Isospin breaking in octet baryon mass splittings

R. Horsley, J. Najjar, Y. Nakamura, D. Pleiter, P. E. L. Rakow, G. Schierholz and J. M. Zanotti

– QCDSF-UKQCD Collaboration –

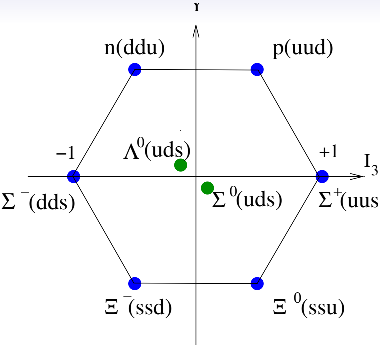
Edinburgh – Regensburg – RIKEN (Kobe) – FZ (Jülich) – Liverpool – DESY (Hamburg) – Adelaide

[Lattice 2012, Cairns, Australia]

QCDSF related talks with $2 + 1$ flavours:

- Paul Rakow
  The effects of flavour symmetry breaking on hadron matrix elements: I
- Ashley Cooke
  The effects of flavour symmetry breaking on hadron matrix elements: II

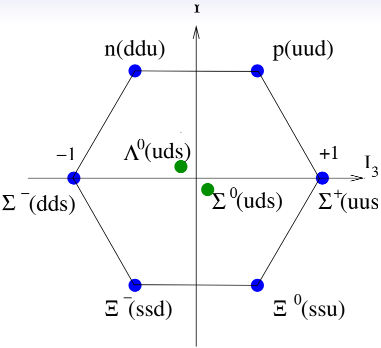
Introduction



$$
\begin{aligned}
(M_n - M_p)^{\text{exp}} &= 1.293333(33)\,\text{MeV} \\
(M_{\Sigma^-} - M_{\Sigma^+})^{\text{exp}} &= 8.079(76)\,\text{MeV} \\
(M_{\Xi^-} - M_{\Xi^0})^{\text{exp}} &= 6.85(21)\,\text{MeV}\,.
\end{aligned}
$$

- Small differences [expt precision way beyond what we achieve here]
- Isospin breaking effects:
  - QED component [not considered here]
  - $m_d - m_u$ component — pure QCD
- $M^{\text{exp}} = M^{QCD} + M^{QED}$
  interplay between effects:
  EM tends to make p heavier than n, but $m_d - m_u$ works in opposite direction

Introduction



$$
\begin{aligned}
(M_n - M_p)^{\text{exp}} &= 1.293333(33)\,\text{MeV} \\
(M_{\Sigma^-} - M_{\Sigma^+})^{\text{exp}} &= 8.079(76)\,\text{MeV} \\
(M_{\Xi^-} - M_{\Xi^0})^{\text{exp}} &= 6.85(21)\,\text{MeV}\,.
\end{aligned}
$$

- $\Lambda(uds)$, $\Sigma^0(uds)$ have the same quark content (quantum numbers) but different wavefunctions
- Can mix (if isospin broken)
- So only consider 'outer' ring here

QCDSF strategy

- develop $SU(3)$ flavour symmetry breaking expansion [arXiv:1102.5300]
- baryon octet [LO + NLO ie linear+quadratic]

$$
\begin{aligned}
M^2(aab) &= M_0^2 + A_1(2\delta m_a + \delta m_b) + A_2(\delta m_b - \delta m_a) \\
&\quad + B_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
&\quad + B_1(2\delta m_a^2 + \delta m_b^2) + B_2(\delta m_b^2 - \delta m_a^2) + B_3(\delta m_b - \delta m_a) \\
&\quad + \ldots
\end{aligned}
$$

- with quarks $q = a, b, \ldots$ from (u, d, s), so

$$M(uud) = p\,, \quad M(dds) = \Sigma^-\,, \ldots$$

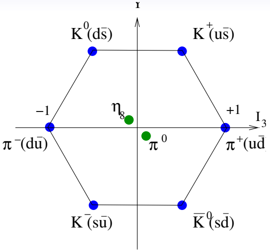
- $SU(3)$ flavour symmetric point $\delta m_q = 0$ where [$\delta m_u + \delta m_d + \delta m_s = 0$]

$$\delta m_q = m_q - \overline{m}\,, \quad \overline{m} = \tfrac{1}{3}(m_u + m_d + m_s)$$

-

$$M_0^2 = M_0^2(\overline{m})\,, \quad A_i = A_i(\overline{m})\,, \quad B_i = B_i(\overline{m})$$

Similarly for pseudoscalar meson octet



- $$\begin{aligned} M^2(a\overline{b}) &= M_{0\pi}^2 + \alpha(\delta m_a + \delta m_b) \\ &\quad + \beta_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &\quad + \beta_1(\delta m_a^2 + \delta m_b^2) + \beta_2(\delta m_a - \delta m_b)^2 + \ldots \end{aligned}$$

- with quarks $q = a,\ b,\ \ldots$ from (u, d, s), so
$$M(d\overline{s}) = K^0\,, \quad M(d\overline{u}) = \pi^-\,, \ldots$$

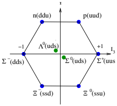
- $SU(3)$ flavour symmetric point $\delta m_q = 0$ where
$$\delta m_q = m_q - \overline{m}\,, \quad \overline{m} = \tfrac{1}{3}(m_u + m_d + m_s)$$

Defining the scale I – many possibilities (using singlet quantities)

- Octet baryons: (centre of mass)

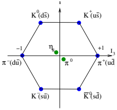
stable under strong ints.

$$X_N^2 = \tfrac{1}{6}(M_p^2 + M_n^2 + M_{\Sigma^+}^2 + M_{\Sigma^-}^2 + M_{\Xi^0}^2 + M_{\Xi^-}^2) = (1.160\,\mathrm{GeV})^2$$



- pseudoscalar mesons: (centre of mass)

$$X_\pi^2 = \tfrac{1}{6}(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\overline{K}^0}^2 + M_{K^-}^2) = (0.4116\,\mathrm{GeV})^2$$



- Some other possibilities

$$X_S^2 = \begin{cases} \tfrac{1}{2}(M_\Sigma^2 + M_\Lambda^2) & \\ M_{\Sigma^*}^2,\ \tfrac{1}{2}(M_\Delta^2 + M_{\Xi^*}^2) & \text{baryon decuple} \\ \tfrac{1}{6}(M_{K^{*+}}^2 + M_{K^{*0}}^2 + M_{\rho^+}^2 + M_{\rho^-}^2 + M_{\overline{K}^{*0}}^2 + M_{K^{*-}}^2) & \text{vector octet} \\ 1/r_0^2 & r_0 = 0.5\,\mathrm{fm}\,? \end{cases}$$

Defining the scale II – $SU(3)$ flavour symmetry expansions

- Octet baryons

$$\begin{aligned} X_N^2 &= M_0^2 + (\tfrac{1}{6}B_0 + B_1 + B_3)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &= M_0^2 + O(\delta m_q^2) \end{aligned}$$

- pseudoscalar mesons

$$\begin{aligned} X_\pi^2 &= M_{0\pi}^2 + (\tfrac{1}{6}\beta_0 + \tfrac{2}{3}\beta_1 + \beta_2)\,(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &= M_{0\pi}^2 + O(\delta m_q^2) \end{aligned}$$

- All singlet quantities

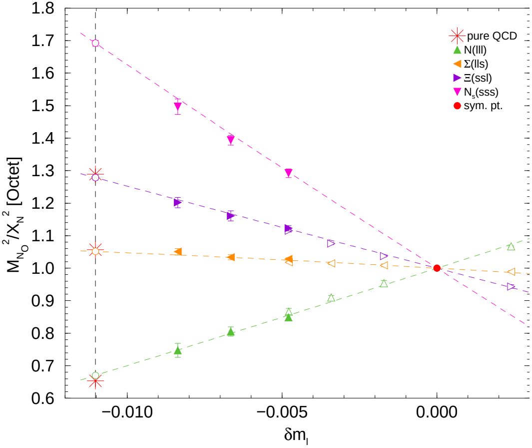
$$X_S = \# + \#(\delta m_q^2)$$

(almost) constant

- Form ratios:

$$\tilde{M} \equiv \frac{M}{X_S}\,, \quad S = N, \pi \qquad \tilde{A}_i \equiv \frac{A_i}{M_0^2}\,, \ldots\,, \quad \text{in expansions}$$

‘Fan’ plot



- $2+1$, $q = l, s$, $\delta m_u = \delta m_d = \delta m_l$ $\delta m_s = -2\delta m_l$
- $O(a)$-improved clover fermions; $32^3 \times 64$ lattices [fitted, filled pts]
- $\delta m_l = m_l - \overline{m}_0$
- $\overline{m}_0 = \text{const.}$ [to find need to tune]
- $M_N = M(lll)$, $M_\Sigma = M(lls)$, $M_\Xi = M(ssl)$, $M_{N_s} = M(sss)$ [PQ]

Use the pseudoscalar fan plot to determine the physical quark mass: δm_l^*

Main observation:

- Provided $\overline{m}$ kept constant, then expansion coefficients remain unaltered whether
  - $1 + 1 + 1$
  - $2 + 1$
- Opens possibility of determining quantities that depend $1 + 1 + 1$ from just $2 + 1$ simulations
- eg use for (pure QCD) isospin breaking effects

- eg (pure QCD) isospin breaking effects [$\sqrt{\ldots}$ $A'_i = A_i/2, \ldots$]

$$\begin{aligned}
\tilde{M}_n - \tilde{M}_p &= \tilde{M}(ddu) - \tilde{M}(uud) \\
&= (\delta m_d - \delta m_u)\left[\tilde{A}'_1 - 2\tilde{A}'_2 + (\tilde{B}'_1 - 2\tilde{B}'_2)(\delta m_d + \delta m_u)\right] \\
\tilde{M}_{\Sigma^-} - \tilde{M}_{\Sigma^+} &= \tilde{M}(dds) - \tilde{M}(uus) \\
&= (\delta m_d - \delta m_u)\left[2\tilde{A}'_1 - \tilde{A}'_2 + (2\tilde{B}'_1 - \tilde{B}'_2 + 3\tilde{B}'_3)(\delta m_d + \delta m_u)\right] \\
\tilde{M}_{\Xi^-} - \tilde{M}_{\Xi^0} &= \tilde{M}(ssd) - \tilde{M}(ssu) \\
&= (\delta m_d - \delta m_u)\left[\tilde{A}'_1 + \tilde{A}'_2 + (\tilde{B}'_1 + \tilde{B}'_2 + 3\tilde{B}'_3)(\delta m_d + \delta m_u)\right]
\end{aligned}$$

- programme:
  1. determine A, B expansion coefficients
  2. parallel procedure for pseudoscalar meson octet
     - determine α, β coefficients
     - determine physical $\delta m_u \pm \delta m_d, \ldots$ by matching to physical $K^0 - K^+$, $K^+ - \pi^+$ mass splitting [using Dashen's thm]
  3. with X_N scale gives MeV results

Expansion coefficients – improvements in determination

- expansion NLO $\to$ NNLO

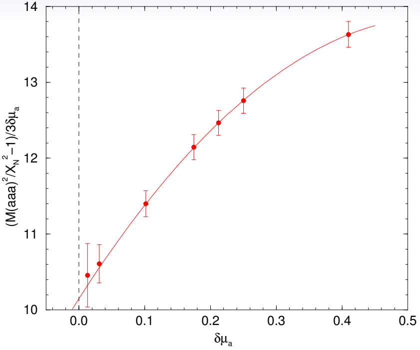
NNLO used as 'control'

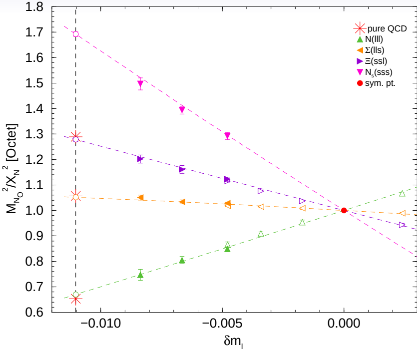
- δm_l range rather small $\sim [-0.01, 0]$ so PQ partially quenching (ie valence quark masses $\neq$ sea quark masses)

$$\begin{aligned} M^2(aab) &= M_0^2 + A_1(2\delta\mu_a + \delta\mu_b) + A_2(\delta\mu_b - \delta\mu_a) \\ &\quad + \tfrac{1}{6}B_0(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &\quad + B_1(2\delta\mu_a^2 + \delta\mu_b^2) + B_2(\delta\mu_b^2 - \delta\mu_a^2) + B_3(\delta\mu_b - \delta\mu_a)^2 \\ &\quad + C_0\delta m_u\delta m_d\delta m_s \\ &\quad + [C_1(2\delta\mu_a + \delta\mu_b) + C_2(\delta\mu_b - \delta\mu_a)](\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\ &\quad + C_3(\delta\mu_a + \delta\mu_b)^3 + C_4(\delta\mu_a + \delta\mu_b)^2(\delta\mu_a - \delta\mu_b) \\ &\quad + C_5(\delta\mu_a + \delta\mu_b)(\delta\mu_a - \delta\mu_b)^2 + C_6(\delta\mu_a - \delta\mu_b)^3 \end{aligned}$$

- a, b denote three valence quarks of arbitrary mass, μ_q
- $\delta\mu_q = \mu_q - \overline{m} \quad q \in \{a, b, \ldots\}$
- expansion coefficients unchanged $M_0^2(\overline{m})$, $A_i(\overline{m})$, $B_i(\overline{m})$, $C_i(\overline{m})$
- mixed sea/valence mass terms
- $2+1$: $\delta m_u = \delta m_d = \delta m_l$; $\delta m_s = -2\delta m_l$

$2 + 1$ joint fits





- PQ data [$\delta m_l = 0$, 46 points]
- illustration, to avoid 3-dim plot

$$\frac{\tilde{M}^2(aaa) - 1}{3\delta\mu_a} = \tilde{A}_1 + \tilde{B}_1\delta\mu_a + \tfrac{8}{3}\,\tilde{C}_3\delta\mu_a^2$$

- unitary data [$\mu_q \to m_q$, 13 points]

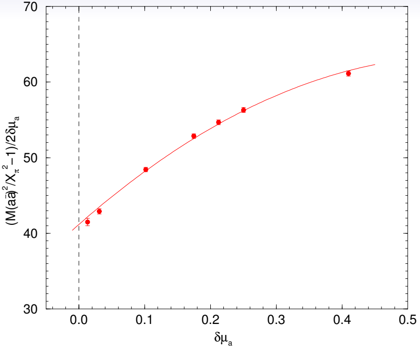
Very different x-scales involved

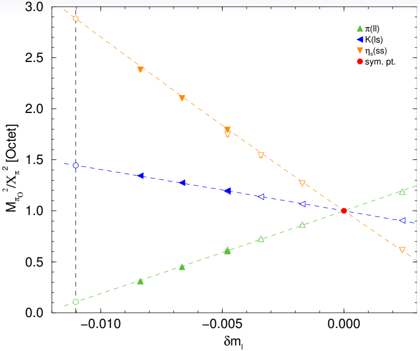
Pseudoscalar meson expansion coefficients

$$\begin{aligned}
M^2(a\bar{b}) \quad = \quad & M_{0\pi}^2 + \alpha(\delta\mu_a + \delta\mu_b) \\
& + \beta_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \beta_1(\delta\mu_a^2 + \delta\mu_b^2) + \beta_2(\delta\mu_a - \delta\mu_b)^2 \\
& + \gamma_0 \delta m_u \delta m_d \delta m_s + \gamma_1(\delta\mu_a + \delta\mu_b)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) \\
& + \gamma_2(\delta\mu_a + \delta\mu_b)^3 + \gamma_3(\delta\mu_a + \delta\mu_b)(\delta\mu_a - \delta\mu_b)^2
\end{aligned}$$

- similar procedure

$2+1$ joint fits





- PQ data [$\delta m_l = 0$, 27 points]
- illustration, to avoid 3-dim plot

$$\frac{\tilde{M}^2(a\bar{a}) - 1}{2\delta\mu_a} \quad = \quad \tilde{\alpha}_1 + \tilde{\beta}_1\delta\mu_a + 4\tilde{\gamma}_2\delta\mu_a^2$$

- unitary data [$\mu_q \to m_q$, 10 points]

Very different x-scales involved

- Have determined all LO+NLO coefficients
  - baryon octet – A, B
  - meson pseudoscalar octet – α, β
- programme:
  - Invert meson pseudoscalar expansions

$$\delta m_d - \delta m_u = \frac{\tilde{M}^2_{K^0} - \tilde{M}^2_{K^+}}{\tilde{\alpha}} \left(1 + \frac{2(\tilde{\beta}_1 + 3\tilde{\beta}_2)}{3\tilde{\alpha}^2}\left(\tfrac{1}{2}(\tilde{M}^2_{K^0} + \tilde{M}^2_{K^+}) - \tilde{M}^2_{\pi^+}\right)\right)$$

$$\delta m_d + \delta m_u = -\frac{2}{3\tilde{\alpha}} \left(\tfrac{1}{2}(\tilde{M}^2_{K^0} + \tilde{M}^2_{K^+}) - \tilde{M}^2_{\pi^+}\right)$$

  - determines physical δm_u^*, δm_d* (or indeed any other value)
- Substitute in baryon expansion

Final/Main result:

$$
\begin{aligned}
\tilde{M}_n - \tilde{M}_p &= 0.0789(41)\left(\tilde{M}^2_{K^0} - \tilde{M}^2_{K^+}\right) \\
&\quad \times \left[1 + 0.0817(92)\left(\tfrac{1}{2}(\tilde{M}^2_{K^0} + \tilde{M}^2_{K^+}) - \tilde{M}^2_{\pi^+}\right)\right] \\
\tilde{M}_{\Sigma^-} - \tilde{M}_{\Sigma^+} &= 0.2243(35)\left(\tilde{M}^2_{K^0} - \tilde{M}^2_{K^+}\right) \\
&\quad \times \left[1 + 0.0077(30)\left(\tfrac{1}{2}(\tilde{M}^2_{K^0} + \tilde{M}^2_{K^+}) - \tilde{M}^2_{\pi^+}\right)\right] \\
\tilde{M}_{\Xi^-} - \tilde{M}_{\Xi^0} &= 0.1455(24)\left(\tilde{M}^2_{K^0} - \tilde{M}^2_{K^+}\right) \\
&\quad \times \left[1 - 0.0324(50)\left(\tfrac{1}{2}(\tilde{M}^2_{K^0} + \tilde{M}^2_{K^+}) - \tilde{M}^2_{\pi^+}\right)\right]
\end{aligned}
$$

- $\tilde{M} = M/X_S$, $S = N, \pi$
- ‘pure’ QCD result
- NLO corrections small from $+10\% \sim -5\%$
- $SU(3)$ flavour symmetry breaking expansion appears highly convergent

Physical values

- What are the physical values of

$$M^2_{K^0}\,,\quad M^2_{K^+}\,,\quad M^2_{\pi^+}\qquad \text{in particular}\quad M^2_{K^0}-M^2_{K^+}\quad ??$$

- $M^{\exp} = M^{QCD} + M^{QED}$
- Dashen's theorem:
  [EM effects for charged mesons K^+, π^+ same; for neutral mesons π^0, K^0 vanish]

$$\begin{aligned} M^{\exp\,2}_{\pi^+} &= M^{*\,2}_{\pi^+} + \mu_\gamma\,, \qquad M^{\exp\,2}_{\pi^0} = M^{*\,2}_{\pi^0} \approx M^{*\,2}_{\pi^+}\,, \\ M^{\exp\,2}_{K^+} &= M^{*\,2}_{K^+} + \mu_\gamma\,, \qquad M^{\exp\,2}_{K^0} = M^{*\,2}_{K^0} \end{aligned}$$

- Sometimes violations to Dashen's theorem ($\epsilon_\gamma = 0$) given as

$$M^{*\,2}_{K^0} - M^{*\,2}_{K^+} = \left(M^2_{K^0} - M^2_{K^+}\right)^{\exp} + (1+\epsilon_\gamma)\left(M^2_{\pi^+} - M^2_{\pi^0}\right)^{\exp}$$

- Regard ϵ_γ here as a possible further error

Physical values

- This gives final results ('pure' QCD)

$$M_n^* - M_p^* = 3.13(15)(76|\epsilon_\gamma|)\,\mathrm{MeV}$$
$$M_{\Sigma^-}^* - M_{\Sigma^+}^* = 8.10(14)(193|\epsilon_\gamma|)\,\mathrm{MeV}$$
$$M_{\Xi^-}^* - M_{\Xi^0}^* = 4.98(10)(120|\epsilon_\gamma|)\,\mathrm{MeV}$$

- PDG:

$M_n^* - M_p^* = 1.29(0)\,\mathrm{MeV}\,,\ M_{\Sigma^-}^* - M_{\Sigma^+}^* = 8.08(8)\,\mathrm{MeV}\,,\ M_{\Xi^-}^* - M_{\Xi^0}^* = 6.9(2)\,\mathrm{MeV}$

ie EM effects:

$n(ddu) - p(uud) < 0\,, \quad \Sigma^-(dds) - \Sigma^+(uus) \approx 0\,, \quad \Xi^-(ssd) - \Xi^0(ssu) > 0$

- Walker-Loud et al, 1203.0254 [nucl-th] gives a determination of EM effects of $n - p$ of $-1.30(47)\,\mathrm{MeV}$, so

$$(M_n - M_p)^{*\ QCD+QED} = 1.83(49)(76|\epsilon_\gamma|)\,\mathrm{MeV}$$

Might indicate that violations of Dashen's theorem are small

Conclusions

- Introduced a method here to determine 'pure' QCD isospin effects in

$$n - p\,, \quad \Sigma^- - \Sigma^+\,, \quad \Xi^- - \Xi^0$$

  due to difference in $u - d$ quark masses
- Developed a $SU(3)$ flavour symmetry breaking expansion keeping the average quark mass $\overline{m}$ constant
  advantages:
  - can use $2+1$ simulations, ie $m_u = m_d = m_l$
  - can use cheap PQ results
- Expansion appears to be highly convergent
- Encouraging first results