Status of the MILC calculation of electromagnetic contributions to pseudoscalar masses

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Motivation

- Quark masses are fundamental parameters in the Standard Model and their nonperturbative lattice determination is important for phenomenology.

- With the advance of high-precision spectroscopy the electromagnetic contributions to the quark masses become important. In fact, they constitute the largest uncertainty in the calculation of m_u/m_d (arXiv:0903.3598):

	m_u	m_d	m_u/m_d
value	1.9	4.6	0.42
statistics	0.0	0.0	0.00
lattice systematics	0.1	0.2	0.01
perturbative	0.1	0.2	–
EM	0.1	0.1	0.04

- The MILC collaboration has been working on reducing the EM uncertainties and progress has been reported previously: PoS(LATTICE 2008)127, PoS(Lattice 2010)084, PoS(Lattice 2010)127.

Theory

- Observation made by Dashen (1960)

$$M^2_{K^\pm} - M^2_{K^0} = M^2_{\pi^\pm} - M^2_{\pi^0}$$

LO electromagnetic effects in the kaon and the pion system are identical.

- NLO corrections to Dashen's theorem:

$$M^2_{K^\pm} - M^2_{K^0} = (1 + \Delta_{EM})(M^2_{\pi^\pm} - M^2_{\pi^0})$$

- Direct measurement of NLO should allow for better accuracy. Need rSχPT, developed in arXiv:1011.3994:

$$\begin{aligned} \Delta M^2_{xy,5} \;=\; & q^2_{xy}\delta_{EM} - \frac{1}{16\pi^2} e^2 q^2_{xy} M^2_{xy,5}\left[3\ln(M^2_{xy,5}/\Lambda^2_\chi) - 4\right] \\ & - \frac{2\delta_{EM}}{16\pi^2 f^2}\frac{1}{16}\sum_{\sigma,\xi}\left[q_{x\sigma}q_{xy}l(M^2_{x\sigma,\xi}) - q_{y\sigma}q_{xy}l(M^2_{y\sigma,\xi})\right] \\ & + \sum_{i=1}^{6} c_i \times (\text{analytic terms}) \end{aligned}$$

where σ are the sea quarks, and ξ is the staggered taste.

Additional terms in the fitting form

- Included a parameter which would allow a lattice spacing dependence in δ_{EM}:

$$\delta_{EM}(1 + C\alpha_s a^2/(\alpha_s a^2)^{\mathrm{fid}}).$$

- Included finite volume effects in the form (BMW arXiv:1201.2787):

$$f_v q_{xy}^2/L_s^2,$$

  where L_s is the spatial lattice size, in addition to standard $m_\pi L_s$ dependent terms from EM tadpoles.

- NNLO analytic terms.(Further checks needed to ensure that all possible NNLO terms have been included.)

Effects of the photon quenching

- We have pseudoscalar spectroscopy data calculated on the MILC asqtad $N_f = 2+1$ lattices using $SU(3)\times U(1)$ gauge fields (the $U(1)$ are independently generated – "sea charges are quenched").
- Bijnens & Danielsson (Phys.Rev.D75, 014505 (2007)) showed that the EM contribution to the square of a pseudoscalar masses can be completely calculated at NLO even with quenched photons.
- For example, in $M^2_{K^\pm} - M^2_{K^0}$, all terms with LECs multiplying sea-quark charges cancel, provided the valence quark masses are the same. The remaining sea-quark charge dependence is in the chiral logs, which are completely calculable at NLO.
- We define the neutral pion on the lattice $'\pi^{0\prime}$ to have the mass:

$$m^2_{'\pi^{0\prime}} = (m^2_{u\bar{u}} + m^2_{d\bar{d}})/2,$$

where $u\bar{u}$ and $d\bar{d}$ are states with charges (2/3, -2/3) and a (1/3, -1/3). I.e., there are no disconnected diagrams included, but their effect is expected to be small.

Strategy

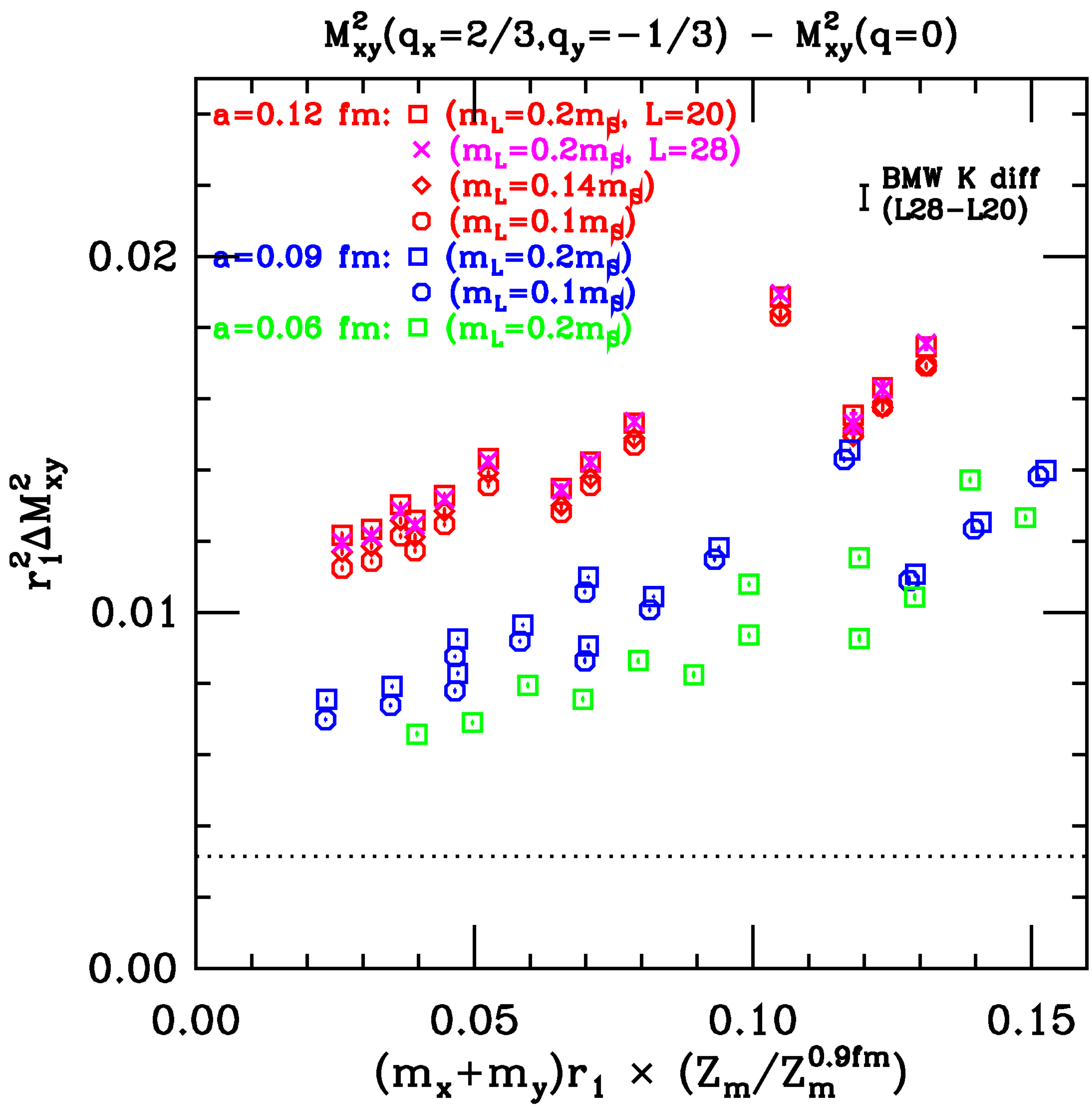
- We calculate the charged meson spectrum on the asqtad ensembles:

a[fm]	Volume	β	m_l/m_s	# configs.
0.12	$20^3 \times 64$	6.76	0.01/0.05	2254
	$28^3 \times 64$	6.76	0.01/0.05	274
	$20^3 \times 64$	6.76	0.007/0.05	1261
	$24^3 \times 64$	6.76	0.005/0.05	2099
0.09	$28^3 \times 96$	7.09	0.0062/0.031	1930
	$40^3 \times 96$	7.08	0.0031/0.031	1015
0.06	$48^3 \times 144$	7.47	0.0036/0.018	670

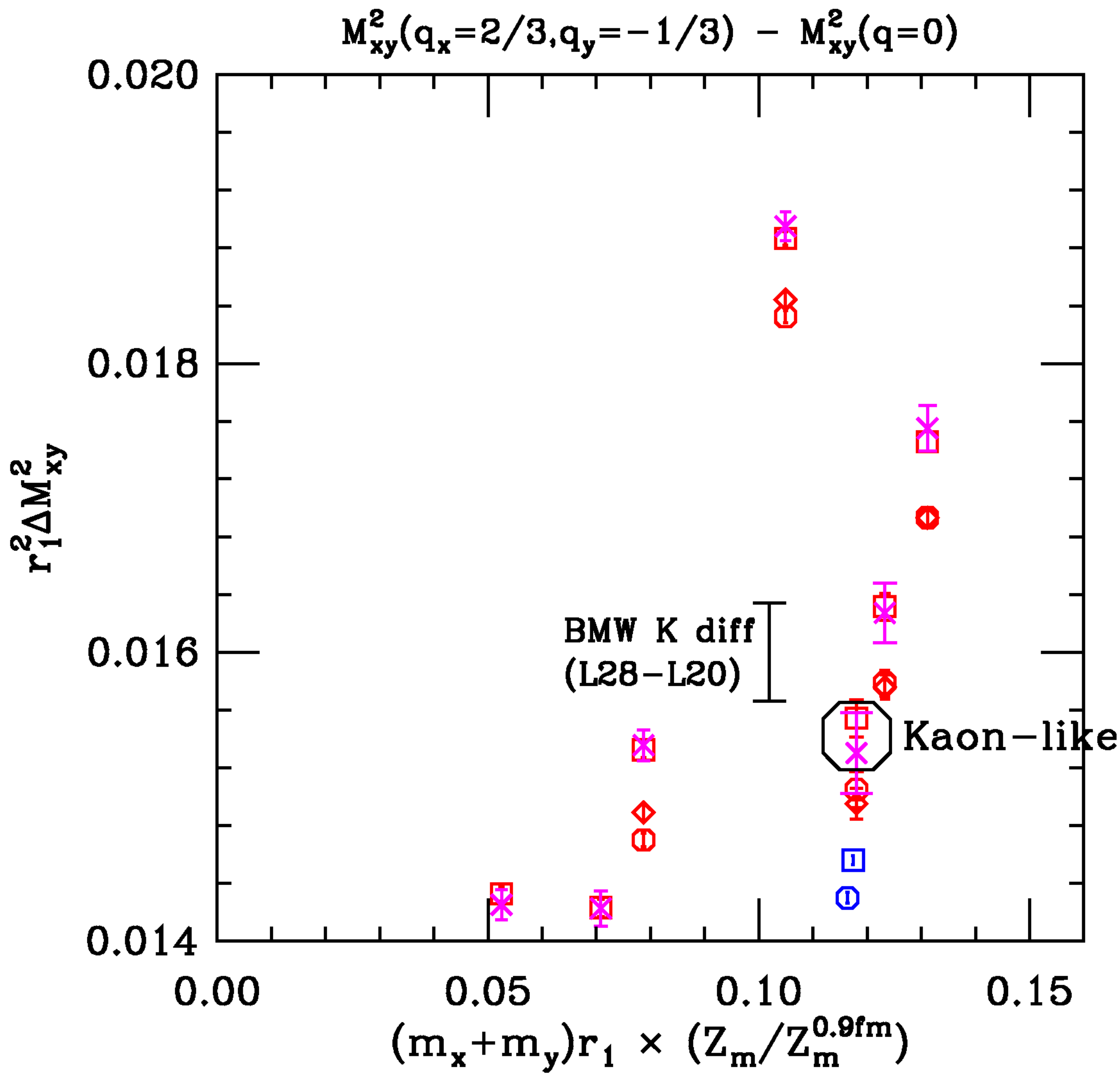
- Spectrum is measured with $e = \pm 0.909, \pm 0.606, \pm 0.303, 0,$ corresponding to $9\alpha_{phys}$, $4\alpha_{phys}$, α_{phys} and 0.

- Goal: To fit the data using rSχ-perturbation theory with EM effects to extract the NLO corrections to Dashen theorem. One of the applications is to reduce the electro-magnetic uncertainty in m_u/m_d which is currently dominant.

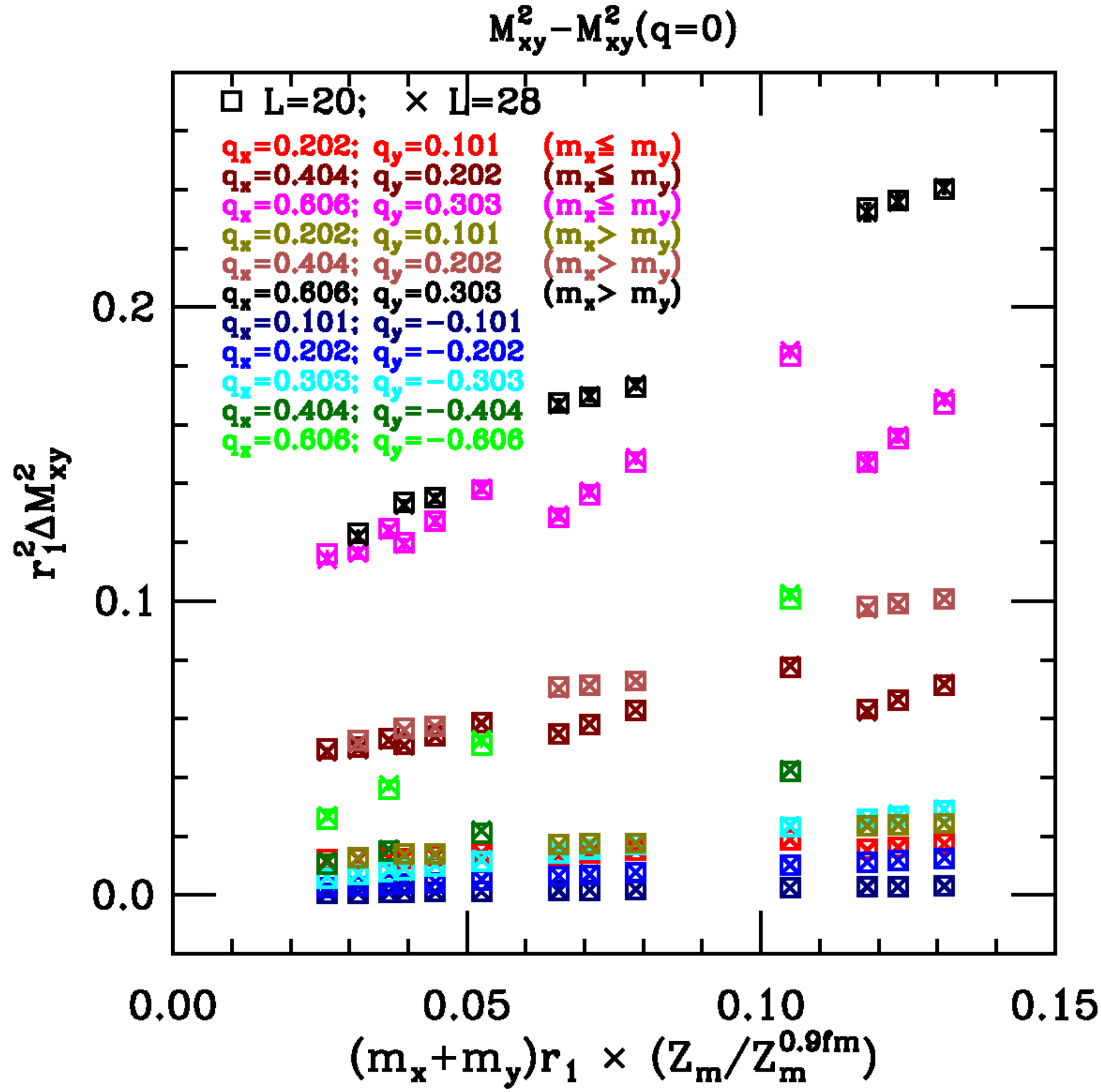
Pseudoscalar EM splittings



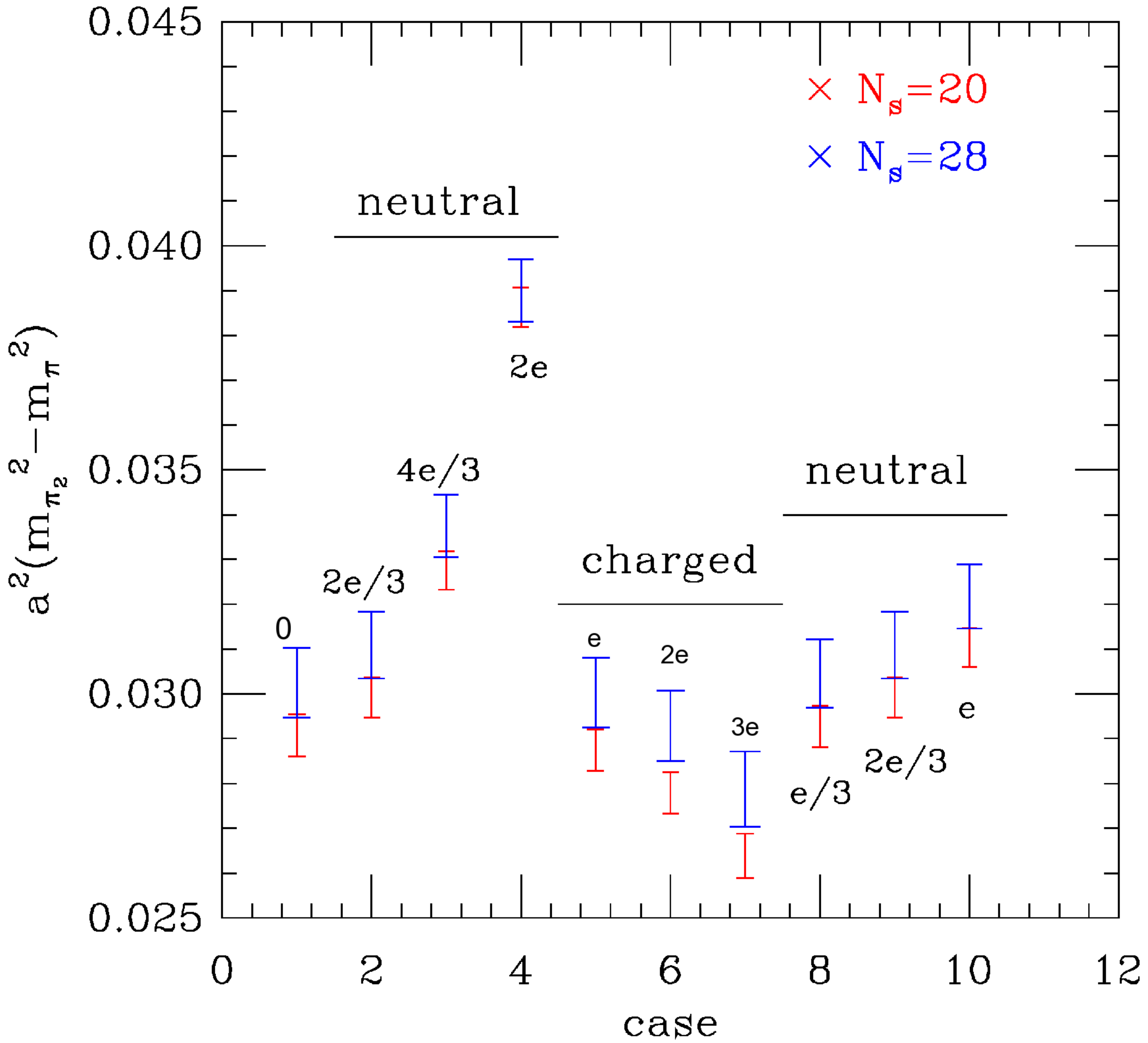
Kaon EM splittings



Finite volume effects at coarse lattice spacing



Pion taste splitting



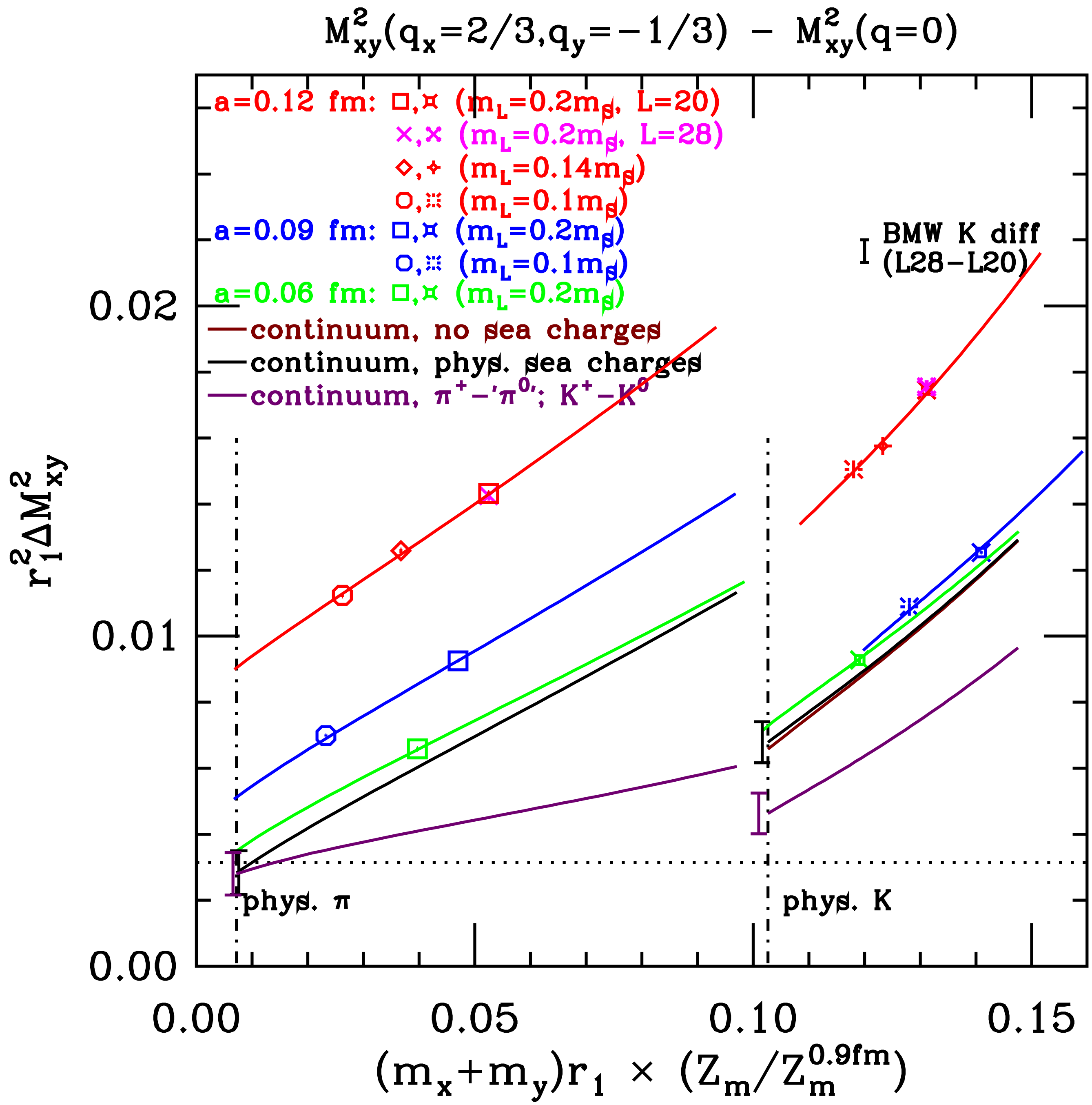
Fitting strategy

- We fit the pseudoscalar meson EM splittings using NLO rSχ-perturbation theory with NNLO analytic terms also included.

- Correlated fits to data do not work well, need data thinning/larger statistics. Large charges ($e > 0.303$) also do not work well (effects on taste splittings).

- Uncorrelated fits fare better; they mostly pass through all the points, when charges > 0.303 are removed.

Chiral fits



$M^2_{xy}(q_x=2/3, q_y=-1/3) - M^2_{xy}(q=0)$
a=0.12 fm: (m_L=0.2m_s, L=20)
(m_L=0.2m_s, L=28)
(m_L=0.14m_s)
(m_L=0.1m_s)
a=0.09 fm: (m_L=0.2m_s)
(m_L=0.1m_s)
a=0.06 fm: (m_L=0.2m_s)
continuum, no sea charges
continuum, phys. sea charges
continuum, π+−'π0'; K+−K0
BMW K diff
(L28−L20)
phys. π
phys. K
0.00
0.01
0.02
0.00
0.05
0.10
0.15
$r_1^2 \Delta M^2_{xy}$
$(m_x+m_y)r_1 \times (Z_m/Z_m^{0.9fm})$

Results and conclusions

- Based on the fits we determine the NLO correction to the Dashen theorem in the continuum limit and physical quark mass limit to be:

$$\Delta_{EM} = 0.65(17).$$

where the error is statistical only. The systematic effects are under investigation; currently they appear to be about 0.15 to 0.20 in size.
Compare with other recent lattice results for Δ_{EM}:

  - $0.60(14)$[statistics only], Portelli et al. (2010), arXiv:1011.4189
  - $0.628(59)$[statistics only], Blum et al. (2010), arXiv:1006.1311
  - $0.70(4)(8)(?)$, Portelli et al. (2012), arXiv:1201.2787

- This leads to the (very) preliminary result (more details in D. Toussaint talk):

	m_u/m_d
central value	0.508
statistics	0.010
excited states	0.003
finite size	0.0005
scale setting	0.001
a^2 extrapol	0.010
EM	0.022

Results and conclusions contd.

- We do not see large finite volume effects in the EM splittings calculated on the coarse ($a = 0.12$ fm) ensembles with $L_s = 20$ and 28. Effects are smaller than, but possibly consistent with, those seen by BMW collaboration. More statistics should help clarify this issue.

- The taste splitting between the Goldstone pion and the second local pion grows with the quark charge for the neutral pions. It appears that for larger charges than the physical, the quark taste changing by high momentum photons starts to become important.

- Currently we are accumulating more statistics with the intention of quadrupling them for the coarse $L_s = 28$ ensemble. Also measurements are in progress for a superfine ensemble with $m_l/m_s = 0.0025/0.018$.