CONTINUUM RESULTS FROM LATTICE V-A DATA

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OUTLINE

- *The V-A correlator*
- *Some chiral LECs*
- *Constraints on excited PS decay constants*

THE V-A CORRELATOR

- Objects of interest: $\Pi^{(J)}_{V-A}(Q^2)$, $J = 0, 1$

- Minkowski space:

$$\Pi^{\mu\nu}_{V/A}(q^2) \equiv i \int d^4x\, e^{iq\cdot x}\langle 0|T\left(J^{\mu}_{V/A}(x)J^{\dagger\,\nu}_{V/A}(0)\right)|0\rangle$$
$$= \left(q^\mu q^\nu - q^2 g^{\mu\nu}\right)\Pi^{(1)}_{V/A;ij}(q^2) + q^\mu q^\nu\,\Pi^{(0)}_{V/A;ij}(q^2)$$

- Euclidean version:

$$\Pi^{\mu\nu}_{V/A}(Q^2) = \left(Q^2\delta^{\mu\nu} - Q^\mu Q^\nu\right)\Pi^{(1)}_{V/A}(Q^2) - Q^\mu Q^\nu\,\Pi^{(0)}_{V/A}(Q^2)$$

General Properties

- $q^2 = 0$ kinematic singularities for $\Pi^{(0,1)}_{V-A}$, none for $s\Pi^{(0)}_{V-A}$, $\Pi^{(0+1)}_{V-A}$

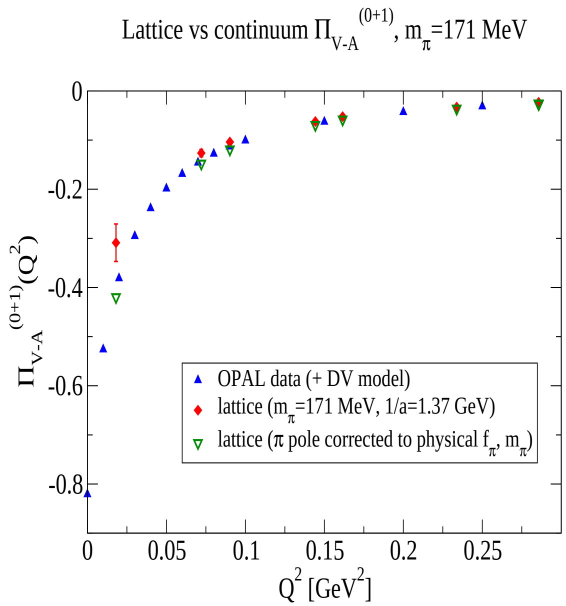
- Spectral functions, $\rho^{(J)}_{V/A}(s)$:

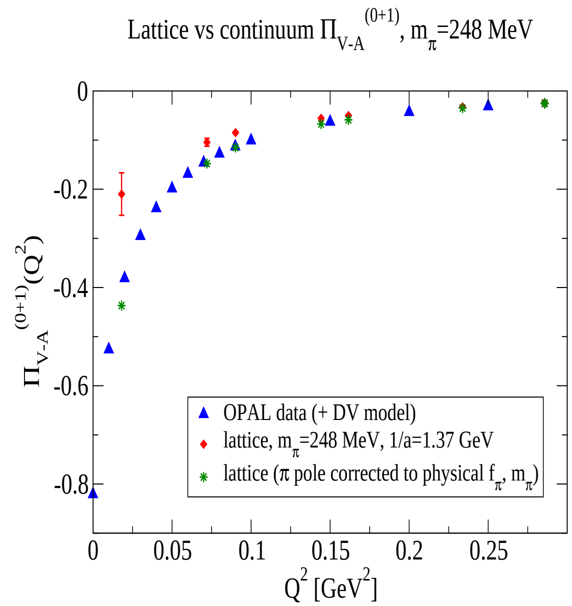
  * $\rho^{(1)}_{V/A}(s)$, π pole term of $\rho^{(0)}_A$ both $O(m^0_{u,d})$

  * Remainder of $\rho^{(0)}_A(s)$: $O[(m_d + m_u)^2]$

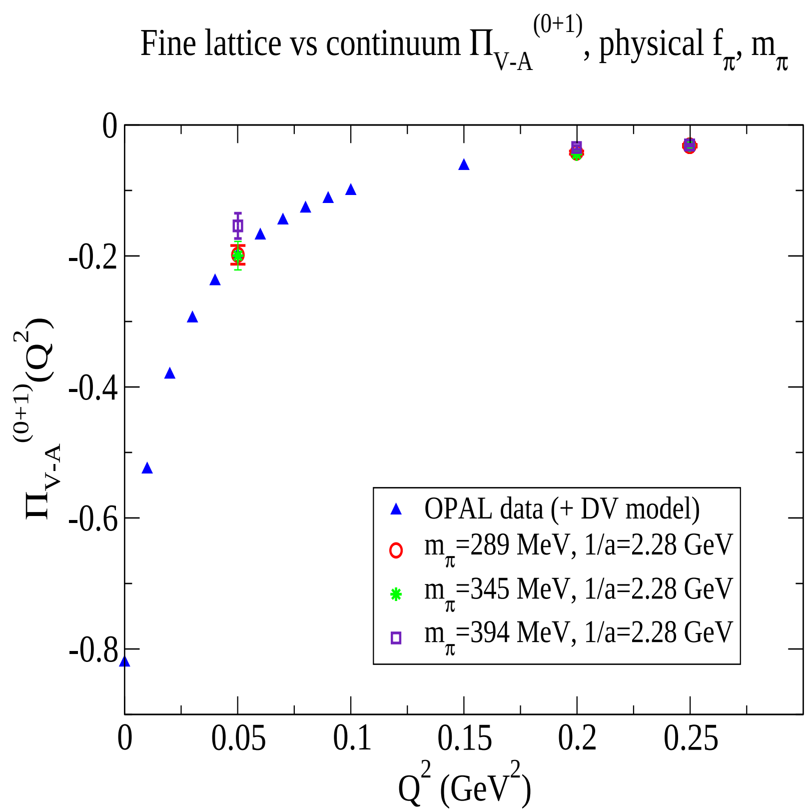
  * $\rho^{(0)}_V(s) \equiv 0$ for $m_u = m_d$ simulations

Lattice data c.f. continuum $\Pi^{(0+1)}_{V-A}(Q^2)$





“Continuum” vs π-pole-corrected, $m_\pi = 289, 345, 394$ MeV



The ChPT LECs ℓ_5^r, L_{10}^r

- Previous analyses

  * 1-loop ChPT fits to $Q^2\,\Pi^{(1)}_{V-A}(Q^2)$

  * JLQCD PRL 101 (2008) 242001: $16^3\times 32$, $n_f=2$, overlap, $1/a=1.67$ GeV, $L\sim 1.9\ fm$, $m_\ell\sim\frac{m_s}{6}\to\frac{m_s}{2}$

  * RBC/UKQCD PR D81 (2010) 014504: $32^3\times 64\times 16_5$, $n_f=2{+}1$, DWF, $1/a=2.33$ GeV, $L\sim 2.7\ fm$, $m_\ell\sim\frac{m_s}{6}\to\frac{m_s}{3}$

  * fits for both limited to single lowest Q^2 [JLQCD: $\sim(320\ MeV)^2$, RBC/UKQCD: $\sim(230\ MeV)^2$]

- Ingredients of the current update

  * More RBC/UKQCD data:

    - $m_\pi = 171, 244$ MeV Iwaskai+DSDR $32^3 \times 64 \times 32_5$, $n_f = 2+1$, $1/a = 1.37$ GeV, $L \sim 4.6\ fm$ [See RBC/UKQCD talks for details, e.g. R. Mawhinney, TH 2:30]

    - Doubled statistics for $m_\pi \sim 290$ MeV

    - Larger $L \Rightarrow$ more low-Q^2 points

  * Analyze $\Pi^{(0+1)}_{V-A}(Q^2)$ rather than $\Pi^{(1)}_{V-A}(Q^2)$

- Here: 1-loop ChPT fit results (2-loop soon)

- Why $\Pi^{(0+1)}_{V-A}$ rather than $\Pi^{(1)}_{V-A}(Q^2)$?
  - * ChPT for $Q^2\,\Pi^{(1)}_{V-A}(Q^2)$ at 1-loop

$$Q^2\,\Pi^{(1)}_{V-A}(Q^2) = -2\left(f_\pi^{1-loop}\right)^2 + Q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5 - \frac{1}{3}\right) + B_{\pi\pi}(Q^2)\right]$$

    with $B_{\pi\pi}(Q^2)$ known (fixed by m_π)

  - * First term is kinematic pole contribution

- Why $\Pi^{(0+1)}_{V-A}$ rather than $\Pi^{(1)}_{V-A}(Q^2)$?

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  * ChPT for $Q^2\,\Pi^{(1)}_{V-A}(Q^2)$ at 2-loops :

$$Q^2\,\Pi^{(1)}_{V-A}(Q^2) = \left[-2\left(f_\pi^{2-loop}\right)^2 + O(p^4)\right] + Q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5 - \frac{1}{3}\right) + B_{\pi\pi}(Q^2) + \cdots\right]$$

- $O(p^4)$ in first line: additional kinematic pole contributions at 4th order in chiral expansion

- $\cdots$: known 2-loop integral contributions, 1-loop contributions proportional to $O(p^2)$ LECs, contributions proportional to $O(p^4)$ LECs

* $\Rightarrow$ Potential systematic complication at the low Q^2 wanted for good convergence of truncated expansion (numerical enhancement of 2-loop kinematic pole contributions relative to $\bar{\ell}_5$ term of interest)

* Nearness $Q^2 = 0$ kinematic pole $\Rightarrow$ tighter cancellation (hence larger relative errors) in residual which determines LEC

* The alternate $Q^2\,\Pi^{(0+1)}_{V-A}(Q^2)$ case

  ◦ At 1-loop

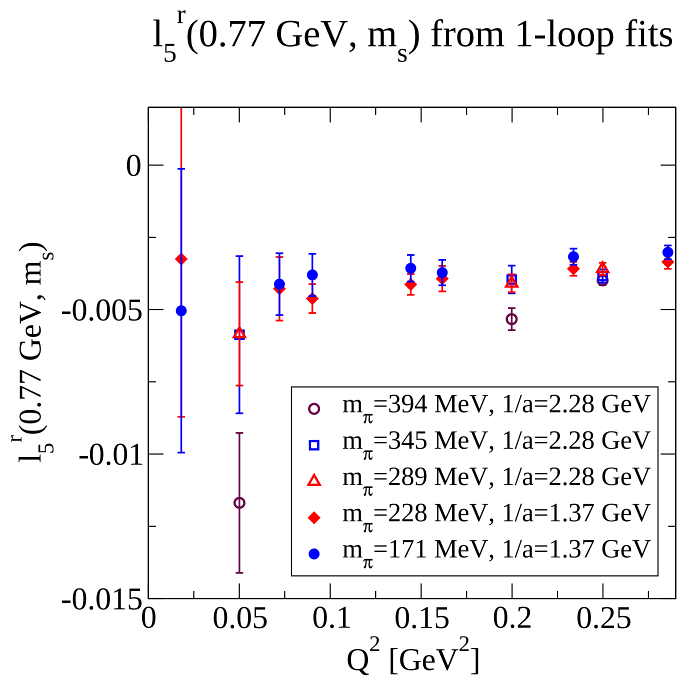
$$Q^2\,\Pi^{(0+1)}_{V-A}(Q^2) = -\frac{2Q^2\left(f_\pi^{1-loop}\right)^2}{Q^2+m_\pi^2} + Q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5-\frac{1}{3}\right)+B_{\pi\pi}(Q^2)\right]$$

  ◦ At 2-loops

$$Q^2\,\Pi^{(0+1)}_{V-A}(Q^2) = -\frac{2Q^2\left(f_\pi^{2-loop}\right)^2}{Q^2+m_\pi^2} + Q^2\left[\frac{1}{24\pi^2}\left(\bar{\ell}_5-\frac{1}{3}\right)+B_{\pi\pi}(Q^2)+\cdots\right]$$

  ◦ No kinematic pole $\Rightarrow$ no potential relative enhancement of unconstrained 2-loop terms, reduced cancellation in residual

$\ell_5^r(m_\rho)$ from $\Pi_{V-A}^{(0+1)}$ (fit for each m_π, Q^2 separately)



LEC results

- Non-trivial shift in relation of ℓ_5^r and L_{10}^r between 1-loop and 2-loop, so quote only 1-loop $\ell_5^r(m_\rho)$ at present

- Good consistency for all Q^2 shown, all but largest m_π

- For maximum safety use only m_π closest to physical ($m_\pi = 171$ MeV), $Q^2 < 0.15$ GeV^2 (acceptability of Q^2 range confirmed by continuum study)

- Compare to 1-loop analysis of continuum "data" (actually OPAL data + DV model for small spectral contributions above $s = m_\tau^2$ [see PR D85 (2012) 093015])

- Small corrections to shift $\ell_5^r(m_\rho, m_K^{sim})$ to $\ell_5^r(m_\rho, m_K^{phys})$

- Lattice result (c.f. result of same 1-loop analysis of OPAL+DV continuum “data”)

$$\ell_5^r(m_\rho) \;=\; -0.0037 \pm 0.0004\ (lattice)$$
$$\ell_5^r(m_\rho) \;=\; -0.0035 \pm 0.0001\ (continuum)$$

- *Comment: continuum error includes contribution from fitted DV model parameter uncertainties; however, DV contributions small, and even expanding the DV error to* 100% *expands continuum error only to* ± 0.0002

Constraints on the π' and π'' Decay Constants

- Basic idea

  * $P(Q^2) \equiv Q^2\,\Pi^{(0)}_{V-A}(Q^2) = -Q^2\,\Pi^{(0)}_A(Q^2)$ satisfies the once-subtracted dispersion relation

$$
\begin{aligned}
P(Q^2) &= P(Q_0^2) + (Q^2 - Q_0^2)\int_0^\infty ds\,\frac{s\,\rho^{(0)}_{V-A}(s)}{(s+Q^2)(s+Q_0^2)} \\
&= P(Q_0^2) - \frac{(Q^2-Q_0^2)\,2f_\pi^2 m_\pi^2}{(s+Q^2)(s+Q_0^2)} \\
&\quad - (Q^2 - Q_0^2)\int_{9m_\pi^2}^\infty ds\,\frac{s\,\rho^{(0)}_A(s)}{(s+Q^2)(s+Q_0^2)}
\end{aligned}
$$

* Rearranged form providing constraints on chirally suppressed continuum spectral contributions (hence on excited PS state decay constants) in terms of quantities measurable on the lattice:

$$P(Q^2) - P(Q_0^2) + \frac{(Q^2 - Q_0^2)\, 2 f_\pi^2 m_\pi^2}{(s+Q^2)(s+Q_0^2)} =$$

$$- (Q^2 - Q_0^2) \int_{9m_\pi^2}^{\infty} ds\, \frac{s\, \rho_A^{(0)}(s)}{(s+Q^2)(s+Q_0^2)}$$

* Spectral positivity $\Rightarrow$ constraints on individual excited PS resonance contributions

* Linearity of excited state f_P with $m_u + m_d$ allows scaling of constraint bounds to physical m_q

- Analysis strategy

  * NWA for π', π''

  $$\rho_A^{(0);cont}(s) \quad = 2f_{\pi'}^2\delta(s - m_{\pi'}^2) + 2f_{\pi''}^2\delta(s - m_{\pi''}^2) + \cdots$$

  * Excited state masses $\simeq$ physical masses (since $O(m_\ell^0)$ to LO in the chiral expansion)

  * Spectral positivity, LHS of rearranged dispersion relation $\Rightarrow$ upper bound for sum of π', π'' contributions *for each* Q^2, Q_0^2 *pair*

  * Each such bound linear in the $f_{\pi'}^2$, $f_{\pi''}^2$ plane

* Final combined constraint = inner envelope of the collection of the constraint lines from the full set of Q^2, Q_0^2 pairs

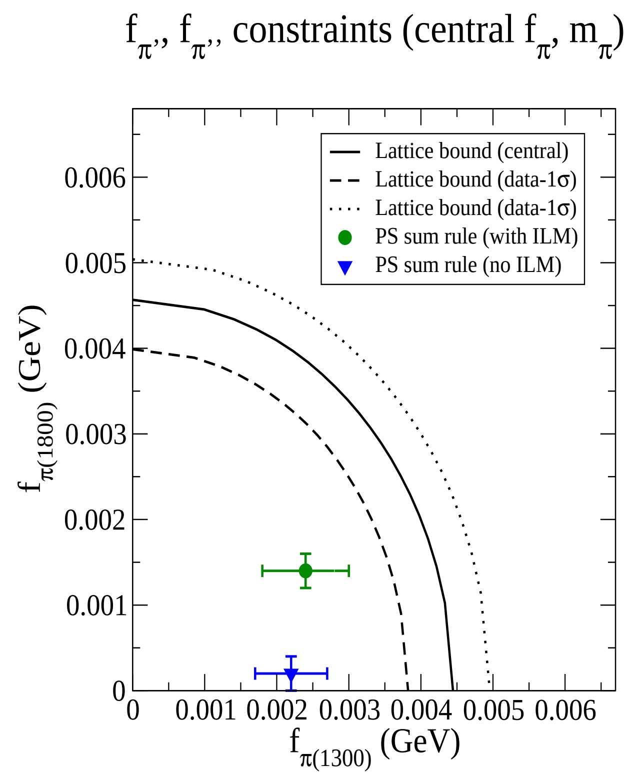
* Q_0^2 small (to improve spectral integral convergence), not too small (to avoid large low-Q_0^2 lattice errors)

* $Q^2 < 3.8\ GeV^2$ (to avoid lattice artefacts)

* Need am_q large enough to see signal in lattice data

* $am_q = 0.004$ ($m_\pi \sim 289$ MeV) (now with increased statistics) the Goldilocks am_q in this case

$am_q = 0.004$ constraints scaled down to physical m_q



SUMMARY

- Successful 1-loop determination $\ell_5^r(m_\rho) = -0.0037(4)$

  * Excellent agreement with same (mildly model-dependent) 1-loop analysis of continuum data

  * 2-loop analysis in progress

  * Other ($O(p^6)$) LECs likely to be determinable in the 2-loop analysis (precision?)

- Non-trivial constraint obtained on π', π'' decay constants
  * NOTE: These enter the most reliable sum rule determinations of $m_u + m_d$ (via FESR, BSR analyses of the $\partial_\mu A^\mu$ 2-point function)
  * Resulting f'_π, f''_π constraints compatible with existing SR results [PR D65 (2002) 074013]
  * Constraint could be fed into SR analysis to produce *upper bounds* on $m_u + m_d$ (c.f. the lower bounds obtained by including only the known π pole contribution on the spectral integral side)