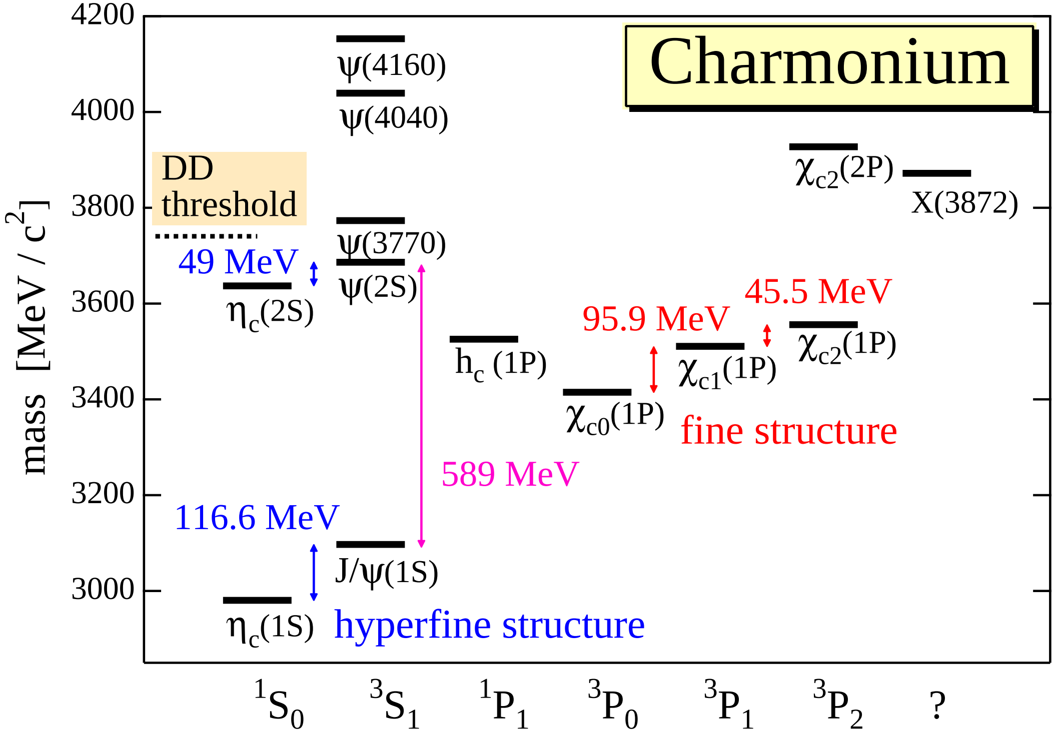
Heavy quarkonium spectroscopy in pNRQCD with lattice QCD input

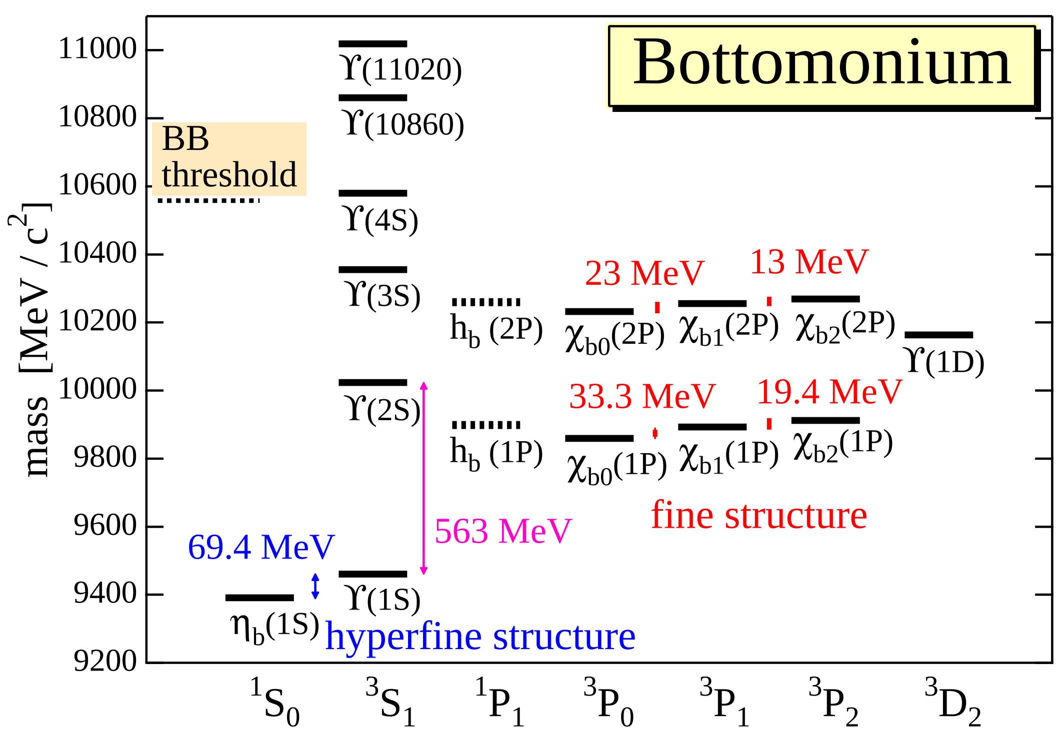
Yoshiaki Koma and **Miho Koma**
Numazu College of Technology

— Lattice 2012 —

Quarkonium

▶ **Bound state of heavy quark and anti-quark**





- **Systematic patterns, HFS and FS (remnant of degeneracy)**

  $\Rightarrow$ **The challenge in QCD**

  $\Rightarrow$ **A promising approach: pNRQCD with lattice QCD input**

Contents

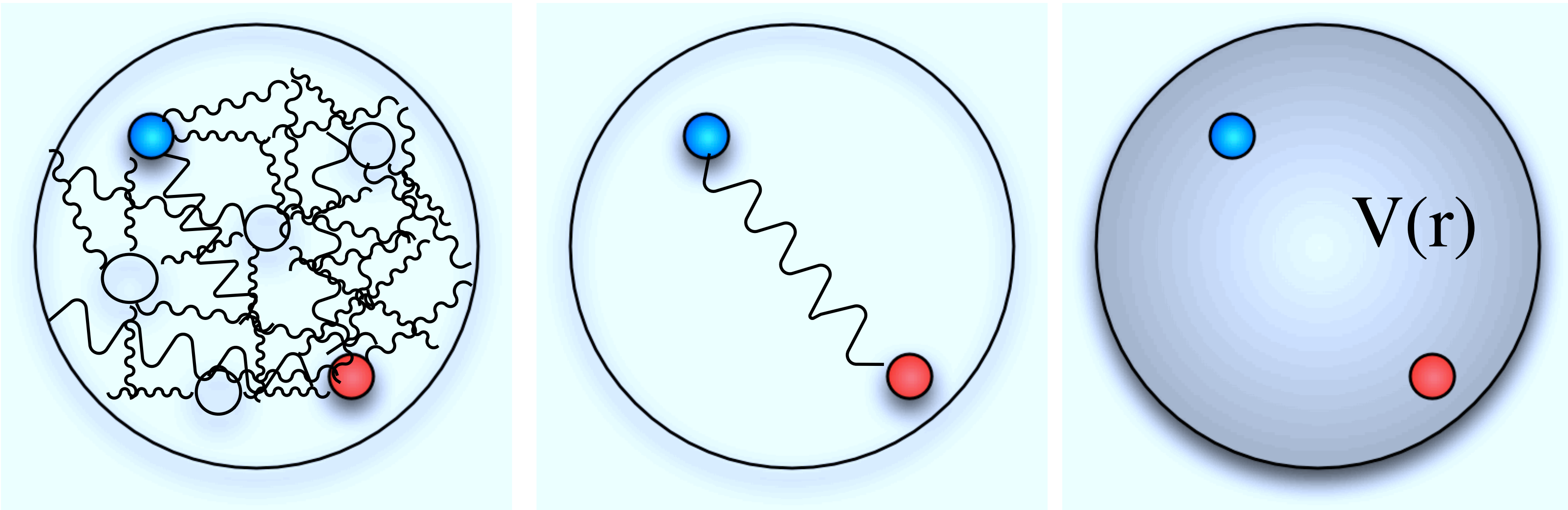
- **pNRQCD ?**
- **Lattice QCD input ?**
- **Quarkonium spectroscopy ?**

pNRQCD ?

- **Potential NRQCD is proposed to make nonrelativistic nature of the quarkonium manifest by paying attention to the hierarchy of energy scales $m \gg mv \gg mv^2$** [Brambilla, Pineda, Soto&Vairo('99ff)]

- **integrate out the scale above m (NRQCD) and mv (pNRQCD)**

QCD $\Rightarrow$ NRQCD $\Rightarrow$ pNRQCD



- **High energy contribution is inherited in the effective interaction in the resulting effective field theories**

pNRQCD ?

▶ **NRQCD** [Caswell&Lepage('86), Manohar('97), Pineda&Soto('98), ...]

$$\mathcal{L} = \psi^\dagger \left(c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2m} + c_D g \frac{\boldsymbol{D}\cdot\boldsymbol{E} - \boldsymbol{E}\cdot\boldsymbol{D}}{8m^2} + i c_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8m^2} + \cdots \right)\psi$$

$$+(\psi \to \chi)$$

$$+\frac{d_{ss}}{m^2}\psi^\dagger\psi\chi^\dagger\chi + \frac{d_{sv}}{m^2}\psi^\dagger\boldsymbol{\sigma}\psi\cdot\chi^\dagger\boldsymbol{\sigma}\chi$$

$$+\frac{d_{vs}}{m^2}\psi^\dagger T^a\psi\cdot\chi^\dagger T^a\chi + \frac{d_{vv}}{m^2}\psi^\dagger T^a\boldsymbol{\sigma}\psi\cdot\chi^\dagger T^a\boldsymbol{\sigma}\chi + \cdots$$

- **Strength of interaction is controlled by effective couplings called the matching coefficients, which are determined to reproduce the same scattering amplitude both in QCD and NRQCD**
  $\Rightarrow$ no need to use bound state information

pNRQCD ?

- **Matching coefficients in NRQCD (leading order)**
  - **Bilinear term c_F, c_D, c_S** [Manohar ('97)]

$$c_F = 1 + \frac{\alpha_s}{2\pi}(C_F + C_A) + O(\alpha_s^2)$$

$$c_D = 1 + \frac{\alpha_s}{2\pi} C_A + O(\alpha_s^2)$$

$$c_S = 2c_F - 1 \qquad (C_F \text{ and } C_A \text{ are Casimir factor})$$

  - **Contact term d_{ss}, d_{sv}, d_{vs}, d_{vv}** [Pineda & Soto ('98)]

$$d_{ss} = \frac{2}{3}\pi\alpha_s + O(\alpha_s^2)\ , \quad d_{sv} = -\frac{2}{9}\pi\alpha_s + O(\alpha_s^2)$$

$$d_{vs} = -\frac{1}{2}\pi\alpha_s + O(\alpha_s^2)\ , \quad d_{vv} = \frac{1}{6}\pi\alpha_s + O(\alpha_s^2)$$

pNRQCD ?

- **Since mv can be $O(\Lambda_{\rm QCD})$, the matching between NRQCD and pNRQCD must be performed nonperturbatively**
  $\Rightarrow$ **quantum mechanical matching** [Brambilla etal. ('99ff)]

1. **write NRQCD hamiltonian**

$$\mathcal{L}_{\rm NRQCD} \longrightarrow H = H^{(0)} + \frac{1}{m} H^{(1)} + \frac{1}{m^2} H^{(2)} + \cdots$$

2. **compute expectation values of $H^{(i \geq 1)}$ with the eigenstate of $H^{(0)}$ for q-$\bar{q}$ system and match them to pNRQCD matching coefficients**

$$V_{\rm pNRQCD}(r) = V^{(0)}(r) + \frac{1}{m} V^{(1)}(r) + \frac{1}{m^2} V^{(2)}(r) + \cdots$$

$\Rightarrow$ **local in time but nonlocal in space (potential)**

$\Rightarrow$ **typical scale of binding energy mv^2**

Lattice QCD input ?

- **pNRQCD potential**

$$V(r) = V^{(0)}(r) + \frac{1}{m}V^{(1)}(r) + \frac{1}{m^2}V^{(2)}(r) + \cdots$$

  - $V^{(0)}$ is the static potential
  - $V^{(1)}$ is typical in QCD which originates from 3-gluon vertex
  - $V^{(2)}$ contains spin-dependent and spin-independent corrections

- **Corrections are represented using the FSCs on the q-$\bar{q}$ source:**

$$V^{(1)} = -\int dt\, t \langle\langle \boldsymbol{E}(0) \cdot \boldsymbol{E}(0) \rangle\rangle$$

$$V_{ls}^{(2)} = \frac{c_S}{2r}\frac{dV^{(0)}}{dr} + \frac{c_F}{r}\Big(\underbrace{\epsilon_{ijk}\hat{r}_i \int dt\, t \langle\langle B^j(0) E^k(0) \rangle\rangle}_{V_1'} + \underbrace{\epsilon_{ijk}\hat{r}_i \int dt\, t \langle\langle B^j(0) E^k(r) \rangle\rangle}_{V_2'}\Big)$$

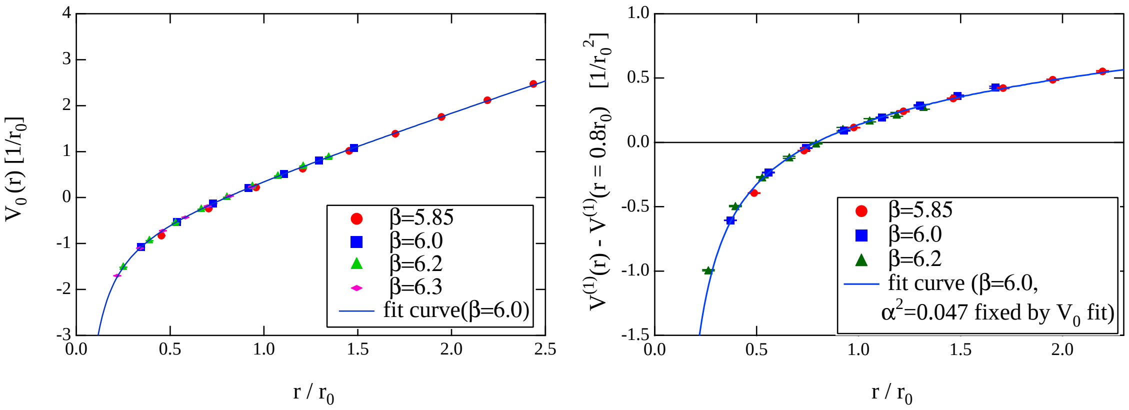
$\Rightarrow$ **compute by lattice QCD simulations**

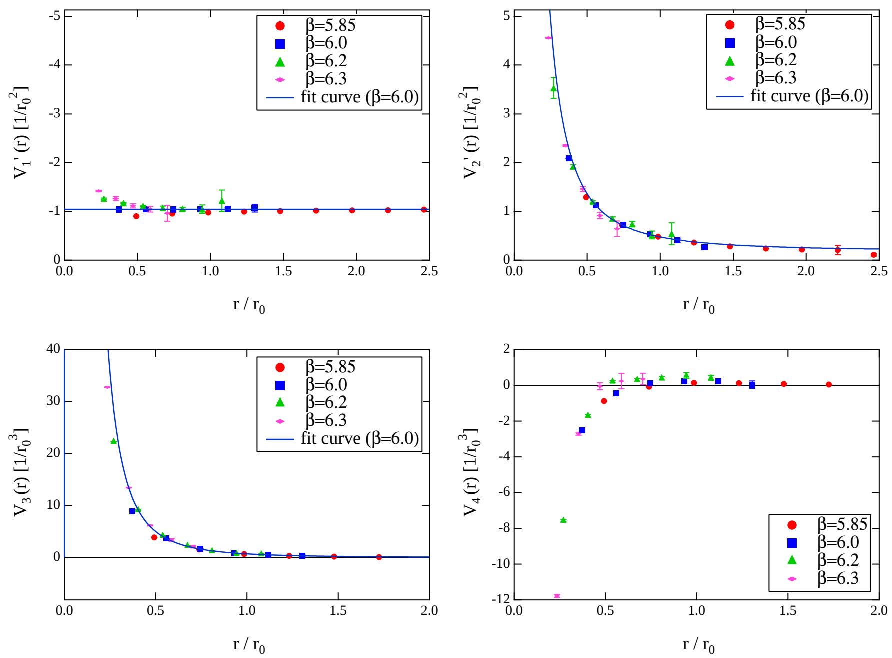
Lattice QCD input ?

▶ **Lattice QCD potentials up to $O(1/m^2)$** [Koma, Koma & Wittig('06ff)]

$$V(r) = V^{(0)} + \frac{1}{m}V^{(1)} + \frac{1}{m^2}V^{(2)}$$

$\Rightarrow$ **compute the FSCs on the PLCF with the multilevel algorithm**





Quarkonium spectroscopy ?

- **Solve the Schrödinger equation**

$$H^{(0)}\psi = \left(\frac{p^2}{m} + V^{(0)}\right)\psi = E^{(0)}\psi$$

- **Add the corrections in the first order perturbation theory**

$$E = E^{(0)} + \delta E = E^{(0)} + \langle\psi|\delta V|\psi\rangle$$

- **Investigate the effects of $V^{(1)}$ and $V_{ls}^{(2)}$ (others are to come...)**

$$\delta E^{(1)} = \frac{1}{m}\langle\psi|V^{(1)}|\psi\rangle$$

$$\delta E_{ls}^{(2)} = \frac{1}{m^2}\langle\psi|\left(\frac{c_s}{2r}\frac{dV^{(0)}}{dr} + \frac{c_F}{r}(V_1' + V_2')\right) l\cdot s|\psi\rangle$$

Quarkonium spectroscopy ?

- **Parameters are fixed by lattice QCD except for the quark masses (and constant shift)**

  - $r_0 = 0.50$ fm **to fix lattice spacing**

  - $V^{(0)} = -\frac{\alpha}{r} + \sigma r \quad (\alpha = 0.297,\ \sigma = 1.06\ \text{GeV/fm})$

  - $\delta E^{(1)} = \frac{1}{m}\langle\psi| -\frac{9\alpha^2}{8r^2} + \sigma^{(1)}\ln r|\psi\rangle \quad (\alpha = 0.297,\ \sigma^{(1)} = 0.142\ \text{GeV}^2)$

  - $\delta E^{(2)}_{ls} = \frac{1}{m^2}\langle\psi|\left(\frac{3\alpha}{2r^3} - \frac{(1-4\epsilon)\sigma}{2r}\right) l\cdot s|\psi\rangle \quad (c_F = c_S = 1,\ \epsilon = 0.2)$

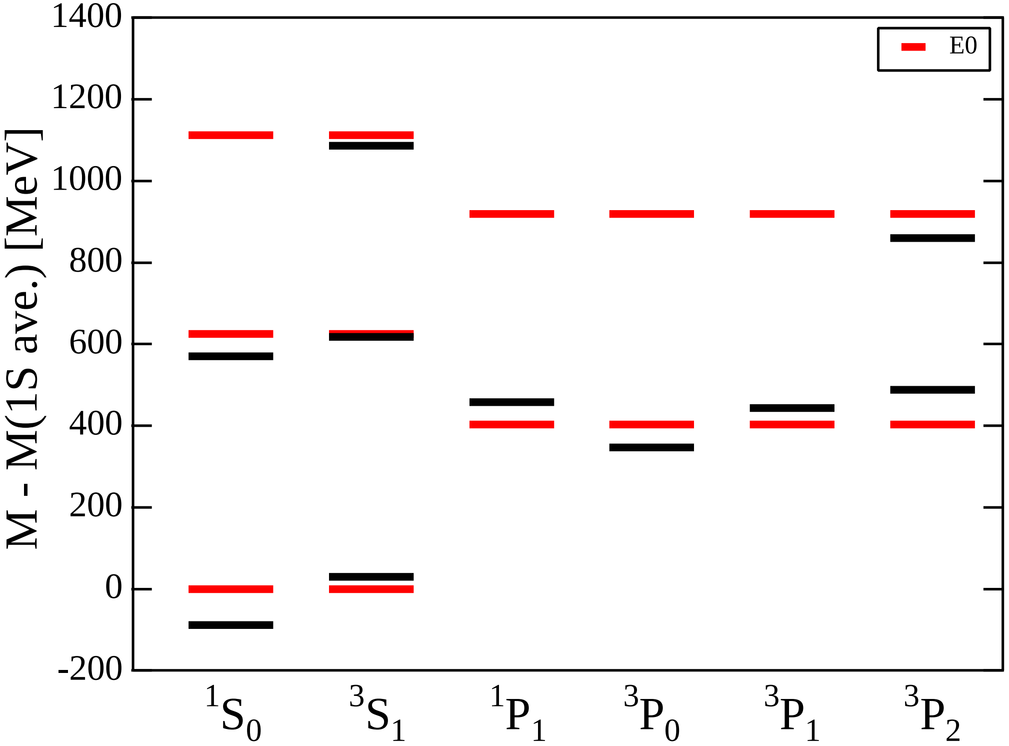
    $\Rightarrow$ ϵ **characterizes long-range contribution in** V_2'

  - $m_c = 1.27$ GeV **(charm),** $\quad m_b = 4.20$ GeV **(bottom)**

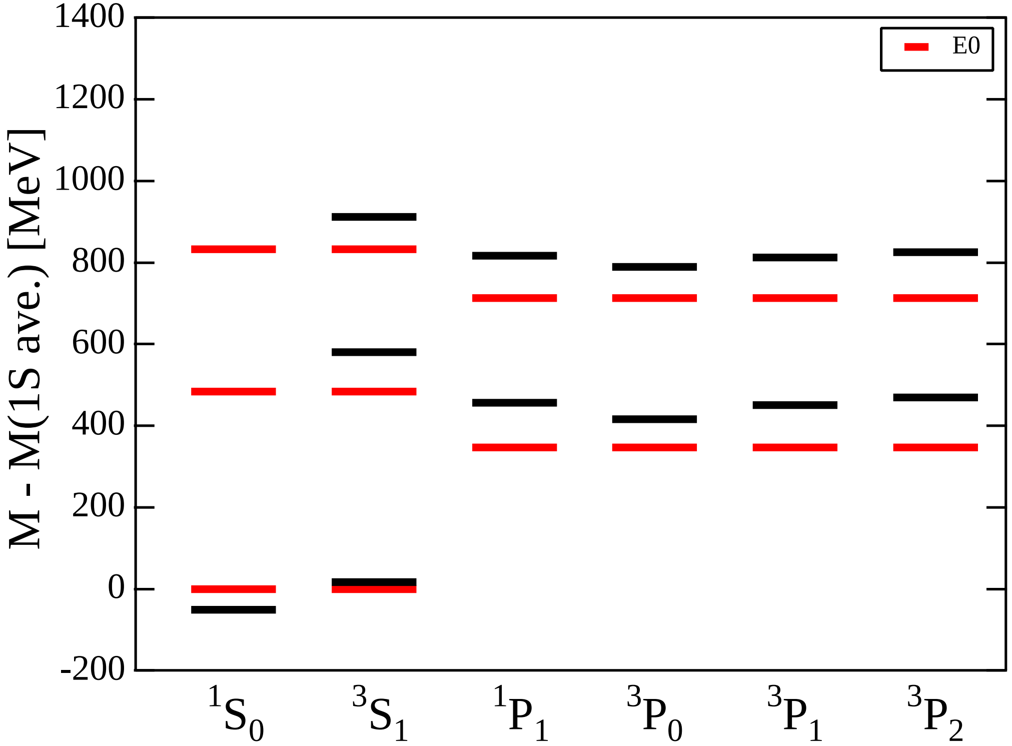
Quarkonium spectroscopy ?

- $E = E^{(0)}$

charmonium

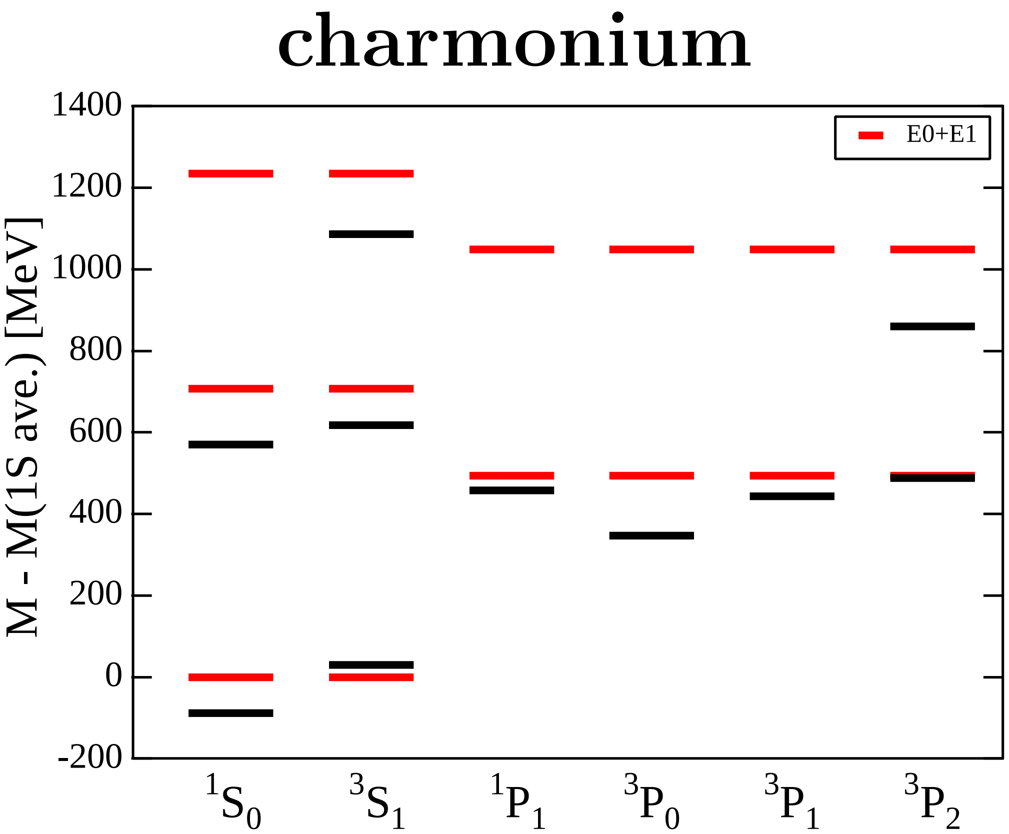


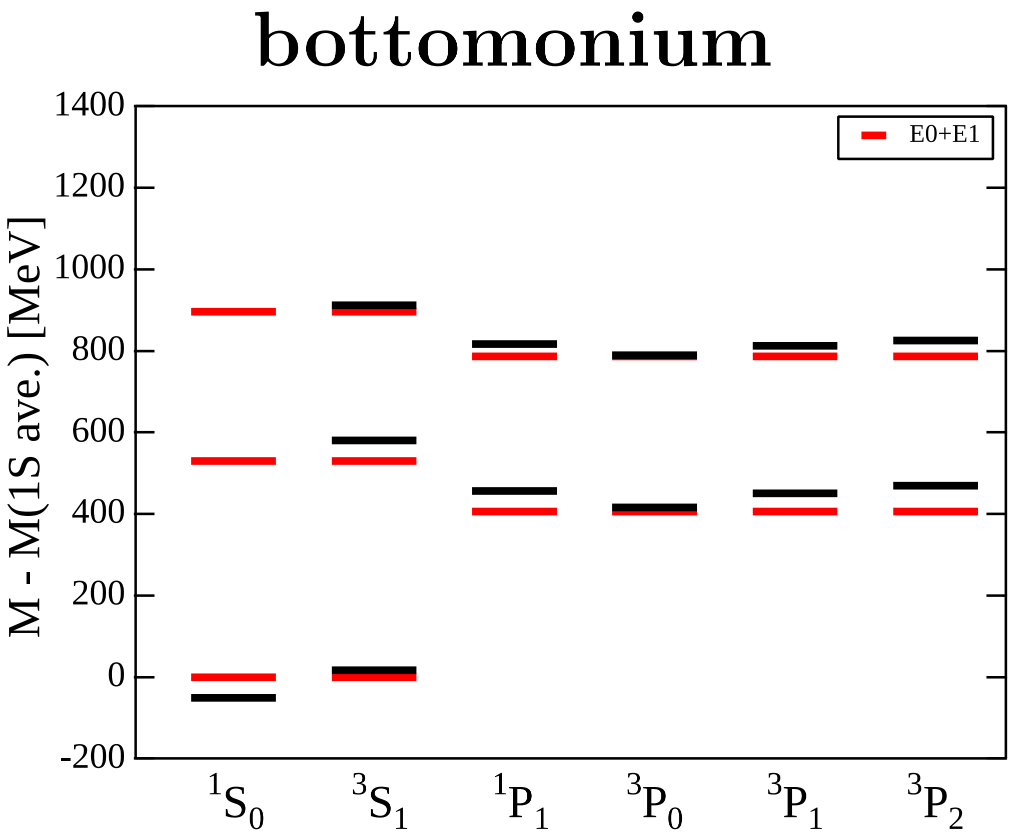
bottomonium



Quarkonium spectroscopy ?

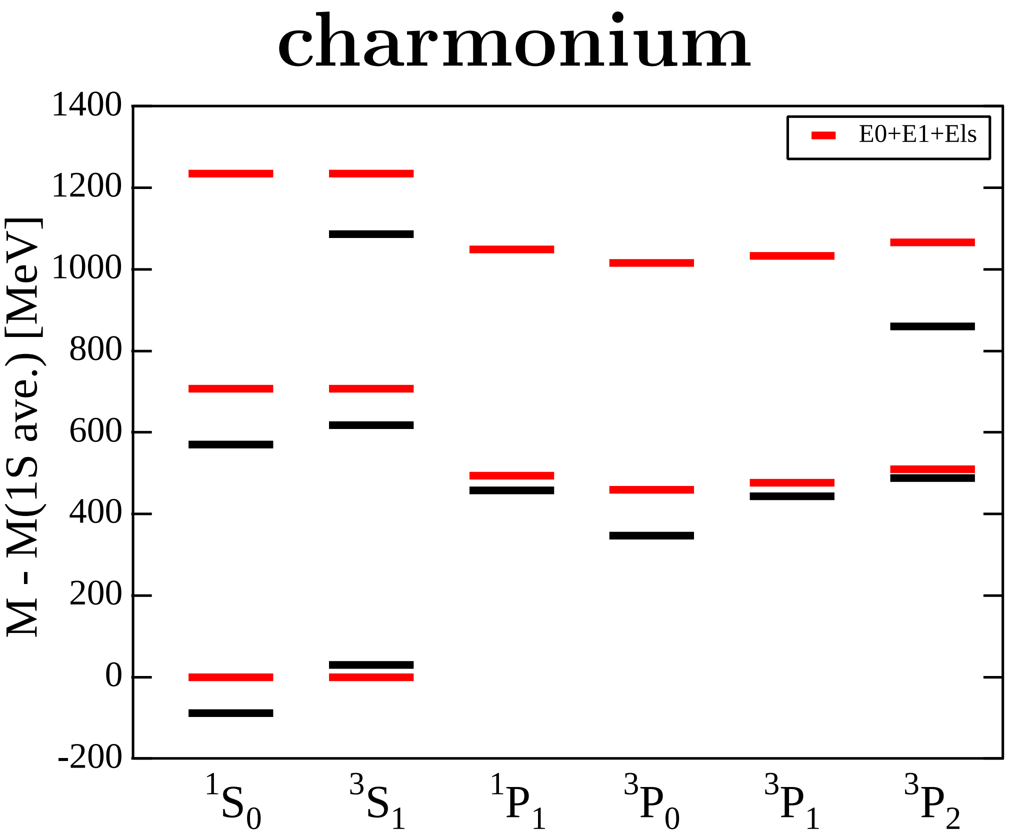
- $E = E^{(0)} + \delta E^{(1)}$

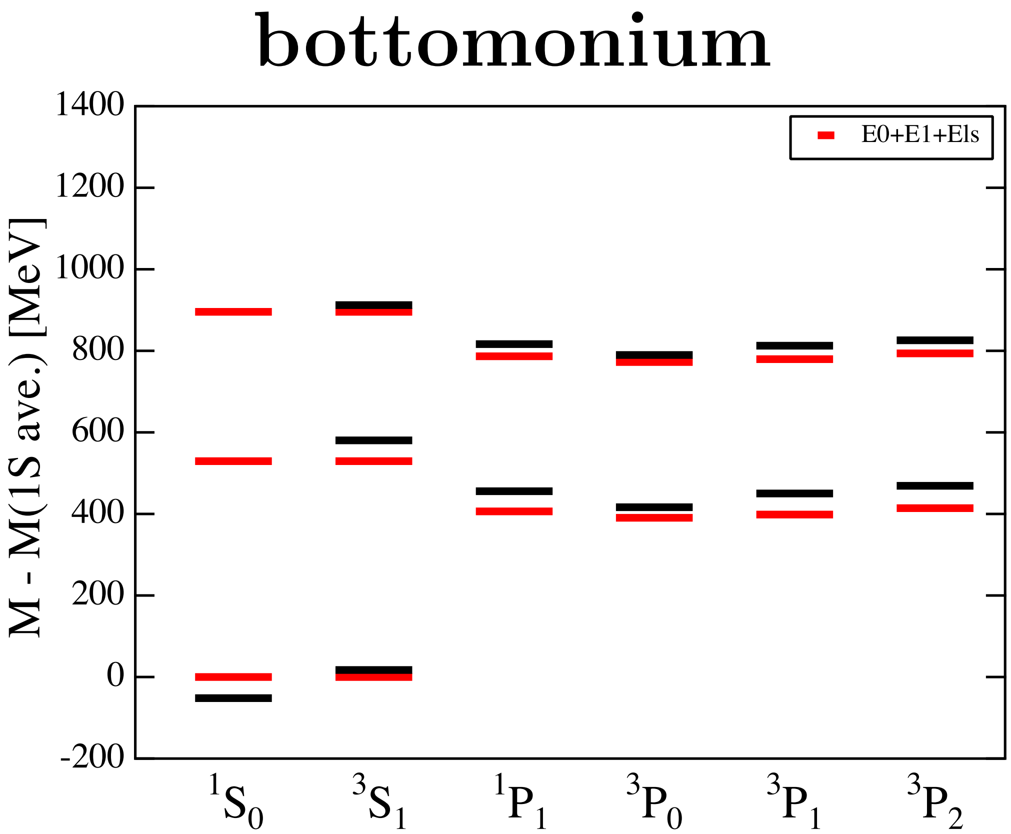




Quarkonium spectroscopy ?

- $E = E^{(0)} + \delta E^{(1)} + \delta E^{(2)}_{ls}$





Summary and outlook

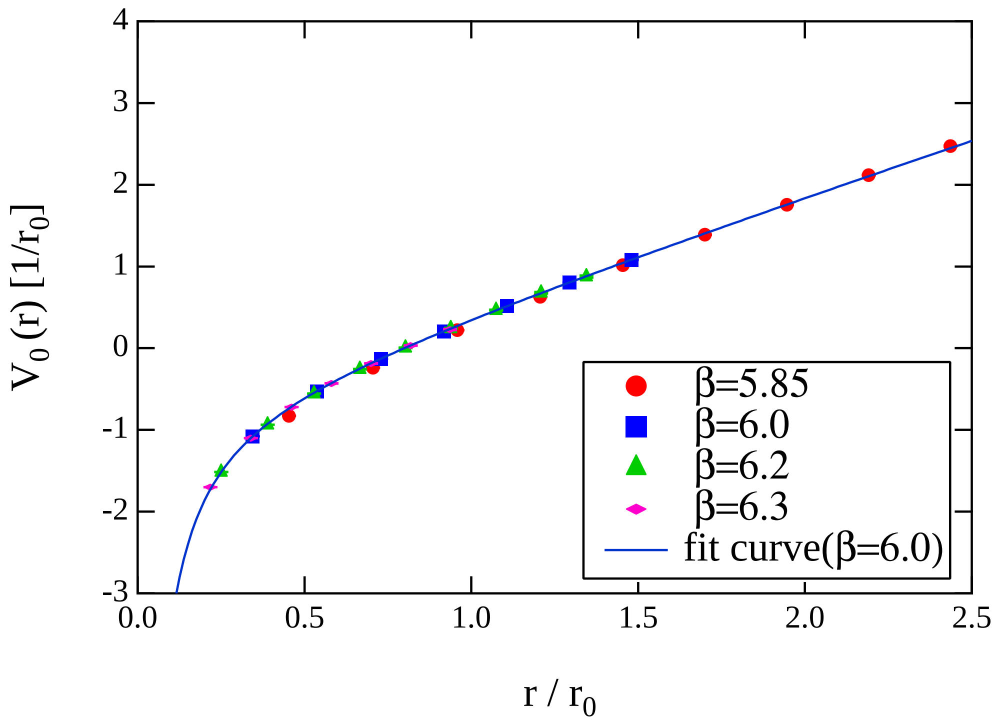
- **We have studied heavy quarkonium spectra systematically based on pNRQCD with lattice QCD input**
  - **pNRQCD is derived by paying attention to the hierarchy of energy scales in the heavy quarkonium system $m \gg mv \gg mv^2$**
  - **Nonperturbative input in pNRQCD (potentials) can be computed by using lattice QCD**
    [cf. LECs in χPT]
- **Compute the spectra with all $O(1/m^2)$ corrections**
  - **Short distance corrections to the lattice potentials**
  - **NLO (and further) matching coefficients in NRQCD**

Appendix

- $V^{(0)}$
- $V^{(1)}$
- $V_{ls}^{(2)}$
- **Effect of** $V^{(1)}$
- **Effect of** $V_{ls}^{(2)}$
- **Effect of** $V_{ls}^{(2)}$ **with tensor term**

$V^{(0)}$

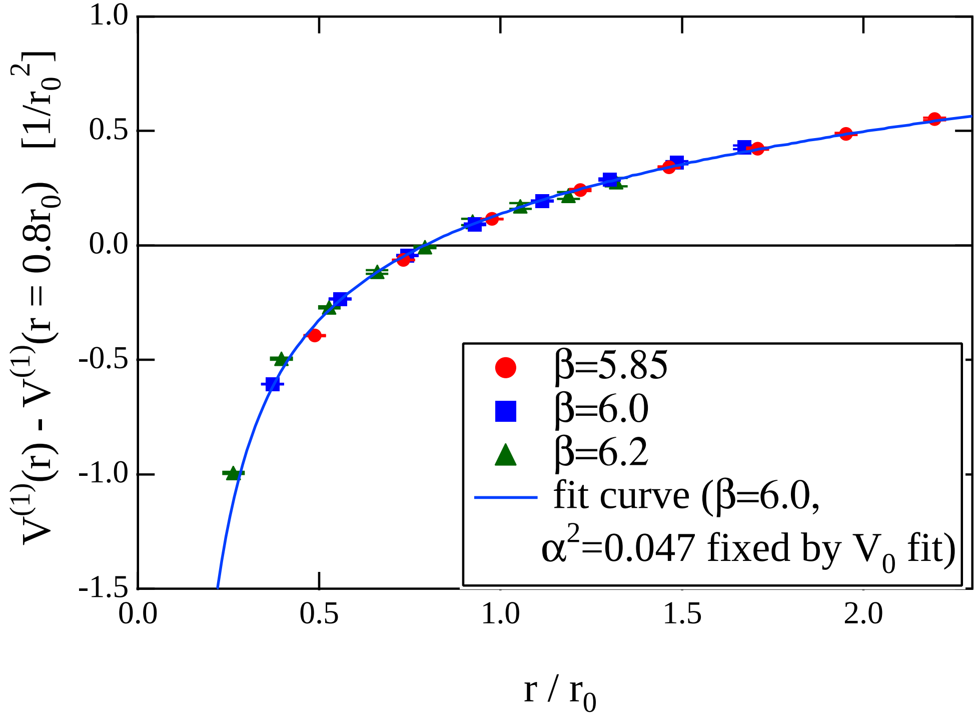
- $V(r) = V^{(0)}(r) + \delta V(r)$



Pert.: $V^{(0)} = -\frac{\alpha}{r} \; (\alpha \equiv C_F \alpha_s) \quad \Rightarrow \quad$ **NP:** $V^{(0)} = -\frac{\alpha}{r} + \sigma r + c^{(0)}$

$V^{(1)}$

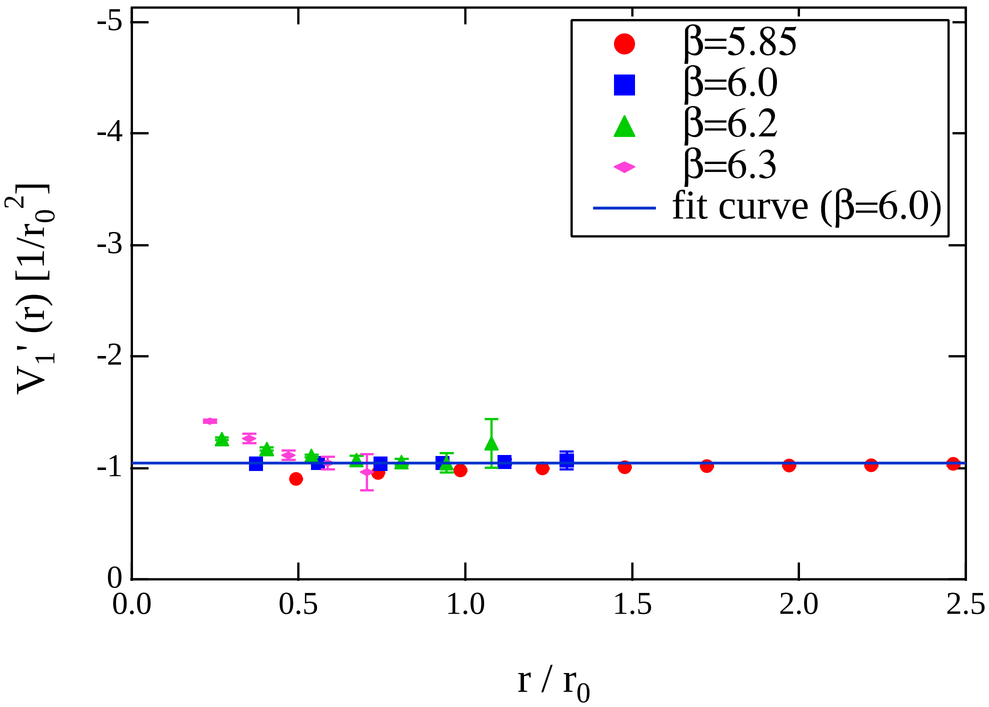
- $\delta V(r) = \dfrac{1}{m} V^{(1)}(r)$ **(y-axis of fig is $V^{(1)}/2$ in this definition)**

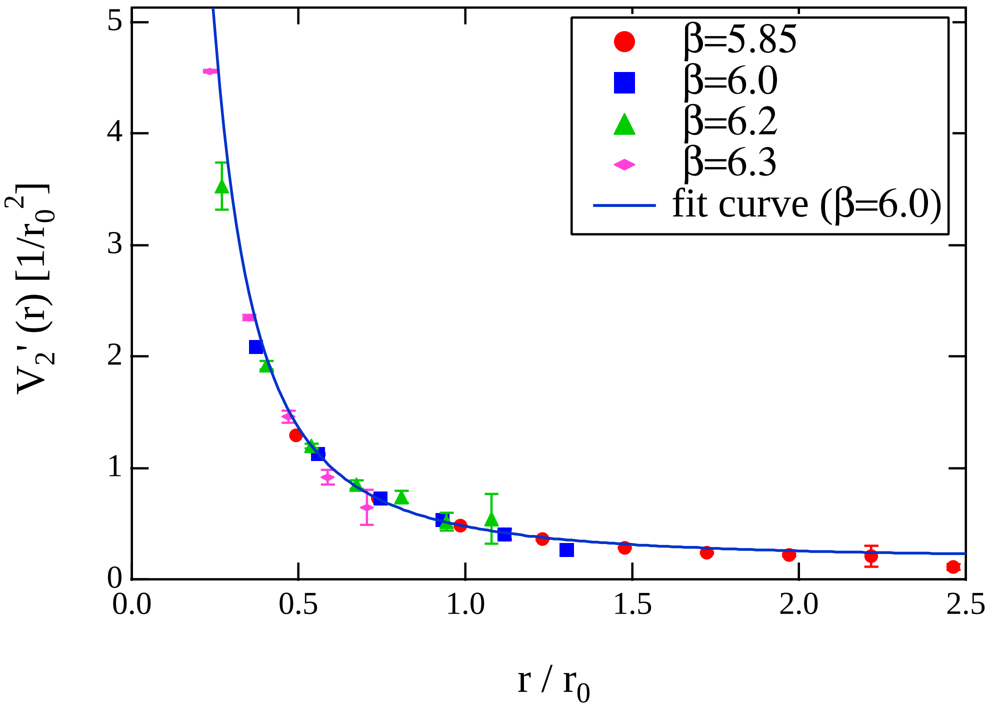


$$\textbf{Pert.:}\ V^{(1)} = -\frac{9\alpha^2}{8r^2} \quad \Rightarrow \quad \textbf{NP:}\ V^{(1)} = -\frac{9\alpha^2}{8r^2} + \sigma^{(1)} \ln r + c^{(1)}$$

$V_{ls}^{(2)}$

- $\delta V(r) = \frac{1}{m^2}\left(\frac{c_s}{2r}\frac{dV^{(0)}(r)}{dr} + \frac{c_F}{r}(V_1'(r)+V_2'(r))\right) l\cdot s$

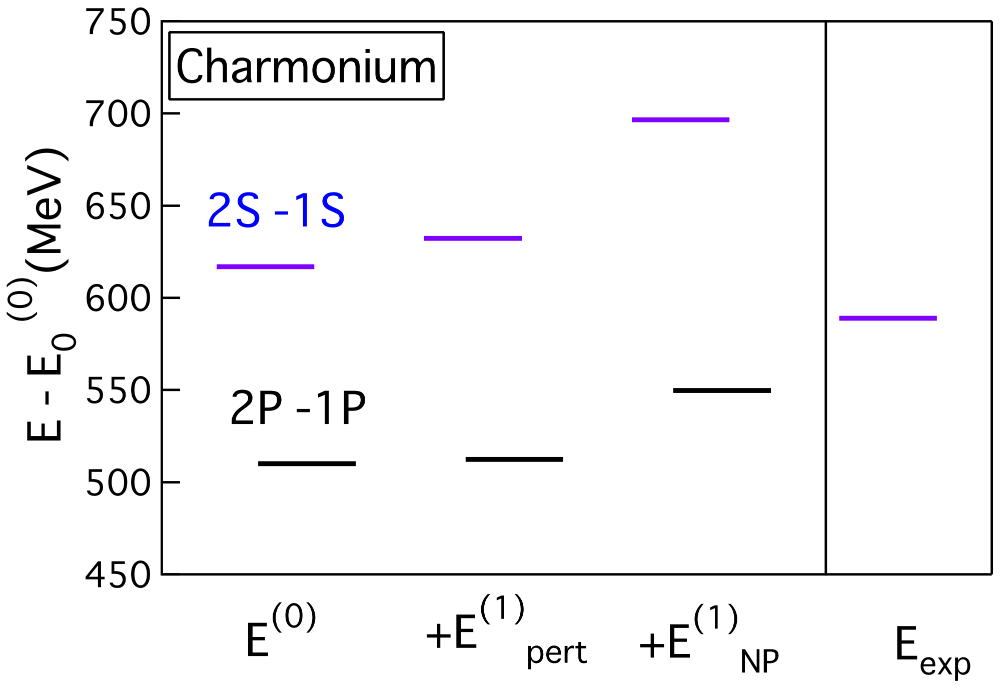


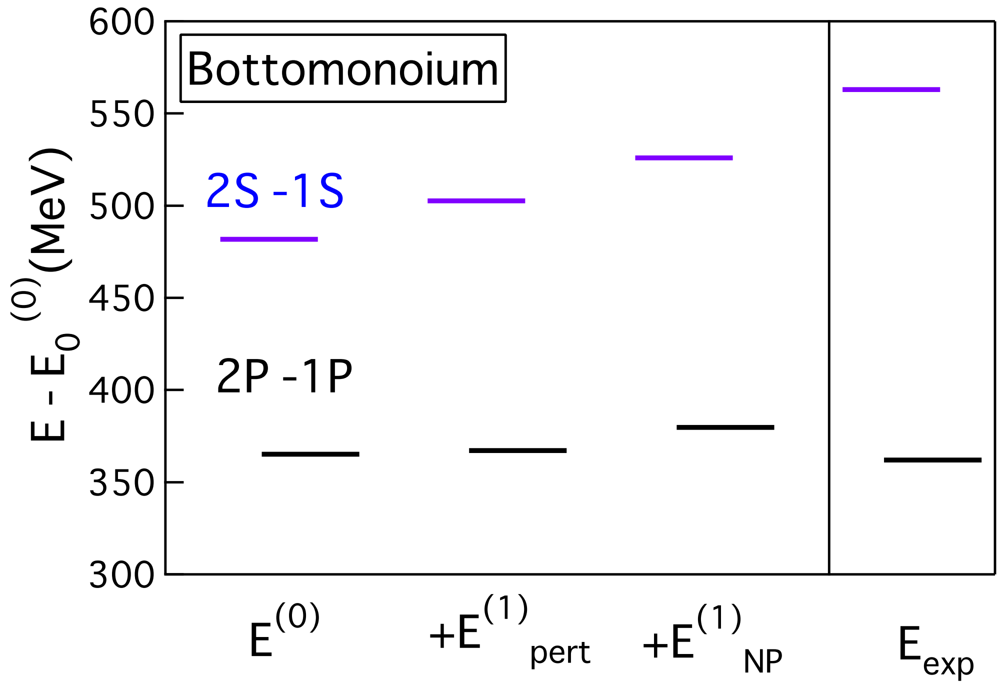


Pert.: $V_1'=0,\ V_2'=\alpha/r^2 \quad \Rightarrow \quad$ **NP:** $V_1'=-(1-\epsilon)\sigma,\ V_2'=\alpha/r^2+\epsilon\sigma$

Effect of $V^{(1)}$

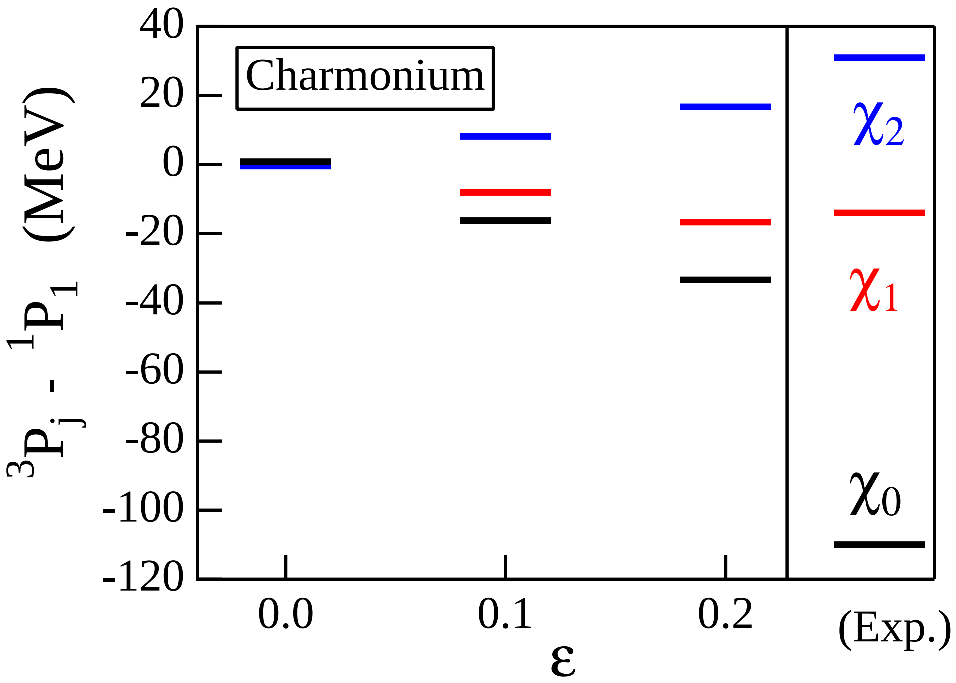
- $\delta E^{(1)} = \frac{1}{m}\langle\psi| -\frac{9\alpha^2}{8r^2} + \sigma^{(1)}\ln r|\psi\rangle = \delta E^{(1)}_{\rm pert} + \delta E^{(1)}_{\rm NP}$

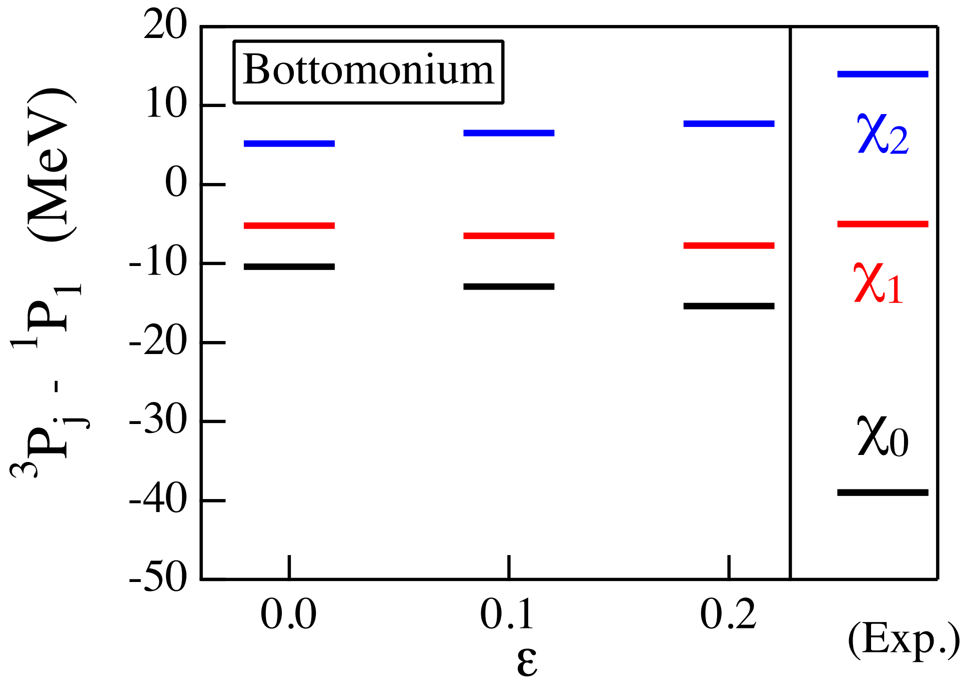




Effect of $V_{ls}^{(2)}$

- $\delta E_{ls}^{(2)} = \frac{1}{m^2}\langle\psi|\left(\frac{3\alpha}{2r^3} - \frac{(1-4\epsilon)\sigma}{2r}\right) l\cdot s|\psi\rangle$, $\langle {}^3P_J | l\cdot s | {}^3P_{J=0,1,2}\rangle = -2, -1, 1$
- ϵ depencence of the level

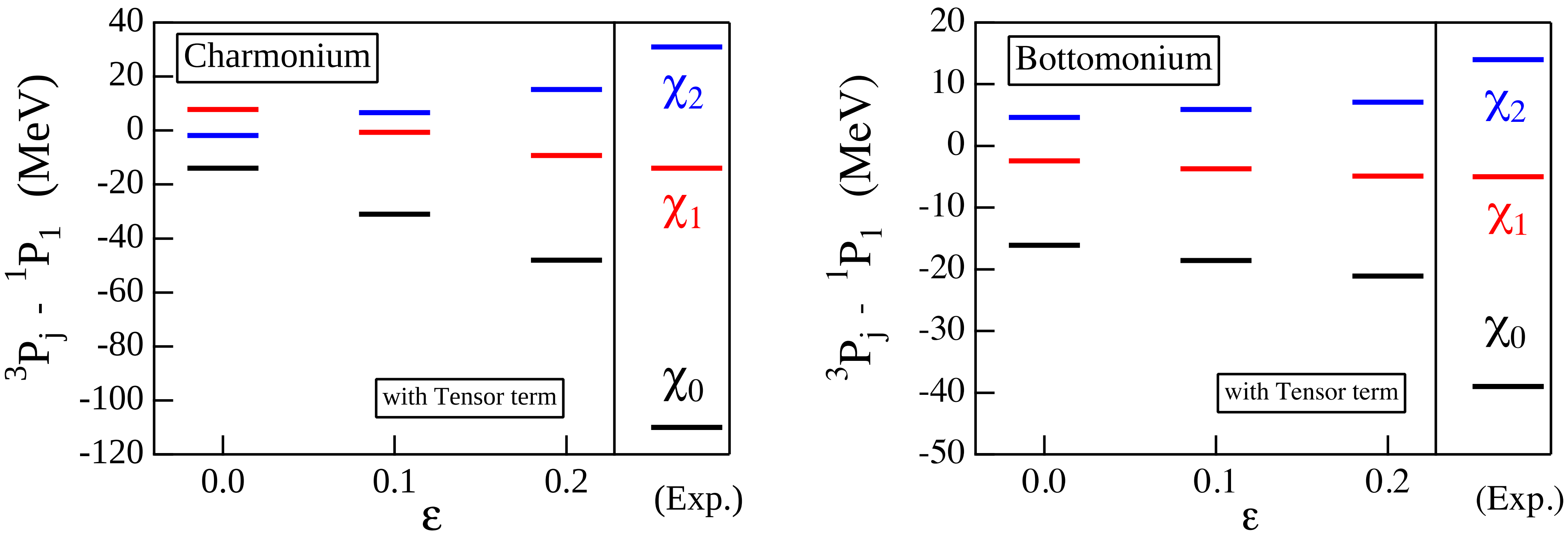




- from lattice QCD $\Rightarrow \epsilon \sim 0.2$

Effect of $V_{ls}^{(2)}$

- $\delta E_{ls}^{(2)} = \frac{1}{m^2}\langle\psi|\left(\frac{3\alpha}{2r^3} - \frac{(1-4\epsilon)\sigma}{2r}\right) l\cdot s|\psi\rangle, \quad \langle {}^3P_J|l\cdot s|{}^3P_{J=0,1,2}\rangle = -2, -1, 1$
- ϵ depencence of the level (include tenser term)



- $V_3 = 3\alpha/r^3, \quad \langle {}^3P_J|s_{12}|{}^3P_{J=0,1,2}\rangle = -1/3, 1/6, -1/30$