block	N_2	N_{trj}	τ
D_1	$32^3 \times 64$	0.1543	$8^3 \times 16$	8	9600	0.5
D_2	,,	0.15455	$8^3 \times 16$	8	12160	0.5
D_3	,,	0.15462	$8^3 \times 16$	24	7776	0.5
D_5	,,	0.15475	$8^3 \times 16$	24	7336	0.5

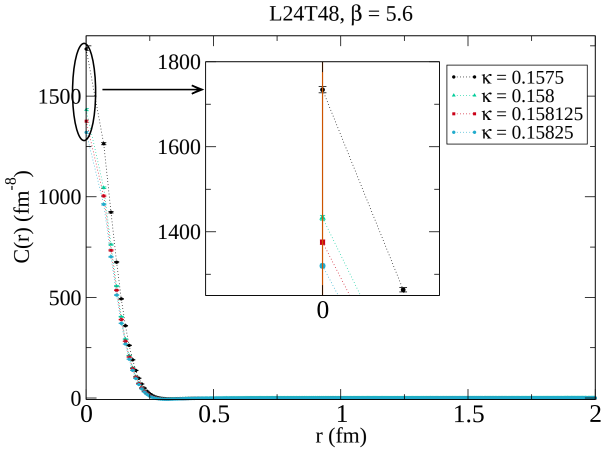
The negativity of the correlator

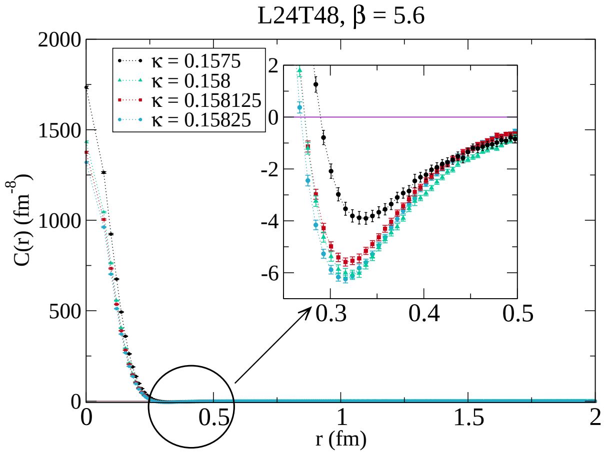


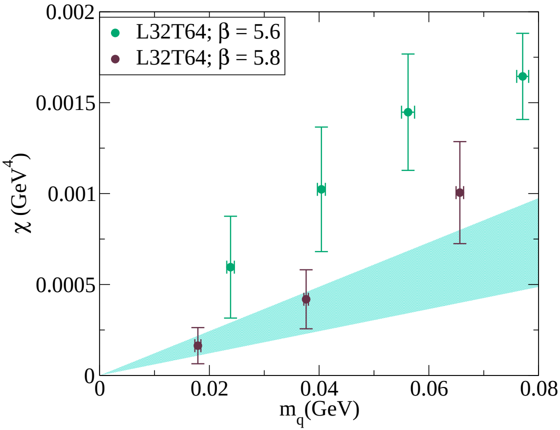
Radius of the positive core: continuum limit



Dependence on the quark mass

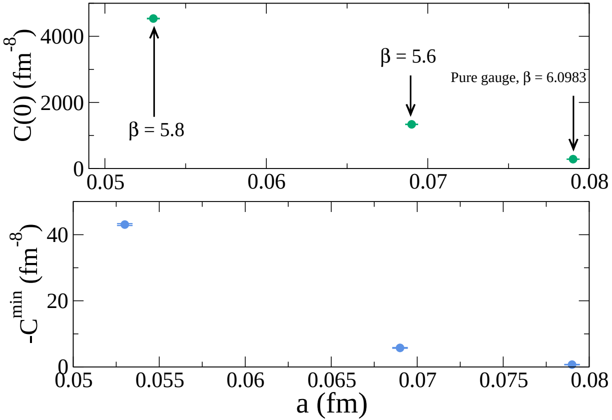



L24T48, β = 5.6
κ = 0.1575
κ = 0.158
κ = 0.158125
κ = 0.15825
C(r) (fm^-8)
r (fm)
2000
1500
1000
500
0
0
0.5
1
1.5
2
2
0
-2
-4
-6
0.3
0.4
0.5



L32T64; β = 5.6
L32T64; β = 5.8
0.002
0.0015
0.001
0.0005
0
χ (GeV^4)
0
0.02
0.04
0.06
0.08
m_q(GeV)

Contact term and short distance divergence

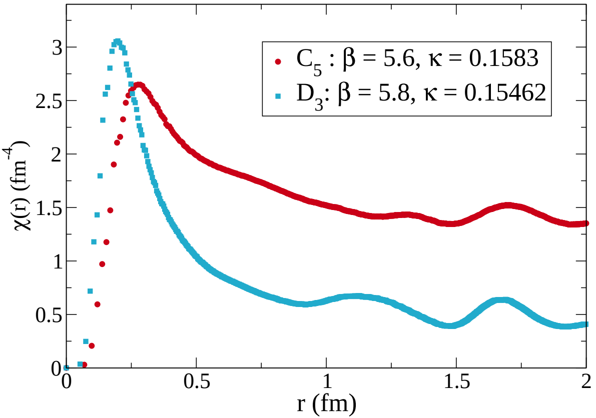


Nonintegrability of divergences

$$\chi = \int d^4 r C(r) = \int 2\pi^2 (r^3) C(r) dr$$

Define

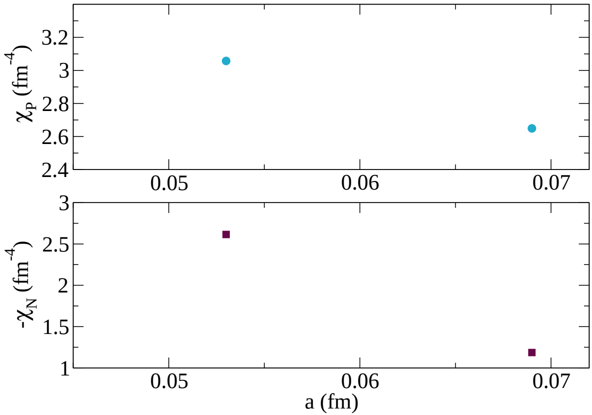
$$\chi(r) = \int_0^r 2\pi^2 (r'^3) C(r') dr'.$$



C5 : β = 5.6, κ = 0.1583
D3: β = 5.8, κ = 0.15462
χ(r) (fm-4)
r (fm)
0
0.5
1
1.5
2
2.5
3

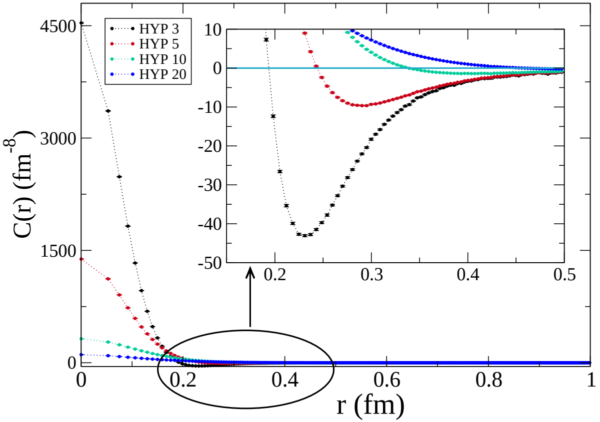
The contributions to the susceptibility from positive and negative parts of $C(r)$,

$$\chi_P(a) = \int_0^{r_c} 2\pi^2 (r'^3) C(r') dr'$$
$$\chi_N(a) = \int_{r_c}^{\infty} 2\pi^2 (r'^3) C(r') dr'$$

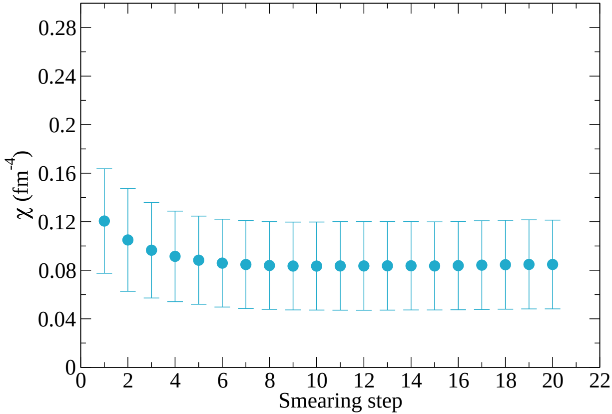


χ_P (fm^{-4})
3.2
3
2.8
2.6
2.4
0.05
0.06
0.07
$-\chi_N$ (fm^{-4})
3
2.5
2
1.5
1
0.05
0.06
0.07
a (fm)

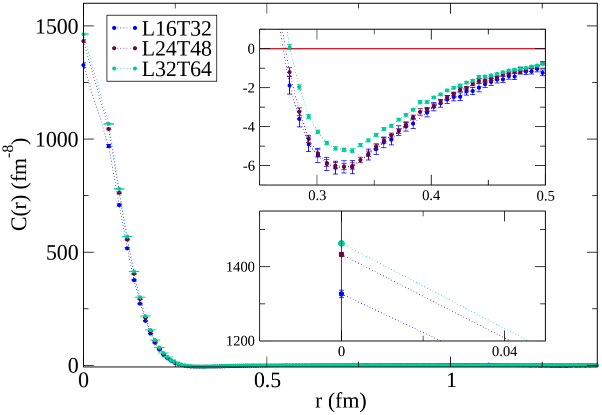
Effect of Smearing



Effect of Smearing



Volume Dependence



Conclusions

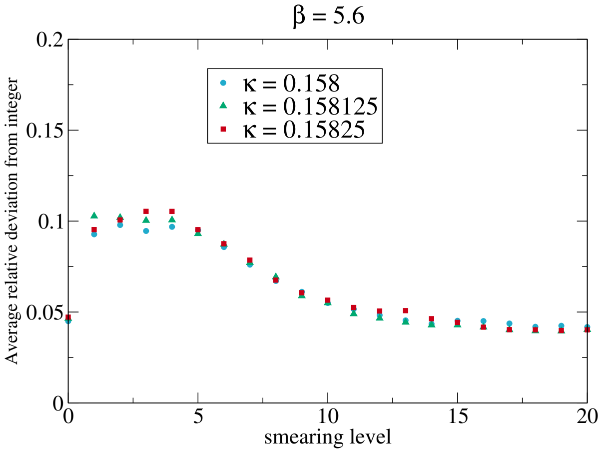
With naive Wilson fermion,

- the topological charge density correlator is negative beyond a positive core and radius of the core shrinks as lattice spacing decreases
- the contact term and radius of the positive core are decreasing with decreasing quark mass at a given lattice spacing, the negative peak increases with decreasing quark mass resulting in the suppression of the topological susceptibility with decreasing quark mass
- both the contact term and the negative peak diverge in nonintegrable fashion
- increasing levels of smearing suppresses the contact term and the negative peak keeping the susceptibility intact.
- C(r) is affected less by critical slowing down than topological suceptibility (Talk on Mon., Session: Algoritms and Machines)

THANK YOU

$\beta = 5.6$	MeV	
κ	m_q	m_{pi}
0.1575	123	790
0.15755	95	684
0.158	65	562
0.158125	51	499
0.15815	49	483
0.15825	35	416
0.1583	28	378
0.1584	21	315

$\beta = 5.8$, Volume=$32^3 64$	MeV	
κ	m_q	m_{pi}
0.1543	76	600
0.15455	42	453
0.15462	31	400
0.1547	18	317
0.15475	16	275



β = 5.6
κ = 0.158
κ = 0.158125
κ = 0.15825
Average relative deviation from integer
smearing level
0.2
0.15
0.1
0.05
0
0
5
10
15
20