Quark mass dependence of s-wave meson-baryon interactions in strangeness sector

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HAL QCD Collaboration
(Hadrons to Atomic nuclei from Lattice QCD)

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Hadrons to Atomic nuclei
HAL
from Lattice QCD

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Contents

- Introduction
- Formula of lattice QCD potentials
- Numerical results of s-wave meson-baryon potentials
- Quark mass dependence
- Summary & Future targets

K-meson and kaon-nucleon interaction

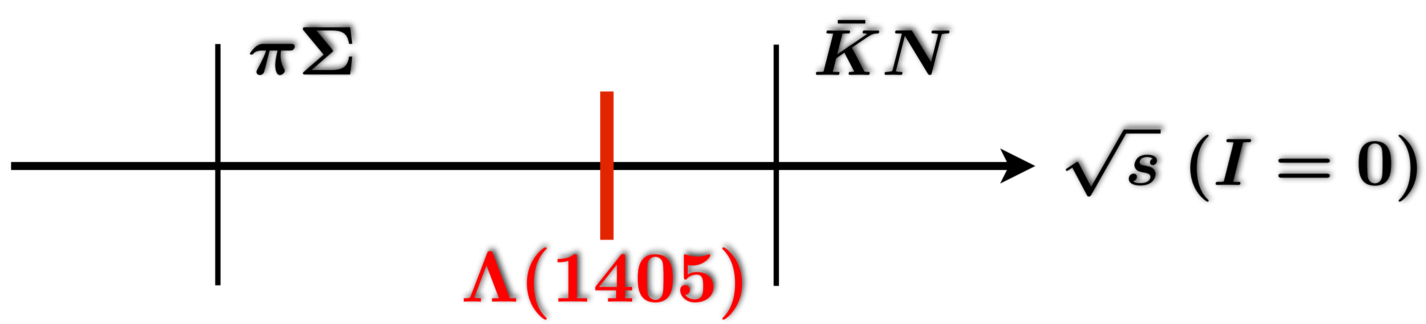
Two aspects of K-meson

- Nambu-Goldstone boson of chiral SU(3)xSU(3) --> SU(3)
- relatively heavy mass: $M_K \sim 495$ MeV

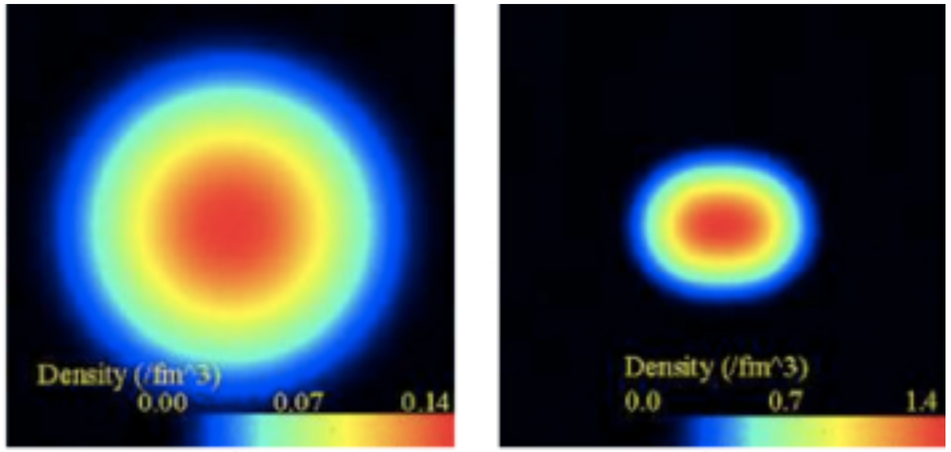
--> peculiar role in hadron physics

Anti-kaon -- nucleon interaction in I=0 is...

- enough attractive to make bound state: Λ(1405)
- coupled to πΣ channel ($K^{bar}N$ channel is not lowest channel)



Phenomenological $K^{bar}N$ interaction leads to dense K^{bar}-nuclei



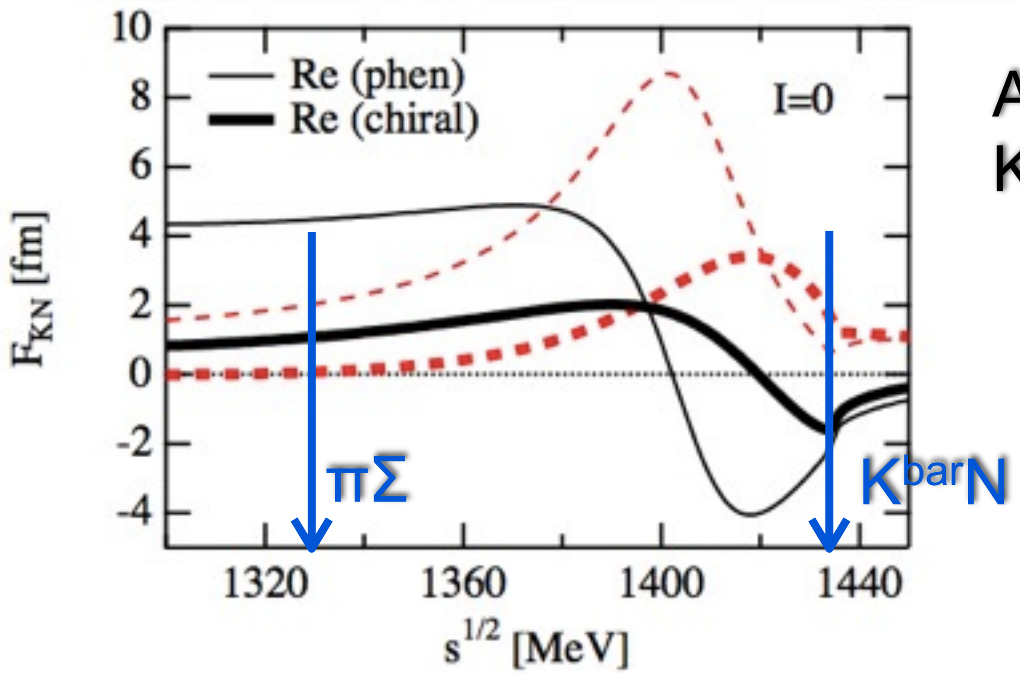
(a) ^{3}He (b) ^{3}HeK$^-$

Central density is much larger than normal nuclei
<- Λ(1405) doorway process to dense matter

Akaishi, Yamazaki, PRC65 (2002).
Dote, Horiuchi, Akaishi, Yamazaki, PRC70 (2004).

Importance of πΣ scattering data

However, $K^{bar}N$ interaction is model dependent



Above $K^{bar}N$ threshold : good agreement
$K^{bar}N$ subthreshold : large deviation

πΣ scattering data based on QCD can eliminate such model ambiguities, although there do NOT exist experimental data.

Hyodo, Weise, PRC77 (2008).
Y.I., Hyodo, Jido, Kamano, Sato, Yazaki, PTP125 (2011).

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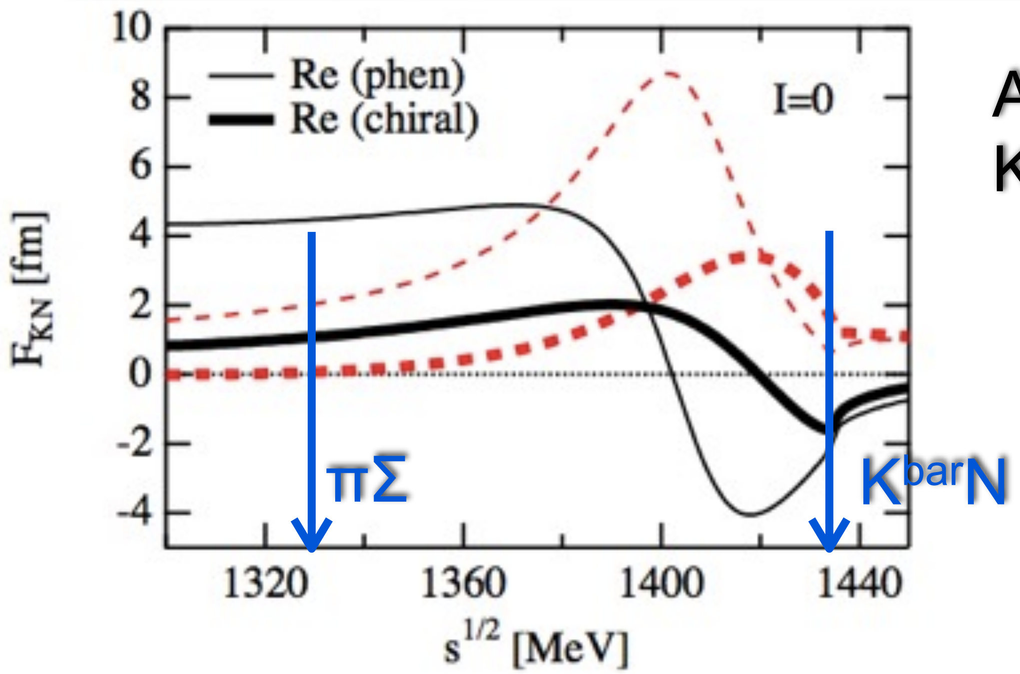

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Possible strategy to determine πΣ scattering length: Λc decay modes

Λ_c decay modes

$$a^{xx}: \Lambda_c \to \pi^+(\pi^+\Sigma^-) \to \pi^+(\pi^-\Sigma^+)$$
$$a^{00}: \Lambda_c \to \pi^+(\pi^+\Sigma^-) \to \pi^+(\pi^0\Sigma^0)$$
$$a^{x}: \Lambda_c \to \pi^0(\pi^+\Sigma^0) \to \pi^0(\pi^0\Sigma^+)$$

Isospin decomposition

$$a^{xx} = \frac{1}{3}a^0 - \frac{1}{2}a^1 + \frac{1}{6}a^2$$
$$a^{00} = \frac{1}{3}a^0 - \frac{1}{3}a^2$$
$$a^{x} = -\frac{1}{2}a^1 + \frac{1}{2}a^2$$

constraint

$$a^{xx} - a^{00} = a^{x}$$

Hyodo & Oka, PRC84 (2011).

Two linearly independent equations for three unknown scattering lengths

--> We investigate I=2 πΣ and I=1 KN scattering in this study

LQCD potential (HAL QCD method)

Aoki, Hatsuda, Ishii, PTP123, 89 (2010).
Ishii et al.(HAL QCD Coll.), PLB712, 437 (2012).

1) Start with meson-baryon correlation functions (R-correlators)

$$R(\vec{r},t) \equiv e^{(m_M+m_B)t} \sum_{\vec{x}} \langle 0|\phi_M(\vec{x}+\vec{r},t)\phi_B(\vec{x},t)\overline{\mathcal{J}}_{\rm src}(t=0)|0\rangle$$

$$= \sum_{\vec{k}} A_{\vec{k}} \exp\left[-\Delta W(\vec{k})t\right] \psi_{\vec{k}}(\vec{r})$$

Interaction energy

$$\Delta W(\vec{k}) = \sqrt{m^2+\vec{k}^2} + \sqrt{M^2+\vec{k}^2} - (m+M)$$

Nambu-Bethe-Salpeter wave function (large r) --> $\psi^{(l)}_{\vec{k}}(\vec{r}) \sim \frac{e^{i\delta_l(k)}}{kr} \sin\left(kr + \delta_l(k) - l\pi/2\right)$

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2) Construct potentials through time-dependent Schrödinger-type equation

$$\left(-\frac{\partial}{\partial t} - H_0\right)R(\vec{r},t) = \int d\vec{r}' U(\vec{r},\vec{r}')R(\vec{r}',t)$$

$$H_0 = -\frac{\nabla_r^2}{2\mu}$$

Energy independent

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Energy independent

3) Velocity expansion:

$$U(\vec{r},\vec{r}') = V(\vec{r},\nabla)\delta(\vec{r}-\vec{r}') \Rightarrow V_{MB}(\vec{r},\nabla) = V_C(\vec{r}) + \vec{L}\cdot\vec{S}V_{LS}(\vec{r}) + \mathcal{O}(\nabla^2)$$

(LO) (NLO)

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(LO) (NLO)

Advantage of this method:

We obtain potentials without ground state saturations

Lattice QCD setup for MB system

Full QCD configurations generated by PACS-CS Coll.

PACS-CS Coll., S. Aoki et al., PRD79, 034503, (2009).

- Gauge coupling : β=1.90
- Iwasaki gauge & Wilson clover
- Lattice spacing : a=0.091 (fm)
- Box size : 32^3x64 -> L=2.9 (fm)
- Conf. # :
  399 (κ_{ud}=0.13700,κ_s=0.13640) / 400 (κ_{ud}=0.13727,κ_s=0.13640)
- Wall source

(κ_{ud}=0.13700,κ_s=0.13640)

Hadron masses (MeV)

M_π~705 (0.8)
M_K~794 (0.7)
M_N~1594 (7.5)
M_Σ~1664 (6.6)

(κ_{ud}=0.13700,κ_s=0.13640)

Hadron masses (MeV)

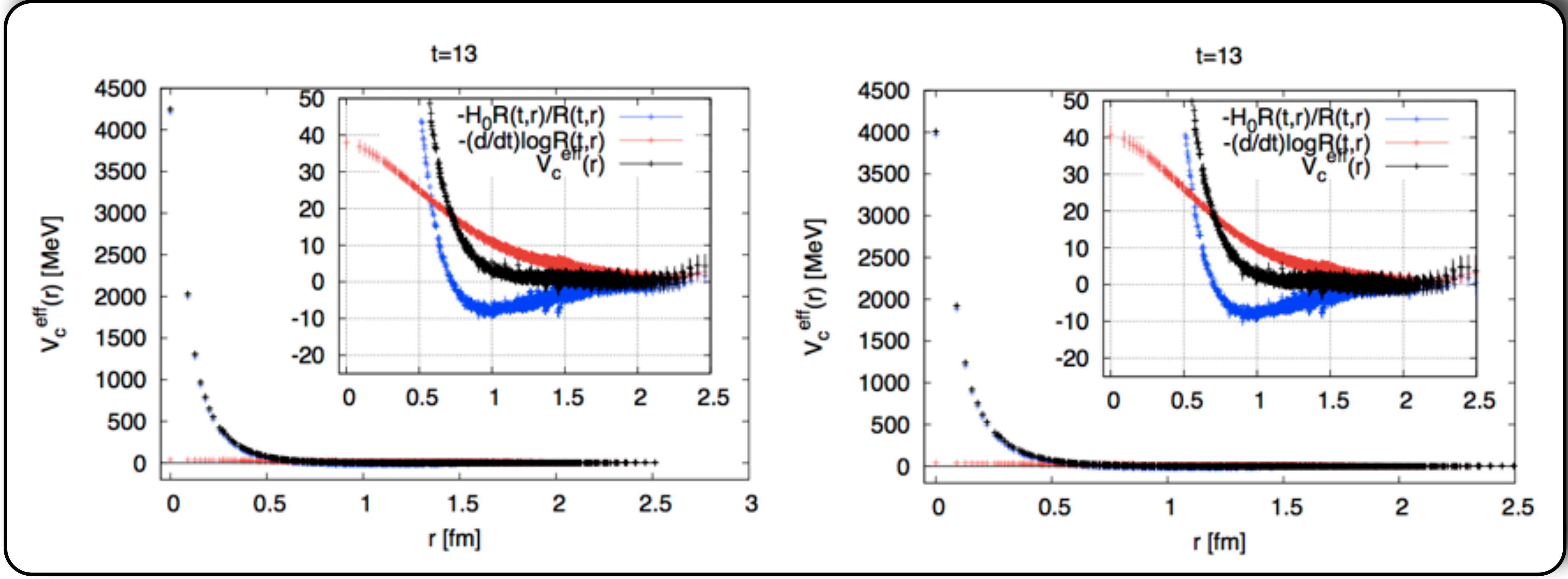
M_π~574 (1.0)
M_K~718 (0.8)
M_N~1400 (7.8)
M_Σ~1527 (6.8)

S-wave meson-baryon potentials@m_π=700MeV

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

Y. Ikeda et al.[HAL QCD], arXiv:1111.2663[hep-lat].

I=2 $\pi\Sigma = \pi^+\Sigma^+$ -> (ud^{bar})(uus) I=1 KN = K^+p -> (us^{bar})(uud)



- Potentials constructed from time-dependent Schrödinger-type equation
- Strong repulsive core near origin (Pauli principle)
- Mid-range attraction from 1st term disappears

Pauli blocking in MB systems

Pauli blocking meson-baryon system (Quark model expectation)

Machida, Namiki, PTP33 (1965).

The hard core appears in

$$NN,\ I=(3/2)\ \pi N,\ I=1\ KN,\ N\Lambda,\ N\Sigma,\ I=1\ N\Xi,\ \text{etc.}$$

and does not appear in

$$\bar{B}B,\ I=(1/2)\ \pi N,\ I=0\ KN,\ \bar{K}N,\ \pi\pi,\ I=0\ N\Xi,\ \text{etc.}$$

I=1 KN = K^+p -> $(us^{bar})(uud)$

I=0 KN ~ K^+n -> $(us^{bar})(udd)$

I=1 KN state : one up-quark cannot be in s-state

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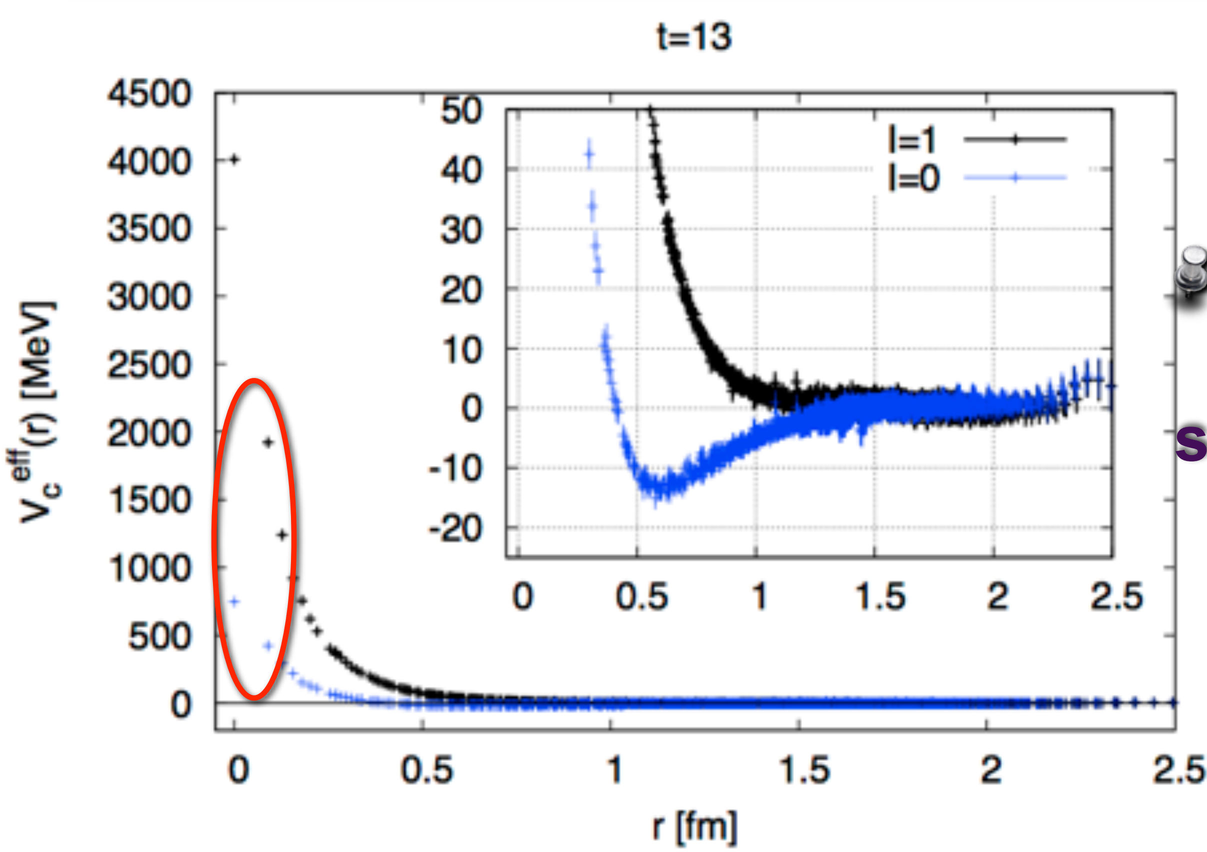
I=1 KN state : one up-quark cannot be in s-state



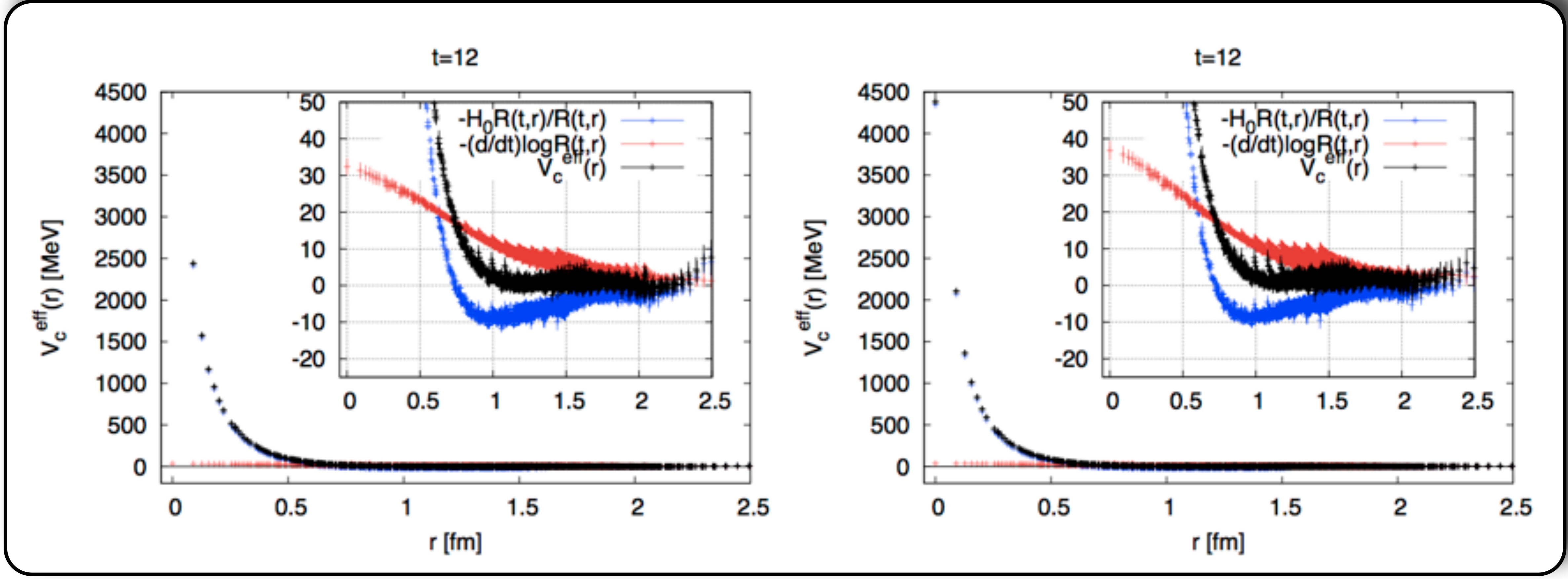
Hard core does NOT appear in I=0 KN potential

see also, Baryon-Baryon potentials in SU(3) limit,
Inoue et al [Hal QCD]., PTP124, 591 (2010).

S-wave meson-baryon potentials@m_π=570MeV

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

I=2 $\pi\Sigma = \pi^+\Sigma^+$ -> $(ud^{bar})(uus)$ I=1 KN = K^+p -> $(us^{bar})(uud)$

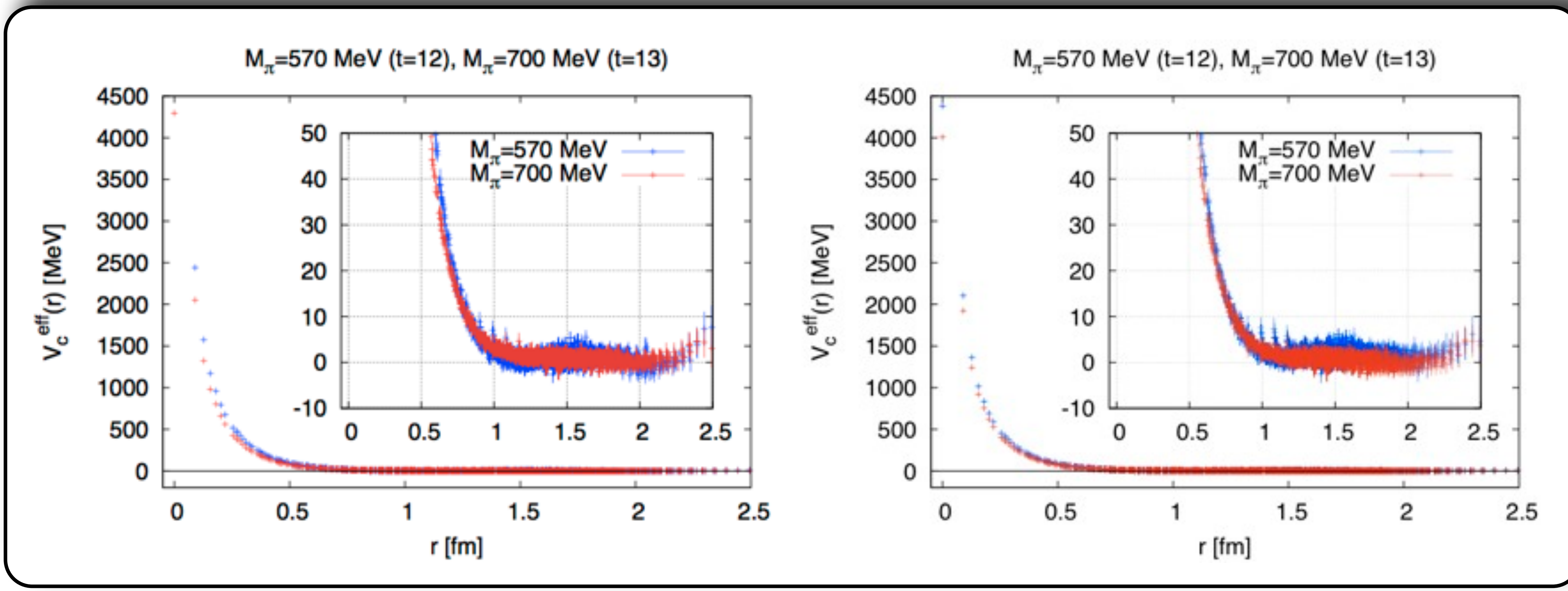


- Potentials constructed from time-dependent Schrödinger-type equation
- Strong repulsive core near origin (Pauli principle at work)

Quark mass dependence of potentials

$$V_C(\vec{r}) = -\frac{H_0 R(\vec{r},t)}{R(\vec{r},t)} - \frac{\partial}{\partial t}\log R(\vec{r},t)$$

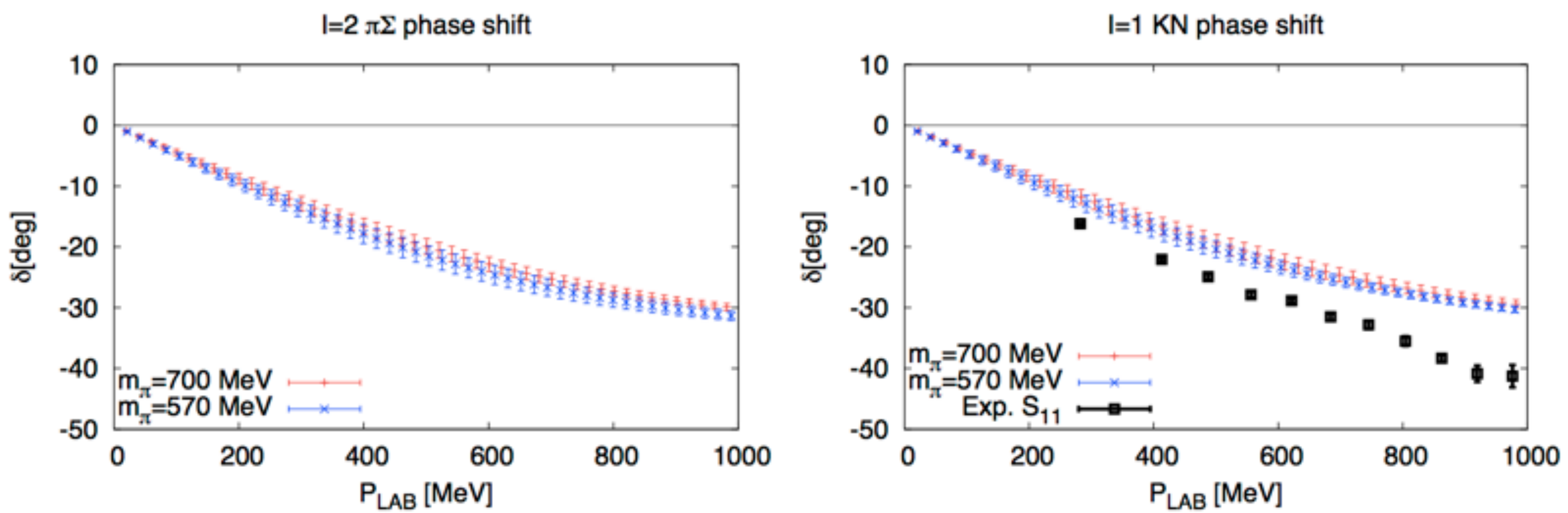
I=2 πΣ = $\pi^+\Sigma^+$ -> ($u\bar{d}$)(uus) I=1 KN = K^+p -> ($u\bar{s}$)(uud)



- **Very weak quark mass dependence**
- **Short range repulsions become a little bit large as decreasing m_q**

Quark mass dependence of phase shifts

Fit potential data --> Solve Schrödinger equations --> phase shifts



Scattering length (fm):

	M_π=570MeV	M_π=700MeV
$a_{\pi\Sigma}$	0.229(25)	0.218(25)
a_{KN}	0.241(33)	0.221(26)

- I=2 πΣ phase shift is **repulsive** (No experimental data)
- I=1 KN phase shift is **qualitatively consistent with experimental data**
- Little quark mass dependences are observed

Summary

I=2 $\pi\Sigma$ & I=1 KN scattering on lattices

- **We study s-wave MB interactions in full QCD simulations**
- **I=2 $\pi\Sigma$ potential reveals scattering phase shift as repulsive**
- **Calculated I=1 KN phase shift is qualitatively consistent with experimental data**
- **Weak quark mass dependences on potentials and observable**

Future targets

- **Non-locality of meson-baryon potentials**
- **Heavy meson - baryon ($D^{bar}N$, ...) system**
- **Coupled-channel (e.g., $K^{bar}N$-$\pi\Sigma$ system is challenging)**
- **Physical point simulation**

Lattice QCD potential (strategy)

Full details, see, Aoki, Hatsuda, Ishii, PTP123, 89 (2010).

Helmholtz equation of NBS wave function:

$$(\nabla_r^2 + \vec{k}^2)\psi_{MB}(\vec{r}; W) = 0 \quad (r > R)$$

$$W = \sqrt{m^2 + \vec{k}^2} + \sqrt{M^2 + \vec{k}^2}$$

Asymptotic form of NBS wave function:

$$\psi^{(l)}(\vec{r}; W) \sim \frac{e^{i\delta_l(k)}}{kr} \sin\left(kr + \delta_l(k) - l\pi/2\right)$$

-> faithful to scattering phase shift

Define half off-shell T-matrix in interacting region:

$$(E - H_0)\psi_{MB}(\vec{r}; W) = \mathcal{K}_W(\vec{r}) \quad (r < R)$$

$$E = \frac{\vec{k}^2}{2\mu}, \quad H_0 = -\frac{\nabla_r^2}{2\mu}$$

Plane wave components are projected out

Derive potential: non-local, energy-independent potential by construction

$$U(\vec{r}, \vec{r}') = \int \frac{dW}{2\pi} \mathcal{K}_W(\vec{r})\psi^*_{MB}(\vec{r}'; W)$$

Non-local potential satisfies time-independent Schrödinger-type equation:

$$(E - H_0)\psi_{MB}(\vec{r}; W) = \int d\vec{r}' U(\vec{r}, \vec{r}')\psi_{MB}(\vec{r}'; W)$$

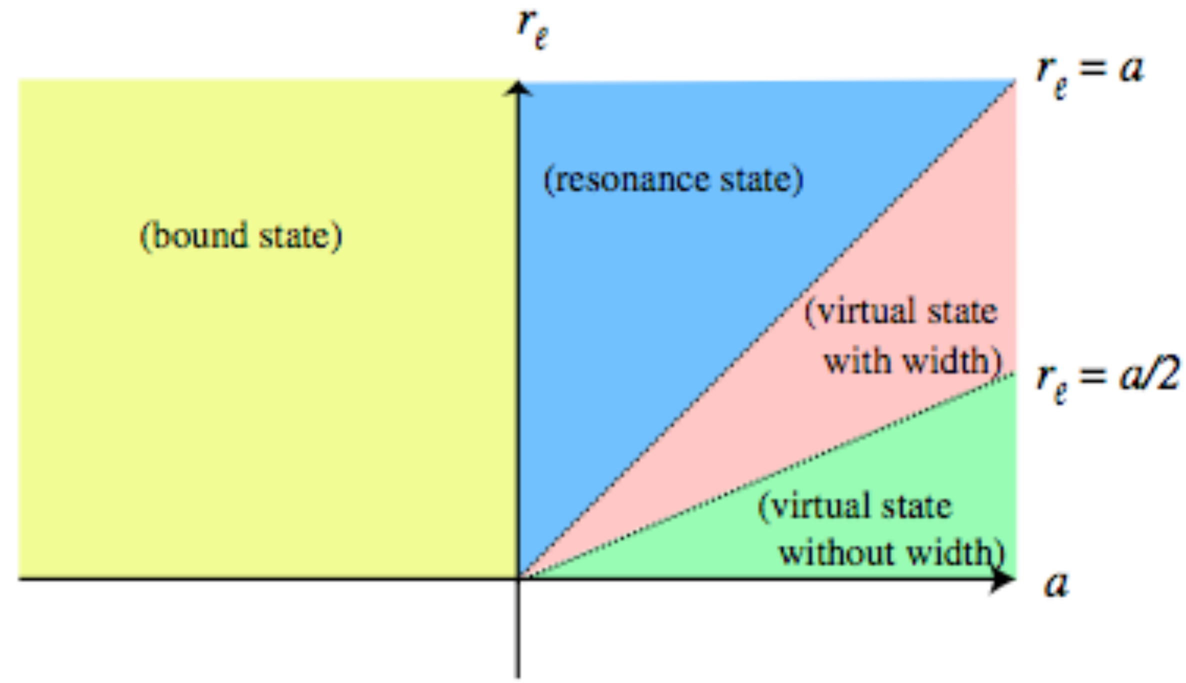
Threshold behavior of πΣ scattering

Scattering amplitude near threshold

$$f(k) = \frac{1}{k\cot\delta(k) - ik}$$

$$\left(k\cot\delta(k) = \frac{1}{a} - \frac{1}{2} r_e k^2 + \cdots \right)$$

Scattering length & effective range
-> nature of pole position



Y.I., Hyodo, Jido, Kamano, Sato, Yazaki, PTP125 (2011).

$\pi\Sigma$ scattering length from Λ_c decay

$$a^{xx} = \frac{1}{3}a^0 - \frac{1}{2}a^1 + \frac{1}{6}a^2$$

$$a^{00} = \frac{1}{3}a^0 - \frac{1}{3}a^2$$

$$a^{x} = -\frac{1}{2}a^1 + \frac{1}{2}a^2$$

$$a^{xx} - a^{00} = a^{x}$$

$$a^{xx} : \Lambda_c \to \pi^+(\pi^+\Sigma^-) \to \pi^+(\pi^-\Sigma^+)$$

$$a^{00} : \Lambda_c \to \pi^+(\pi^+\Sigma^-) \to \pi^+(\pi^0\Sigma^0)$$

$$a^{x} : \Lambda_c \to \pi^0(\pi^+\Sigma^0) \to \pi^0(\pi^0\Sigma^+)$$

Hyodo & Oka, PRC84 (2011).

One of $\pi\Sigma$ scattering lengths is important input

(Clear signal in I=2 $\pi\Sigma$ scattering is expected from LQCD)

We examine s-wave I=2 $\pi\Sigma$ scattering from LQCD potentials