Radiative improvement of spin and Darwin terms in the NRQCD action

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Introduction

- NRQCD has been used successfully to describe both quarkonia and B_q mesons
- Until recently, only tree-level action known and used
- Radiative improvement of $\sigma \cdot B$ operator in NRQCD action known to have significant effect on bottomonium hyperfine splitting [H^4M, arXiv:1105.5309]
- Here:
  - Update on $\sigma \cdot B$
  - Radiative improvement of Darwin term
  - Update on four-fermion operators

NRQCD

NRQCD action used is

$$S = \sum_{\vec{x},\tau} \psi^\dagger(\vec{x},\tau)\left[\psi(\vec{x},\tau) - K(\tau)\psi(\vec{x},\tau)\right]$$

with the kernel

$$K(\tau) = \left(1 - \frac{\delta H|_\tau}{2}\right)\left(1 - \frac{H_0|_\tau}{2n}\right)^n U_4^\dagger(\tau-1)\left(1 - \frac{H_0|_{\tau-1}}{2n}\right)^2\left(1 - \frac{\delta H|_{\tau-1}}{2}\right)$$

where

$$H_0 = \frac{\Delta^{(2)}}{2M_0}\,, \quad \delta H = -c_1\frac{(\Delta^{(2)})^2}{8M_0^3} + c_2\frac{ig}{8M_0^2}\left(\vec{\Delta}^\pm\cdot\vec{E} - \vec{E}\cdot\vec{\Delta}^\pm\right)$$

$$-c_3\frac{g}{8M_0^2}\vec{\sigma}\cdot\left(\vec{\Delta}^\pm\times\vec{E} - \vec{E}\times\vec{\Delta}^\pm\right) - c_4\frac{g}{2M_0}\vec{\sigma}\cdot\vec{B} + c_5 a^2\frac{\Delta^{(4)}}{24M_0} + c_6 a\frac{(\Delta^{(2)})^2}{16nM_0^2}$$

and $n \geq 3/(M_0 a)$ is a stability parameter. At tree level $c_i = 1$.

Matching

Determine radiative corrections to c_i by demanding that QCD and NRQCD give the same effective potential after non-relativistic reduction, i.e. the following diagram commutes:

$$\begin{tikzcd} \mathrm{QCD} \arrow[r, "\text{NR reduction}"] \arrow[d, "\text{1PI}"'] & \begin{array}{c}\text{tree-level}\\ \text{NRQCD}\end{array} \arrow[d, "\text{1PI}"] \\ \Gamma \arrow[r, "\text{NR reduction}"'] & \Gamma^{\mathrm{NR}} \end{tikzcd}$$

$\rightsquigarrow$ Need effective potential for gauge theory.

We will encounter the limits of this approach soon!

Background Field Method

Effective potential is defined by

$$\Gamma[\Phi] = \int_{\mathrm{1PI}} D\phi \, \mathrm{e}^{S[\Phi+\phi]}$$

and only makes sense in perturbation theory restricted to 1PI diagrams.
In gauge theories,

- decompose $A_\mu = B_\mu + g q_\mu$.
- BRST invariance guarantees $D \leq 4$ operators in Γ are gauge covariant $\rightsquigarrow$ renormalizability, but
- $D > 4$ operators are not necessarily gauge covariant

NRQCD is an effective theory with $D > 4$ operators. Gauge covariance

- can be imposed at tree level,
- must be retained at the loop level to avoid serious complications,
- is preserved by using background field gauge
  [DeWitt 1981; Barvinsky, Vilkovitsky 1983; ...]

Lattice Implementation

On the lattice decompose the link as the ordered product

$$U_\mu(x) = \mathrm{e}^{g_0 q_\mu(x+\frac{1}{2}\hat{\mu})}\mathrm{e}^{B_\mu(x+\frac{1}{2}\hat{\mu})}$$

leading to a dependence of the Feynman rules on the number of background and quantum fields (qqq, Bqq, BBq, etc.), and different terms for different orderings (Bqq, qBq, qqB, etc.)

Background field gauge (BFG) is defined by the gauge fixing function

$$f(A) = D_\mu^B q^\mu = (\partial_\mu + iB_\mu)q^\mu$$

which on the lattice affects all vertices with exactly two quantum gluons

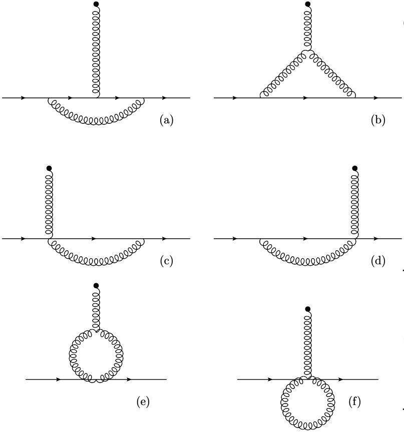
Implemented in HiPPY and HPsrc for automated lattice perturbation theory [Hart, Horgan, vH et al., arXiv:1011.2696]

Checking that

- gauge dependences match for individual terms,
- sum is non-trivially gauge independent,

gives confidence in the correctness of the results.

The $\sigma \cdot B$ and Darwin Terms



Continuum QCD effective action contains

$$\Gamma[\Psi, \overline{\Psi}, A] = \overline{\Psi} F_1(q^2)\, \not{D} \Psi + \overline{\Psi} \frac{F_2(q^2) \sigma^{\mu\nu} F_{\mu\nu}}{2M} \Psi + \ldots$$

Renormalization and NR reduction gives

$$(1 + \underbrace{F_2(0) F_1(0)^{-1}}_{=b_\sigma}) \psi_R^\dagger \frac{\vec{\sigma} \cdot \vec{B}}{2M_R} \psi_R$$

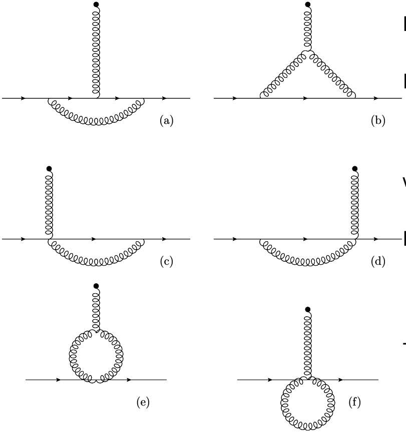
for $\sigma \cdot B$ term,

$$(1 \underbrace{- 8 M_R^2 F_1'(0) + 2F_2(0)}_{=b_D}) \psi_R^\dagger \left(-\frac{g q^2 A_0}{8 M_R^2} \right) \psi_R$$

for Darwin term.

Straightforward continuum calculation:

$$b_\sigma^{(1)} = \left(\frac{3}{2\pi} \log \frac{\mu}{M} + \frac{13}{6\pi} \right) \alpha \qquad b_D^{(1)} = \left(-\frac{M^2}{\pi \mu^2} - \frac{7M}{4\mu} - \frac{1}{\pi} - \frac{50}{9\pi} \log \frac{\mu}{M} \right) \alpha$$



Effective action for NRQCD contains

$$\begin{aligned}\Gamma_\sigma[\psi,\psi^\dagger,A] &= c_4 Z_\sigma^{\mathrm{NR}} \psi^\dagger \frac{i\sigma\cdot(q\wedge A)}{2M}\psi \\ &- c_2 Z_D^{\mathrm{NR}} \psi^\dagger \frac{gq^2A_0}{8M^2}\psi + \ldots\end{aligned}$$

which after renormalization gives

$$\begin{aligned}\Gamma_\sigma[\psi_R,\psi_R^\dagger,A] &= c_4 Z_\sigma^{\mathrm{NR}} Z_2^{\mathrm{NR}} Z_m^{\mathrm{NR}} \psi_R^\dagger \frac{i\sigma\cdot(q\wedge A)}{2M_R}\psi_R \\ &- c_2 Z_D^{\mathrm{NR}} Z_2^{\mathrm{NR}} (Z_m^{\mathrm{NR}})^2 \psi_R^\dagger \frac{gq^2A_0}{8M_R^2}\psi_R +\end{aligned}$$

This gives matching conditions

$$\begin{aligned}c_4 Z_\sigma^{\mathrm{NR}} Z_2^{\mathrm{NR}} Z_m^{\mathrm{NR}} &= 1 + b_\sigma \\ c_2 Z_D^{\mathrm{NR}} Z_2^{\mathrm{NR}} (Z_m^{\mathrm{NR}})^2 &= 1 + b_D\end{aligned}$$

At tree level $c_i^{(0)} = 1$, and at one-loop order we find $(Z = 1 + \delta Z)$

$$\begin{aligned}c_2^{(1)} &= b_D^{(1)} - \delta Z_D^{\mathrm{NR},(1)} - \delta Z_2^{\mathrm{NR},(1)} - 2\delta Z_m^{\mathrm{NR},(1)} \\ c_4^{(1)} &= b_\sigma^{(1)} - \delta Z_\sigma^{\mathrm{NR},(1)} - \delta Z_2^{\mathrm{NR},(1)} - \delta Z_m^{\mathrm{NR},(1)}\end{aligned}$$

Also need to take into account mean-field improvement $U \mapsto U/u_0$ in $\delta Z_{\sigma,D,m}^{\mathrm{NR},(1)}$

Four-Fermion Terms

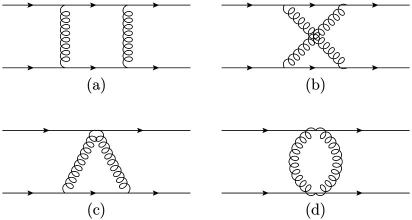
Beyond tree level, NRQCD action contains four-fermion terms

$$\begin{aligned}\mathcal{L}_{4f} &= d_1 \frac{\alpha_s}{M^2}(\psi^\dagger\chi^*)(\chi^t\psi) + d_2 \frac{\alpha_s}{M^2}(\psi^\dagger\sigma\chi^*)(\chi^t\sigma\psi) \\ &+ d_3 \frac{\alpha_s}{M^2}(\psi^\dagger t^a\chi^*)(\chi^t t^a\psi) + d_4 \frac{\alpha_s}{M^2}(\psi^\dagger\sigma t^a\chi^*)(\chi^t\sigma t^a\psi)\end{aligned}$$

or, after Fierz transformations:

$$\begin{aligned}\mathcal{L}_{4f} &= a_1 \frac{g^2}{M^2}(\chi^\dagger\chi)(\psi^\dagger\psi) + a_8 \frac{g^2}{M^2}(\chi^\dagger t_a^t\chi)(\psi^\dagger t_a\psi) \\ &+ b_1 \frac{g^2}{M^2}(\chi^\dagger\sigma^*\chi)(\psi^\dagger\sigma\psi) + b_8 \frac{g^2}{M^2}(\chi^\dagger\sigma^* t_a^t\chi)(\psi^\dagger\sigma t_a\psi)\end{aligned}$$

d_i linear combinations of a_i, b_i computable from box diagrams.



Additional contribution from $Q\overline{Q}$ annihilation in QCD:

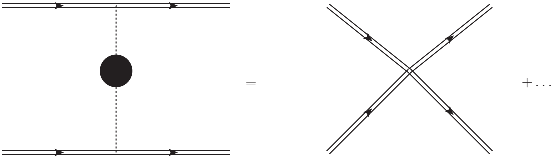
$$d_1^{ann} = -\frac{2\alpha_s}{9M^2}(2 - 2\log 2)$$

IR Divergences

- NRQCD contribution contains IR logarithm $\log(\mu a)$, combines with QCD IR logarithm $\log(\mu/M)$ to give $\log(Ma)$
- any power divergences must match between QCD and NRQCD.
- Spin-independent part: IR divergences up to M^3/μ^3
- Spin-dependent part: IR divergences up to M/μ
- Subtract known QCD divergences analytically from NRQCD integrands.
- This is enough to make spin-dependent part well-defined.
- In the spin-independent part, lattice artifacts can move leading IR divergences to lower order, leaving apparently meaningless $(Ma)^2 \log(\mu a)$ divergence.

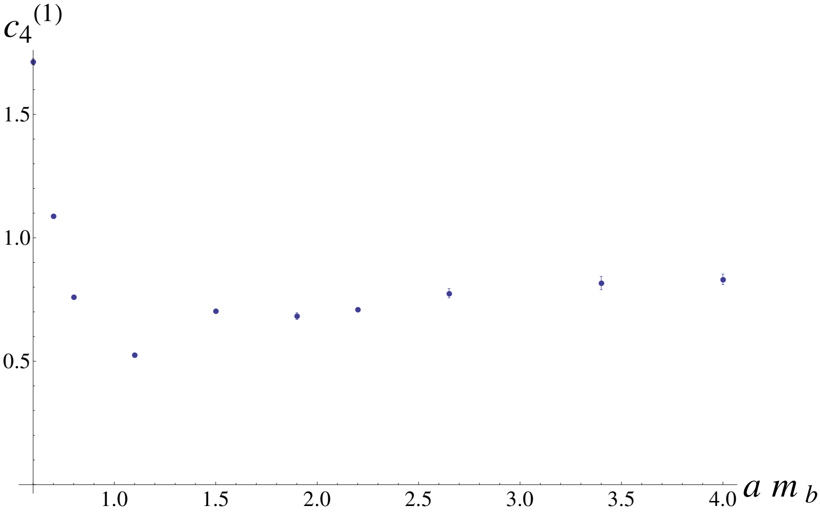
Artifact IR Divergences

- There are no improvements missing in the NRQCD action.
- Origin of artifact divergence is from the octet exchange.
- Cancels against artifact divergence from inserting self-energy a^2 correction on a Coulomb exchange line
- Limits of effective action matching
- Numerically difficult to evaluate – results for spin-independent part still too preliminary to quote

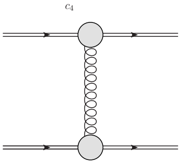


PRELIMINARY Results

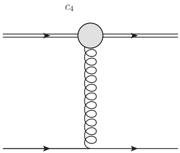
Ma	$c_4^{(1)}$	$c_2^{(1)}$	b_1	b_8
		v^4 NRQCD, $n = 2$		
1.95	0.728(7)	–	0.0080(3)	0.0274(8)
2.8	0.799(7)	–	0.0319(4)	0.0436(15)
4.0	0.842(6)	–	0.0608(5)	0.1451(26)
		v^4 NRQCD, $n = 4$		
1.9	0.68(1)	1.33(8)	0.0043(1)	-0.0378(5)
2.65	0.78(2)	0.08(10)	0.0266(6)	0.0266(1)
3.4	0.82(3)	-1.20(12)	0.0459(2)	0.0909(8)
		v^6 NRQCD, $n = 4$		
1.9	0.78(8)	wip	0.0341(2)	0.0899(9)
2.65	0.76(7)	wip	0.0478(2)	0.1243(7)
3.4	0.83(5)	wip	0.0622(2)	0.1690(9)


$c_4^{(1)}$
1.5
1.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
$a\ m_b$

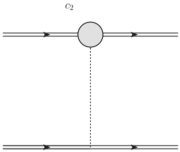
Physical Impact



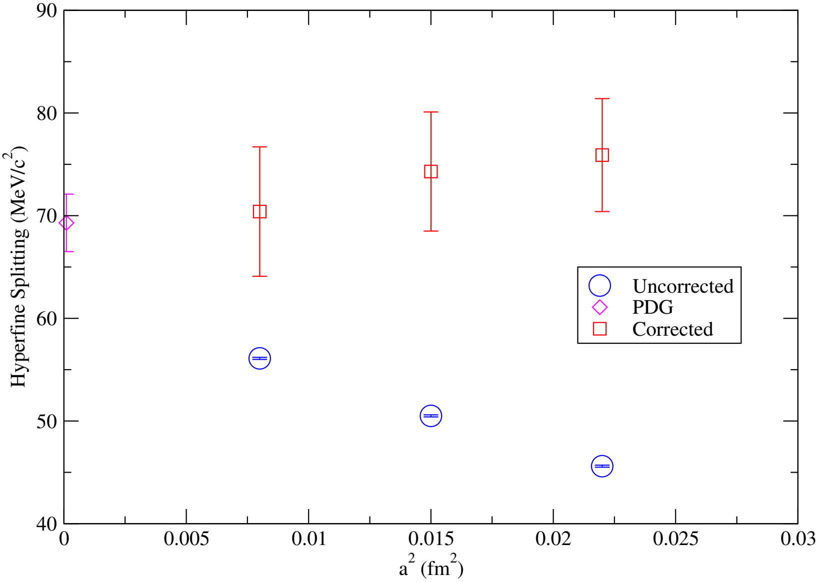
- Bottomonium hfs $\sim c_4^2$, four-fermion term reduces a dependence, compensates smaller hfs for v^6
- Improvement has impact: ~ 60 MeV corrected to 70 MeV in agreement with experiment
- HPQCD prediction [Dowdall et al., arXiv:1110.6887] of 35(3)(1) MeV for 2S hfs agrees with newest Belle results



- B hfs $\sim c_4$, no four-fermion contribution
- good agreement with experiment for B_d, B_s hfs
- allows prediction for B_c hfs . . . [Dowdall et al., arXiv:1111.0449]



- S-wave energy shift $\sim c_2$
- Very small effect except on very coarse lattices



Data from [Dowdall et al., arXiv:1110.6887]

Summary

- Updated radiative improvement of $\sigma \cdot B$ term and spin-dependent four-fermion terms in NRQCD action for $n = 4$ v^4 and v^6 actions
- Radiatively improved NRQCD Darwin term for $n = 4$ v^4 action
- Computing spin-independent four-fermion terms in NRQCD action
- Spin-dependent terms have significant impact on hfs: agreement with experiment depends on radiative improvement
- Spin-independent four-fermion terms require full physical process to be matched properly

The End

Thank you for your attention

Backup Slides

– BACKUP SLIDES –

More on BFG

Background field gauge (BFG) is defined by the gauge fixing function

$$f(A) = D_\mu^B q^\mu = (\partial_\mu + iB_\mu)q^\mu$$

and hence qqB and $qqBB$ vertices are gauge-parameter dependent.
Lattice gauge theories in BFG are renormalizable [Lüscher, Weisz, 1995].
BFG leads to

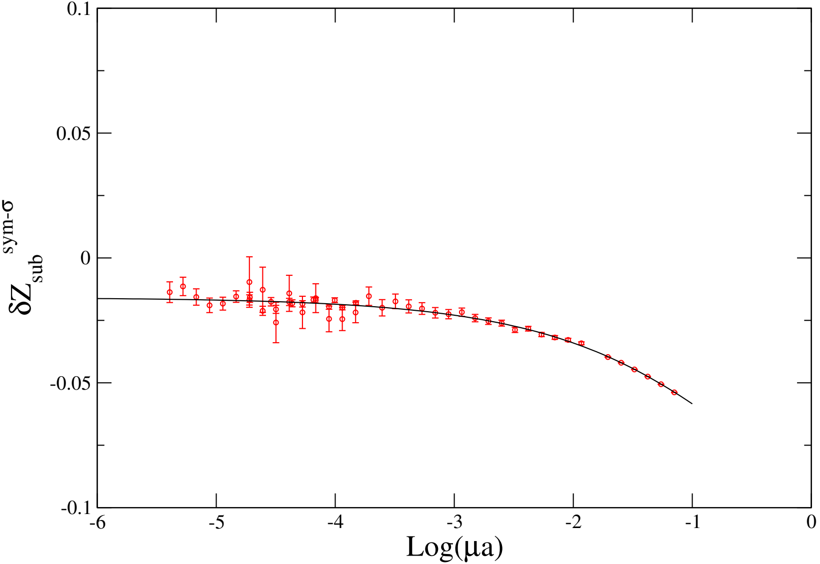
- QED-like Ward identities, and
- finite counterterms.

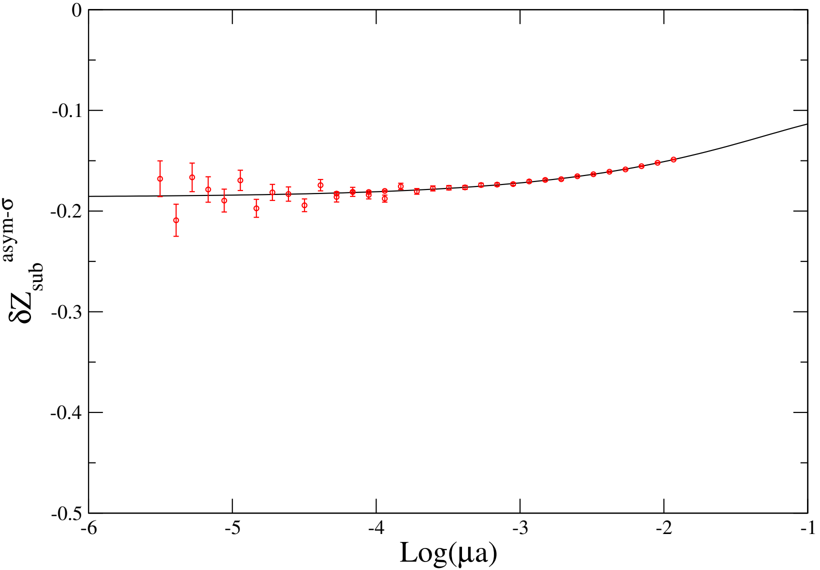
and hence we

- can compute all diagrams numerically, and
- do not need to calculate gauge field renormalization.

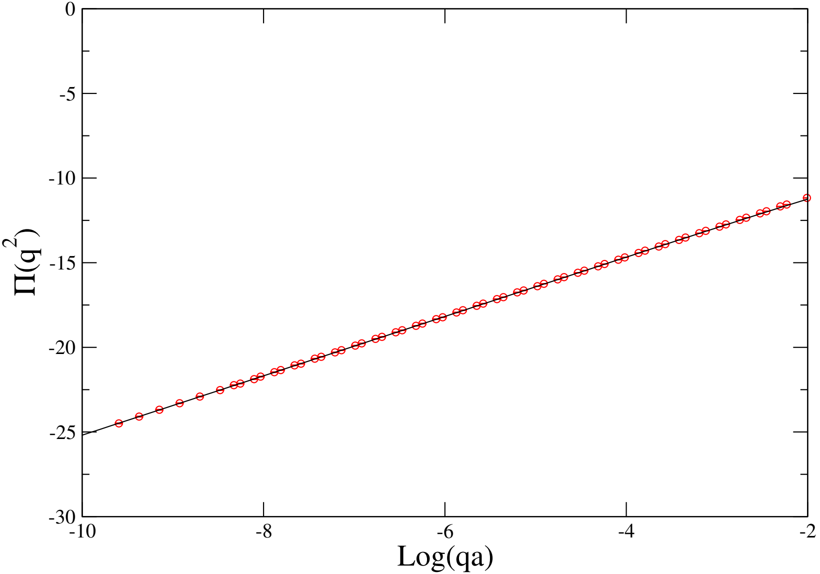
Practical for checking gauge invariance of results.

c_4, c_2 gauge-parameter independent (splittings are physical)

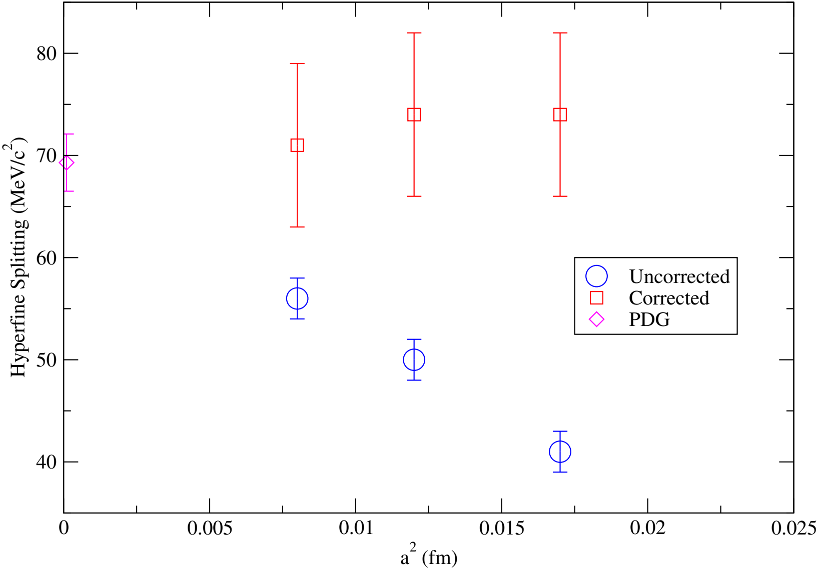

0.1
0.05
0
-0.05
-0.1
-6
-5
-4
-3
-2
-1
0
$\delta Z_{sub}^{sym\text{-}\sigma}$
Log(μa)

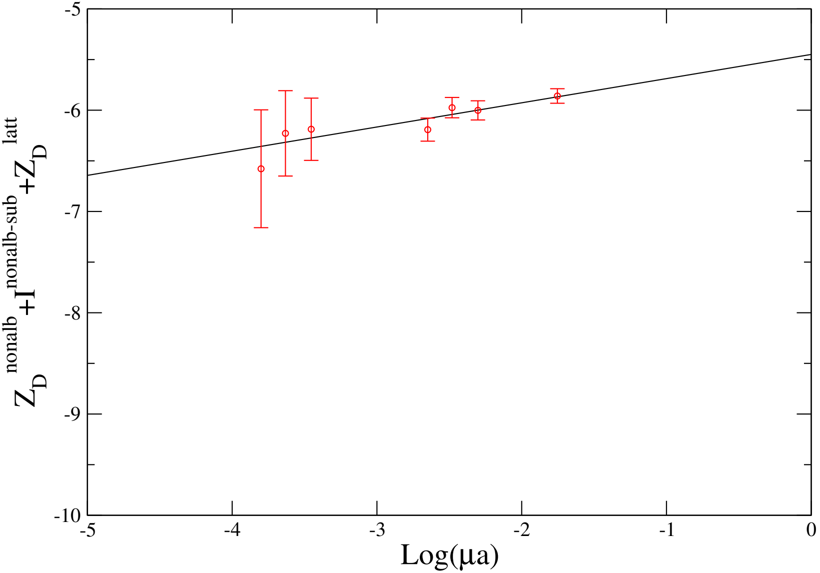


0
-0.1
-0.2
-0.3
-0.4
-0.5
-6
-5
-4
-3
-2
-1
$\delta Z_{sub}^{asym-\sigma}$
Log(μa)

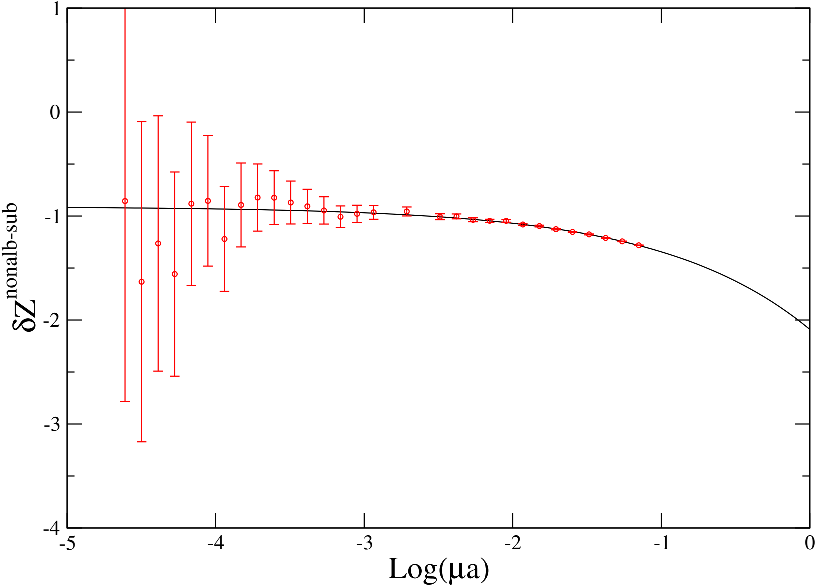


0
-5
-10
-15
-20
-25
-30
-10
-8
-6
-4
-2
Log(qa)
Π(q²)


Hyperfine Splitting (MeV/c^2)
80
70
60
50
40
0
0.005
0.01
0.015
0.02
0.025
a^2 (fm)
Uncorrected
Corrected
PDG



-5
-6
-7
-8
-9
-10
-5
-4
-3
-2
-1
0
$Z_D^{nonalb}+I^{nonalb-sub}+Z_D^{latt}$
Log(μa)



1
0
-1
-2
-3
-4
-5
-4
-3
-2
-1
0
$\delta Z^{\text{nonalb-sub}}$
Log(μa)