First calculation of the spin- orbit force in the parity odd sector in NN system from Lattice QCD

K. Murano for HAL QCD Collaboration
RIKEN

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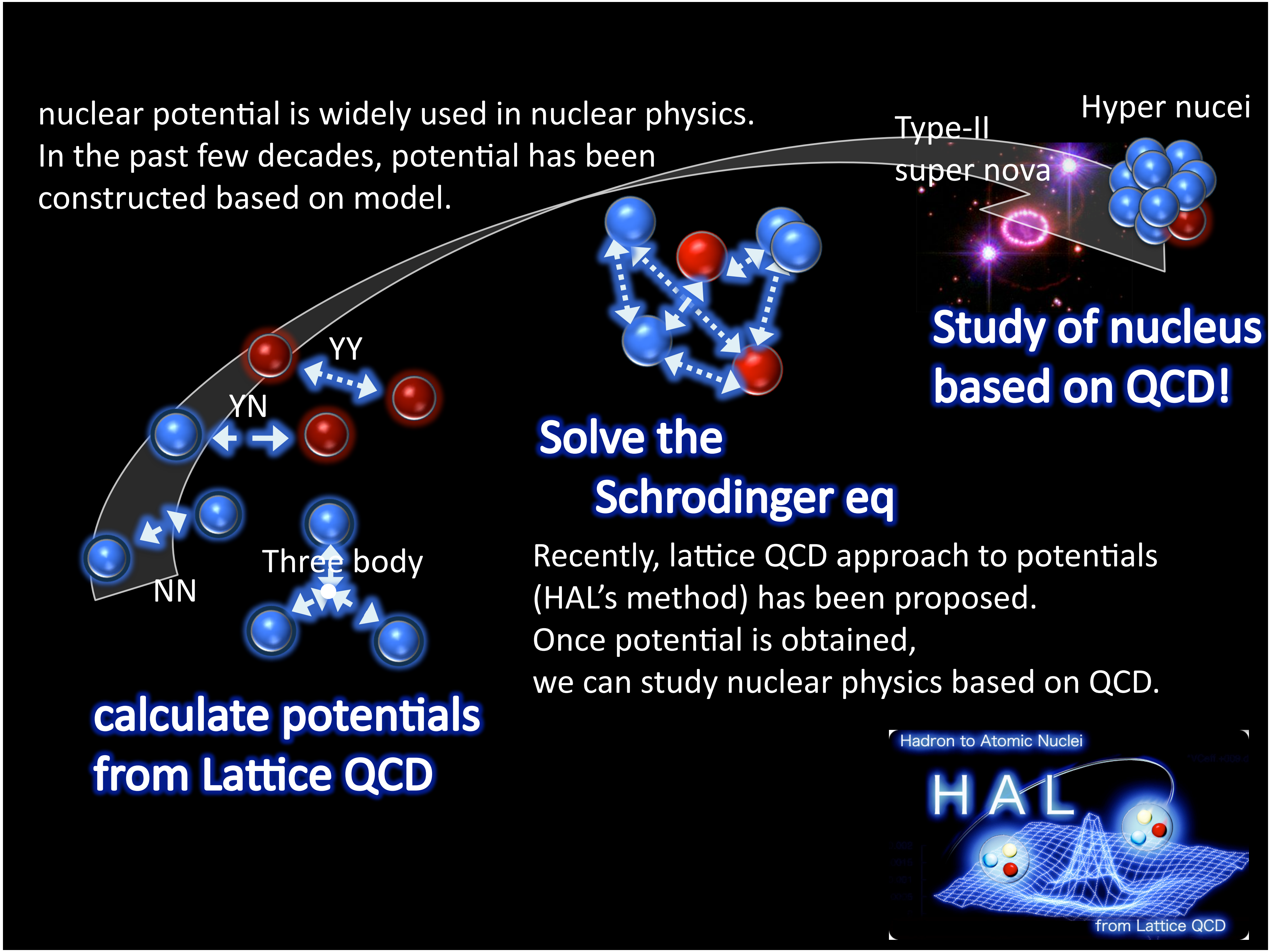
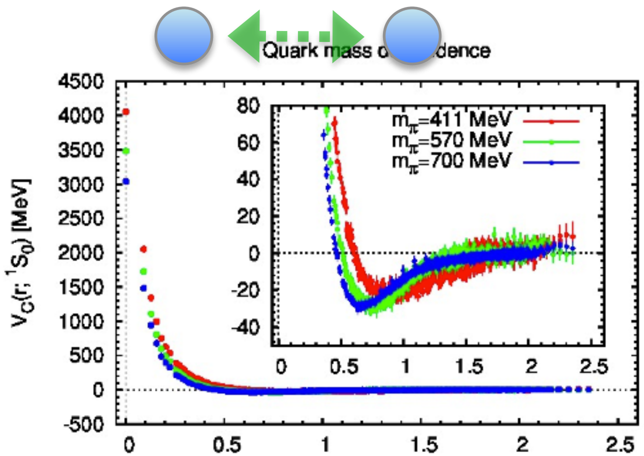

nuclear potential is widely used in nuclear physics.
In the past few decades, potential has been
constructed based on model.
Type-II
super nova
Hyper nucei
YY
YN
NN
Three body
calculate potentials
from Lattice QCD
Solve the
Schrodinger eq
Study of nucleus
based on QCD!
Recently, lattice QCD approach to potentials
(HAL's method) has been proposed.
Once potential is obtained,
we can study nuclear physics based on QCD.
Hadron to Atomic Nuclei
HAL
from Lattice QCD

HAL's method was extended to various system

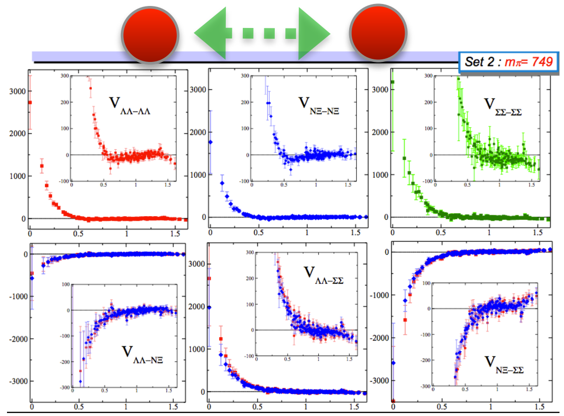
central force and tensor force in parity even sector
has been calculated in various system

N-N potential



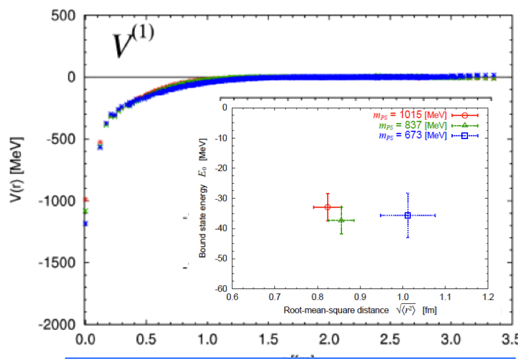
by N. Ishii Wednesday June 27

Y-N, Y-Y potential



K. Sasaki Thursday June 28

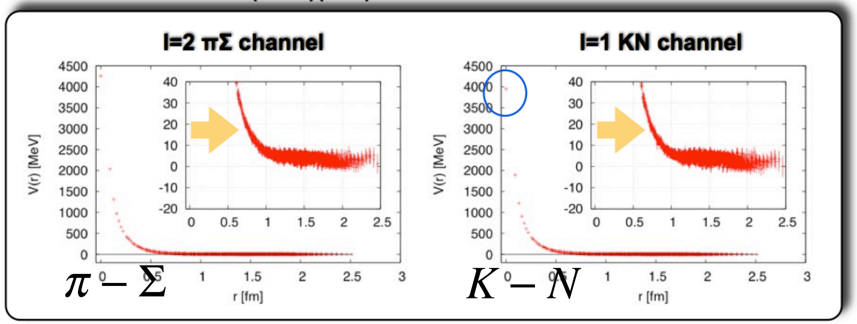
H dibaryon in SU(3)



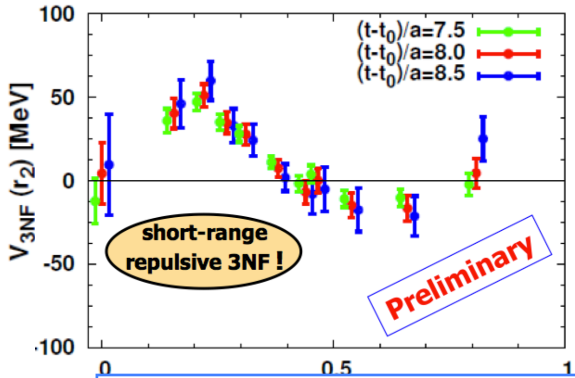
Inoue Wednesday June 27

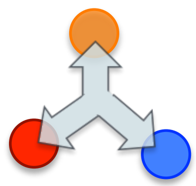
meson-baryon potential

I=2 πΣ = π⁺Σ⁺ -> (udbar)(uus)   I=1 KN = K⁺p -> (usbar)(uud)



Y. Ikeda Tuesday June 26





three-body force

T.Doi Tuesday June 26 Plenarry 11:30 -

[redacted]

central force and tensor force in parity even sector
in various system has been calculated

However, Potentials in parity-odd sector (LO)
and LS force(NLO) is still missing.

already calculated

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc^{(+)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]P^+\phi(\vec{r};E)$$

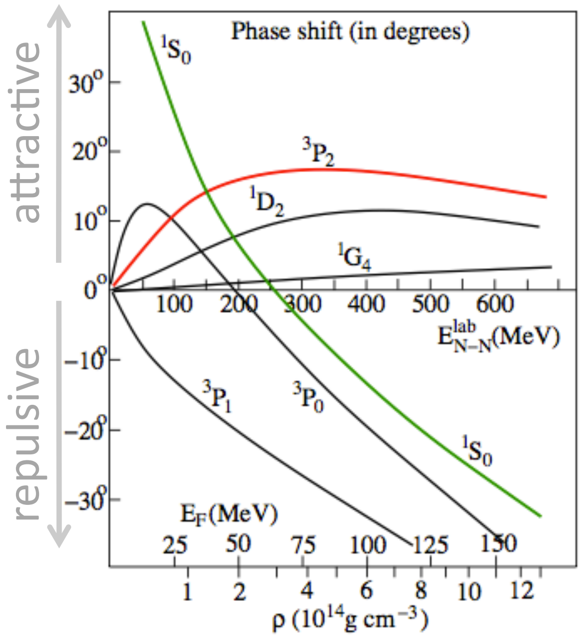
$$+\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]P^-\phi(\vec{r};E)+\mathcal{O}(\nabla^2)$$

Not yet

$V_{LS}^{(+)}$, $Vc^{(-)}$, $V_T^{(-)}$, $V_{LS}^{(-)}$ are needed for the complete determination of
NN potentials (up to the next-to- leading order)

J.L. Gaammel, R.M. Thaler Phys.Rev. 107 (1957)

Strong Spin-Orbit force is required for the NN system



phase shift analyses indicate that there should be a strong spin-orbit force between nucleons.

$\delta(^3P_0) > \delta(^3P_2) > 0 > \delta(^3P_1)$

low Energy

(long range)

explained with tensor force

(positive at long range)

⇨

$\delta(^3P_2) > 0 > \delta(^3P_0) > \delta(^3P_1)$

high Energy

(short range)

explained with Spin-Orbit force

(negative at short range)

$$V(r) = Vc(r) + S_{12}V_T(r) + \vec{L}\cdot\vec{S}\ V_{LS}(r)$$

$$\vec{L}\cdot\vec{S} = \frac{1}{2}(J^2 - L^2 - S^2) = \frac{1}{2}\left(J(J+1) - L(L+1) - S(S+1)\right)$$

p-wave: L=1
triplet: S=1

$$V(r;{}^3P_0) = Vc(r) - 4V_T(r) - 2V_{LS}(r)$$

$$V(r;{}^3P_1) = Vc(r) + 2V_T(r) - V_{LS}(r)$$

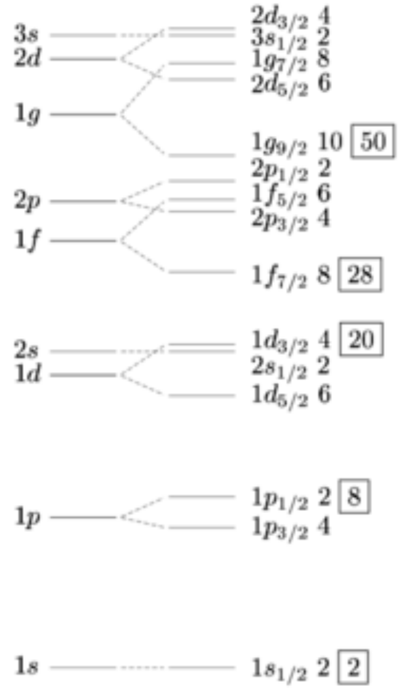
$$V(r;{}^3P_2) = Vc(r) - 0.4V_T(r) + V_{LS}(r) + (\text{coupled term with } {}^3F_2)$$

3P0 phase shift rapidly decrease at high E.

🡐 Spin-Orbit force should be strongly negative at short range.

Spin-Orbit force is important for..

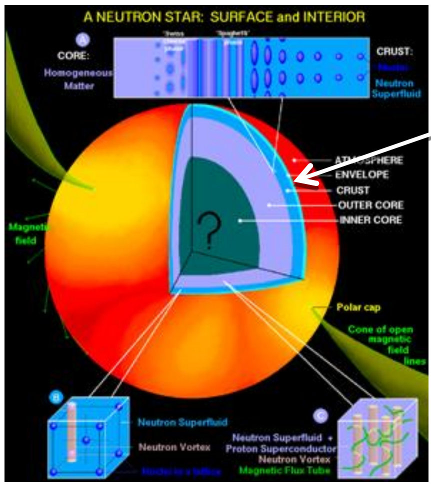
Magic number of atomic nuclei



In 1948, Mayer & Jensen introduce spin-orbit force (in atomic nuclei) to shell model ➔ Magic Number .

Spin-orbit force in atomic nuclei is closely related to the spin-orbit force of two-nucleon interaction.

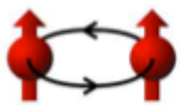
Structure of neutron star



$^3P_2$ neutron superfluidity in neutron star

Spin-Orbit force make that whereas $^3P_0, {}^3P_1$ state is negative, $^3P_2$ state is atractive in high density.

➔ $^3P_2$ nucleon pairing (cooper pair) ➔ ($^3P_2$ superfluid)



Purpose of this work:

In order to complete NN potential from Lattice QCD,
we extend the method to Spin-Orbit force.

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc^{(+)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]P^{+}\phi(\vec{r};E)$$

$$+\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]P^{-}\phi(\vec{r};E)+\mathcal{O}(\nabla^2)$$

This work

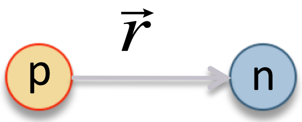
First calculation of Spin-Orbit force
with parity minus sector in NN system

How to construct the potential from Lattice QCD

Construction of the potential

To construct potential, Nambu-Bethe-Salpeter (NBS) wave function is introduced:

NBS wave function



$$\equiv \phi(\vec{r};E)$$

Nucleon interpolating field

$$\phi(\vec{r};E) \equiv \langle 0 | N(\vec{x},0) N(\vec{y},0) | B=2;E \rangle \qquad \vec{r} \equiv \vec{x} - \vec{y}$$

Baryon number = 2 state

NBS wf. has a same asymptotic form as quantum mechanics.
(NBS wave function contains information of phase shift.)

ex) S-wave

$$\Psi(r) \xrightarrow{r\to\infty} Z e^{i\delta(p)} \frac{\sin(pr+\delta(p))}{pr}$$

[C.-J.D.Lin et al., NPB619(2001)467.]

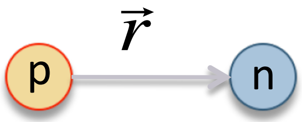
The potential is constructed through the Schrodinger equation:

$$\left(\frac{\nabla^2}{m_N} + E\right)\phi(\vec{r};E) = \left[Vc\ (r) + V_T\,(r) S_{12} + V_{LS}(r)\vec{L}\cdot\vec{S}\right]\phi(\vec{r};E)$$

derivation : [S. Aoki, T. Hatsuda, N. Ishii, Prog.Theor.Phys. 123 (2010) 89-128]

NBS wave function can be calculated from Lattice QCD

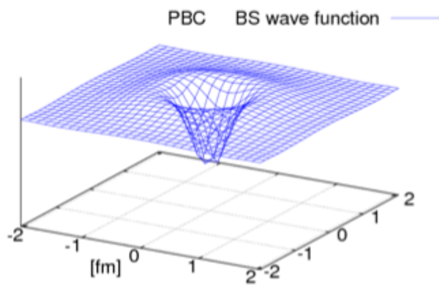
NBS wave function



$$\equiv \phi(\vec{r};E) \qquad \phi(\vec{r};E) \equiv \langle 0 | N(\vec{x},0) N(\vec{y},0) | B=2;E \rangle$$

Here, NBS wave function can be obtained from nucleon 4-point function on the lattice:

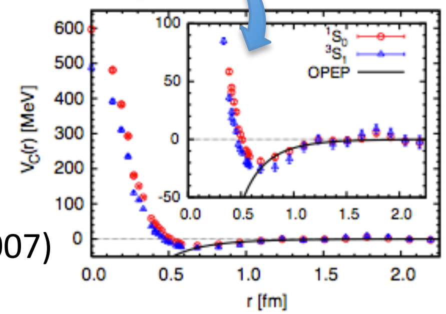
$$\sum_{\vec{x}} \langle 0 | \hat{N}^i{}_\alpha(\vec{x}+\vec{r},t) \hat{N}^j{}_\beta(\vec{x},t) \overline{NN}(t_0) | 0 \rangle \xrightarrow[(t-t_0)\to\infty]{} \phi(\vec{r};E_0)\; e^{-E_0(t-t_0)}$$



ex) S-wave:

$$\left(\frac{\nabla^2}{m_N} + E \right) \phi^{{}^1S_0}(\vec{r};E) = \left[Vc\ (r) \right] \phi^{{}^1S_0}(\vec{r};E)$$

Ishii,Aoki,Hatsuda
Phys. Rev. Lett. 99, 022001 (2007)



After calculation of NBS wave function, we can obtain potential from solving SE eq with using it.

In order to extract Spin-Orbit force,
we will extend HAL method to..

* Parity Odd sector
* higher angular momentum

$$\left(\frac{\nabla^2}{m_N}+E\right)\,{}^3P_X(\vec{r};E)=\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]\,{}^3P_X(\vec{r};E)+\mathcal{O}(\nabla^2)$$

$X=0,1,2$

1. Calculate the 3P0, 3P1 and 3P2 NBS wave function from Lattice QCD
2. Solve the Schrodinger-like equation about potentials, with using NBS wave functions

How to construct Parity odd, with $L \neq 0$

Parity :

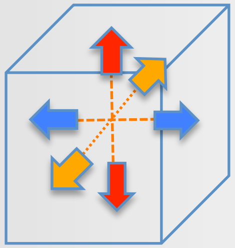
➢ Inject the momentum to the nucleon (quark)

$$N(+\vec{k}) = \epsilon_{abc}\left(q_a^t C\gamma_5 q_b\right)\sum_{\vec{x}} q_c(\vec{x})\exp(+\vec{k}\cdot\vec{x}) \qquad J = N(+\vec{k})N(-\vec{k})$$

parity even $\quad \phi(\vec{r})^{P=+} = \phi(\vec{r};+\vec{k}) + \phi(\vec{r};-\vec{k})$

parity odd $\quad \phi(\vec{r})^{P=-} = \phi(\vec{r};+\vec{k}) - \phi(\vec{r};-\vec{k})$

Angular momentum:



➢ construct the L=1 (on the cubic group : T1) state with combining 6-different directions of momentum.

Projection operator based on cubic group: because rotational sym is broken [O(3) ➔ cubic group]

$$\frac{2l+1}{4\pi}\int d\theta\, d\phi \;\; D^{(l=L)}(\theta,\phi)^* \;\psi(R(\theta,\phi)\;\vec{x})$$

$$\frac{d_\Gamma}{24}\sum_{i=0}^{24} D^{(\Gamma)}_{\mu,\nu}(g_i)^*\psi_\nu(R(g_i)\;\vec{x})$$

$\Gamma = A_1, A_2, E, T_1, T_2$

Numerical Results

Set Up

Japan Lattice Data Grid

Nf=2
Iwasaki gauge + clover fermion
beta=0.195
kappa=0.1375

a=0.1555
1/a = 1271 MeV
16^3 x 32 Lattice
mN=2165 MeV
mpi=1136 MeV

The detail of this method: N. Ishii, et.al., Physics Letters B712 (2012) 437-441.

Technical comments

We used "*time-dependent* " method which make possible to obtain potential without relying on the ground state saturation

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc\ (r)+V_T(r)S_{12}+V_{LS}(r)\vec{L}\cdot\vec{S}\right]\phi(\vec{r};E)$$

time dependent method

$$\left(\frac{\nabla^2}{m_N}-\frac{\partial}{\partial t}+\frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right)R(\vec{r};t)=\left[Vc\ (r)+V_T(r)S_{12}+V_{LS}(r)\vec{L}\cdot\vec{S}\right]R(\vec{r};t)$$

$$R(t)=\frac{\psi(\vec{r};t)}{<N\bar{N}>^2}\sim\exp\left[-\left(2\sqrt{m_N^2+k^2}-2m_N\right)t\right]$$

$$\sqrt{m_N^2+\alpha k^2+\mathcal{O}(k^4)}$$

since $m_N a$ is large,
a dispersion relation affect the large lattice artifact.
So we correct the dispersion relation in this set up.

$$E^2=m_N^2+k^2 \Rightarrow m_N^2+\alpha k^2+\mathcal{O}(k^4)$$

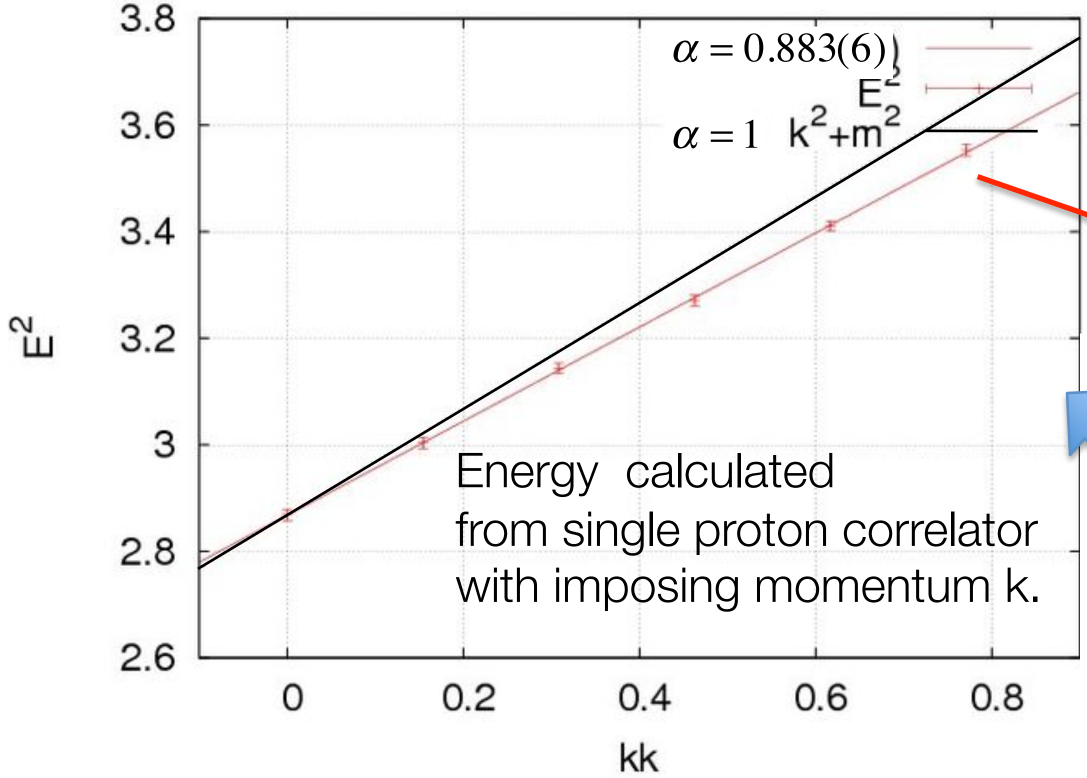
correct the dispersion relation

$$\left(\frac{\nabla^2}{m_N}+\frac{1}{\alpha}\left[-\frac{\partial}{\partial t}+\frac{1}{4m_N}\frac{\partial^2}{\partial t^2}\right]\right)R(\vec{r};E)=\left[Vc\ (r)+V_T(r)S_{12}+V_{LS}(r)\vec{L}\cdot\vec{S}\right]R(\vec{r};E)$$

Technical comments

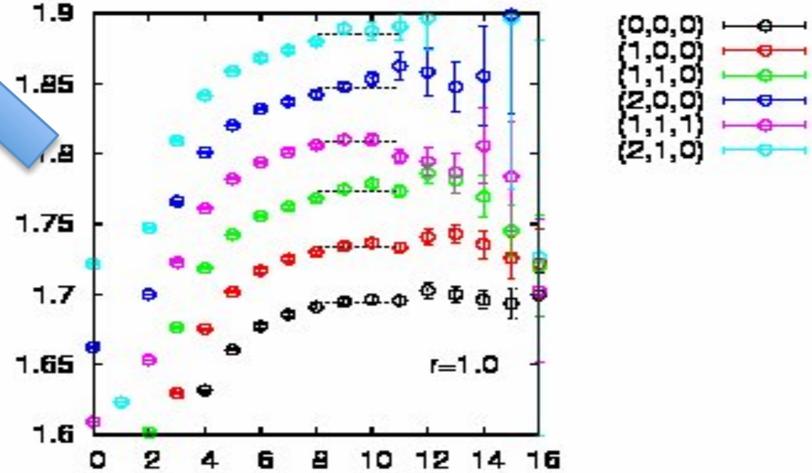
the correction to dispersion relation is estimated from calculation of effective mass of single-proton correlator which has momentum.

$$E^2 = m_N^2 + \alpha k^2 + \mathcal{O}(k^4)$$



α is estimated from linear fit of E^2

$\alpha = 0.883(6)$

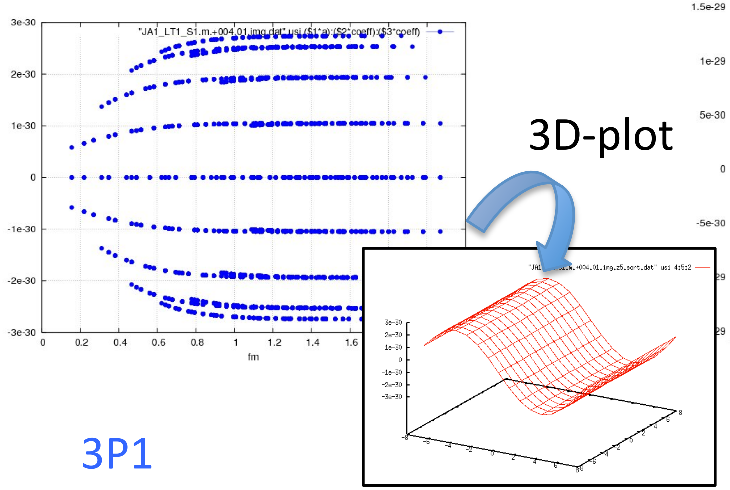


We also confirm that k^4 term in the corrected dispersion relation is negligible.

results : NBS wave function for S=1

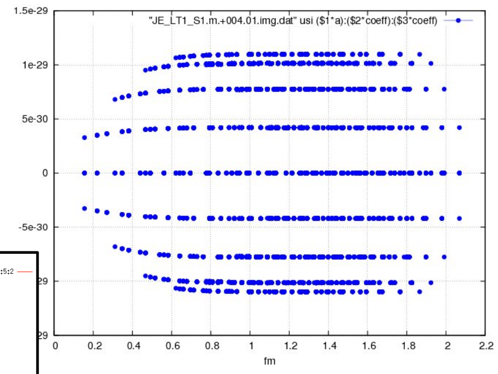
3P0

J=1 (A1) L=1 (T1) (imaginary part)



3P2

J=2 (E) L=1 (T1) (imaginary part)

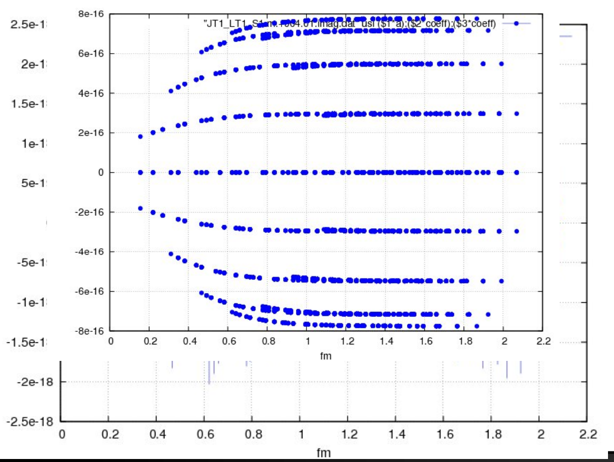


a=0.1555
1/a = 1271 MeV
L^3 x T = 16 ^3 x 32
mN=2165 MeV

$$\psi(\vec{r}) = R(r) Y_{1m}(\theta,\phi)$$

3P1

J=1 (T1) L=1 (T1)



$$\left(\frac{\nabla^2}{2m}+E\right)\psi^{3P0}(\vec{r}) = Vc^{(-)}\ \psi^{3P0}(\vec{r}) + V_T^{(-)} S_{12}\ \psi^{3P0}(\vec{r}) + V_{LS}^{(-)}\ \vec{L}\cdot\vec{S}\ \psi^{3P0}(\vec{r})$$

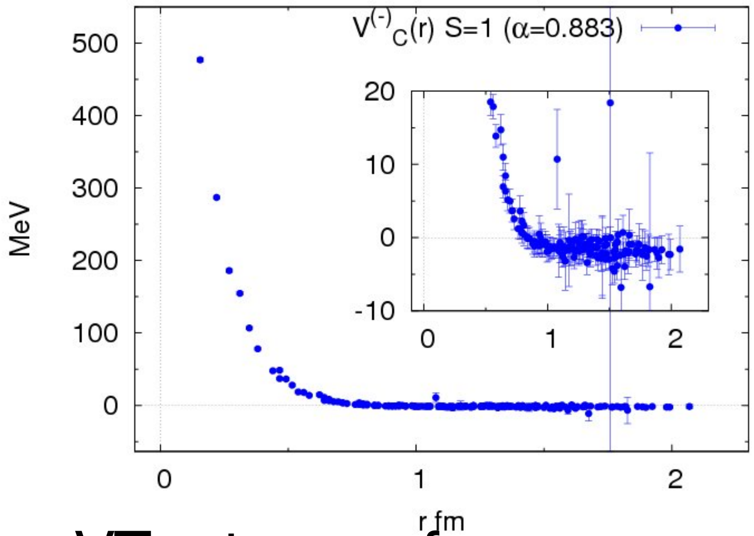
$$\left(\frac{\nabla^2}{2m}+E\right)\psi^{3P1}(\vec{r}) = Vc^{(-)}\ \psi^{3P1}(\vec{r}) + V_T^{(-)} S_{12}\ \psi^{3P1}(\vec{r}) + V_{LS}^{(-)}\ \vec{L}\cdot\vec{S}\ \psi^{3P1}(\vec{r})$$

$$\left(\frac{\nabla^2}{2m}+E\right)\psi^{3P2}(\vec{r}) = Vc^{(-)}\ \psi^{3P2}(\vec{r}) + V_T^{(-)} S_{12}\ \psi^{3P2}(\vec{r}) + V_{LS}^{(-)}\ \vec{L}\cdot\vec{S}\ \psi^{3P2}(\vec{r})$$

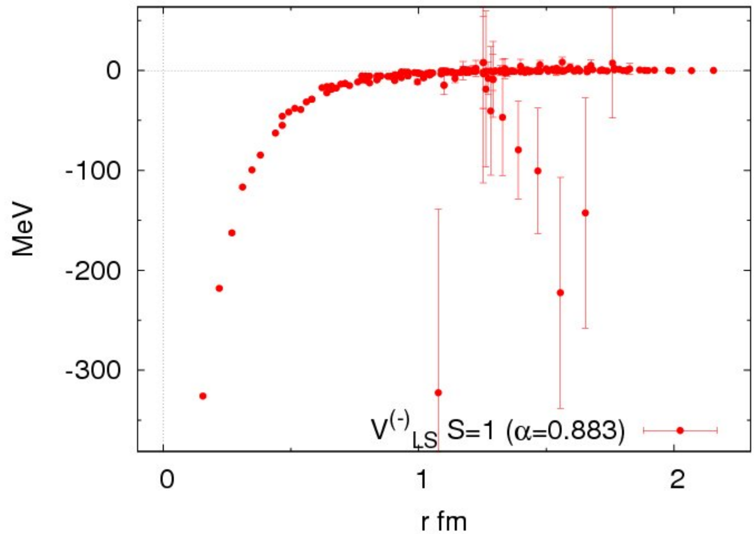
using these NBS wf. , Vc, VT and VLS are solved

Numerical Results

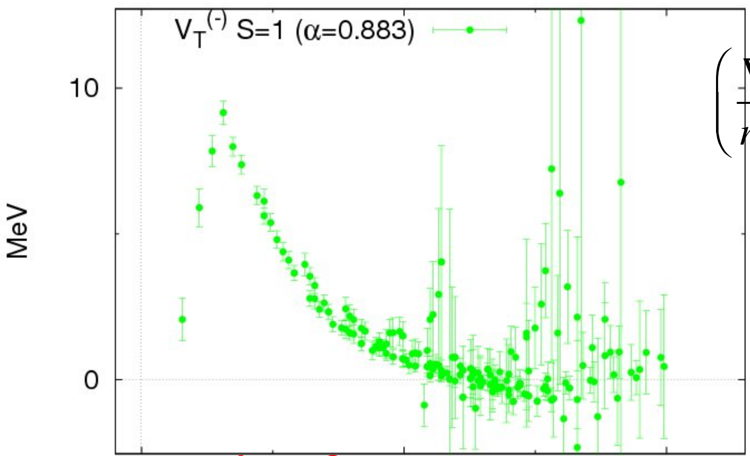
Vc : center force



VLS : spin-orbit force



VT : tensor force



$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[Vc^{(+)}(r)+V_T^{(+)}(r)S_{12}+V_{LS}^{(+)}(r)\vec{L}\cdot\vec{S}\right]P^{+}\phi(\vec{r};E)$$

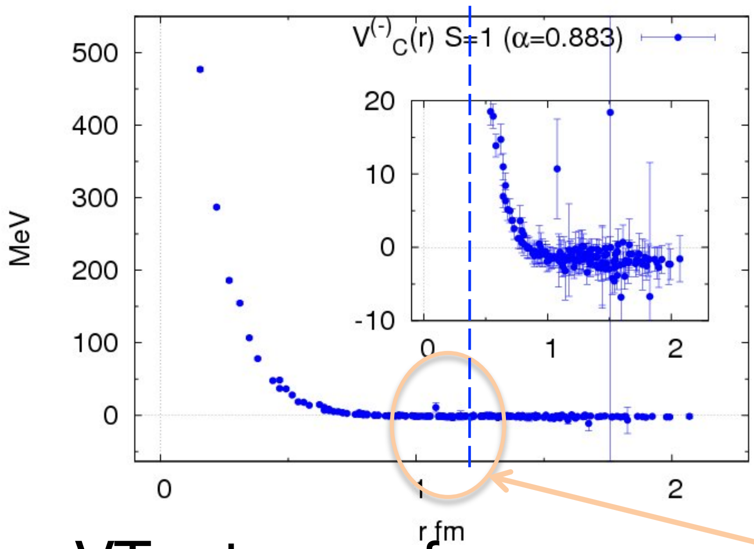
$$+\left[Vc^{(-)}(r)+V_T^{(-)}(r)S_{12}+V_{LS}^{(-)}(r)\vec{L}\cdot\vec{S}\right]P^{-}\phi(\vec{r};E)+\mathcal{O}(\nabla^2)$$

Spin-Orbit force is strong negative value at short range.

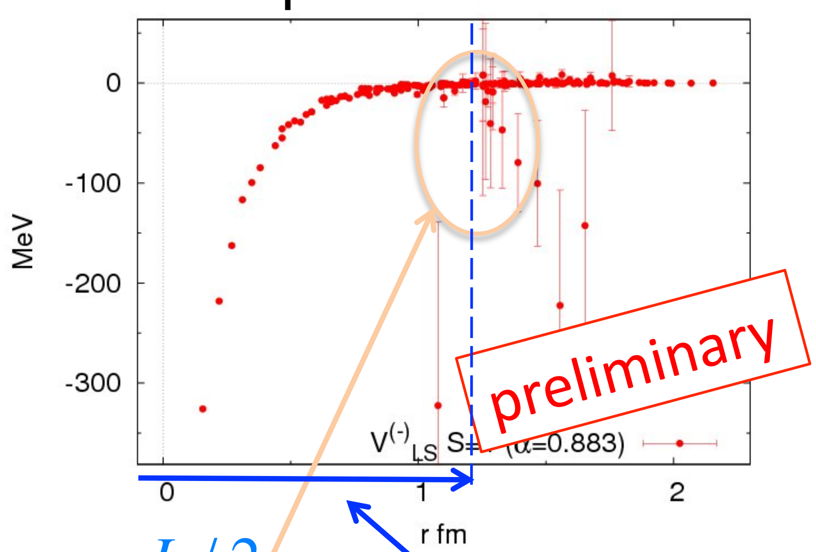
Tensor force is positive value.

← consistent with expectation from phase shift analysis.

Vc : center force



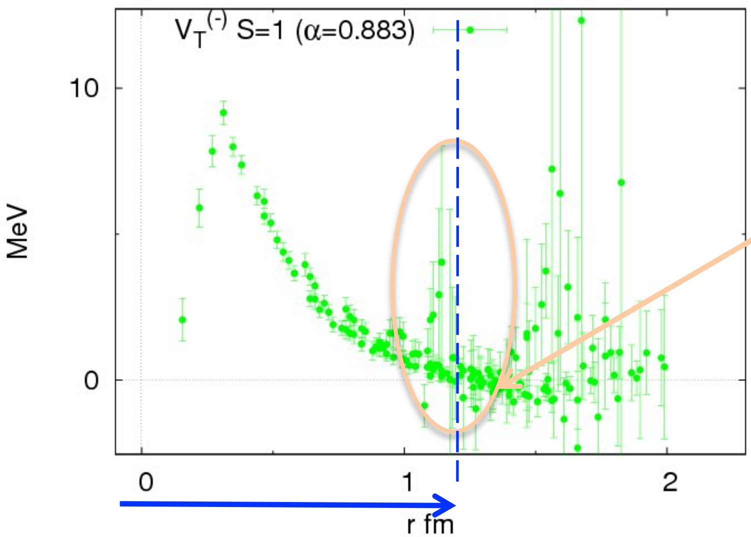
VLS : spin-orbit force

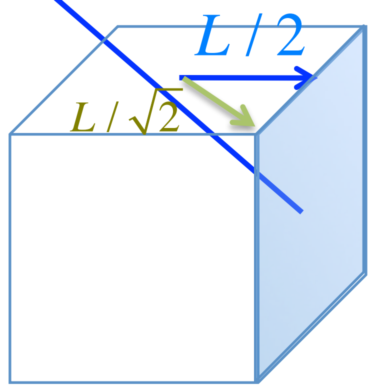


$L/2$

there are strong fluctuation.

VT : tensor force





← caused by boundary.

NBS wf. have strong finite size effect

Conclusion

The first calculation of Central, Tensor and Spin-orbital forces, in parity odd sector from Lattice QCD.

We extended HAL method to parity odd sector and higher angular momentum.

Central, Tensor and Spin-orbital forces are successfully calculated in NN system at mpi=1100 MeV with Nf=2 dynamical calculation.

Future work

➔ Extend to Hyperon system
Calculation of Symmetric and AntiSymmetric Spin-Orbit force.

Back Up Slides

NBS wave-> potential

more strict discussion is shown in:
S. Aoki, T. Hatsuda, N. Ishii, arXiv:0909.5585 [hep-lat]

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=K(E;\vec{r})$$

construct a orthogonal complete set from NBS wave functions obtained.
(for simplify, we don't take in account degeneracy)

$$\int d^3r\,\tilde{\phi}(\vec{r};E')\phi(\vec{r};E)=\delta_{E,E'}$$

$$U(r,r')\equiv\int dE' K(E',\vec{r})\tilde{\phi}(\vec{r}';E')$$

$$\Rightarrow K(E;\vec{r})=\int dr'\,U(\vec{r};\vec{r}')\,\phi(\vec{r}';E)$$

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\int dr' U(\vec{r};\vec{r}')\phi(\vec{r}';E)$$

$$k^2=m_N E$$

non-local
Energy independent !

U is Energy independent by definition !!

$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\int dr' U(\vec{r};\vec{r}')\,\phi(\vec{r}';E)$$

$$U(r,r')\equiv\int dE' K(E',\vec{r})\,\tilde{\phi}(\vec{r}';E')$$

※ In order to construct the potential U,
We need to obtain $\phi(\vec{r};E)$ set (up tp E_th) .

In practice, it is difficult to calculate all set of $\phi(\vec{r};E)$.

Derivative expansion approximation

$$S_{12}=\frac{3}{r^2}(\vec{\sigma}_1\cdot\vec{r})(\vec{\sigma}_2\cdot\vec{r})-(\vec{\sigma}_1\cdot\vec{\sigma}_2)$$

expantion arounx r

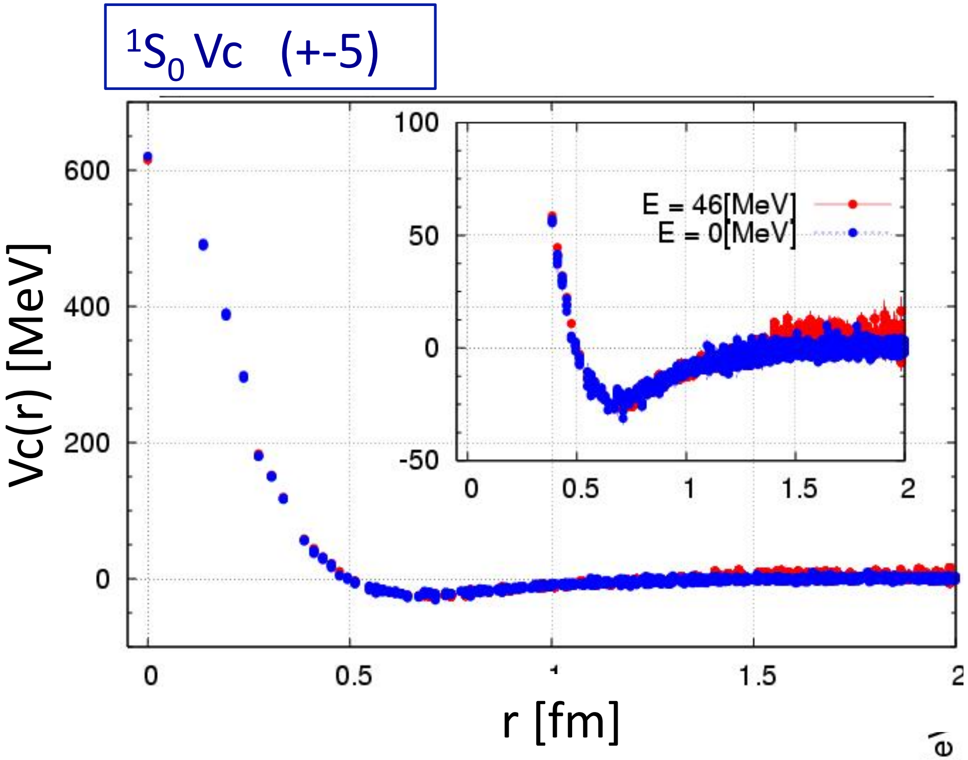
$$\left(\frac{\nabla^2}{m_N}+E\right)\phi(\vec{r};E)=\left[V_0(\vec{r})+V_\sigma(\vec{r})(\vec{\sigma}_1\cdot\vec{\sigma}_2)+V_T(r)S_{12}+V_{LS}(r)\vec{L}\cdot\vec{S}+O(\nabla^2)\right]\phi(\vec{r};E)$$

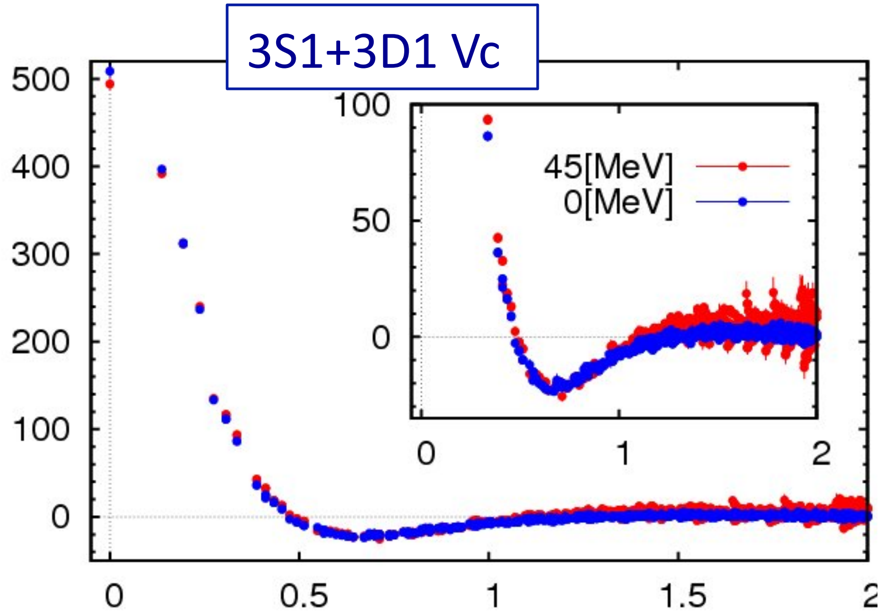
NLO

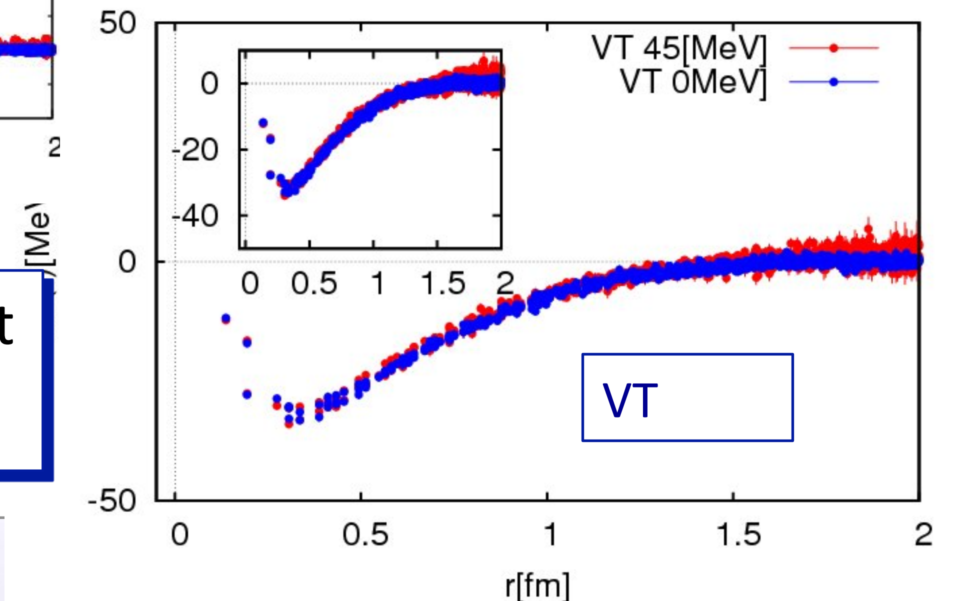
Okubo-Marshak

In this approximation, we can calculate potential U easily.

comparison of potentials : 0 MeV and 45 MeV







45MeV and 0MeV are consistent

Energy dependence is weak.

Above the threshold

$$\left(\frac{\nabla^2}{m_N} + E\right)\phi_{NN}(\vec{r};E) = V(r)\ \phi_{NN}(\vec{r};E)$$

$$\left(\frac{\nabla^2}{m_N} + E\right)\phi_{NN}(\vec{r};E) = V_N(r)\ \phi_{NN}(\vec{r};E) + V_{\pi NN}(r)\phi_{\pi}$$

Projection operator

$$G_{\alpha,\beta;\alpha'\beta'}(\vec{x},\vec{y},t) \equiv \langle 0 | T[N_\alpha(\vec{x},t) N_\beta(\vec{y},t) \bar{N}_{\alpha'} \bar{N}_{\beta'}] | 0 \rangle \xrightarrow{t\to\infty} \psi_{\alpha,\beta;\alpha',\beta'}(\vec{r})$$

$$\phi^{J=1,S=0,L=\Gamma}(\vec{r}) = P^{S=0}_{\alpha',\beta'} P^{L=\Gamma} P^{J=1} \ \psi_{\alpha,\beta;\alpha',\beta'}(\vec{r})$$

projection of L = Γ

$$P^{(L=\Gamma)} = \frac{d_\Gamma}{24} \sum_{i=1}^{24} \chi^{(\Gamma)}(R_i)^* \ \widehat{R}_i$$

$\exp(-i\omega L)$

projection of J = Γ

$$P^{(J=\Gamma)} = \frac{d_\Gamma}{24} \sum_{i=1}^{24} \chi^{(\Gamma)}(R_i)^* \ \widehat{R}_i$$

$\exp(-i\omega J)$

Projection of total angular momentum J

projection of J（J=T1)

Projection operator

$$P^{(\Gamma)} = \frac{d_\Gamma}{24}\sum_{i=1}^{24}\chi^{(\Gamma)}(R_i)^* \hat{R}$$

$\exp(-i\omega J)$

$$P^{(J=T_1)} = \frac{3}{24}\sum_{i=1}^{24}\chi^{(T_1)}(R_i)^* \widehat{R_i}$$

$$P^{J=T_1}G_{\alpha,\beta;\alpha'\beta'}(\vec{x},\vec{y},t) \equiv \langle 0 | T[N_\alpha(\vec{x},t)N_\beta(\vec{y},t)P^{J=T_1}\bar{N}_{\alpha'}\bar{N}_{\beta'}] | 0\rangle$$
$$= \frac{3}{24}\sum_{i=1}^{24}\chi^{(T_1)}(R_i)^* S_{\alpha',\bar{\alpha}'}(g_i)S_{\beta',\bar{\beta}'}(g_i)\, G_{\alpha,\beta:\bar{\alpha}'\bar{\beta}'}(\vec{x},\vec{y},t)$$

Here, we use..

$$S(g) \equiv \exp(\frac{i}{4}\sigma_{ij}\omega_{ij}), \qquad \sigma_{ij} \equiv -\frac{i}{2}[\gamma_i,\gamma_j], \quad g \in SO(3) \qquad \hat{R}(g)\, q(\vec{x}) = S(g)\, q(g^{-1}\vec{x})$$

$$\hat{R}(g)\, N_\beta = \sum_{\vec{y}_1,\vec{y}_2,\vec{y}_3} \epsilon_{abc}(q_a^T(g^{-1}\vec{y}_1)S(g)^T C\gamma_5 S(g) q_b(g^{-1}\vec{y}_2))S(g)\, q_{c,\beta}(g^{-1}\vec{x})$$
$$= S_{\beta,\bar{\beta}}(g)N_{\bar{\beta}}$$

$S(g)^{-1}$

$$\hat{R}(g)\,\bar{q}(x) = \bar{q}(gx)S^{-1}(g^{-1})$$

$$\bar{N}_{\alpha'}(\vec{x}_0,\vec{x}_1,\vec{x}_2) = \left(\bar{q}^t(x_0)C\gamma_5\gamma_0\bar{q}(x_1)\right)\bar{q}_{\alpha'}(x_2) = \sum_{i,j}\bar{q}_i(x_0)\bar{q}_j(x_1)\bar{q}_{\alpha'}(x_2)\left[C\gamma_5\gamma_0\right]_{i,j}$$

$$\hat{R}(g)\,\bar{N}_{\alpha'}(x_0,x_1,x_2) = \sum_{\alpha''}\left(\bar{q}^t(gx_0)C\gamma_5\gamma_0\bar{q}(gx_1)\right)\,\bar{q}_{\alpha''}(gx)S^{-1}_{\alpha'',\alpha'}(g^{-1})$$

$$\hat{R}(g)\left[\bar{N}_{\alpha'}(x_0,x_1,x_2)\bar{N}_{\beta'}(x_3,x_4,x_5)\right]$$

$$= \sum_{\alpha''}\sum_{\beta''}\left[S^{-1}(g^{-1})\right]^t_{\alpha',\alpha''}\left[S^{-1}(g^{-1})\right]^t_{\beta',\beta''}\bar{q}_i(gx_0)\bar{q}_j(gx_1)\bar{q}_{\alpha''}(gx_2)\,\bar{q}_k(gx_3)\bar{q}_l(gx_4)\bar{q}_{\beta''}(gx_5)$$

$$\times\left[C\gamma_5\gamma_0\right]_{i,j}\left[C\gamma_5\gamma_0\right]_{k,l}$$

$$= \sum_{\alpha''}\sum_{\beta''}\left[S^{-1}(g^{-1})\right]^t_{\alpha',\alpha''}\left[S^{-1}(g^{-1})\right]^t_{\beta',\beta''}\bar{N}_{\alpha''}(gx_0,gx_1,gx_2)\bar{N}_{\beta''}(gx_3,gx_4,gx_5)$$

$$S^{-1}(g) = S^{\dagger}(g)$$ ※g is spatial rotation

$$\left[S^{-1}(g^{-1})\right]^t = S^*(g^{-1})$$

$$G^{(n)}_{\alpha,\beta,\alpha'',\beta''} \equiv \langle 0 | T[N_\alpha(\vec{x},t) N_\beta(\vec{y},t) \sum_{x_0,\ldots,x_5} \bar{N}_{\alpha'}(\vec{x}_0,\vec{x}_1,\vec{x}_2)\bar{N}_{\beta'}(\vec{x}_3,\vec{x}_4,\vec{x}_5)] | 0 \rangle f^{(n)}(\vec{x}_0,\cdots,\vec{x}_5)$$

$$f^{(n)}(\vec{x}_0,\ldots,\vec{x}_5) = \exp(i\vec{p}^{(n)}\vec{x}_0) \cdots \exp(i\vec{p}^{(n)}\vec{x}_5)$$

projection of J

$$P^{J=\Gamma} G^{(n)}_{\alpha,\beta,\alpha'',\beta''} \equiv \langle 0 | T[N_\alpha(\vec{x},t) N_\beta(\vec{y},t) P^{J=\Gamma} \left[\sum_{x_0,\ldots,x_5} \bar{N}_{\alpha'}(\vec{x}_0,\vec{x}_1,\vec{x}_2)\bar{N}_{\beta'}(\vec{x}_3,\vec{x}_4,\vec{x}_5) \right]] | 0 \rangle f^{(n)}(\vec{x}_0,\cdots,\vec{x}_5)$$

$$P^{J=\Gamma} \left[\sum_{x_0,\ldots,x_5} \bar{N}_{\alpha'}(\vec{x}_0,\vec{x}_1,\vec{x}_2)\bar{N}_{\beta'}(\vec{x}_3,\vec{x}_4,\vec{x}_5) \right] f^{(n)}(\vec{x}_0,\cdots,\vec{x}_5)$$

$$\equiv \frac{d^\Gamma}{24} \sum_{g\in O} \chi^{*(\Gamma)}(g)\ \hat{R}(g) \left[\sum_{x_0,\ldots,x_5} \bar{N}_{\alpha'}(\vec{x}_0,\vec{x}_1,\vec{x}_2)\bar{N}_{\beta'}(\vec{x}_3,\vec{x}_4,\vec{x}_5) \right] f^{(n)}(\vec{x}_0,\cdots,\vec{x}_5)$$

$$= \frac{d^\Gamma}{24} \sum_{g\in O} \chi^{*(\Gamma)}(g) \sum_{\alpha''} \sum_{\beta''} \left[S^{-1}(g^{-1}) \right]^t_{\alpha',\alpha''} \left[S^{-1}(g^{-1}) \right]^t_{\beta',\beta''}$$

$$\sum_{x_0,\ldots,x_5} \bar{N}_{\alpha''}(gx_0,gx_1,gx_2)\bar{N}_{\beta''}(gx_3,gx_4,gx_5) f^{(n)}(\vec{x}_0,\cdots,\vec{x}_5)$$

$x \longrightarrow gx$ の和のとりかえ

$$= \frac{d^\Gamma}{24} \sum_{g \in O} \chi^{*(\Gamma)}(g) \sum_{\alpha''} \sum_{\beta''} \left[S^{-1}(g^{-1}) \right]^t_{\alpha',\alpha''} \left[S^{-1}(g^{-1}) \right]^t_{\beta',\beta''}$$

$$\sum_{x_0,\ldots,x_5} \bar{N}_{\alpha''}(x_0,x_1,x_2)\bar{N}_{\beta''}(x_3,x_4,x_5) f^{(n)}(g^{-1}\vec{x}_0,\cdots,g^{-1}\vec{x}_5)$$

$$= \frac{d^\Gamma}{24} \sum_{g \in O} \chi^{*(\Gamma)}(g) \sum_{\alpha''} \sum_{\beta''} \left[S^{-1}(g^{-1}) \right]^t_{\alpha',\alpha''} \left[S^{-1}(g^{-1}) \right]^t_{\beta',\beta''}$$

$$\sum_{m} \sum_{x_0,\ldots,x_5} \bar{N}_{\alpha''}(x_0,x_1,x_2)\bar{N}_{\beta''}(x_3,x_4,x_5)\, U(g)_{nm} f^{(m)}(\vec{x}_0,\cdots,\vec{x}_5)$$

projection of J

$$P^{J=\Gamma} G^{(n)}_{\alpha,\beta,\alpha'',\beta''} = \frac{d^\Gamma}{24} \sum_{g \in O} \chi^{*(\Gamma)}(g) \sum_{\alpha''} \sum_{\beta''} \left[S^{-1}(g^{-1}) \right]^t_{\alpha',\alpha''} \left[S^{-1}(g^{-1}) \right]^t_{\beta',\beta''} \sum_{m} U_{n,m}(g) G^{(m)}_{\alpha,\beta,\alpha'',\beta''}$$