Phase shifts in $I = 2$ $\pi\pi$ heavy-pion-scattering from two approaches

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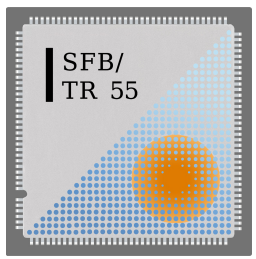
Lattice 2012, Cairns, June 28th 2012

All results are preliminary



Introduction

- nuclear physics: much empirical modeling, not based on fundamental principles $\Rightarrow$ *put on "solid ground" of QCD*
- interesting questions: understanding multi-nucleon systems (*binding energies, scattering phases, etc.*)
- however: direct computation of multi-nucleon scattering complicated (large number of contractions, finite chemical potential, etc.)
- potential method aims at circumventing these problems (HAL-QCD 2012 [arXiv:1203.3642])
- this talk: test and compare this method to the Lüscher method (Lüscher 1986 [Commun.Math.Phys. 105]) in case of less complicated I=2 $\pi\pi$-scattering problem

Ingredients

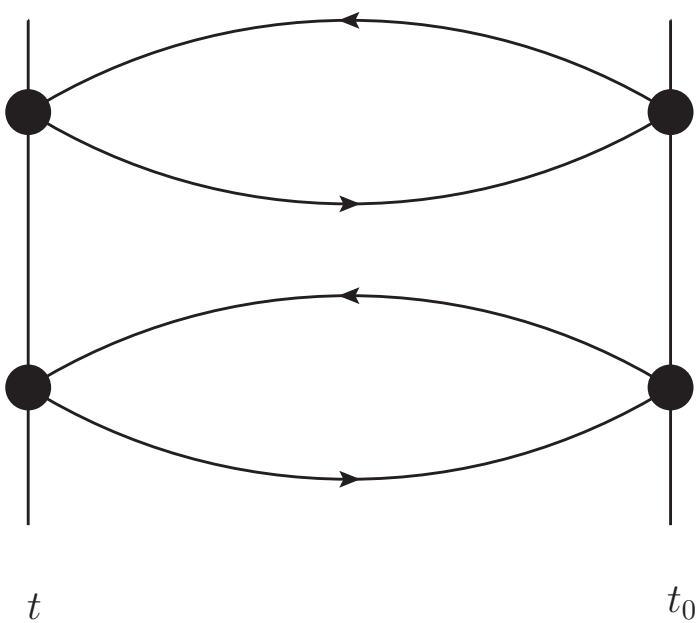
- compute two- and single-pion correlators:

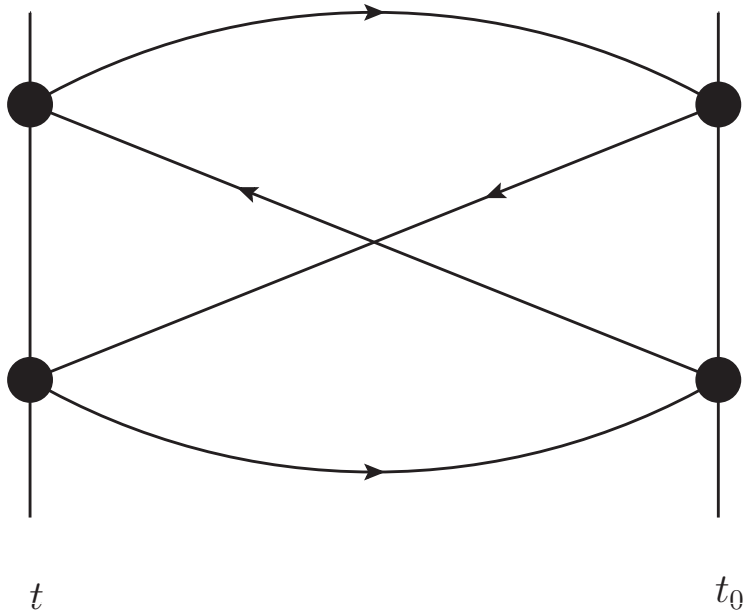
$$C_{\pi\pi}(t,\mathbf{r},\mathbf{P}) \equiv \sum_{\mathbf{R}} e^{-i\mathbf{RP}} \langle \pi^+\big(t,(\mathbf{R}+\mathbf{r})/2\big)\pi^+\big(t,(\mathbf{R}-\mathbf{r})/2\big) J_{\pi-}(0,\mathbf{P}) J_{\pi-}(0,\mathbf{P})\rangle,$$

$$C_{\pi}(t,\mathbf{P}) \equiv \sum_{\mathbf{R}} e^{-i\mathbf{RP}} \langle \pi^+(t,\mathbf{R}) J_{\pi-}(0,\mathbf{P})\rangle$$

- $J_{\pi-}$ are Gaussian or wall sources with momenta $\mathbf{P}$
- applying Dirichlet boundary conditions, i.e. $\bar{\psi}(t^*,\mathbf{x}) = \psi(t^*,\mathbf{x}) = 0$ or anti-periodic boundary conditions
- sources are gauge dependent $\Rightarrow$ fixing to Coulomb gauge

Contractions





- contractions necessary for computing $C_{\pi\pi}(t, \mathbf{r})$
- no disconnected contributions in I=2 channel

Time dependent potential method I

- Nambu-Bethe-Salpeter (NBS) wave-function with asymptotic momentum **k** given by (HAL-QCD 2012 [arXiv:1203.3642])

$$\psi_{\mathbf{k}}(\mathbf{r}) \equiv \sum_{\mathbf{x}} \langle 0|\pi^+(\mathbf{x})\pi^+(\mathbf{x}+\mathbf{r})|\pi^-(\mathbf{k})\pi^-(-\mathbf{k})\rangle$$

- NBS-WF satisfies Schroedinger-equation (note: $E<E_{\mathrm{th}}$)

$$\left(\mathbf{k}^2+\nabla^2\right)\psi_{\mathbf{k}}(\mathbf{r}) = m_\pi \int_{\mathbb{R}^3} \mathrm{d}^3 r'\, U(\mathbf{r},\mathbf{r}')\,\psi_{\mathbf{k}}(\mathbf{r}')$$

and the asymptotic behavior

$$\psi_{\mathbf{k}}(\mathbf{r}) \overset{|\mathbf{r}|=r\to\infty}{\longrightarrow} e^{i\delta(k)}\frac{\sin\left(kr+\delta(k)\right)}{kr}+\ldots$$

Time dependent potential method II

- define

$$R(t,\mathbf{r}) \equiv C_{\pi\pi}(t,\mathbf{r},0)/C_{\pi}^{2}(t,0)$$

- inserting complete set of states yields

$$R(t,\mathbf{r}) = \sum_{\mathbf{k}} \psi_{\mathbf{k}}(\mathbf{r})\, a_{\mathbf{k}}\, e^{-t\Delta E(k)}$$

where $\Delta E(k) = 2\sqrt{k^2 + m_\pi^2} - 2m_\pi$ and

$$a_{\mathbf{k}} = \langle \pi^-(\mathbf{k})\pi^-(-\mathbf{k})|J_{\pi^-}(0,0)J_{\pi^-}(0,0)|0\rangle$$

- use $\Delta E(k) = k^2/m_\pi - \Delta E^2(k)/4m_\pi$ to write

$$-\frac{\partial}{\partial t}R(t,\mathbf{r}) = \sum_{\mathbf{k}} \left(\frac{\mathbf{k}^2}{m_\pi} - \frac{\Delta E^2(k)}{4m_\pi}\right) \psi_{\mathbf{k}}(\mathbf{r})\, a_{\mathbf{k}}\, e^{-t\Delta E(k)}$$

Time dependent potential method III

- this yields

$$\left(\frac{1}{4m_\pi}\frac{\partial^2}{\partial t^2} - \frac{\partial}{\partial t} + \frac{\nabla^2}{m_\pi}\right) R(t,\mathbf{r}) = \int_{\mathbb{R}^3} \mathrm{d}^3 r'\, U(\mathbf{r},\mathbf{r}') R(t,\mathbf{r}')$$

- expand non-local potential for $I{=}2$ case:

$$U(\mathbf{r},\mathbf{r}') \to V_C(\mathbf{r})\,\delta(\mathbf{r}-\mathbf{r}')$$

- this allows us to compute LO potential V_C:

$$V_C(\mathbf{r}) = \frac{\nabla^2 R(t,\mathbf{r})}{m_\pi R(t,\mathbf{r})} - \frac{(\partial/\partial t) R(t,\mathbf{r})}{R(t,\mathbf{r})} + \frac{1}{4m_\pi}\frac{(\partial/\partial t)^2 R(t,\mathbf{r})}{R(t,\mathbf{r})}$$

Time dependent potential method IV

- compute $R(t, r)$ on the lattice
- compute LO potential $V_C(r)$
- model potential using suitable fits
- solve SE for arbitrary k (ext. parameter) and obtain $\psi_k(r)$
- compute scattering phases using $\beta \equiv [r\,\mathrm{d}\ln\psi_k/\mathrm{d}r]_{r=R}$

$$\tan\delta(k) = \frac{kR\,j_0'(kR) - \beta\,j_0(kR)}{kR\,n_0'(kR) - \beta\,n_0(kR)}$$

where $j_0(\rho) = \sin\rho/\rho$ and $n_0(\rho) = -\cos\rho/\rho$.

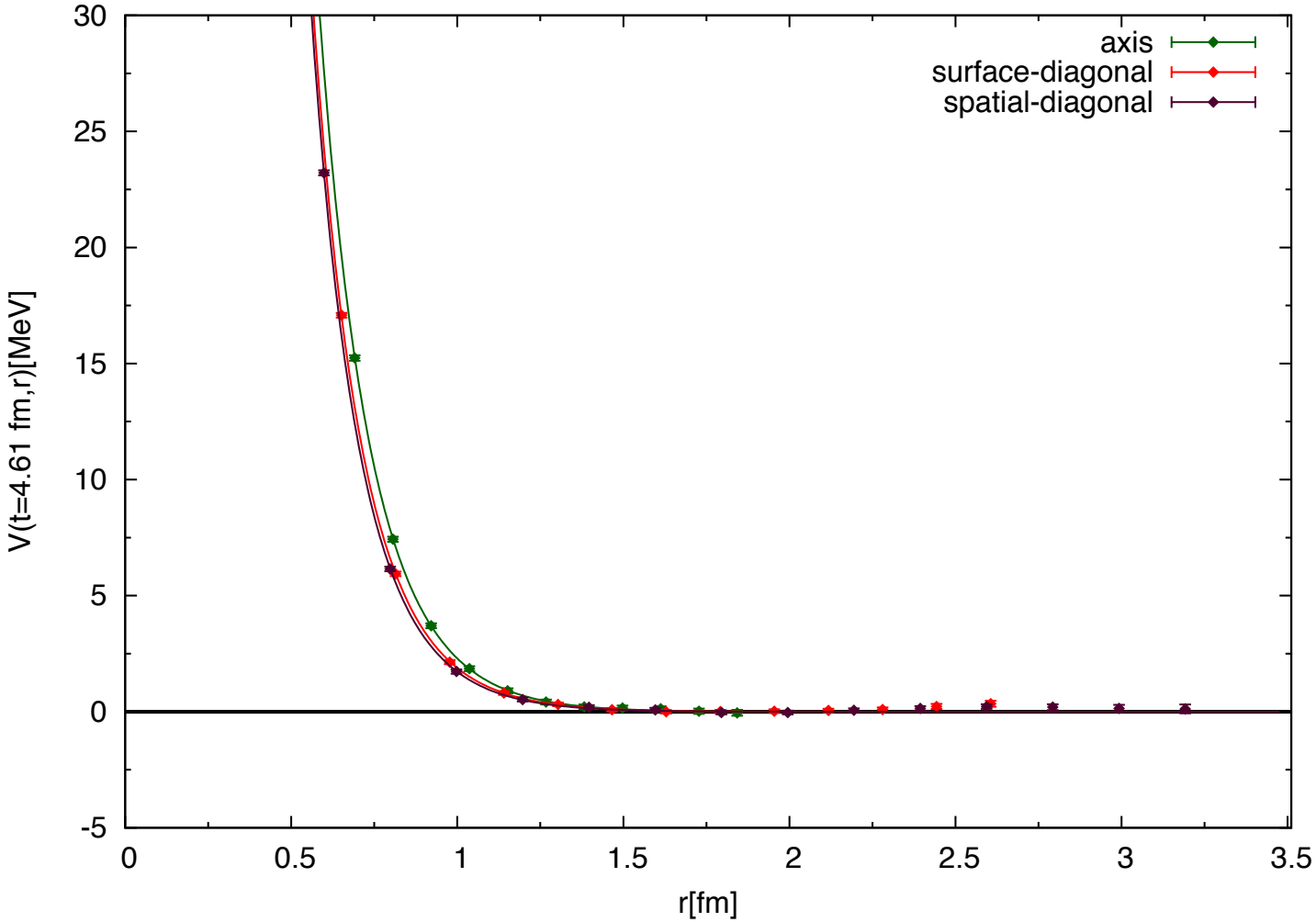
- compute scattering length $a_{\pi\pi}^{I=2}$ by fitting $\delta(k)$ to ERE:

$$\frac{k\cot\delta(k)}{m_\pi} = -\frac{1}{m_\pi a_{\pi\pi}^{I=2}} + \frac{1}{2}m_\pi r\left(\frac{k^2}{m_\pi^2}\right) + P(m_\pi r)^3\left(\frac{k^2}{m_\pi^2}\right)^2 + \ldots$$

Results time dependent potential method

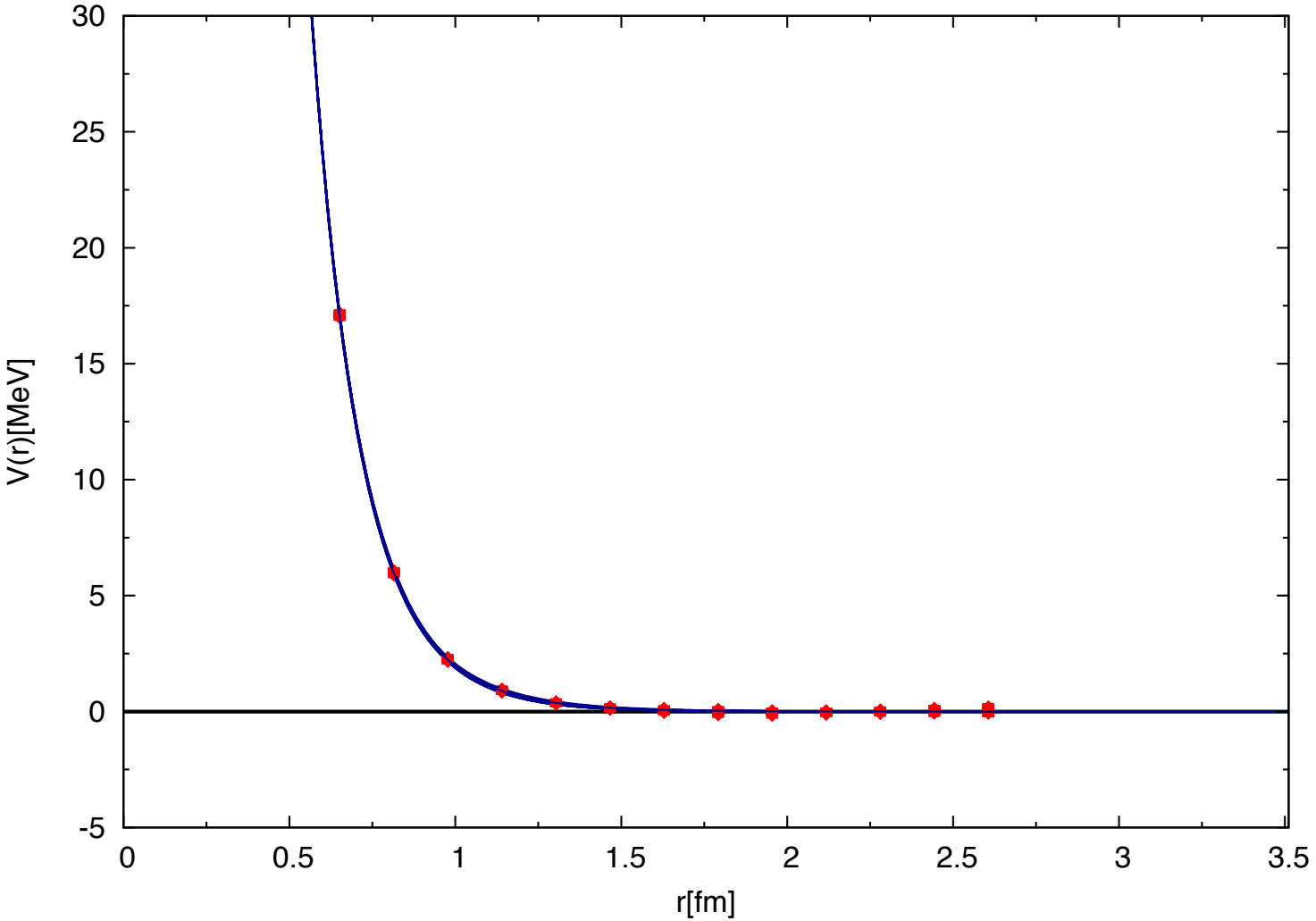
- test method on quenched setup using $M_\pi \sim (700 - 940)\,\mathrm{MeV}$ with 2 HEX smeared tree-level improved clover-Wilson quarks
- $a \approx 0.115\,\mathrm{fm}$ and $L \approx 3.7\,\mathrm{fm}$
- statistical error from 2000 bootstrap samples
- systematic uncertainties:
  - rotational invariance breaking $\Rightarrow$ perform analysis using data along axis, surface-diagonals and space diagonals
  - source dependence $\Rightarrow$ use wall and gauss sources ($r \approx 0.3\,\mathrm{fm}$)
  - ground state saturation (energy dependence) $\Rightarrow$ use different time-slices
  - potential modeling $\Rightarrow$ use different potential models (empirical), additionally apply cutoffs on largest available distance r
  - asymptotic region of ψ_k $\Rightarrow$ different distances R
- estimate systematic error using histogram method (exception: sources handled separately)

Potentials I



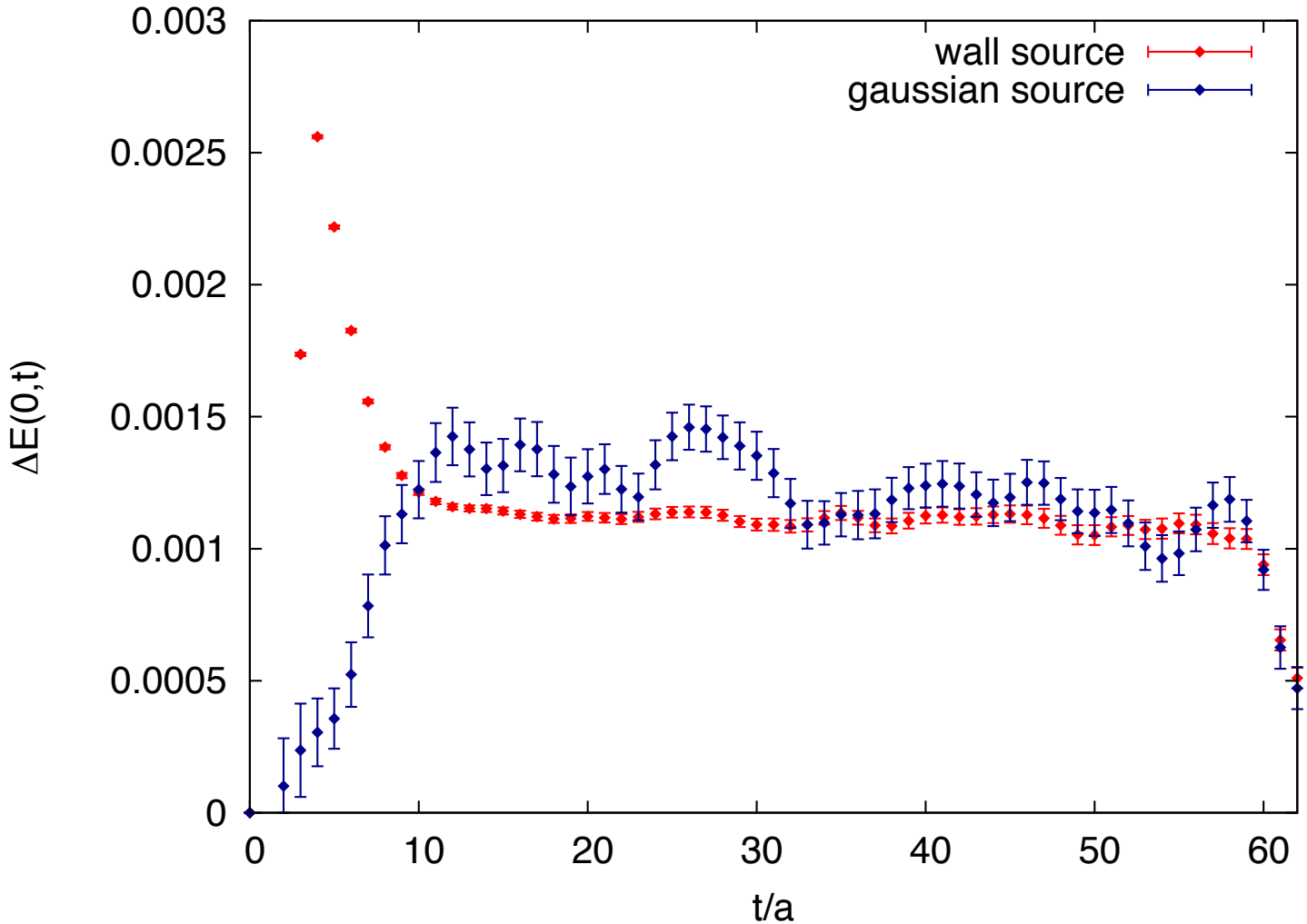
- largest systematic uncertainty stems from breaking of rotational invariance

Potentials II



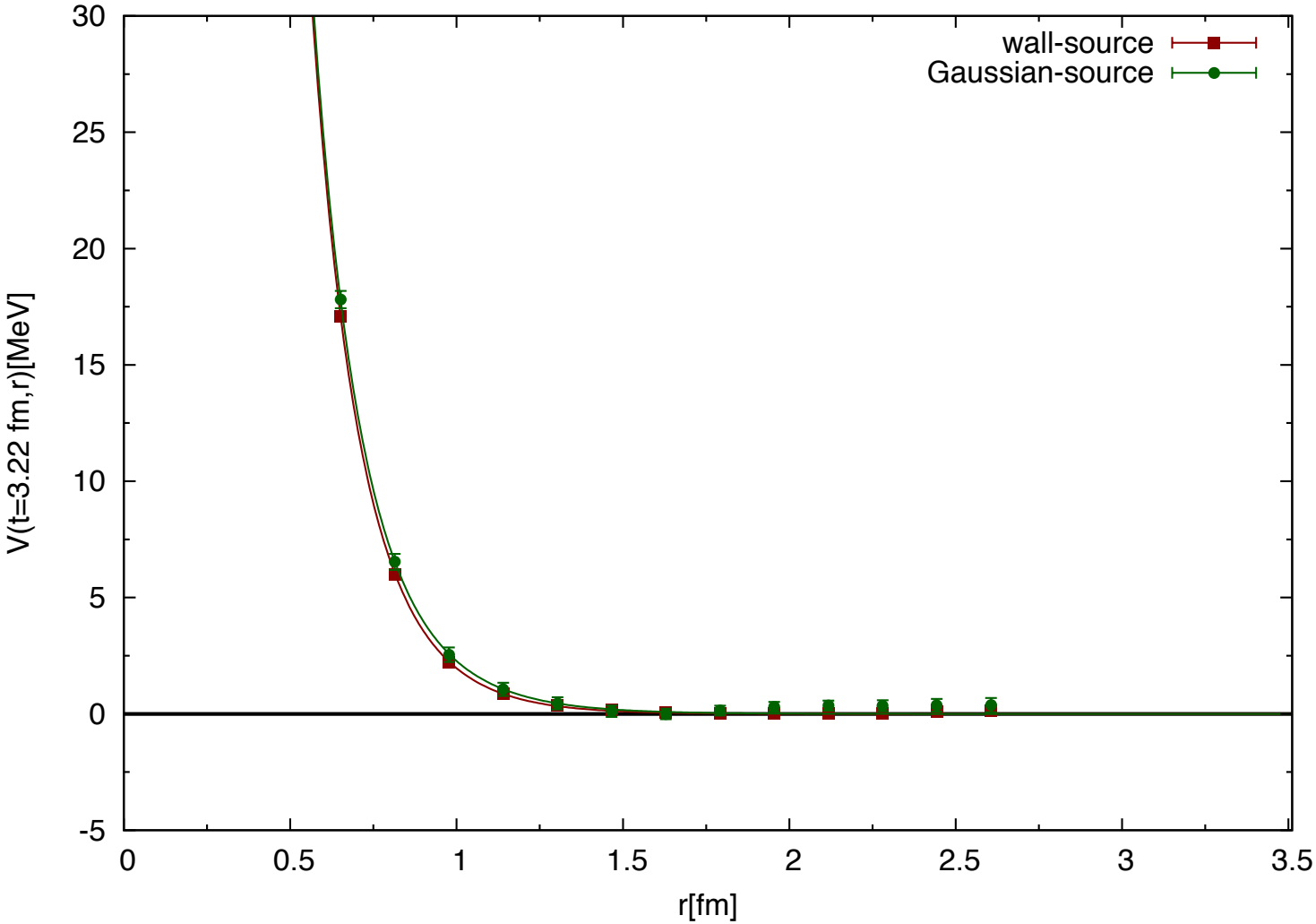
- no dependence on time-slice (for $t \gtrsim 1.45\,\mathrm{fm}$)

Potentials III-A



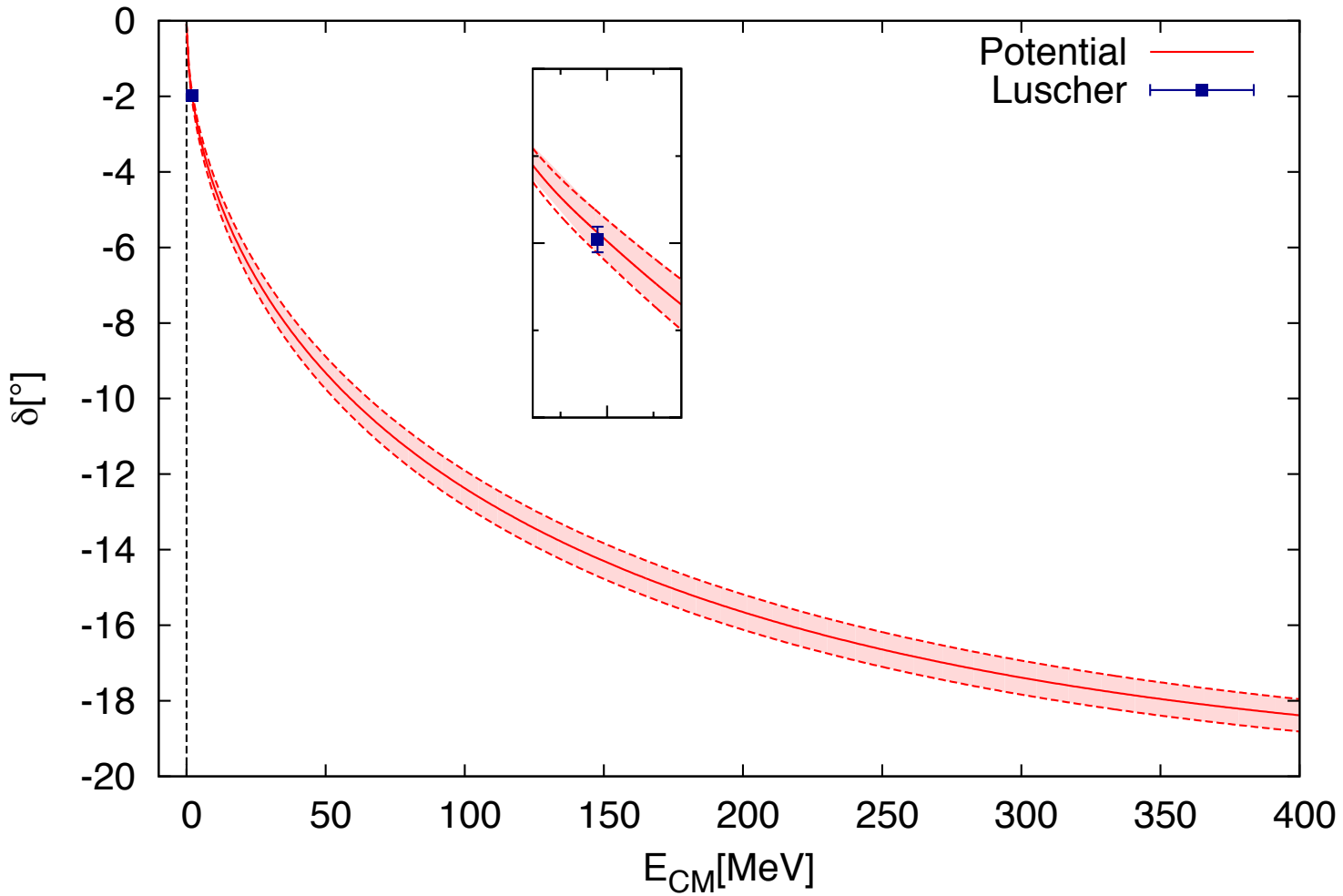
- which one is the correct pleteau?

Potentials III-B



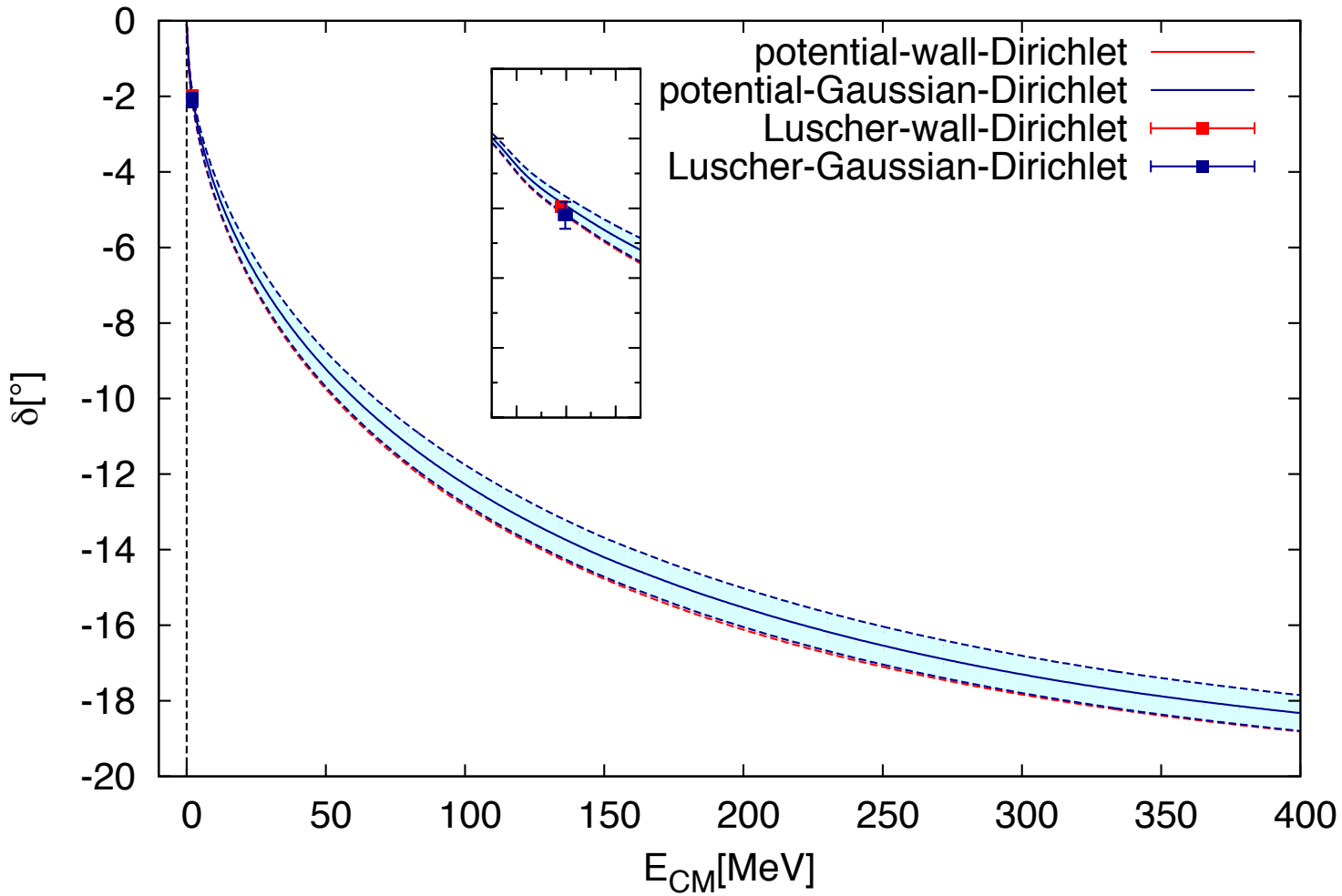
- noise weakened in potential method

Phases I



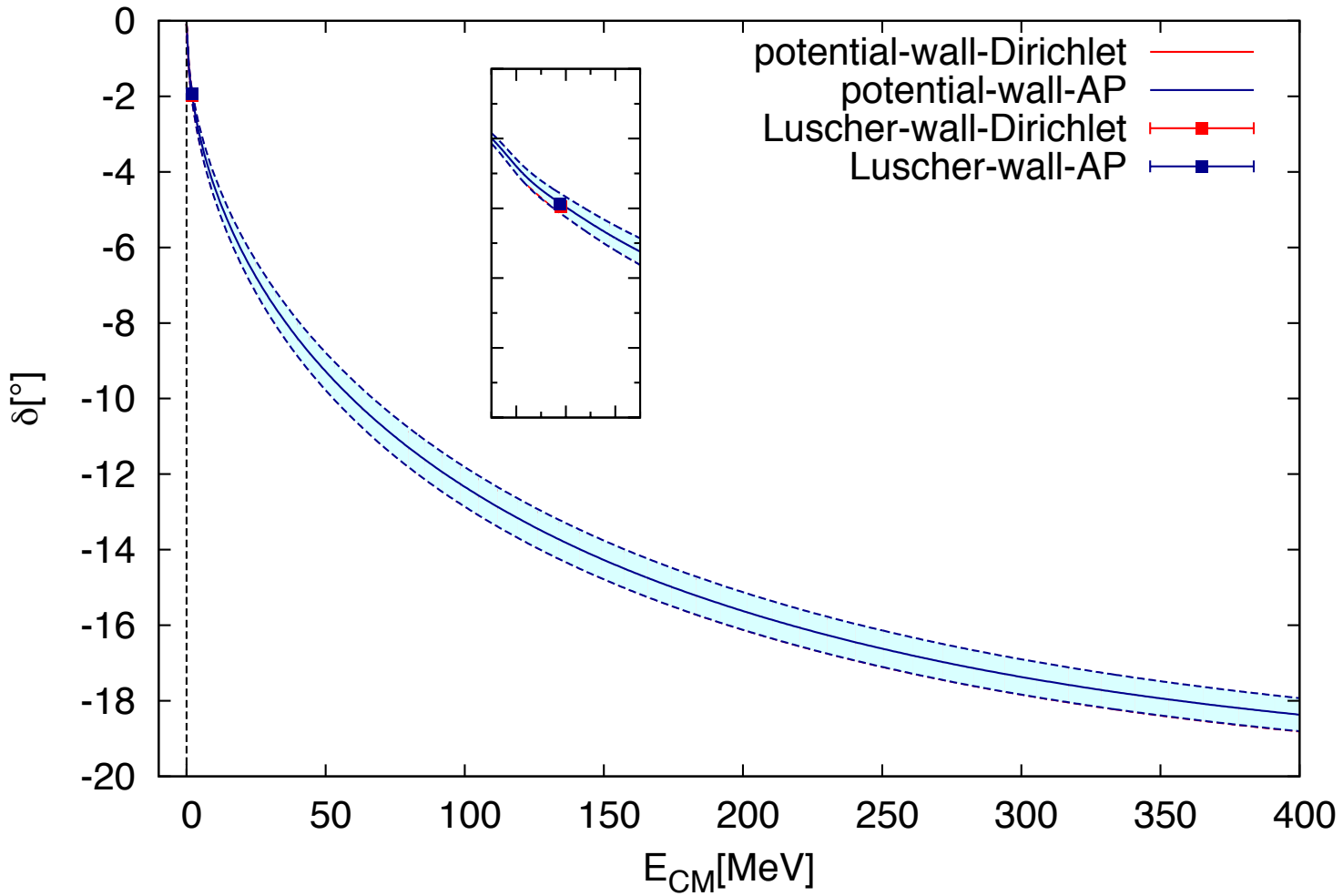
- phase dependence on E_{CM} for wall source

Phases III



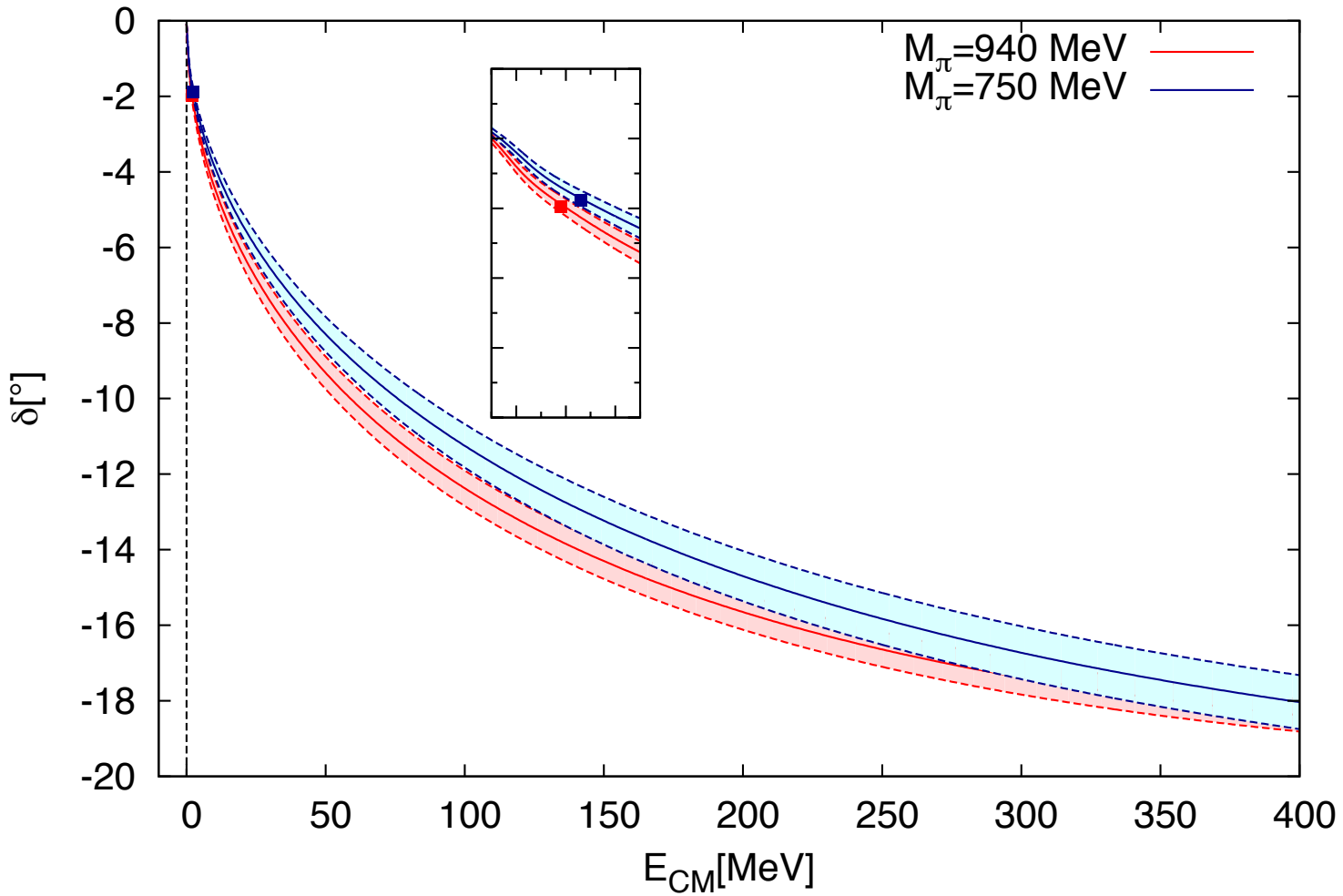
- phase dependence on E_{CM} for both source types in comparison

Phases IV



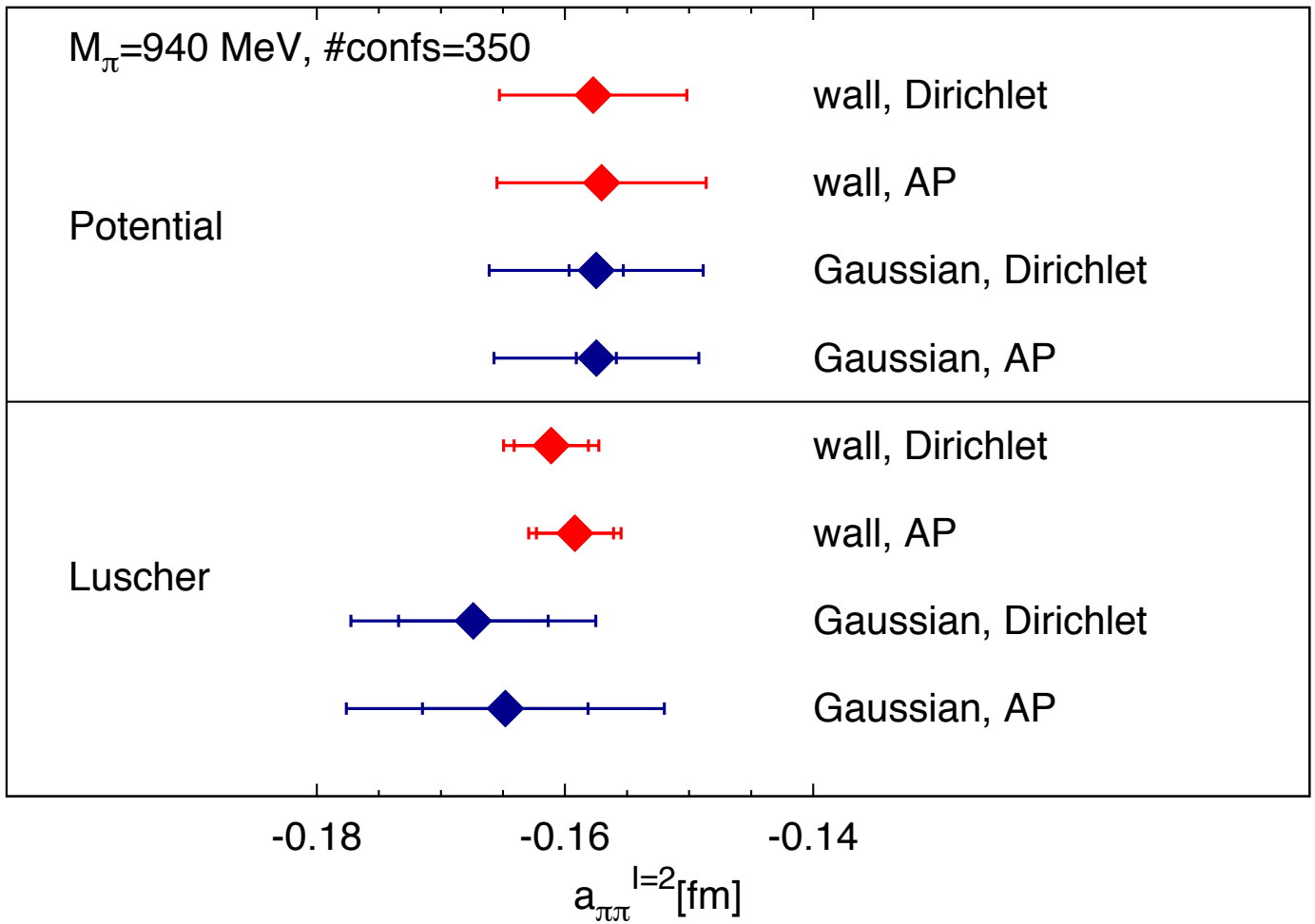
- phase dependence on E_{CM} for both source types in comparison

Phases V



- mass dependence of $\delta(k)$

Scattering lengths



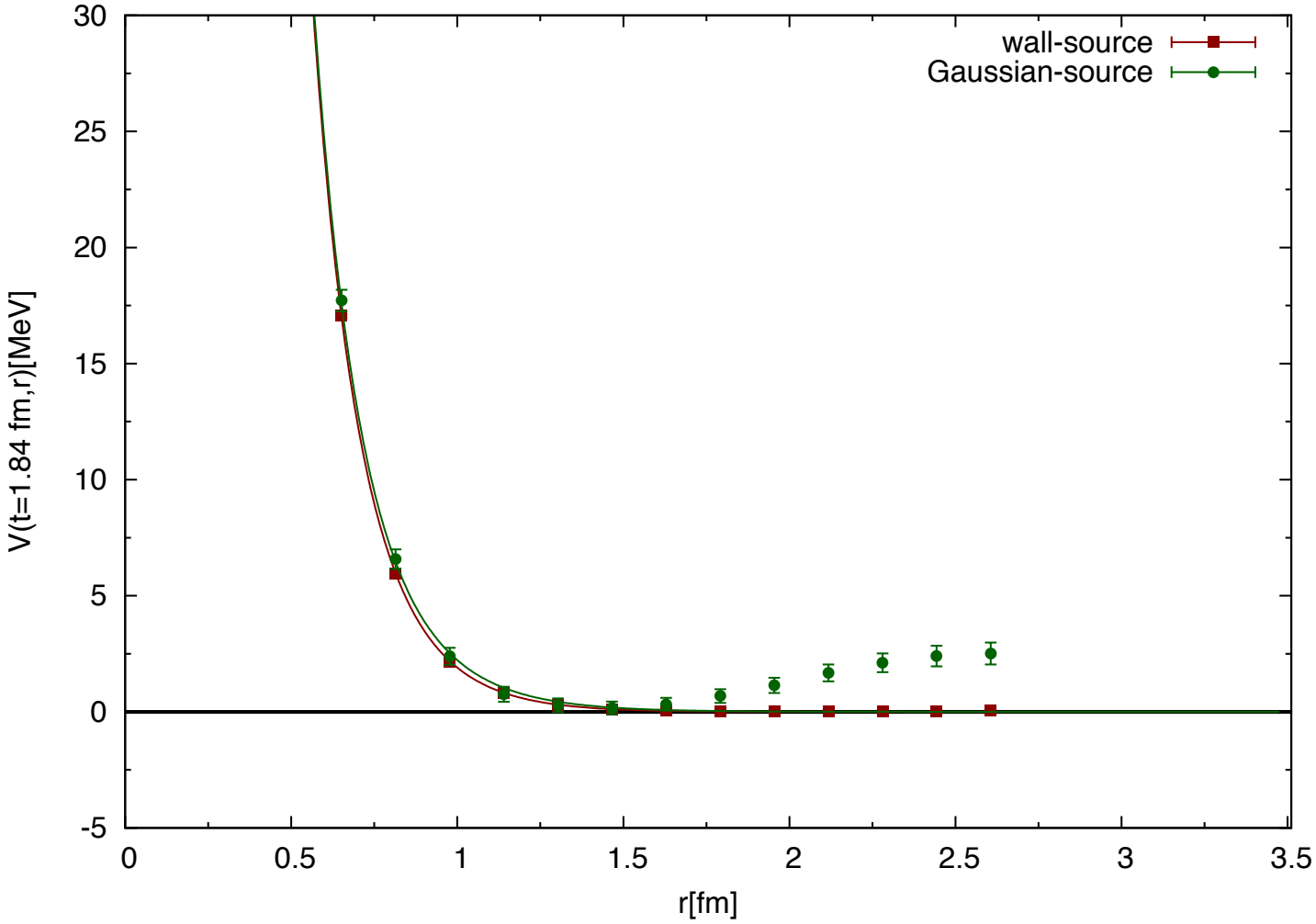
- comparison of scattering lengths

Summary

- compared time-dependent potential method to Lüscher method for treating QCD scattering problems
- both methods allow for extracting scattering phases and lengths
- Lüscher method: higher momenta (than ground-state momenta) have to be computed by fitting excited states, using initially boosted pions, etc.
- potential method: relative momentum is free parameter
- both methods yield compatible results for ground-state energies
- excited states need still to be computed $\Rightarrow$ improve check for energy dependence of potential
- challenge in case of nucleons: ground-state saturation (relaxed by time-dependent potential-method)

BACKUP

Scattering lengths



- immediate recognition of contribution of states above some threshold