Bound states of multi-nucleon channels in $N_f = 2 + 1$ lattice QCD

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Refs: PRD81:111504(R)(2010); PRD84:054506(2011)

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Outline

1. Introduction
2. Simulation parameters
3. Preliminary results
   - ^{4}He and ^{3}He channels
   - NN channels
4. Summary and future work

1. Introduction

Strong interaction

$$\text{Bind}\left\{\begin{array}{rcl}\text{protons and neutrons} & \rightarrow & \text{nuclei}\\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons}\end{array}\right.$$

Spectrum of nuclei

success of Shell model: Jensen and Mayer(1949)

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons}}^{\text{Shell model}}}_{\text{(lattice) QCD}}\rightarrow \text{nuclei}$$

1. Introduction

Strong interaction

$$\text{Bind} \begin{cases} \text{protons and neutrons} & \rightarrow & \text{nuclei} \\ \text{quarks and gluons} & \rightarrow & \text{protons and neutrons} \end{cases}$$

Spectrum of nuclei

success of Shell model: Jensen and Mayer(1949)

degrees of freedom of protons and neutrons

Spectrum of proton and neutron (nucleons)

success of non-perturbative calculation of QCD

such as lattice QCD

degrees of freedom of quarks and gluons

Motivation: Understand property of nuclei from (lattice) QCD

$$\underbrace{\text{quarks and gluons} \rightarrow \overbrace{\text{protons and neutrons} \rightarrow \text{nuclei}}^{\text{Shell model}}}_{\text{(lattice) QCD}}$$

1. Introduction

Motivation :

Understand property and structure of nuclei from QCD

If we can study nuclei from QCD, we may be able to

1. reproduce spectrum of nuclei
2. predict property of nuclei hard to calculate or observe

such as neutron rich nuclei

So far only few works for multi-baryon bound states
Before studying such difficult problems, we should check

$\rightarrow$ Can we calculate known binding energy in a-few-nucleon systems?

Multi-baryon bound state from lattice QCD

Not observed before '09 (except H-dibaryon '88 Iwasaki *et al.*)

Recent studies of lattice QCD for bound state of multi-baryon systems

1. ^{4}He and ^{3}He channels
   '10 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD81:111504(R)(2010)
2. H dibaryon in ΛΛ channel (S=−2, I=0)
   '11 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV
   '11 HALQCD $N_f = 3$ $m_\pi = 0.67$–1.02 GeV
   '11 Luo et al. $N_f = 0$ $m_\pi = 0.5$–1.3 GeV
3. NN channels
   '11 PACS-CS $N_f = 0$ $m_\pi = 0.8$ GeV PRD84:054506(2011)
   '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV (Possibility)
4. ΞΞ channel
   '12 NPLQCD $N_f = 2+1$ $m_\pi = 0.39$ GeV

Other studies: 2- and 3-nucleon forces HALQCD, ΩΩ channel Buchoff *et al.*

Extend our works to $N_f = 2+1$ QCD with smaller m_π and a

c.f. '12 NPLQCD $N_f = 3$ $m_\pi = 0.8$ GeV: ^{4}He, ^{3}He, NN and others

Problems of multi-nucleon bound state

Traditional method for example ^{4}He channel

$$\langle 0|O_{^4\mathrm{He}}(t)O^\dagger_{^4\mathrm{He}}(0)|0\rangle = \sum_n \langle 0|O_{^4\mathrm{He}}|n\rangle\langle n|O^\dagger_{^4\mathrm{He}}|0\rangle e^{-E_n t} \xrightarrow[t\gg 1]{} A_0\, e^{-E_0 t}$$

Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto$ exp $\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

2. Calculation cost

Wick contraction for ^{4}He $= p^2n^2 = (udu)^2(dud)^2$: 518400

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state $\Delta E_0 = E_0 - N_N m_N < 0$

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Difficulties for multi-nucleon calculation

1. Statistical error

Statistical error $\propto \exp\left(N_N\left[m_N - \frac{3}{2}m_\pi\right]t\right)$

$\rightarrow$ heavy quark mass $m_\pi = 0.5$ GeV + large # of measurements

2. Calculation cost PACS-CS PRD81:111504(R)(2010)

Wick contraction for $^4\text{He} = p^2n^2 = (udu)^2(dud)^2$: 518400 $\rightarrow$ 1107

$\rightarrow$ reduction using $p(n) \leftrightarrow p(n)$ $p \leftrightarrow n$, $u(d) \leftrightarrow u(d)$ in $p(n)$

Multi-meson: '10 Detmold and Savage, Multi-baryon: '12 Doi and Endres

3. Identification of bound state on finite volume

Finite volume effect of attractive scattering state $\Delta E_0 = E_0 - N_N m_N < 0$

$\rightarrow$ Volume dependence of ΔE '86,'91 Lüscher, '07 Beane *et al.*

Spectral weight: '04 Mathur *et al.*, Anti-PBC '05 Ishii *et al.*

2. Simulation parameters

$N_f = 2 + 1$ QCD

Iwasaki gauge action at $\beta = 1.90$

$a^{-1} = 2.194$ GeV with $m_\Omega = 1.6725$ GeV '10 PACS-CS

non-perturbative $O(a)$-improved Wilson fermion action

$m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV

$m_s \sim$ physical strange quark mass

Finite volume dependence of ΔE_0 with four volumes

(^{4}He, ^{3}He, $^3S_1$ and $^1S_0$ channels)

L	L [fm]	N_{Traj}	N_{conf}	N_{meas}
32	2.9	4000	200	192
40	3.6	2000	200	192
48	4.3	2000	200	192
64	5.8	1900	190	256

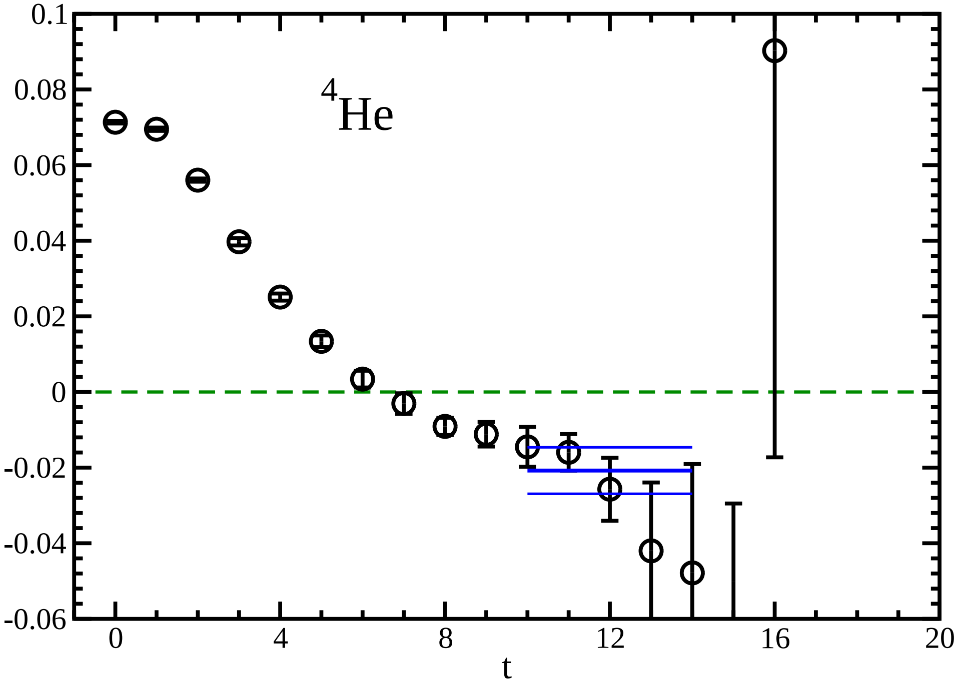
Simulations:

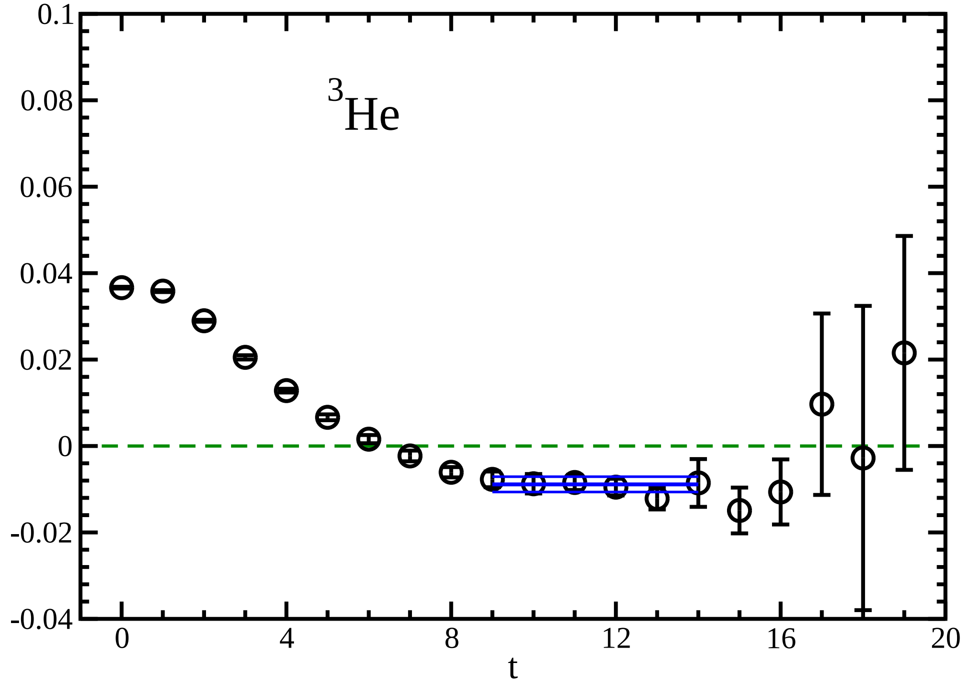
PACS-CS, T2K-Tsukuba, HA-PACS at Univ. of Tsukuba, HA8000 at Univ. of Tokyo and K at AICS

3. Preliminary results

ΔE_L in ^{4}He and ^{3}He channels at $L = 5.8$ fm

$$\Delta E_L = \log\left(\frac{R_{^4\mathrm{He}}(t)}{R_{^4\mathrm{He}}(t+1)}\right) \text{ with } R_{^4\mathrm{He}}(t) = \frac{C_{^4\mathrm{He}}(t)}{(C_N(t))^4}$$



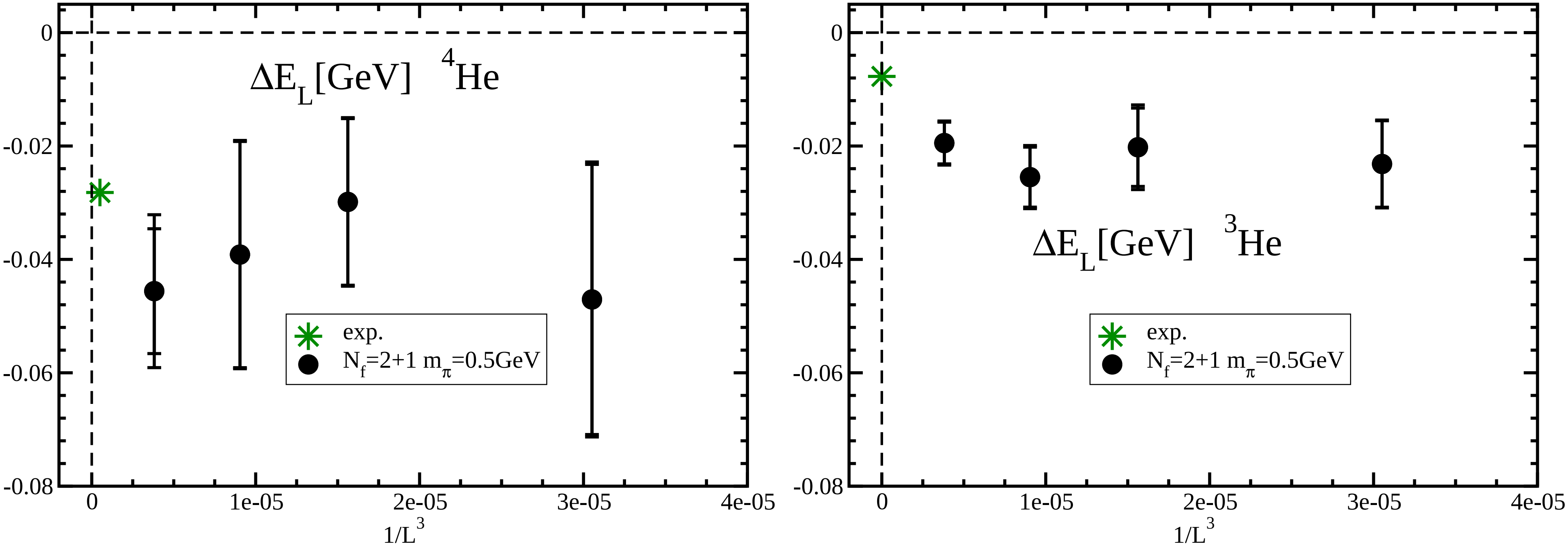


- Statistical error under control in $t < 12$
- Relatively smaller error in ^{3}He channel
- Negative ΔE_L in both channels

3. Preliminary results

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$

Identification of bound state from volume dependence of ΔE

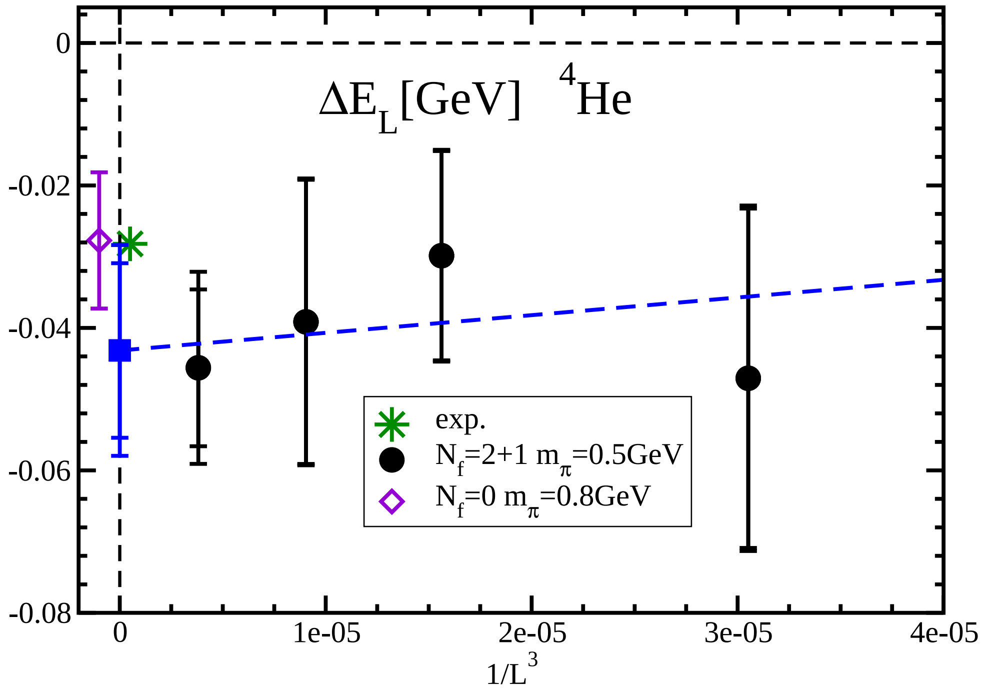


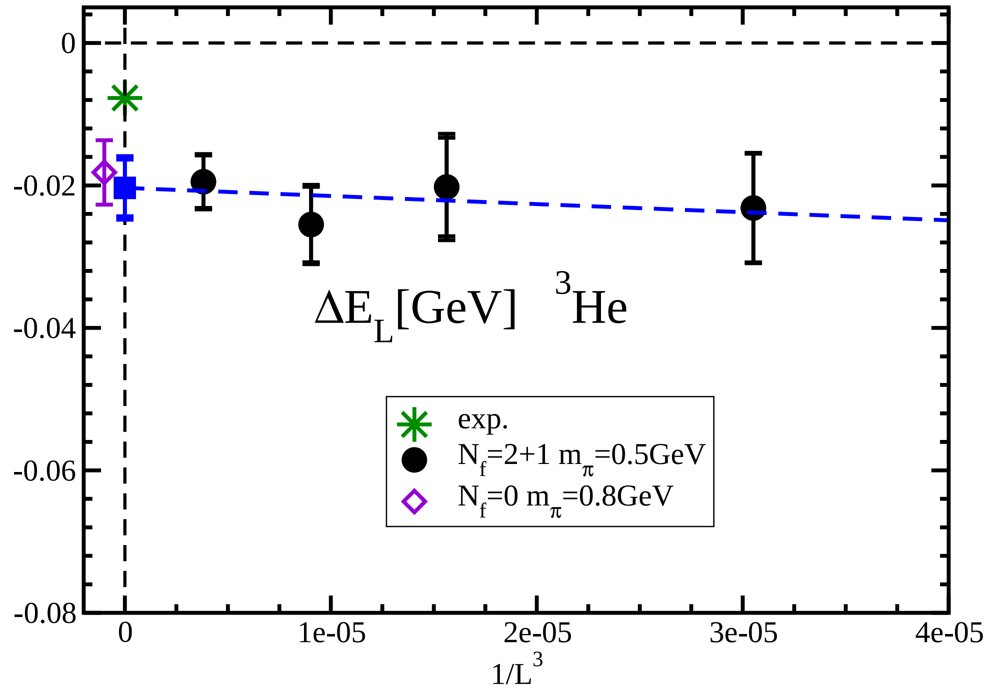
- $\Delta E_L < 0$ and small volume dependence
- Infinite volume extrapolation with $\Delta E_L = -\Delta E_{\text{bind}} + C/L^3$

3. Preliminary results

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$

Identification of bound state from volume dependence of ΔE





$\Delta E_{^4\mathrm{He}} = 43(12)(8)$ MeV $\quad\quad$ $\Delta E_{^3\mathrm{He}} = 20.3(4.0)(2.0)$ MeV

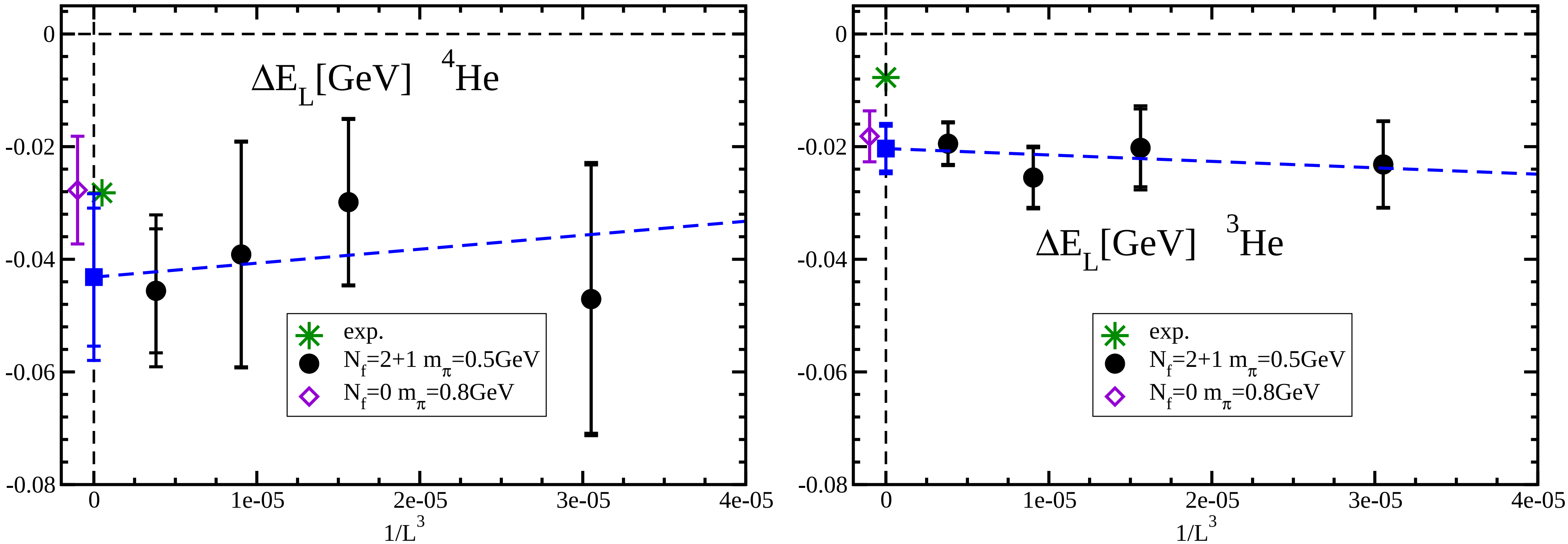
1. Observe bound state in both channels
2. Similar to quenched result

open symbols: quenched at $m_\pi = 0.8$ GeV, PRD81:111504(R)(2010)

3. Preliminary results

^{4}He and ^{3}He channels $\Delta E_L = E_0 - N_N m_N$

Identification of bound state from volume dependence of ΔE



$\Delta E_{^4\mathrm{He}} = 43(12)(8)$ MeV $\quad$ $\Delta E_{^3\mathrm{He}} = 20.3(4.0)(2.0)$ MeV

'12 NPLQCD: $N_f = 3$ $m_\pi = 0.8$ GeV

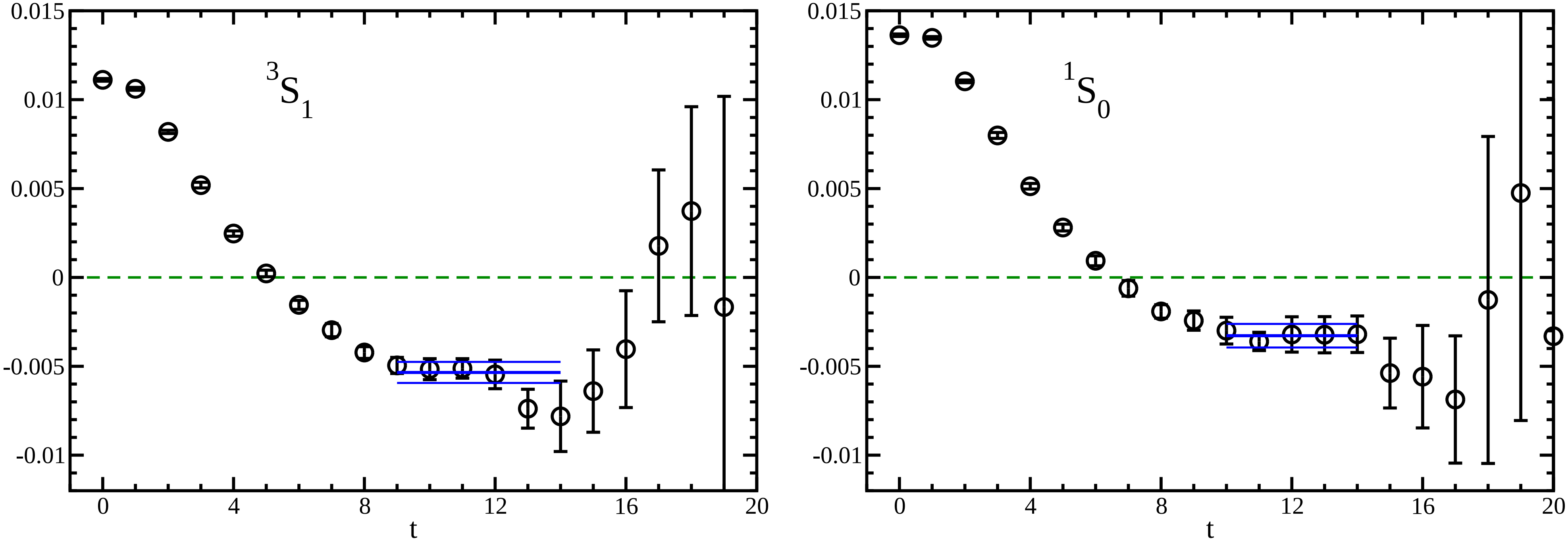
$\Delta E_{^4\mathrm{He}} = 110(20)(15)$ MeV $\quad$ $\Delta E_{^3\mathrm{He}} = 71(6)(5)$ MeV

Large quark mass effect? Further investigation is necessary.

3. Preliminary results

ΔE_L in 2-nucleon channels at $L = 5.8$ fm

$$\Delta E_L = \log\left(\frac{R_{\rm NN}(t)}{R_{\rm NN}(t+1)}\right) \text{ with } R_{\rm NN}(t) = \frac{C_{\rm NN}(t)}{(C_N(t))^2}$$

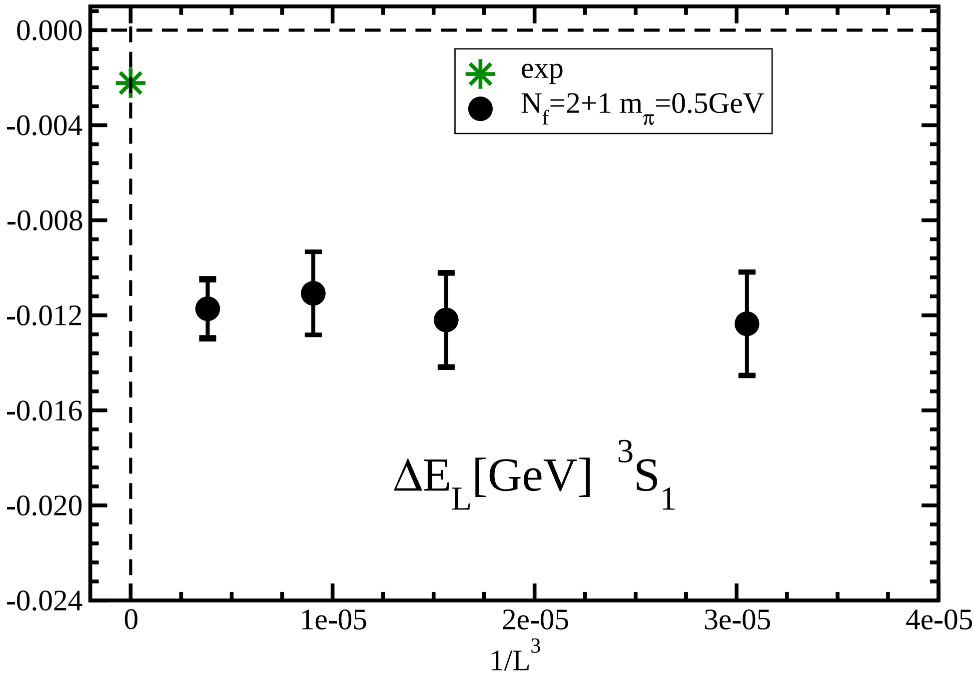


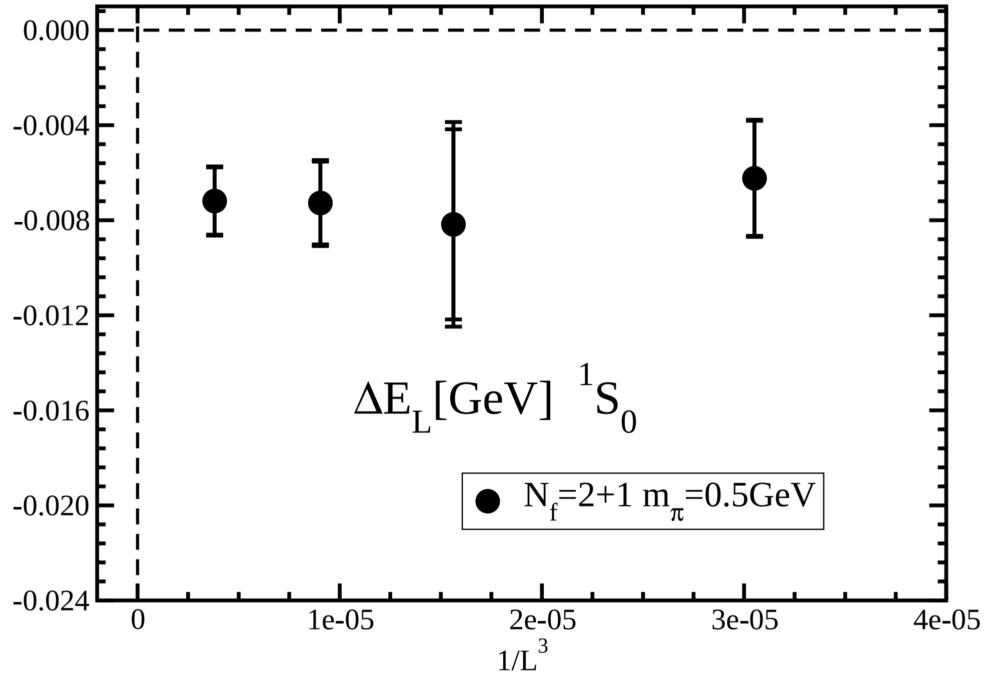
- Statistical error under control in $t \leq 12$
- Relatively smaller error than ^{4}He and ^{3}He channels
- Negative ΔE_L in both channels

3. Preliminary results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

Identification of bound state from volume dependence of ΔE





- Negative ΔE_L
- Infinite volume extrapolation of ΔE_L '04 Beane *et al.*, '06 Sasaki & TY

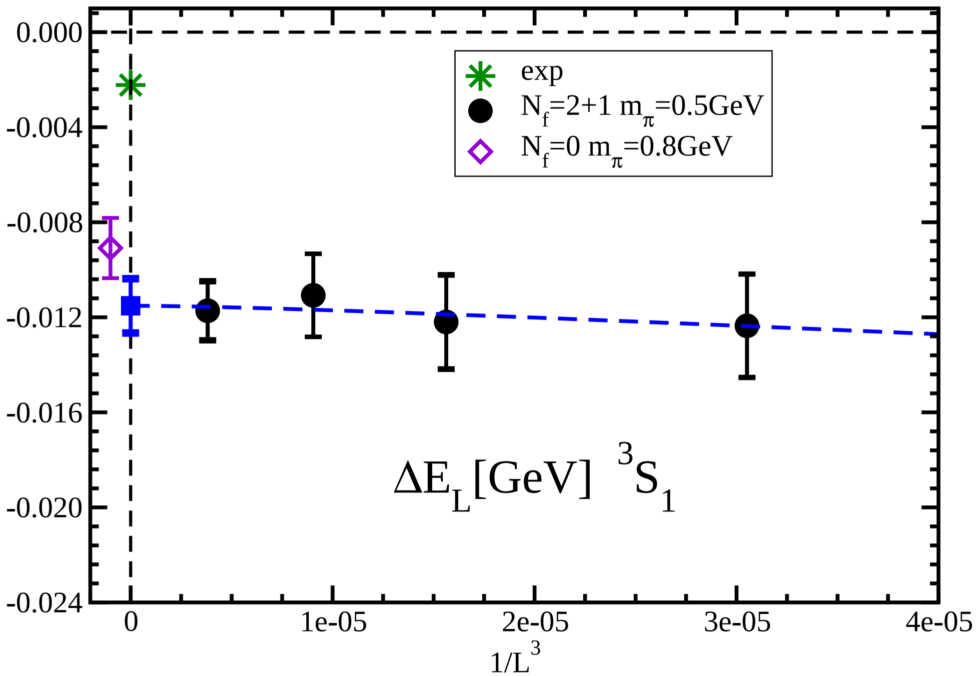
$$\Delta E_L = -\frac{\gamma^2}{m_N}\left\{1 + \frac{C_\gamma}{\gamma L}\sum_{\vec{n}}{}' \frac{\exp(-\gamma L\sqrt{\vec{n}^2})}{\sqrt{\vec{n}^2}}\right\},\ \Delta E_{\text{bind}} = \frac{\gamma^2}{m_N}$$

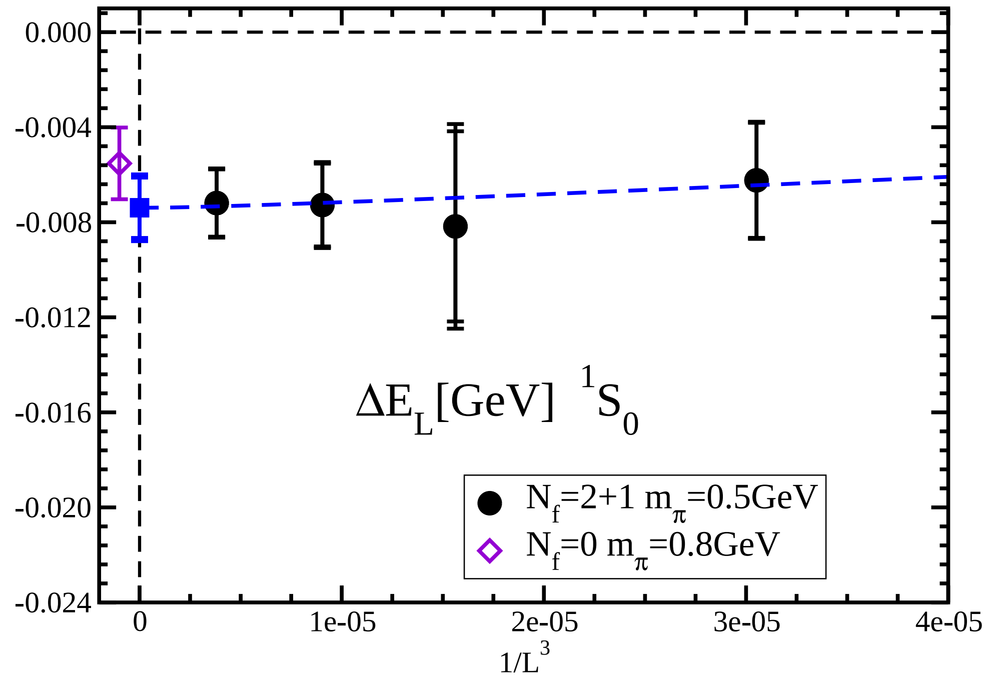
based on Lüscher's finite volume formula

3. Preliminary results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

Identification of bound state from volume dependence of ΔE





Bound state in both channels $\leftarrow$ different from experiment

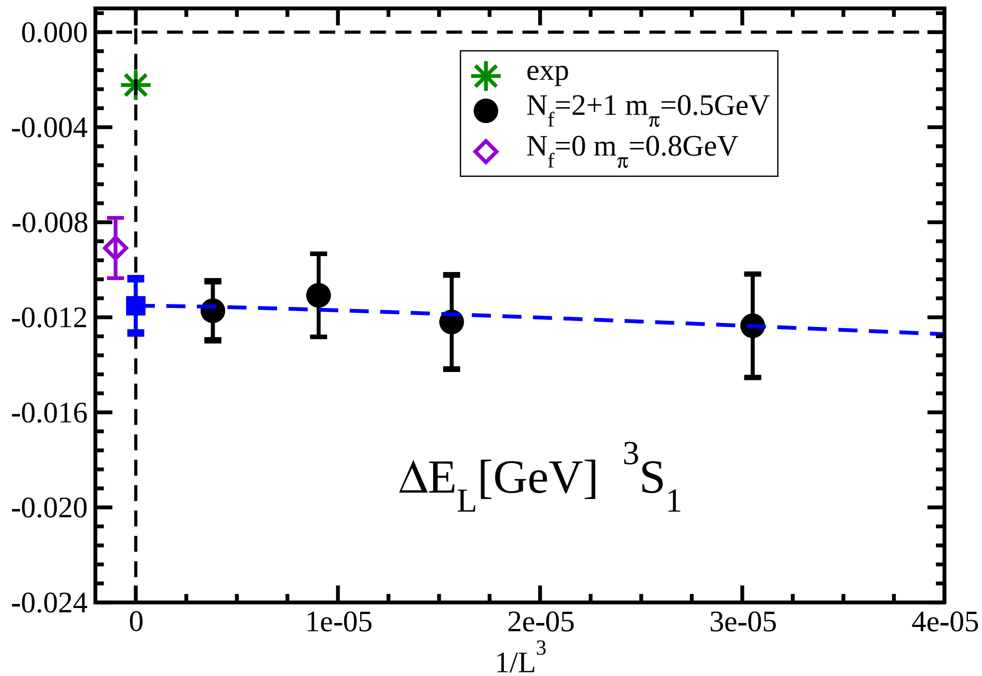
$\Delta E_{^3S_1} = 11.5(1.1)(0.6)$ MeV $\qquad \Delta E_{^1S_0} = 7.4(1.3)(0.6)$ MeV

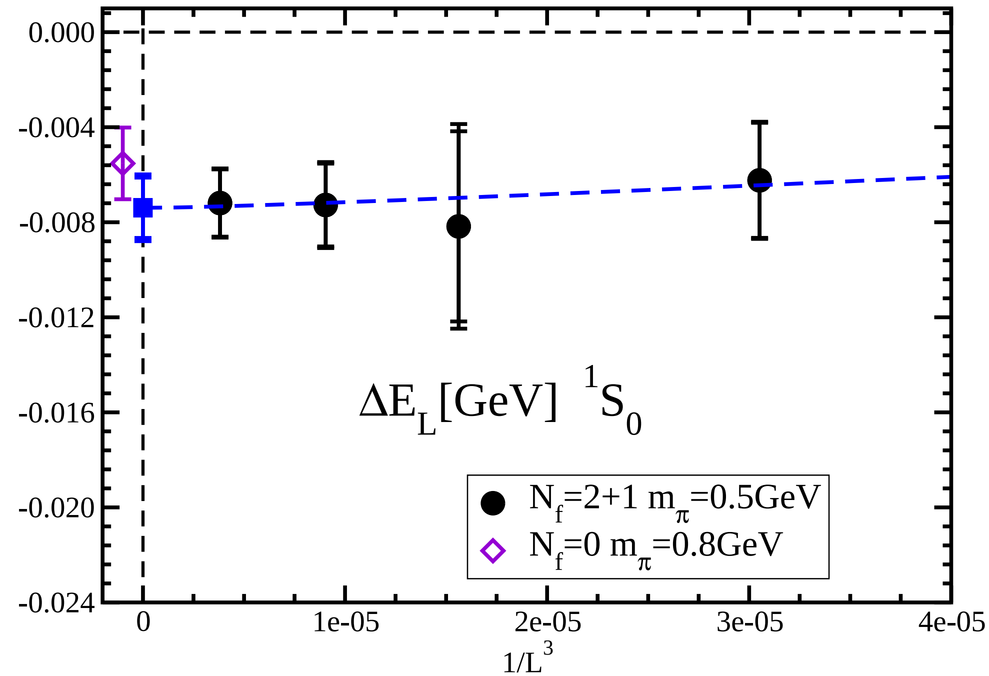
- Similar to quenched result at $m_\pi = 0.8$ GeV PRD84:054506(2011)
- Possibility of bound states $N_f = 2 + 1$ $m_\pi = 0.39$ GeV ('12 NPLQCD)

3. Preliminary results

NN ($^3S_1$ and $^1S_0$) channels $\Delta E_L = E_0 - 2m_N$

Identification of bound state from volume dependence of ΔE





Bound state in both channels ← different from experiment

$\Delta E_{^3S_1} = 11.5(1.1)(0.6)$ MeV $\qquad \Delta E_{^1S_0} = 7.4(1.3)(0.6)$ MeV

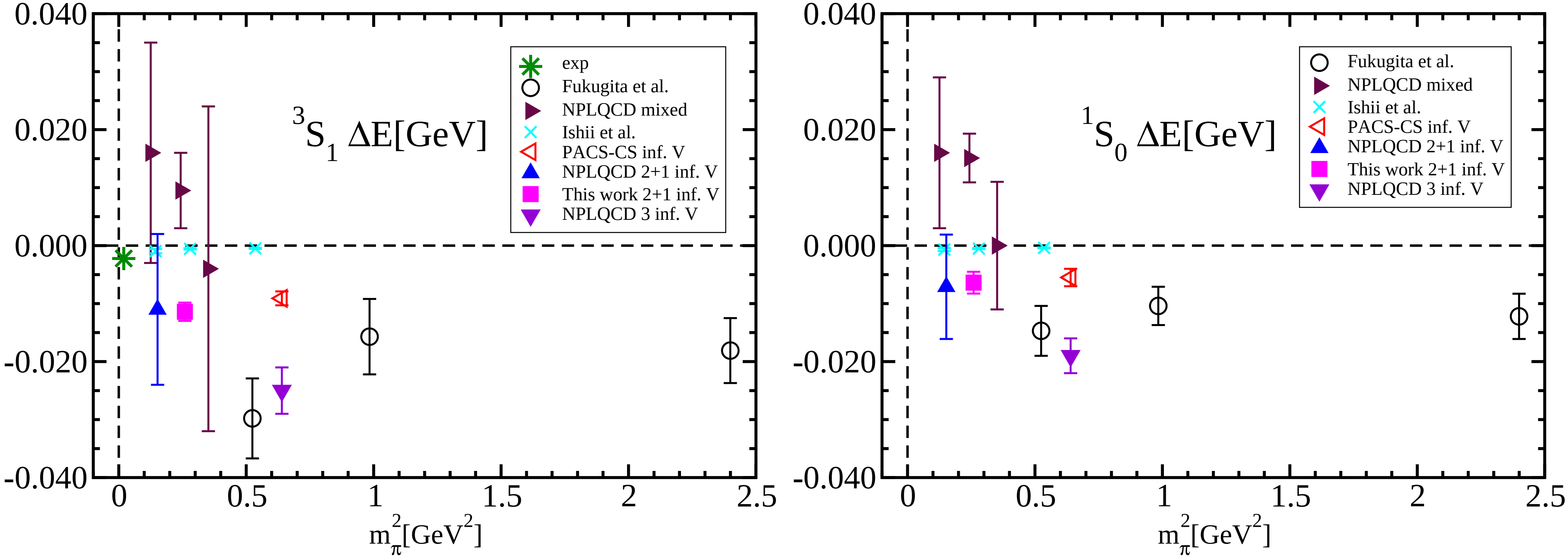
'12 NPLQCD: $N_f = 3$ $m_\pi = 0.8$ GeV

$\Delta E_{^3S_1} = 25(3)(2)$ MeV $\qquad \Delta E_{^1S_0} = 19(3)(1)$ MeV

Large quark mass effect? Further investigation is necessary.

Comparison of ΔE with previous works

NN ($^3S_1$ and $^1S_0$) channels



filled(open) symbols: dynamical(quenched) results

Roughly consistent with other results

Large sea quark effect? $\leftarrow$ Our quenched and NPLQCD $N_f = 3$ results

Large quark mass effect? $\leftarrow$ This work and NPLQCD $N_f = 3$ result

Need further study of quark mass dependence

4. Summary and future work

Extend our exploratory, quenched studies to $N_f = 2 + 1$ lattice QCD

- Heavy quark mass of $m_\pi = 0.5$ GeV
- Volume dependence of ΔE_0

  $\Delta E \neq 0$ of 0th state in infinite volume limit

$\rightarrow$ bound state in ^{4}He, ^{3}He, ^{3}S$_1$ and ^{1}S$_0$ at $m_\pi = 0.5$ GeV

- Similar result to quenched ones $\rightarrow$ ΔE larger than experiment
- Bound state in ^{1}S$_0$ not observed in experiment

  Possibility in $N_f = 2 + 1$ at $m_\pi = 0.39$ GeV ('12 NPLQCD)

  Deep bound state in $N_f = 3$ at $m_\pi = 0.8$ GeV ('12 NPLQCD)

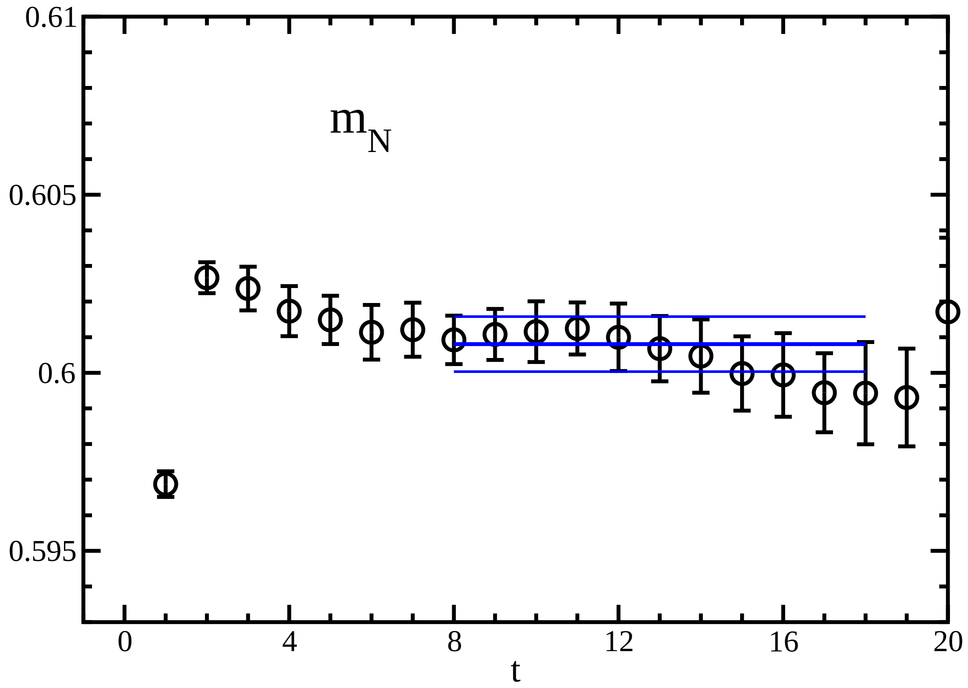
Need further investigations *e.g.* quark mass dependence

Back up

3. Preliminary results

Effective nucleon mass at $L = 5.8$ fm

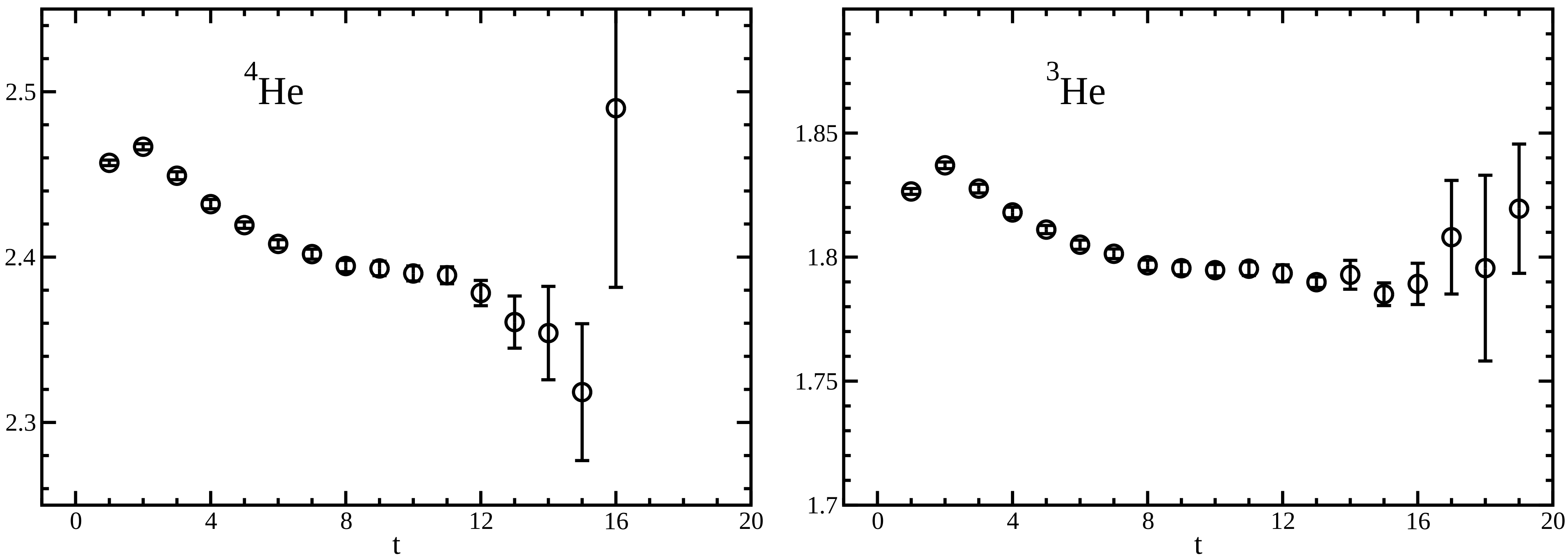
$$m_N = \log\left(\frac{C_{\mathrm{N}}(t)}{C_{\mathrm{N}}(t+1)}\right)$$



3. Preliminary results

Effective energy in ^{4}He and ^{3}He channels at $L = 5.8$ fm

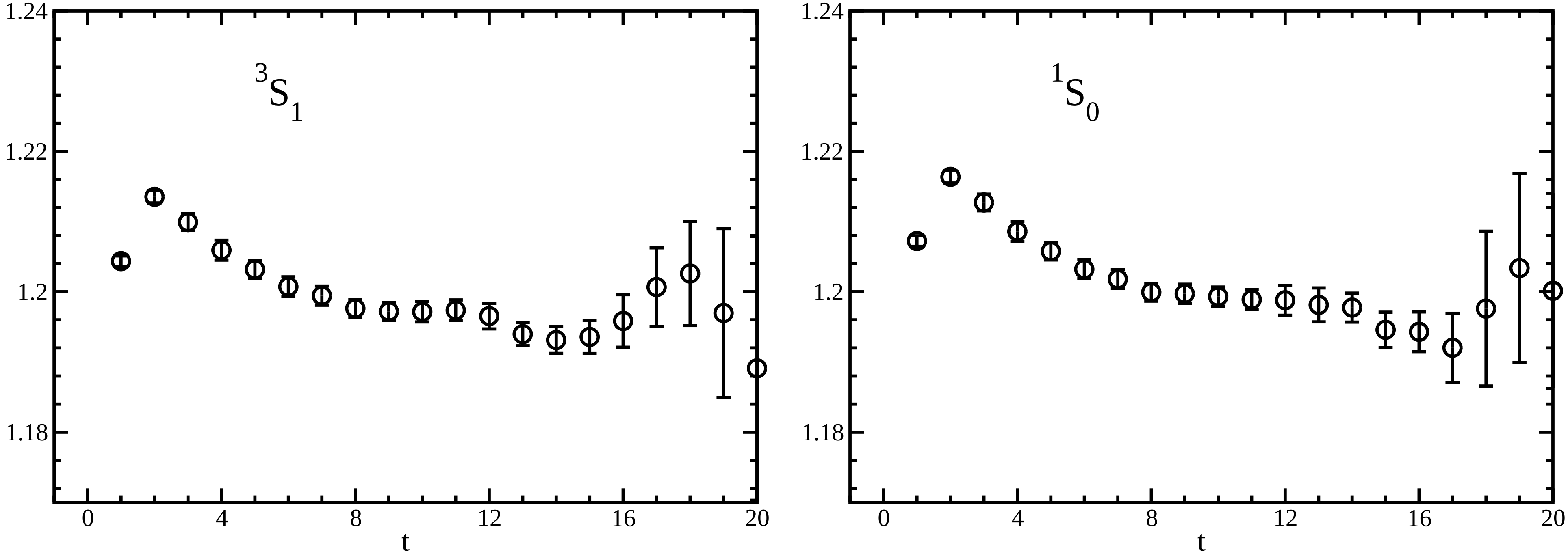
$$E_0 = \log\left(\frac{C_{^4\mathrm{He}}(t)}{C_{^4\mathrm{He}}(t+1)}\right)$$



3. Preliminary results

Effective energy in NN channels at $L = 5.8$ fm

$$E_0 = \log\left(\frac{C_{\mathrm{NN}}(t)}{C_{\mathrm{NN}}(t+1)}\right)$$



Example of large quark mass dependence

rms radii from form factors F_1 and F_2

'09 RBC + UKQCD