η, η' meson masses from a mixed action approach in 2+1+1 twisted mass lattice QCD

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For the ETM Collaboration

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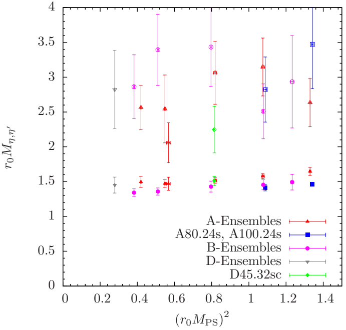
Introduction

In the unitary setup we had calculated the η- and η'-masses for various Ensemble from the ETM Collaboration...



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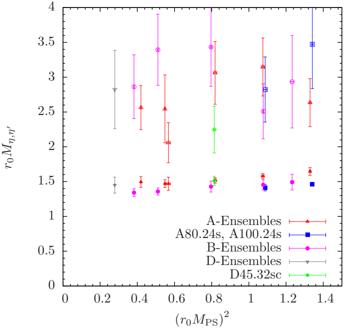

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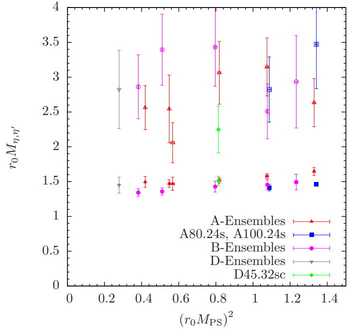

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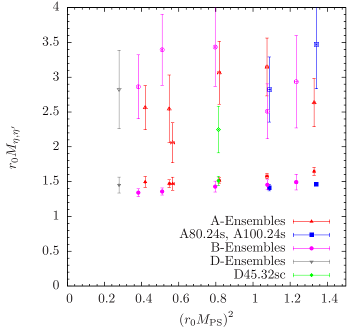

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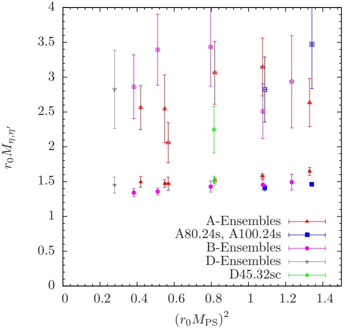

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We are looking for an analysis method...

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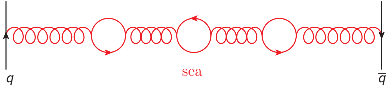
2. and an option to vary the strange quark mass μ_s

This can be achieved by changing the valence quark regularisation!

The mixed action approach

Idea: Use different regularisation in the sea and valence quark sector



- **The sea quarks**:
  obey the standard wTm action for 2+1+1 dynamical quark flavours which was used to generate the configurations:

- **The valence quark action:**
  will be replaced by the Osterwalder-Seiler action ...

And in the valence quark sector ...

We replace the wtm action by the **Osterwalder-Seiler action**:

$$S_{OS}[F, q_f, \bar{q}_f] = a^4 \sum_x \bar{q}_f(x) \left[\frac{1}{2}(D_w(r_f) + m_{cr}(r_f) + i\gamma_5 \mu_f \right] q_f(x) \ ,$$

R. Frezzotti and G. C. Rossi, JHEP 08, 007 (2004), hep-lat/0306014

Features

- Freedom to introduce an arbitrary number of valence quark flavours $q_f = s, s', s'', ..., c, c', c'', ...$
- with an individual value of the quark mass μ_f!
- ... and individual sign, too. *// Will be of later use!*
- No flavour mixing for light flavours as well as for heavy quarks

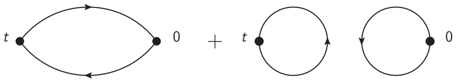
The bare quark mass is the only parameter to be tuned: $m_0 \to m_{cr}$
But: *value known from pure tm + we preserve* $\mathcal{O}(a)$ *improvement*

R. Frezzotti and G. C. Rossi, JHEP 10, 070 (2004), hep-lat/0407002

Correlation Matrix

The η/η' states can be extracted by means of the axial and pseudoscalar interpolating operators.

- $\rightarrow$ 3 pseudoscalars: $\eta_l := \bar{q}_l i\gamma_5 q_l$, $\eta_s := \bar{q}_s i\gamma_5 q_s$, $\eta_c := \bar{q}_c i\gamma_5 q_c$,
- *Correlators* $C_{ij}(t) =< 0|\eta_i(0)\eta_j(t)|0 >$ consist of connected and disconnected diagrams :



$$\Leftrightarrow \quad C_{ij}(t) = c_{ij}(t) + d_{ij}(t) \quad , \; c_{ij} = 0 \; \forall i \neq j \in \{l, s, c\}$$

- **Disconnected pieces are of special importance: responsible for the large η'-mass compared to the rest of the octet mesons.**

  This believed to be caused by the anomalous broken $U(1)_A$-symmetry.

G. 't Hooft, PRL 37, 8 (1976)

Matching of the Valence quark masses (1)

The valence quark masses in the Osterwalder-Seiler setup are free parameters & can be used to match sea and valence quark regularisation.

$\Rightarrow$ *We must tune the the strange valence quark mass* μ_s.

(The light quark mass is fixed by the unitary value)

- Therefore: need of observables to match the unitary and OS setup
- Option: Kaons are the lightest meson with strange quark content

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1. Tune the strange quark mass until we find:
   $m^{\mathrm{OS}}_{s\bar{s}\text{-connected}} = m^{\mathrm{unitary}}_{s\bar{s}\text{-connected}}$
2. Check:
   $m^{\mathrm{OS}}_{\eta} = m^{\mathrm{unitary}}_{\eta}$ & $m'^{\mathrm{OS}}_{\eta} = m'^{\mathrm{unitary}}_{\eta}$

Variance reduction

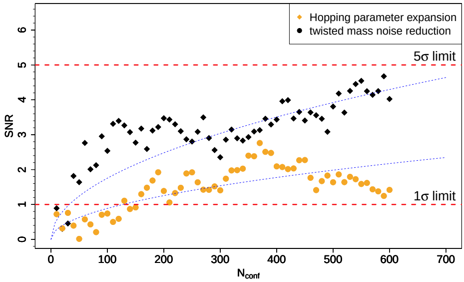
Form pure tm we know a powerful tool - the so-called "VV-method" - to reduce the noise arising from disconnected loops in the light quark sector.

K. Jansen et. al., Eur. Phys. J C58 261-269 (2008)

- The loop $[\bar{u}u - \bar{d}d](t)$ can be evaluated directly: (Note: the $-$ sign arises from tm!)

$$\frac{1}{D_d} - \frac{1}{D_u} = 2i\mu_l \frac{1}{D_d}\gamma_5\frac{1}{D_u}$$

- Very effective: signal/noise $\approx \sqrt{V}$ instead of 1



Dinter, Simon et al. arXiv:1202.1480 [hep-lat]

Variance reduction

How can we apply the "VV-method" to heavy quark flavours, too ?

Trick:

1. Rewrite:
   $s\bar{s} = \frac{1}{2}(s\bar{s} + s\bar{s})$
2. Use freedom of Osterwalder to introduce "new" valence flavour s'...
   $= \frac{1}{2}(s\bar{s} - s'\bar{s}')$
3. Rotate s & s' in the twisted basis:

$\Rightarrow$ This is the same situation as in the light sector.

In the Osterwalder-Seiler framework the VV-method can by applied to all disconnected loops.

Results: Ensemble characteristics

We will consider 2 ensembles provided by the ETM collaboration:

R. Baron et. al., JHEP 06 111 (2010)

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- B85.24 configurations
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  - $V/a^4 = 24^3 x48$
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Results: Ensemble characteristics

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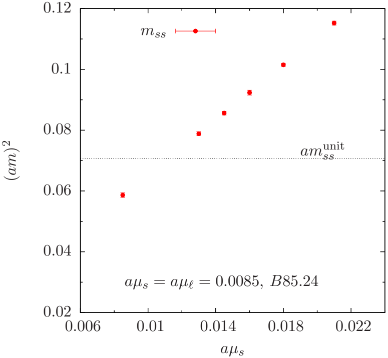
- B85.24 configurations
  - $a \approx 0.078 fm$
  - $V/a^4 = 24^3 x48$
  - $m_{PS\pm} \approx 490 MeV$
- Smaller lattice spacing: D45.32sc
  - $a \approx 0.061 fm$
  - $V/a^4 = 32^3 x64$
  - $m_{PS\pm} \approx 390 MeV$

R. Baron et. al., JHEP 06 111 (2010)

Matching of the Valence quark masses (2)

The mass of the chosen $s\bar{s}$-connected meson in the OS framework as a function of the strange quark mass



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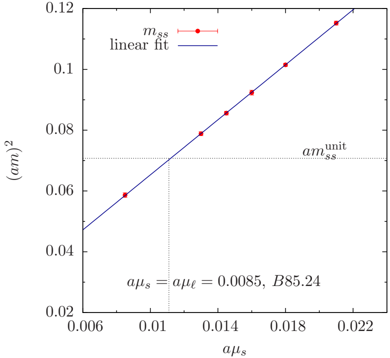

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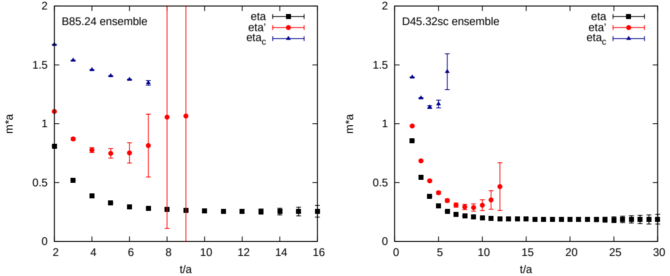
- m_{ss} graph intersects the unitary value at $\mu_s^{ss} \approx 0.011$

Generalised eigenvalue problem

Masses extracted via GEVP application from the 3x3 local correlation matrix:



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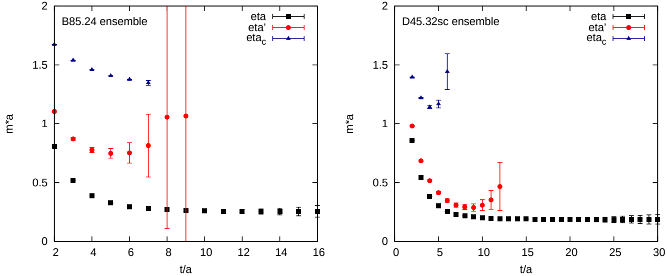

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- We observe 2 separated states corresponding to η/η' $\rightarrow$ $m_{\bar{s}s}$ is an appropriated quantity to determine μ_s

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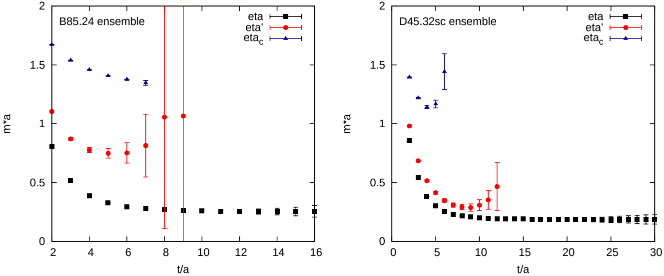

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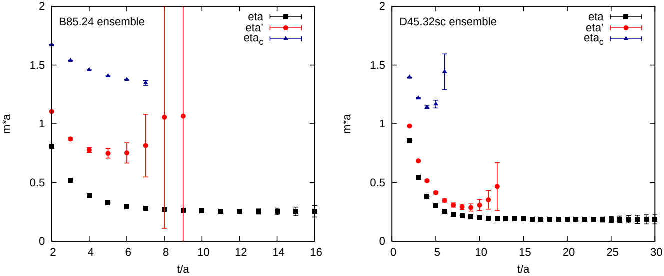

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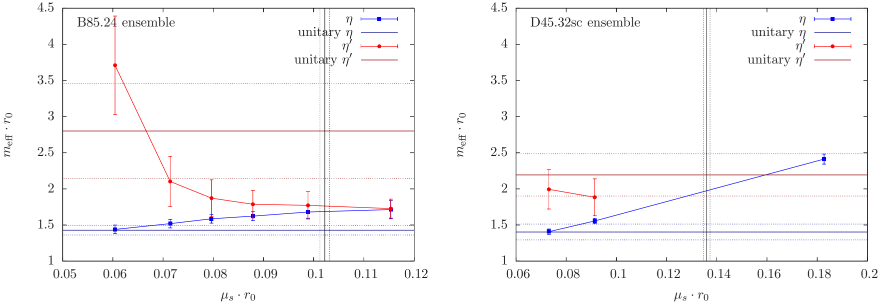
- η: ground state can be extracted very well!
- η': still just a few points $\rightarrow$ fit becomes difficulties
- (η_c: charm dominated state without much disconnected contributions)

Strange quark mass dependence (1)

How does the η/η'-mass evolves as a function of the μ_s mass?



- *Horizontal line corresponds to the unitary result of the η/η'-mass*
- *Vertical line marks μ_s matching via the neutral Kaon*
- *The most left point corresponds to $m_{\bar{s}s}$ matching*

Separation of states becomes more difficult for larger values of μ_s

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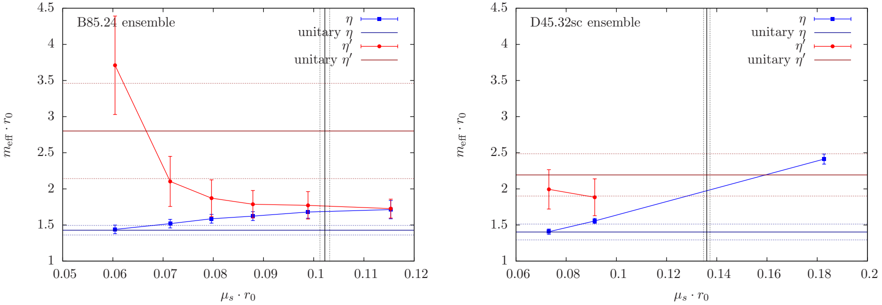

η-state

$\Rightarrow$ best agreement for $m_{\bar{s}s}$ matching & m_η exceeds the unitary value at larger μ_s

η'-state

- Again: good agreement at $m_{\bar{s}s}$ matching
- m_η drops below the unitary range at larger μ_s

Summary

The Osterwalder-Seiler mixed action is a promising tool to...

- Investigate the dependence of m_η on s,c-quark masses
- Reduce the noise in the heavy quark flavour sector

We have tested the setup successfully to reproduce the η-masses from the unitary framework.

$\Rightarrow$ For pseudoscalar flavour singlet quantities the sea loops have significant impact, but evaluation with proper matching is possible.

TODO:

- Increase the statistic of disconnected loops
- Explore further ensembles to asses systematic errors and
- Extrapolate to physical light quark mass.

Thank you for your attention!

Acknowledgments

We like to thank the Jülich Supercomputing Centre (JSC) for providing computation time on JUGENE and JUDGE. We are also grateful for support from ETMC, DFG via SFB/TR16

The Osterwalder-Seiler setup

Idea: Use different regularization in the valence and sea quark sector

$$S = S_G[F] + i \underbrace{\sum_{q\in\{l,h\}} S_{wtm}[F,\chi_q,\bar{\chi}_q]}_{\text{sea quarks } \chi_q} + \underbrace{\sum_f [S_{OS}[F,q_f,\bar{q}_f] + S_{Ghost}[F,\Phi_f]]}_{\text{valence quarks } q_f}$$

We have the Osterwalder-Seiler action for valence quarks

$$S_{OS}[F,q_f,\bar{q}_f] = a^4 \sum_x \bar{q}_f(x) \left[\frac{1}{2}(\nabla_\mu + \nabla^*_\mu) - W_{cr}(r_f) + i\gamma_5\mu_f\right] q_f(x)$$

with $W_{cr}(r_f) = -a\frac{r_f}{2}\sum_\mu \nabla^*_\mu \nabla_\mu + m_{cr}(r_f)$ and valence quark mass μ_f

Assign a ghost field Φ_f (euclidean commuting spinor) to each q_f with the singel purpose of canceling the corresponding valence determinant. The ghost action reads:

$$S_{gh}[F,\Phi_f] = a^4 \sum_x \Phi^\dagger_f sign(\mu_f) \left[\frac{1}{2}(\nabla_\mu + \nabla^*_\mu) - i\gamma_5 W_{cr}(r_f) + \mu_f\right] \Phi_f(x)$$

K. Osterwalder, E. Seiler, Ann. Phys. (NY) 110, 440 (1978)