Excited and Exotic Charmonium Spectroscopy From Lattice QCD

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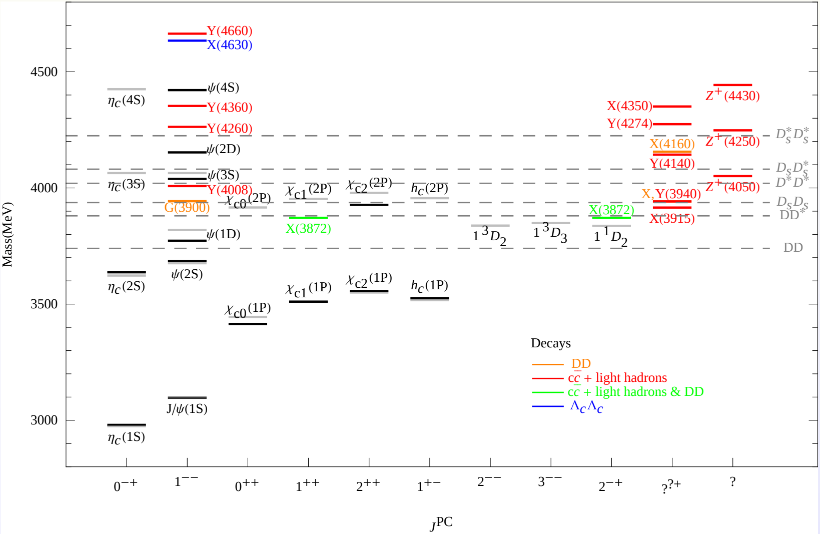
Experimental Overview



Lattice setup

We use the clover configurations generated by Hadron Spectroscopy Collaboration:

- $N_f = 2 + 1$ dynamical flavours
- Anisotropy: $\xi = a_s/a_t = 3.5$
- Lattice spacing: $a_s = 0.12$ fm, $a_t^{-1} = 5.667$ GeV
- Volume: $16^3 \times 128$, $24^3 \times 128$
- $m_\pi = 396$ MeV
- Tree-level Symanzik-improved gauge action
- Wilson clover fermion action.

Phys. Rev. D84, 094506 (2011)

Spectroscopy on lattice

In lattice calculation, meson masses are extracted from two-point correlation functions:

$$C_{ij}(t) = \langle 0|\mathcal{O}_i(t)\mathcal{O}_j^\dagger(0)|0\rangle = \sum_n \frac{Z_i^n Z_j^{n*}}{2E_n} e^{-E_n t}.$$

where $Z_i^n = \langle n|\mathcal{O}_i^\dagger|0\rangle$ are referred as *overlaps.*

- It is essential that the operators overlap well with the states under consideration.
- Distillation quark smearing. Phys.Rev.D80:054506,2009

  Smearing function: $\square_{xy} = \sum_{k=1}^{N_{vecs}} v_x^{(k)} \otimes v_y^{(k)*}$

  meson two-point correlation function:

  $$C(t_1, t_0) = \mathrm{tr}\,[\Phi_1(t_1)\tau(t_1, t_0)\Phi_0(t_0)\tau(t_0, t_1)]$$

  with

  $$\Phi_a^{(i,j)} = v^{(i)*}\Gamma_a v^{(j)} \text{ and } \tau^{(i,j)} = v^{(i)*}(t_1)M^{-1}(t_1, t_0)v^{(j)}(t_0)$$

Variational Method

- Large basis of interpolating operators $\{\mathcal{O}_i\}$ with definite J^{PC}.
- Construct the matrix of correlators $C_{ij}(t)$
- Solve the generalized eigenvalue problem

$$C_{ij}(t)v_j^n = \lambda_n(t)C_{ij}(t_0)v_j^n$$

  - Eigenvalues: $\lambda_n(t) \rightarrow e^{-m_n t}(1 + O(e^{-\Delta m\ t}))$
  - Eigenvectors related to the overlaps:
  $$Z_i^n = \sqrt{2m_n}e^{m_n t_0/2}v_j^{n\,*}C_{ji}(t_0).$$
  - Optimal linear combinations of the operators to overlap on a state : $\Omega^n \sim \sum_i v_i^n \mathcal{O}_i$.

Interpolating operators: in continuum space

- Continuum space: $SO(3)$ symmetry → spin (J, M).
- Simplest meson interpolation operators: local fermion bilinears.
  $\bar{\Psi}_{i\alpha}(\mathbf{x}, t)\Gamma_{\alpha,\beta}\Psi_{i,\beta}(\mathbf{x}, t)$, $J^{PC} = 0^{-+}, 0^{++}, 1^{--}, 1^{++}, 1^{+-}$
- Non-local operators with definite (J, M),

$$\mathcal{O}^{J,M} \sim \bar{\Psi}(x)\Gamma_i \overleftrightarrow{D_i}\,\overleftrightarrow{D_j} \cdots \Psi(x)$$

We use up to 3 derivative operators.

Interpolating operators: on lattice

- Lattice: cubic group $\rightarrow$ irreps Λ (A_1, A_2, T_1, T_2, E).

Λ	d_Λ	J
A_1	1	0, 4, 6, $\cdots$
A_2	1	3, 6 ,7, $\cdots$
T_1	3	1, 3, 4, $\cdots$
T_2	3	2, 3, 4, $\cdots$
E	2	2, 4, 5, $\cdots$

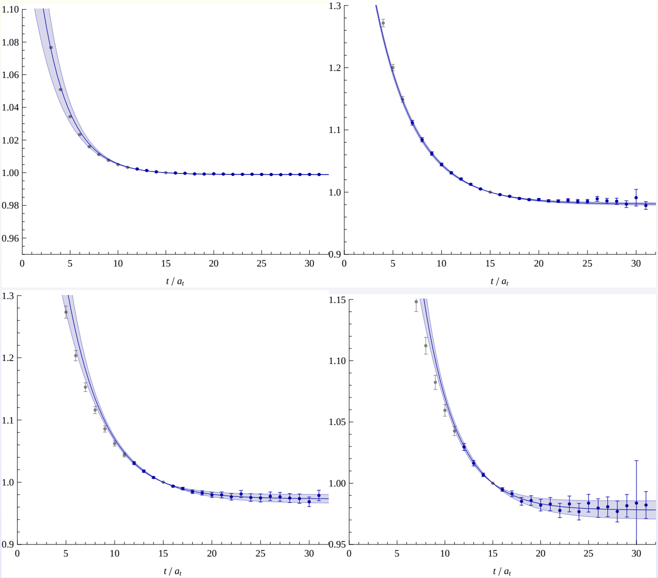
J	Λ
0	$A_1(1)$
1	$T_1(3)$
2	$T_2(3) \bigoplus E(2)$
3	$T_1(3) \bigoplus T_2(3) \bigoplus A_2(1)$
4	$A_1(1) \bigoplus T_1(3) \bigoplus T_2(3) \bigoplus E(2)$

Subduction:

$$\mathcal{O}^{[J]}_{\Lambda,\lambda} = \sum_M \mathcal{S}^{J,M}_{\Lambda,\lambda} \mathcal{O}^{J,M}$$

- Number of operators in each lattice irrep Λ^{PC}

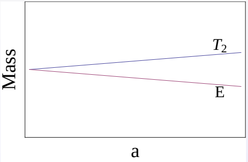
A_1^{++}	13	A_1^{+-}	5	A_1^{-+}	12	A_1^{--}	6
T_1^{++}	22	T_1^{+-}	22	T_1^{-+}	18	T_1^{--}	26
T_2^{++}	22	T_2^{+-}	14	T_2^{-+}	18	T_2^{--}	18
E^{++}	17	E^{+-}	9	E^{-+}	14	E^{--}	12
A_2^{++}	5	A_2^{+-}	5	A_2^{-+}	4	A_2^{--}	6



1.10
1.08
1.06
1.04
1.02
1.00
0.98
0.96
0
5
10
15
20
25
30
t / a_t
1.3
1.2
1.1
1.0
0.9
t / a_t
1.15
1.10
1.05
1.00
0.95

Spin identification

In principle, the spin can be identified by the emergence the energy degenerate between different irreps in the continuum limit.



- Need calculations on different lattice spacing.
- Need high pricision.

Spin identification: overlaps (1)

The operator $\mathcal{O}_{\Lambda}^{[J]}$ carries a “memory" of the continuum spin J, from which it was subduced.

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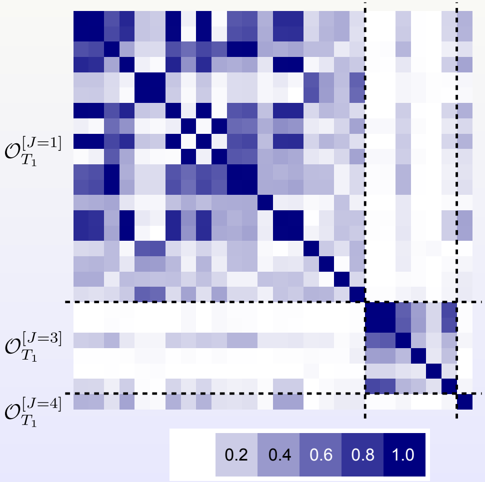

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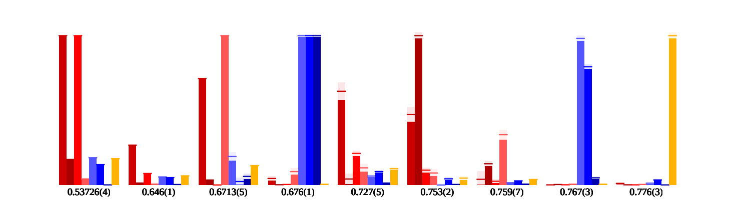
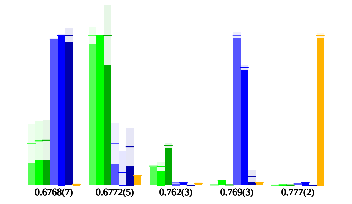
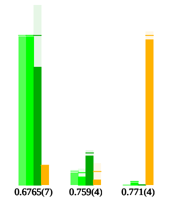


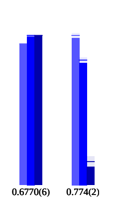
T_1^{--}



T_2^{--}



E^{--}



A_2^{--}

A_1^{--}

Spin identification: overlaps (2)

Comparing the overlaps in different lattice irreps.
In the continuum: $\langle 0|\mathcal{O}^{J,M}|J',M'\rangle = Z^J \delta_{J,J'}\delta_{M,M'}$,
therefore $\langle 0|\mathcal{O}^{[J]}_{\Lambda,\lambda}|J',M\rangle = \mathcal{S}^{J,M}_{\Lambda,\lambda} Z^J \delta_{J,J'}$.
Z^J is common for different irreps.

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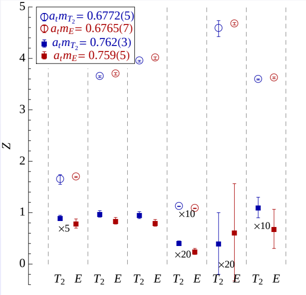
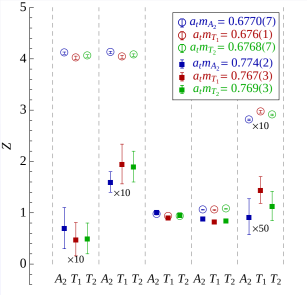
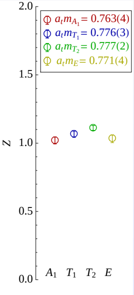


$J = 2$

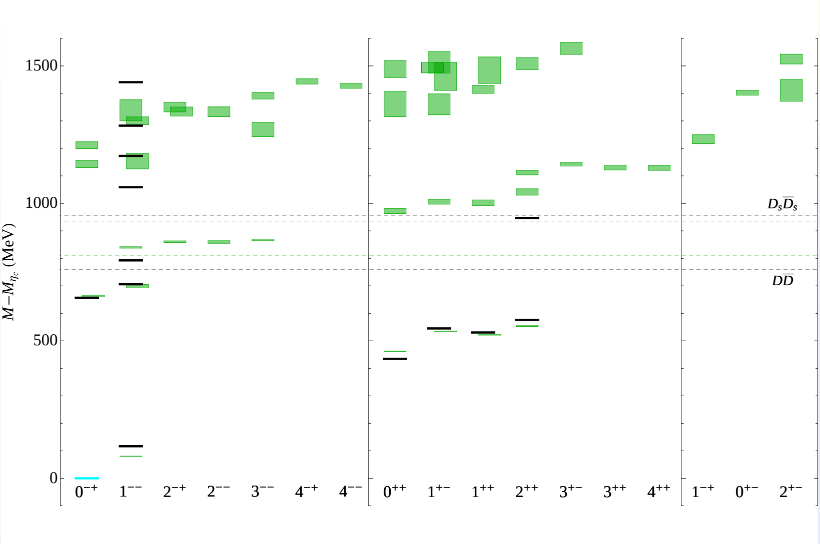


$J = 3$



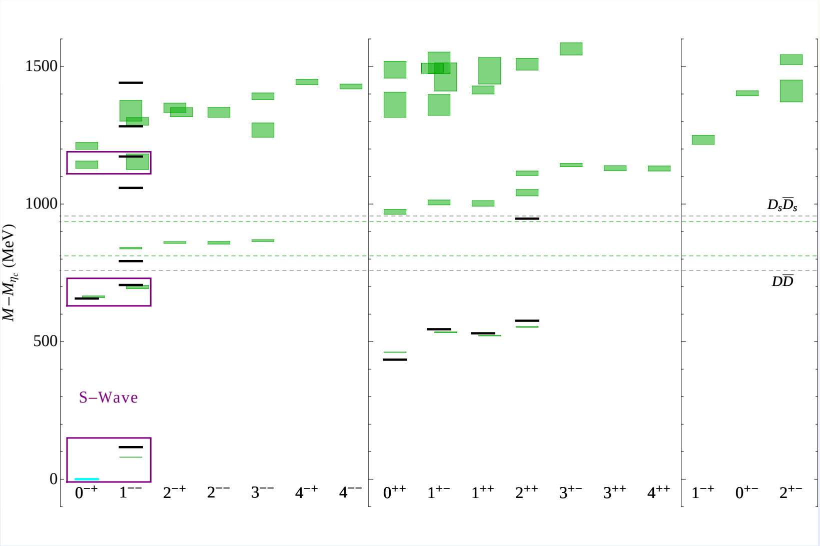
$J = 4$

Results



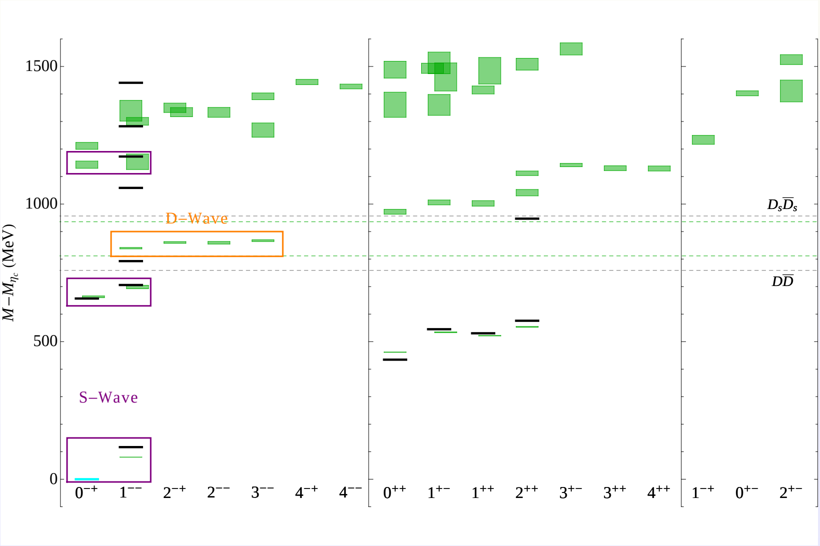
1500
1000
500
0
$M - M_{\eta_c}$ (MeV)
$D_s\overline{D}_s$
$D\overline{D}$
0^{-+}
1^{--}
2^{-+}
2^{--}
3^{--}
4^{-+}
4^{--}
0^{++}
1^{+-}
1^{++}
2^{++}
3^{+-}
3^{++}
4^{++}
1^{-+}
0^{+-}
2^{+-}

Results



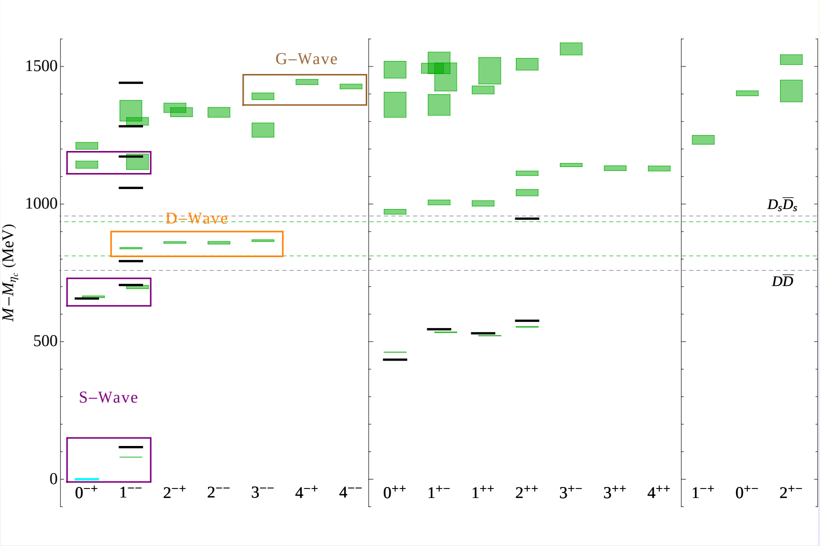
$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
S−Wave
$D_s\overline{D}_s$
$D\overline{D}$
0^{-+} 1^{--} 2^{-+} 2^{--} 3^{--} 4^{-+} 4^{--}
0^{++} 1^{+-} 1^{++} 2^{++} 3^{+-} 3^{++} 4^{++}
1^{-+} 0^{+-} 2^{+-}

Results



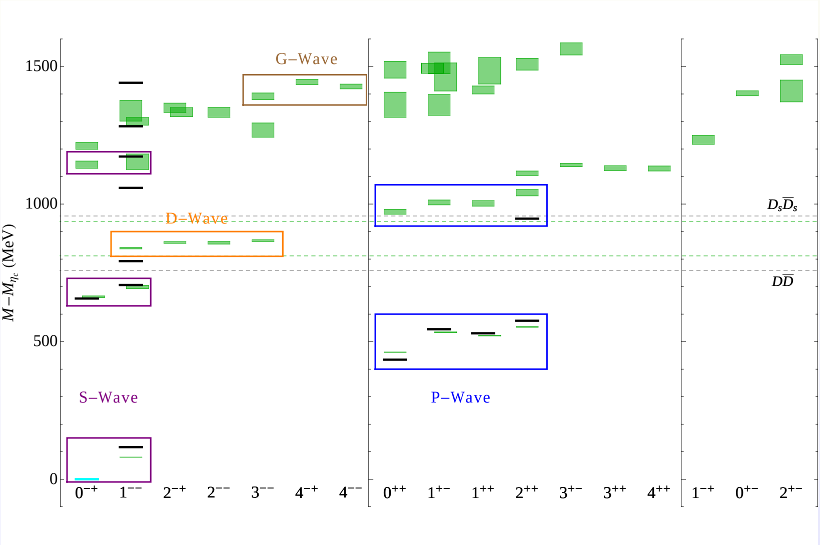
$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
D−Wave
S−Wave
$D_s\overline{D}_s$
$D\overline{D}$
0^{-+} 1^{--} 2^{-+} 2^{--} 3^{--} 4^{-+} 4^{--}
0^{++} 1^{+-} 1^{++} 2^{++} 3^{+-} 3^{++} 4^{++}
1^{-+} 0^{+-} 2^{+-}

Results



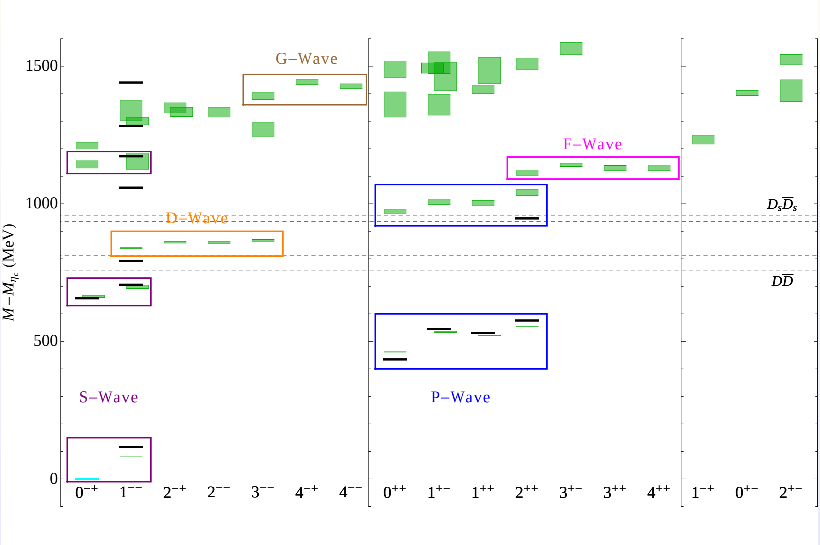
G−Wave
D−Wave
S−Wave
1500
1000
500
0
$M - M_{\eta_c}$ (MeV)
$D_s\overline{D}_s$
$D\overline{D}$
0^{-+} 1^{--} 2^{-+} 2^{--} 3^{--} 4^{-+} 4^{--}
0^{++} 1^{+-} 1^{++} 2^{++} 3^{+-} 3^{++} 4^{++}
1^{-+} 0^{+-} 2^{+-}

Results



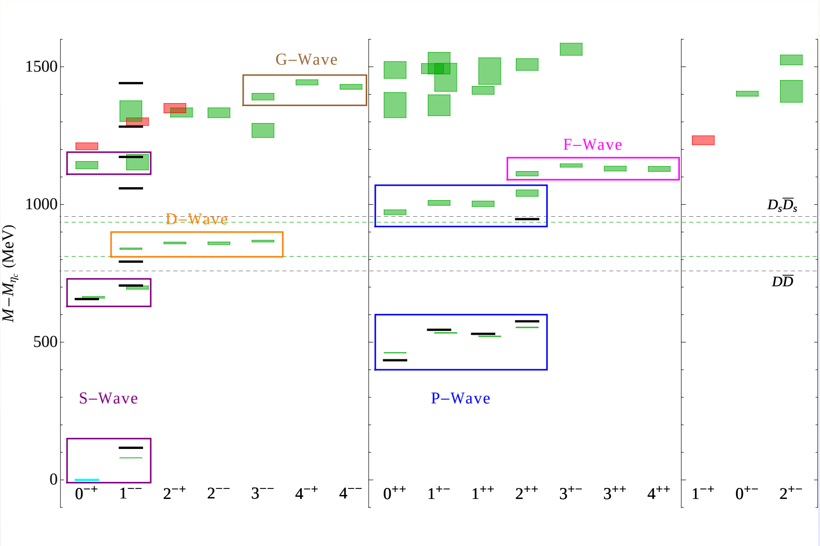
G−Wave
D−Wave
S−Wave
P−Wave
$D_s\overline{D}_s$
$D\overline{D}$
$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
0^{-+}
1^{--}
2^{-+}
2^{--}
3^{--}
4^{-+}
4^{--}
0^{++}
1^{+-}
1^{++}
2^{++}
3^{+-}
3^{++}
4^{++}
1^{-+}
0^{+-}
2^{+-}

Results



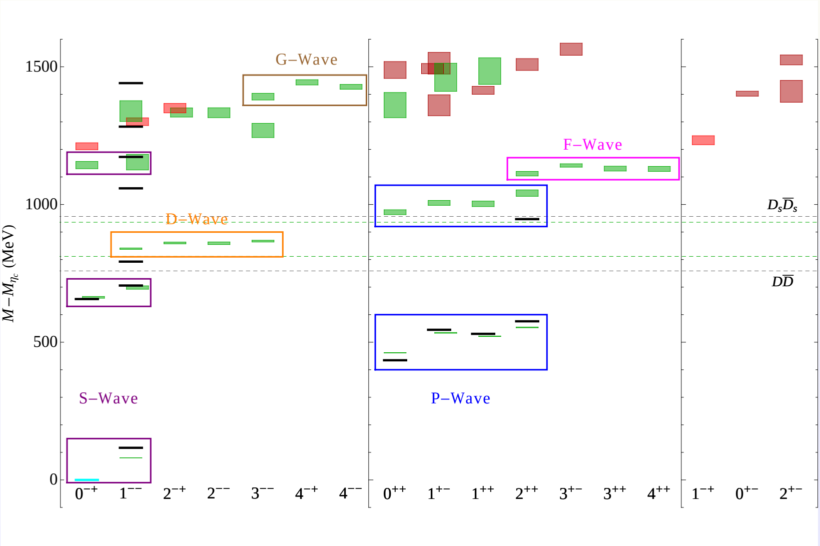
G−Wave
F−Wave
D−Wave
S−Wave
P−Wave
$D_s\overline{D}_s$
$D\overline{D}$
$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
0^{-+} 1^{--} 2^{-+} 2^{--} 3^{--} 4^{-+} 4^{--}
0^{++} 1^{+-} 1^{++} 2^{++} 3^{+-} 3^{++} 4^{++}
1^{-+} 0^{+-} 2^{+-}

Results



G−Wave
F−Wave
D−Wave
S−Wave
P−Wave
$D_s\overline{D}_s$
$D\overline{D}$
$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
0^{-+}
1^{--}
2^{-+}
2^{--}
3^{--}
4^{-+}
4^{--}
0^{++}
1^{+-}
1^{++}
2^{++}
3^{+-}
3^{++}
4^{++}
1^{-+}
0^{+-}
2^{+-}

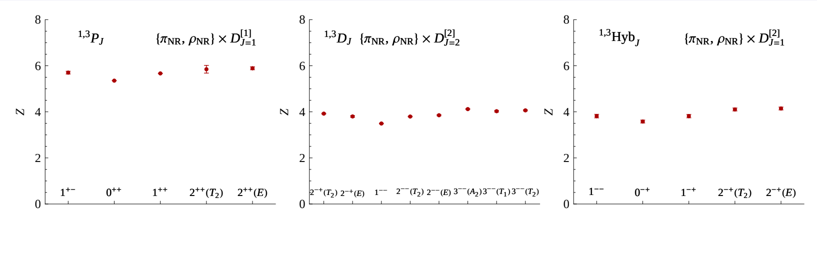
Results



$M - M_{\eta_c}$ (MeV)
1500
1000
500
0
G–Wave
F–Wave
D–Wave
S–Wave
P–Wave
$D_s\overline{D}_s$
$D\overline{D}$
0^{-+} 1^{--} 2^{-+} 2^{--} 3^{--} 4^{-+} 4^{--}
0^{++} 1^{+-} 1^{++} 2^{++} 3^{+-} 3^{++} 4^{++}
1^{-+} 0^{+-} 2^{+-}

Discussions

- Hybrids.
  - Big overlap with the operators $\mathcal{O} \sim [D_i, D_j] \sim F_{i,j}$.
  - Lightest hybrid supermultiplet: $(J_g^{PC} = 1^{+-}) \times (c\bar{c}$ S-wave$) \rightarrow [(0,1,2)^{-+}, 1^{--}]$
  - Exited hybrid supermultiplet: $(J_g^{PC} = 1^{+-}) \times (c\bar{c}$ P-wave$) \rightarrow [0^{+-}, (1^{+1})^3, (2^{+-})^2, 3^{+-}, 0^{++}, 1^{++}, 2^{++}]$
- We see no multi-hadron state in our extracted spectra.
- Supermultiplets.



- $Y(4260), J^{PC} = 1^{--}$; $X(3872), J^{PC} = 2^{-+}$ or 1^{++}.

Outlook

- Analysis of disconnected diagrams and flavour mixing.
- D-meson spectroscopy underway.
- Multi-hadron system.

Thank you!