High-precision scale setting in lattice QCD

Thorsten Kurth for the Budapest-Marseille-Wuppertal Collaboration

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Motivation

- setting the scale should be cheap and precise in order to achieve percent-level accuracy
- common inputs
  - r_0, r_1 [Sommer; Nucl.Phys.B411] [MILC; Phys.Rev. D62] (purely gluonic, complicated analysis, noisy)
  - M_Ω (fitting correlators, precise experimental input, sensitive to m_s, inversions, noisy correlation function, problematic on coarse lattices)
  - f_π, f_K (fitting correlators, precise experimental input, inversions, strong dependence on quark masses, renormalization in case of Wilson quarks)
  - $m_{s\bar{s}}$, $f_{s\bar{s}}$ [HPQCD, Phys.Rev. D81] (fitting correlators, independent of m_l, strong dependence on m_s, renormalization in case of Wilson quarks)
- in this talk
  - w_0 [BMW-c, arXiv:1203.4469] (purely gluonic, uncomplicated analysis, good accuracy, mod. expensive, large autocorrelation)

Introduction

- w_0-scale is based on Wilson flow [Narayanan, Neuberger; JHEP 0603] [Lüscher; Commun.Math.Phys. 293, JHEP 1008]
  - individual configurations are infinitesimally smeared up to scale t in flow-time ($[t] = m^2$)
  - smearing is performed until specific dimensionless observable reaches specific value
  - time t_0 at which that happens can be used to set the scale on the original lattices
- suitable observable is

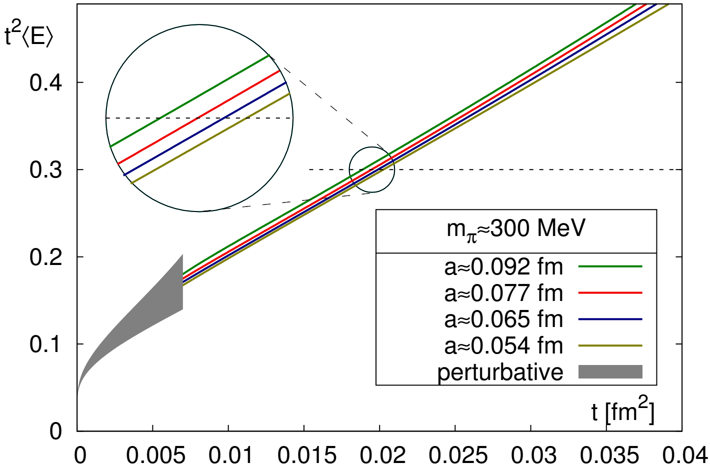
$$\langle E(t)\rangle = (G^a_{\mu\nu} G^{\mu\nu}_a)(t)/4$$

  where $G^a_{\mu\nu}$ is the clover-leaf definition of the QCD field strength tensor
- Lüscher's idea: use

$$T(t) \equiv t^2 \langle E(t)\rangle \sim t$$

  and t_0 defined by $T(s_0) = 0.3$, $t_0 = \sqrt{8 s_0}$

Introduction II



- t_0 scale in fm as determined by M_Ω [BMW-c, arXiv:1203.4469]

Introduction III

- effect of integration: $T(t)$ incorporates informations from all scales $> \mathcal{O}\left(1/\sqrt{t}\right)$ and thus also discretization effects from around $t{\sim}a^2$ (constant of integration) $\Rightarrow$ t_0-scale contains non-universal part and therefore has larger discretization errors
- our proposal:

$$W(t) \equiv t\frac{\mathrm{d}}{\mathrm{d}t}\left[t^2\langle E(t)\rangle\right]$$

  and define w_0 via

$$W(t)\big|_{t=w_0^2} = 0.3$$

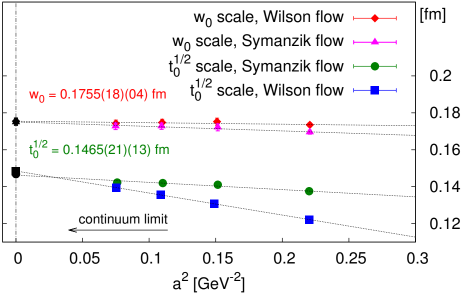
- w_0 depends on scales of $\mathcal{O}(1/\sqrt{t}) \Rightarrow$ non-universal part shrinks and w_0 has thus smaller discretization errors

Symanzik vs. Wilson flow

- infinitesimal smearing equivalent to solving flow-equation

$$\dot{V}_t = Z(V_t)V_t, \quad V_0 = U$$

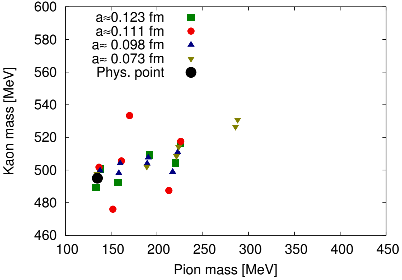
- $Z(V_t)$ is derivative of gauge action $\Rightarrow$ allows to define different flow-types (Symanzik, Wilson)

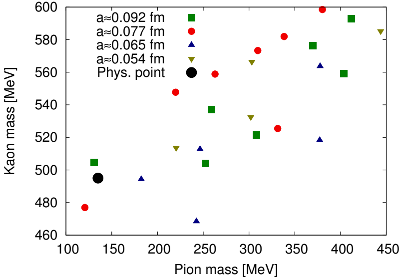


- w_0 very insensitive to different flow definitions
- however: only shuffling $\mathcal{O}(a^2)$ corrections around

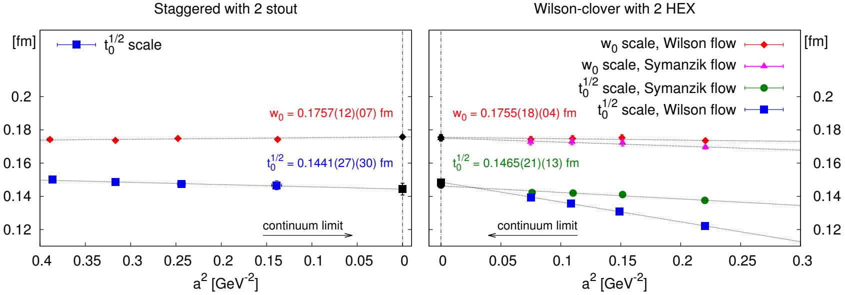
Expressing w_0 in physical units

- w_0 is intermediate scale and needs to be converted into physical units
  - use our 2 HEX smeared tree-level clover improved WIlson ensembles with $0.054\,\mathrm{fm} < a < 0.093\,\mathrm{fm}$, $m_s \simeq m_s^{\mathrm{phys}}$ and M_π even below its physical value
  - cross-check on stout-link staggered ensemble
- use M_Ω scale-setting to determine $w_0[\mathrm{fm}]$



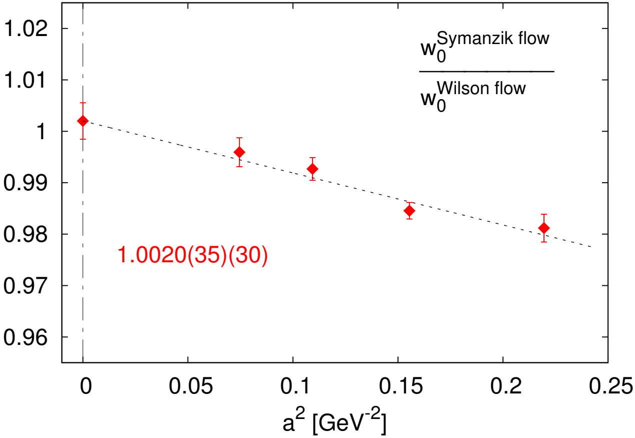


Continuum limit



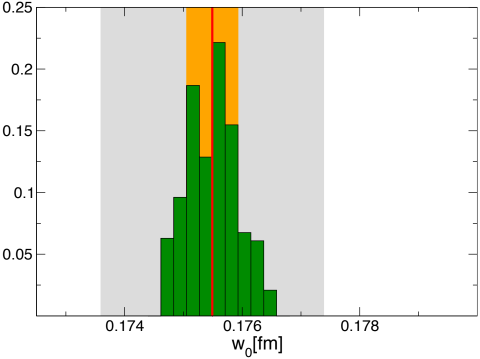
- result: $w_0 = 0.1755(18)(04)\,\mathrm{fm}$ (Wilson)
- all continuum limits agree, but slopes are different $\Rightarrow$ w_0 compensates discretization errors in M_Ω best
- t_0 scale benefits from Symanzik definition of flow

Continuum limit II



- both flow types give the same result but have different $\mathcal{O}(a^2)$
- Wilson flow is computationally cheaper

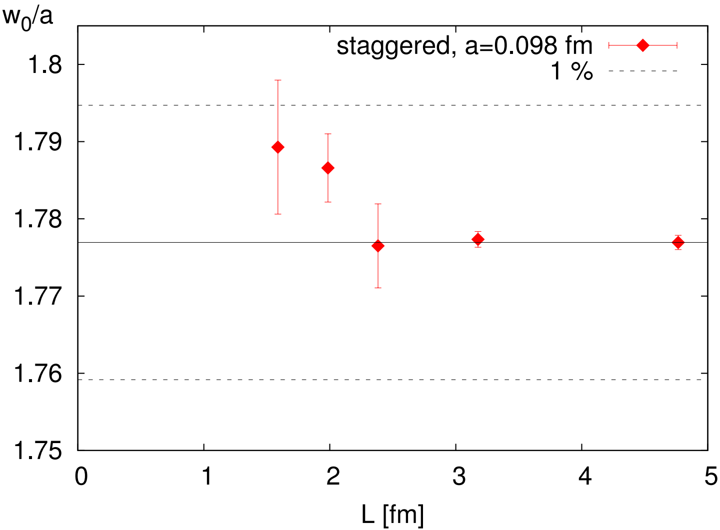
Continuum limit III



- statistical error: 2000 bootstrap samples
- systematic error:
  - 4 fit forms for $w_0(M_\pi, M_K, a)$
  - 2 pion mass cuts ($M_\pi < 300\,\mathrm{MeV}, 350\,\mathrm{MeV}$)
  - 2 M_π-cuts in M_Ω-scale determination ($M_\pi < 380\,\mathrm{MeV}, 480\,\mathrm{MeV}$)
  - 2 fit ranges for extracting masses from correlators
  - 2 scaling assumptions

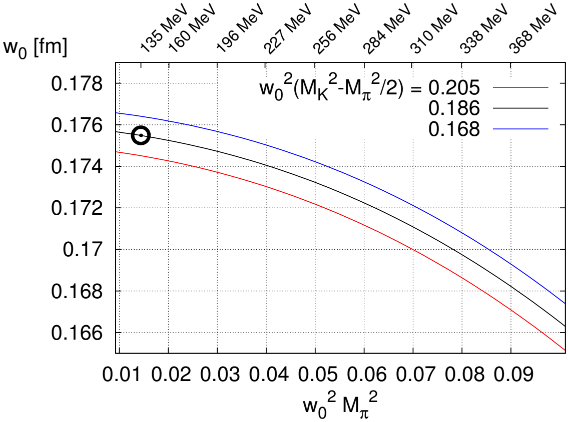
- tiny systematic uncertainty
- statistical uncertainty of $w_0[\mathrm{fm}]$ is $\sim 1\%$ (dominated by aM_Ω)
- w_0/a can be inexpensively determined up to very high precision (per-mil level if necessary)

Finite volume effects



- absence of FV effects for lattices with $L \gtrsim 2\,\mathrm{fm}$

Mass dependence



- 10% change in m_s translates into $\sim$ 0.5% change in w_0
- $\mathcal{O}(10)$ spread in m_{ud} translates into $\sim$ 5% spread in w_0
- detailed dependency in [BMW-c, arXiv:1203.4469]

Price tag

- single config, 1000 ODE steps (time in seconds)

machine	BG/P rack	2 Fermi-GPUs	core i7 (unoptimized)
$16^3 \times 32$	33.6	74.7	5583
$24^3 \times 48$	-	257	9030
$24^3 \times 64$	53.6	329	11758
$32^3 \times 96$	384	1108	39460

- rule of thumb: $2/a^2[\mathrm{fm}^2]$ steps
- public CHROMA implementation
- arXiv:1203.4469: `wilson_flow.c` for generating and `w0_scale.c` for analyzing the flow

Conclusions

- Summary
  - proposed new scale-setting observable w_0 based on Wilson flow [BMW-c,arXiv:1203.4469]
  - obtained $w_0 = 0.1755(18)(04)\,\mathrm{fm}$ in physical units from 2 HEX-smeared Wilson data including physical pion masses
  - crosschecked with staggered
  - mild quark mass dependence (can be corrected for) and very weak volume dependence
- Applications
  - high-precision scale-setting
  - in combination with QCDSF "mass splitting" strategy: parameter tuning becomes much simpler
  - suitable for anisotropic lattices: [BMW-c, arXiv:1205.0781]