Staggered Chiral Perturbation Theory for Neutral B Mixing

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Effective Weak Operators for B Mixing

- Neutral B_q mixing dominated by short distance contributions from the operators:

$$
\left.\begin{array}{lcl}
\mathcal{O}_1^q & = & \bar{b}_i \gamma^\nu L q_i \, \bar{b}_j \gamma^\nu L q_j \\
\mathcal{O}_2^q & = & \bar{b}_i L q_i \, \bar{b}_j L q_j \\
\mathcal{O}_3^q & = & \bar{b}_i L q_j \, \bar{b}_j R q_i
\end{array}\right\} \text{Standard Model}
$$

$$
\left.\begin{array}{lcl}
\mathcal{O}_4^q & = & \bar{b}_i L q_i \, \bar{b}_j R q_j \\
\mathcal{O}_5^q & = & \bar{b}_i L q_j \, \bar{b}_j R q_i
\end{array}\right\} \text{BSM} \, .
$$

- q is either d or s. i, j are color indices. $R = (1+\gamma_5)/2$, $L = (1-\gamma_5)/2$.
- These are a complete set (“SUSY-basis”) [Gabbiani et al., NPB 477 ('96) 321].
  - Matrix elements of all others can be written in terms of these 5 using Fierz, parity,. . .
    [see, e.g., C. Bouchard, FERMILAB-THESIS-2011-32.]

Effective Weak Operators for B Mixing

- $\mathcal{O}_1$ (which mixes with $\mathcal{O}_2$ under renormalization) gives ΔM_q in SM:

$$\langle \bar{B}^0_q | \mathcal{O}^q_1 | B^0_q \rangle(\mu) = \frac{2}{3} M^2_{B_q} f^2_{B_q} \hat{B}_{B_q}(\mu)$$

$$\Delta M^{\mathrm{SM}}_q = |V^*_{tq} V_{tb}|^2 \underbrace{\frac{G^2_F M^2_W}{6\pi^2} \eta^B_2 S_0(x_t) M_{B_q}}_{\text{known stuff}} \underbrace{f^2_{B_q} \hat{B}_{B_q}}_{\text{lattice}}$$

- Quantity ξ is particularly useful in CKM unitarity triangle analysis.

$$\xi \equiv \frac{f_{B_s}\sqrt{\hat{B}_{B_s}}}{f_{B_d}\sqrt{\hat{B}_{B_d}}}$$

$$\left|\frac{V_{td}}{V_{ts}}\right| = \xi \sqrt{\frac{\Delta M_d \, M_{B_s}}{\Delta M_s \, M_{B_d}}}$$

Lattice Calculations of B Mixing

- Calculations with $2+1$ flavors by HPQCD [Gámiz et al., PRD 80 ('09) 014503] and Fermilab/MILC [Bazavov et al., arXiv:1205.7013] use staggered light quarks with NRQCD or Fermilab heavy quarks.
- To make heavy-light bilinears or 4-quark ops., convert staggered fermion $\chi(x)$ to naive fermion $\Psi(x)$ [Wingate et al., PRD 67 ('03) 054505].
- Then couple locally
  - bilinear: $\bar{Q}(x)\Gamma\Psi(x)$.
  - 4-quark operator: $\bar{Q}(x)\Gamma\Psi(x)\ \bar{Q}(x)\Gamma'\Psi(x)$

    (Γ,Γ' some Dirac spin matrices).

Lattice Operators

- Bilinear $\bar{Q}(x)\Gamma\Psi(x)$ works exactly as desired:
  - Sum over $\boldsymbol{x}$ includes sum over spatial cubes.
  - Properly converts χ to spin-taste basis.
- Four quark operators $\bar{Q}(x)\Gamma\Psi(x)\ \bar{Q}(x)\Gamma'\Psi(x)$ are more complicated.
  - Each bilinear not separately summed over spatial cubes, so undesired spin-tastes can enter.

Naive Fermions

- Naive light quark action can be rewritten as four copies of the staggered action:

$$\Psi(x) = \Omega(x)\, \underline{\chi}\,, \qquad \Omega(x) = \gamma_0^{x_0}\, \gamma_1^{x_1}\, \gamma_2^{x_2}\, \gamma_3^{x_3}\ .$$

- $\underline{\chi}$ is a "copied" staggered field, with each Dirac component $\underline{\chi}_i$ separately having the staggered action.
- "Copy symmetry": the $SU(4)$ that acts on copy index i. Copy symmetry an exact lattice symmetry, so

$$\langle \underline{\chi}_i(x)\, \bar{\underline{\chi}}_{i'}(y) \rangle = \delta_{i,i'}\ \langle \chi(x)\, \bar{\chi}(y) \rangle\ .$$

(χ is normal (uncopied) staggered field.)

- Implies

$$\langle \Psi(x)\, \bar{\Psi}(y) \rangle = \Omega(x)\, \Omega^\dagger(y) \langle \chi(x)\, \bar{\chi}(y) \rangle\ .$$

  - Standard way to convert from naive to staggered quarks [Wingate et al, PRD 67 ('03) 054505].
  - In practice, use this for propagators; don't need to construct naive field Ψ itself.

Naive Fermions: Bilinears

- Construct interpolating field $\mathcal{H}(x)$ for a heavy-light pseudoscalar meson by writing

$$\mathcal{H}(x) = \bar{Q}(x)\,\gamma_5\,\Psi(x) = \bar{Q}(x)\,\gamma_5\,\Omega(x)\,\underline{\chi}(x),$$

- $\mathcal{H}(x)$ is always summed over a time-slice (either explicitly, or implicitly by using translation invariance).
- To leading order in a, $Q(x)$ varies smoothly (up to gauge transformation) between neighboring spatial sites, but $\underline{\chi}$ does not, due to taste doubling.
- Staggered fields are smooth in the spin-taste basis on the doubled lattice: need to sum over hypercubes.

Naive Fermions: Bilinears

- Focus on the average of $\mathcal{H}(x)$ over a spatial cube.
  - Let $x = (t, \boldsymbol{x})$ with $\boldsymbol{x} = 2\boldsymbol{y}$ even, & let $\eta = (\eta_0, \boldsymbol{\eta})$ have all components 0 or 1.
- For t even ($t = 2\tau$):

$$\begin{aligned}
\mathcal{H}^{(\mathrm{av})}(t,\boldsymbol{x}) &= \frac{1}{8}\sum_{\boldsymbol{\eta}} \bar{Q}(t,\boldsymbol{x}+\boldsymbol{\eta})\,\gamma_5\,\Omega(2\tau,\boldsymbol{\eta})\,\underline{\chi}(2\tau, 2\boldsymbol{y}+\boldsymbol{\eta}) \\
&\cong \frac{1}{8}\,\bar{Q}(t,\boldsymbol{x})\,\gamma_5 \sum_{\boldsymbol{\eta}} \Omega(\boldsymbol{\eta})\,\underline{\chi}(2\tau, 2\boldsymbol{y}+\boldsymbol{\eta}) \\
&\cong \frac{1}{16}\,\bar{Q}(t,\boldsymbol{x})\,\gamma_5 \sum_{\eta} \Big[\Omega(\eta)\,\underline{\chi}(2\tau+\eta_0, 2\boldsymbol{y}+\boldsymbol{\eta}) + \\
&\qquad +(-1)^{\eta_0}\,\Omega(\eta)\,\underline{\chi}(2\tau+\eta_0, 2\boldsymbol{y}+\boldsymbol{\eta})\Big]
\end{aligned}$$

- (Inserted gauge links for point-split quantities implicit.)

Naive Fermions: Bilinears

- For t odd ($t = 2\tau + 1$), result the same except the term on the last line changes sign. This is usual oscillating state with opposite parity.
- For simplicity, assume from now on that oscillating state is removed by fitting procedure & all components of x are even.
- Then

$$\mathcal{H}^{(\mathrm{av})}(x) \to \frac{1}{16}\,\bar{Q}(x)\,\gamma_5 \sum_\eta \Omega(\eta)\,\underline{\chi}(2y+\eta)$$

- To convert to spin-taste basis, use:

$$q^{\alpha a}(y) = \frac{1}{8}\sum_\eta \Omega^{\alpha a}(\eta)\,\chi(2y+\eta),$$

where α a spin index, and a a taste index.

Naive Fermions: Bilinears

- The copied version of spin-taste basis:

$$q_i^{\alpha a}(y) = \frac{1}{8}\sum_\eta \Omega^{\alpha a}(\eta)\,\underline{\chi}_i(2y+\eta)\;.$$

- Note: $\mathcal{H}^{(\mathrm{av})}(x) \propto \gamma_5 \sum_\eta \Omega(\eta)\,\underline{\chi}(2y+\eta)$ has matrix multiplication $\Rightarrow$ couples taste and copy indices.
- Get (with spin indices implicit from now on):

$$\mathcal{H}^{(\mathrm{av})}(x) \to \frac{1}{2}\,\bar{Q}(x)\,\gamma_5\,q_i^a(y)\,\delta_i^a\;.$$

- Using copy symmetry, contraction of $\mathcal{H}$ with $\mathcal{H}^\dagger$ automatically averaged over tastes:

$$\langle \mathcal{H}(x)\,\mathcal{H}^\dagger(x')\rangle \sim \frac{1}{4}\,\langle \bar{Q}(x)\gamma_5 q^a(y)\;\bar{q}^a(y')\gamma_5 Q(x')\rangle\;.$$

Naive Fermions: Four-Quark Operators

- Two bilinears not separately summed over space.
- Disentangle using:

$$\frac{1}{256}\sum_K tr\Big(\Omega(\eta)\,K\,\Omega^\dagger(\eta)\,K\Big)\;\; tr\Big(\Omega(\eta')\,K\,\Omega^\dagger(\eta')\,K\Big) = \delta_{\eta,\eta'}\ .$$

(K runs over 16 independent Hermitian gamma matrices.)

- Get, for operator $\mathcal{O}_n$ ($n = 1, \ldots, 5$)

$$\mathcal{O}_n^{(av)}(x) \quad\rightarrow\quad \frac{1}{4}\sum_K (\bar{Q}\Gamma_n K q_k^c\ \bar{Q}\Gamma'_n K q_\ell^d)\ K_{ck}K_{d\ell}$$

- Contributions with $K \neq I$ have wrong spin, and funny coupling of taste (c, d) and copy (k, ℓ) indices.
  - Have dropped “wrong parity” part, which doesn’t contribute if oscillating terms are removed from fit.

Naive Fermions: Four-Quark Operators

$$\mathcal{O}_n^{(av)} \to \frac{1}{4}\sum_K (\bar{Q}\Gamma_n K q_k^c\ \bar{Q}\Gamma'_n K q_\ell^d)\ K_{ck}K_{d\ell}$$

- Recall:

$$\mathcal{H}^{(\mathrm{av})}(x) \to \frac{1}{2}\ \bar{Q}(x)\,\gamma_5\, q_i^a(y)\,\delta_i^a\ .$$

- Copy symmetry then implies

$$\langle \mathcal{H}^\dagger \mathcal{O}_n^{(av)} \mathcal{H}^\dagger \rangle \propto \langle D^{ac} D^{ed} \rangle K_{ca} K_{de} + \text{(2nd equivalent contraction)},$$

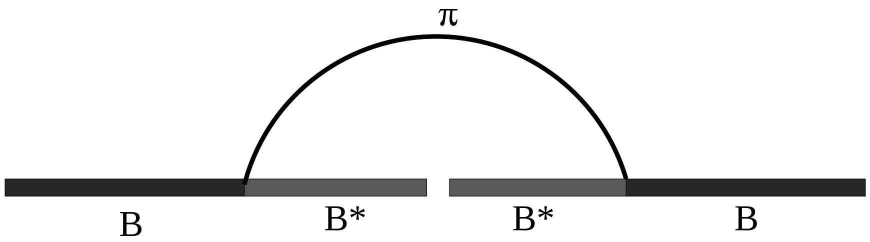
  where D^{ac} is quark propagator (in a given background) for taste a into taste c.
- When taste symmetry exact, $\langle D^{ac} D^{ed} \rangle \propto \delta_{ac}\delta_{ed}\ \Rightarrow$ only correct spin ($K = I$) contributes.
- So continuum limit is correct.

Naive Fermions: Four-Quark Operators

- But at one loop, taste-violations mean that $\langle D^{ac} D^{ed} \rangle$ doesn't have to be proportional to $\delta_{ac}\delta_{ed} \quad \Rightarrow$ wrong spins can contribute.
- For example, the taste-violating hairpin with vector taste can give a term proportional to $\xi^{\mu}_{ac}\, \xi^{\mu}_{ed}$
  - then the spin of the operator is $\Gamma_n \gamma_\mu \otimes \Gamma'_n \gamma_\mu$ instead of $\Gamma_n \otimes \Gamma'_n$
- This is different from wrong spin/wrong taste operators from perturbative corrections. They are already suppressed by $\alpha_S/4\pi$, and one loop makes them $\mathcal{O}(a^2 \alpha_S/4\pi)$, which is effectively NNLO.

SχPT for B Mixing

- Meson Diagrams:
  - Sunset diagram:

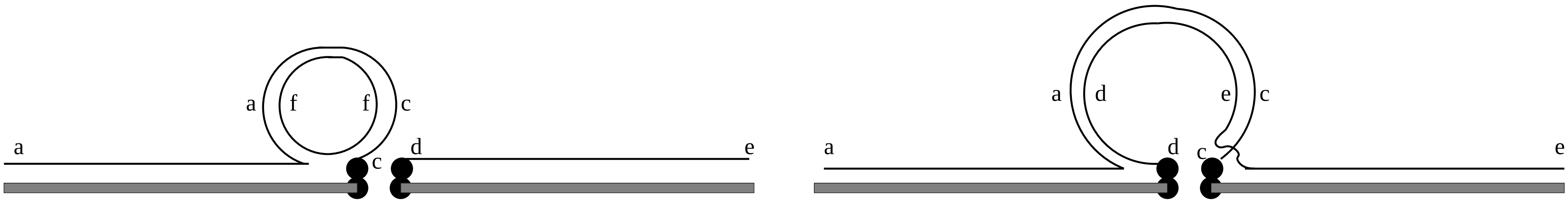


  - Tadpoles:

  - There are also wave-function renormalization diagrams.

Quark Flow Diagrams

- Effect of wrong spins depends quark flow. As example, look at tadpoles with connected (no hairpin) pion propagators.



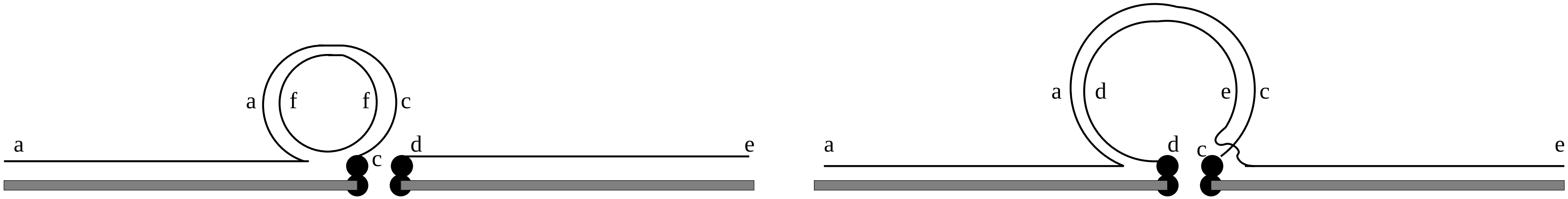
each gets multiplied by $K_{ca}K_{de}$

- Left diagram for pion of taste Ξ (one of 16) is proportional to

$$\Xi^{\mu}_{af}\Xi^{\mu}_{fc}\delta_{ed}K_{ca}K_{de} = [\mathrm{tr(K)}]^2$$

  - Only correct spin/taste ($K = I$) contributes.

Quark Flow Diagrams



each gets multiplied by $K_{ca}K_{de}$

- Right diagram for pion of taste Ξ is proportional to

$$\Xi_{ad}\,\Xi_{ec}\;K_{ca}\,K_{de} = \mathrm{tr}(\Xi\;\mathrm{K}\;\Xi\;\mathrm{K})$$

  - Because of taste violations, rest of diagram (pion propagator) depends on Ξ (i.e., whether it's S,V,T,A or P).
  - $\Rightarrow$ Various K values contribute $\Rightarrow$ various wrong spins.
  - (If propagator were independent of Ξ, sum on tastes would give $K = I$.)

Calculation

- Wrong spin terms $\Rightarrow$ different operators $\Rightarrow$ different chiral representatives.
- Fortunately, all operators can be written as linear combination of "SUSY basis" ($\mathcal{O}_1, \ldots, \mathcal{O}_5$).
- Detmold and Lin give chiral representatives of all five operators [PRD 76 ('07) 014501].
- With P, P^* heavy-light mesons, σ pion field, x light flavor, c, d tastes, get:

$$
\begin{aligned}
O_1^{xc;xd} &= \beta_1 \left[\left(\sigma P^{(b)\dagger}\right)_{x,c} \left(\sigma P^{(\bar b)}\right)_{x,d} + \left(\sigma P_\mu^{*(b)\dagger}\right)_{x,c} \left(\sigma P^{*(\bar b),\mu}\right)_{x,d} \right] \quad [\textbf{or } c \leftrightarrow d], \\
O_{2(3)}^{xc;xd} &= \beta_{2(3)} \left(\sigma P^{(b)\dagger}\right)_{x,c} \left(\sigma P^{(\bar b)}\right)_{x,d} + \beta'_{2(3)} \left(\sigma P_\mu^{*(b)\dagger}\right)_{x,c} \left(\sigma P^{*(\bar b),\mu}\right)_{x,d} \quad [\textbf{or } c \leftrightarrow d], \\
O_{4(5)}^{xc;xd} &= \frac{\beta_{4(5)}}{2} \left[\left(\sigma P^{(b)\dagger}\right)_{x,c} \left(\sigma^\dagger P^{(\bar b)}\right)_{x,d} + \left(\sigma^\dagger P^{(b)\dagger}\right)_{x,c} \left(\sigma P^{(\bar b)}\right)_{x,d} \right] \\
&\quad + \frac{\beta'_{4(5)}}{2} \left[\left(\sigma P_\mu^{*(b)\dagger}\right)_{x,c} \left(\sigma^\dagger P^{*(\bar b),\mu}\right)_{x,d} + \left(\sigma^\dagger P_\mu^{*(b)\dagger}\right)_{x,c} \left(\sigma P^{*(\bar b),\mu}\right)_{x,d} \right] \quad [\textbf{or } c \leftrightarrow d].
\end{aligned}
$$

- Effect of copy indices $\Rightarrow$ external taste-a quark contracts with operator taste c; similarly for e and d.

Calculation

- Write complete answer as

$$\langle \overline{B}^0_x | O^x_n | B^0_x \rangle = \beta_n \left(1 + \mathcal{W}_B + \mathcal{T}^{(n)}_x + \tilde{\mathcal{T}}^{(n)}_x\right) + \beta'_n \left(\mathcal{Q}^{(n)}_x + \tilde{\mathcal{Q}}^{(n)}_x\right) + \text{analytic terms},$$

  where $\mathcal{W}_B$ is B wave function renormalization, $\mathcal{T}$ and $\tilde{\mathcal{T}}$ are the right- and wrong-spin tadpole diagrams, and $\mathcal{Q}$ and $\tilde{\mathcal{Q}}$ are the right- and wrong-spin sunset diagrams.
  - x labels light quark flavor.
  - $\beta'_1 = \beta_1$ by heavy-quark spin argument [Detmold and Lin].
- $\mathcal{W}_B$ is standard — same in naive quark version as in normal SχPT.

Final χPT Results

- For operator $\mathcal{O}_1^x$ (others are similar):

$$\mathcal{T}_x^{(1)} = \frac{-i}{f_\pi^2}\Bigg\{ \frac{1}{16}\sum_{\mathcal{S},\rho} N_\rho\,\mathcal{I}_{x\mathcal{S},\rho} + \frac{1}{16}\sum_{\rho} N_\rho\,\mathcal{I}_{X,\rho} + \frac{2}{3}\Bigg[R^{[2,2]}_{X_I}(\{M^{(5)}_{X_I}\};\{\mu_I\})\,\frac{\partial \mathcal{I}_{X,I}}{\partial m^2_{X_I}}$$

$$- \sum_{j\in\{M_I^{(5)}\}} D^{[2,2]}_{j,X_I}(\{M^{(5)}_{X_I}\};\{\mu_I\})\mathcal{I}_{j,I}\Bigg] + a^2\delta'_V\Bigg[R^{[3,2]}_{X_V}(\{M^{(7)}_{X_V}\};\{\mu_V\})\,\frac{\partial \mathcal{I}_{X,V}}{\partial m^2_{X_V}}$$

$$- \sum_{j\in\{M_V^{(7)}\}} D^{[3,2]}_{j,X_V}(\{M^{(7)}_{X_V}\};\{\mu_V\})\mathcal{I}_{j,V}\Bigg] + (V\to A)\Bigg\},$$

$$\tilde{\mathcal{T}}_x^{(1)} = \frac{-i}{f_\pi^2}\Bigg\{ \frac{1}{16}\Big(-5\mathcal{I}_{X,P} - 4\mathcal{I}_{X,A} + 18\mathcal{I}_{X,T} - 4\mathcal{I}_{X,V} - 5\mathcal{I}_{X,I}\Big)$$

$$+ \frac{2(\beta_2+\beta_3)}{\beta_1}\Bigg(-\mathcal{I}_{X,V} + \mathcal{I}_{X,A}$$

$$+a^2\delta'_V\Bigg[R^{[3,2]}_{X_V}(\{M^{(7)}_{X_V}\};\{\mu_V\})\,\frac{\partial \mathcal{I}_{X,V}}{\partial m^2_{X_V}} - \sum_{j\in\{M_V^{(7)}\}} D^{[3,2]}_{j,X_V}(\{M^{(7)}_{X_V}\};\{\mu_V\})\mathcal{I}_{j,V}\Bigg]$$

$$-a^2\delta'_A\Bigg[R^{[3,2]}_{X_A}(\{M^{(7)}_{X_A}\};\{\mu_A\})\,\frac{\partial \mathcal{I}_{X,A}}{\partial m^2_{X_A}} - \sum_{j\in\{M_A^{(7)}\}} D^{[3,2]}_{j,X_A}(\{M^{(7)}_{X_A}\};\{\mu_A\})\mathcal{I}_{j,A}\Bigg]\Bigg)\Bigg\}$$

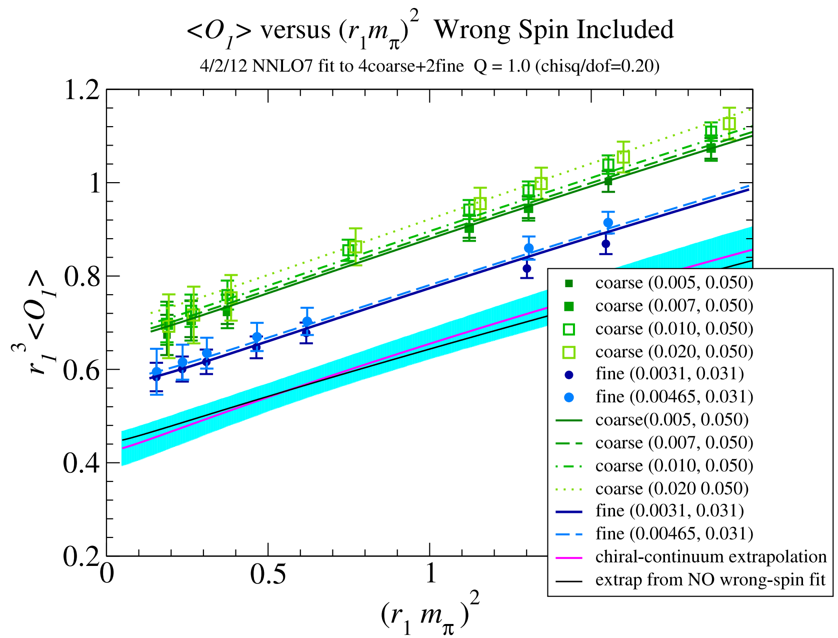
Final χPT Results

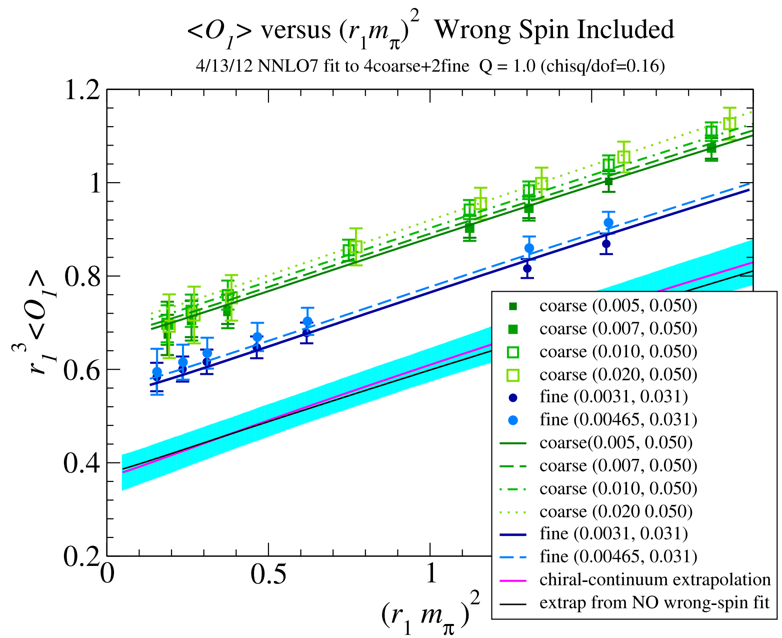
$$\mathcal{Q}_x^{(1)} = \frac{-ig^2_{B^*B\pi}}{f_\pi^2}\left\{\frac{1}{16}\sum_\rho N_\rho\,\mathcal{H}^{\Delta^*}_{X,\rho} + \frac{1}{3}\left[R^{[2,2]}_{X_I}\left(\{M^{(5)}_{X_I}\};\{\mu_I\}\right)\,\frac{\partial\mathcal{H}^{\Delta^*}_{X,I}}{\partial m^2_{X_I}} - \sum_{j\in\{M_I^{(5)}\}} D^{[2,2]}_{j,X_I}\left(\{M^{(5)}_{X_I}\};\{\mu_I\}\right)\mathcal{H}^{\Delta^*}_{j,I}\right]\right\}.$$

$$\tilde{\mathcal{Q}}_x^{(1)} = \frac{-ig^2_{B^*B\pi}}{f_\pi^2}\left\{\frac{1}{16}\left(-5\mathcal{H}^{\Delta^*}_{X,P} - 4\mathcal{H}^{\Delta^*}_{X,A} + 18\mathcal{H}^{\Delta^*}_{X,T} - 4\mathcal{H}^{\Delta^*}_{X,V} - 5\mathcal{H}^{\Delta^*}_{X,I}\right) + \frac{2(\beta_2' + \beta_3')}{\beta_1}\left(-\mathcal{H}^{\Delta^*}_{X,V} + \mathcal{H}^{\Delta^*}_{X,A} + a^2\delta_V'\left[R^{[3,2]}_{X_V}\left(\{M^{(7)}_{X_V}\};\{\mu_V\}\right)\,\frac{\partial\mathcal{H}^{\Delta^*}_{X,V}}{\partial m^2_{X_V}} - \sum_{j\in\{M_V^{(7)}\}} D^{[3,2]}_{j,X_V}\left(\{M^{(7)}_{X_V}\};\{\mu_V\}\right)\mathcal{H}^{\Delta^*}_{j,V}\right] - a^2\delta_A'\left[R^{[3,2]}_{X_A}\left(\{M^{(7)}_{X_A}\};\{\mu_A\}\right)\,\frac{\partial\mathcal{H}^{\Delta^*}_{X,A}}{\partial m^2_{X_A}} - \sum_{j\in\{M_A^{(7)}\}} D^{[3,2]}_{j,X_A}\left(\{M^{(7)}_{X_A}\};\{\mu_A\}\right)\mathcal{H}^{\Delta^*}_{j,A}\right]\right)\right\},$$

Fermilab/MILC, arXiv:1205.7013

- Using final expression (including wrong spins):





- Left and right are two versions of chiral priors.
- Overall effect of wrong spins is smaller than statistical or chiral errors.
- But wrong spins systematically raise $\langle \mathcal{O}_1 \rangle$ for B_s and lower it for $B_{B_d} \Rightarrow$ raise ξ.

Fermilab/MILC, arXiv:1205.7013

Table 1: Complete error budget (in %) for ξ.

Source of uncertainty	Error (%)
Statistics $\oplus$ light-quark disc. $\oplus$ chiral extrapolation	3.7
Mixing with wrong-spin operators	3.2
Heavy-quark discretization	0.3
Scale uncertainty (r_1)	0.2
$g_{BB^*\pi}$	0.7
Light-quark masses	0.5
One-loop matching	0.5
Tuning κ_b	0.4
Finite volume	0.1
Mistuned coarse u_0	0.1
Total Error	5.0

Result: $\xi = 1.268(63)$

Conclusions

- Calculated one-loop B mixing in SχPT.
- Operators are local (i.e., not point split) $\Rightarrow$ contributions from wrong-spin operators.
- Fact that naive field = 4 copies of staggered enters non-trivially.
- Luckily, no new LECs at one loop (if already analyzing complete basis of 5 operators).
- Recent Fermilab/MILC calculation analyzed only $\mathcal{O}_1$ $\Rightarrow$ systematic error associated with wrong spin ops is present (and included in final result).
- Earlier HPQCD calculation did not consider effects of wrong spins $\Rightarrow$ additional systematic error was not included.
- Ongoing Fermilab/MILC calculation looks at all 5 operators; will have no additional error from wrong-sping effects. (Just normal chiral extrap error.) [talk by E. Freeland, Thurs., 3:10 PM]