Isospin Violation in the Light Hadron Spectrum

Lattice 2012
Cairns Australia

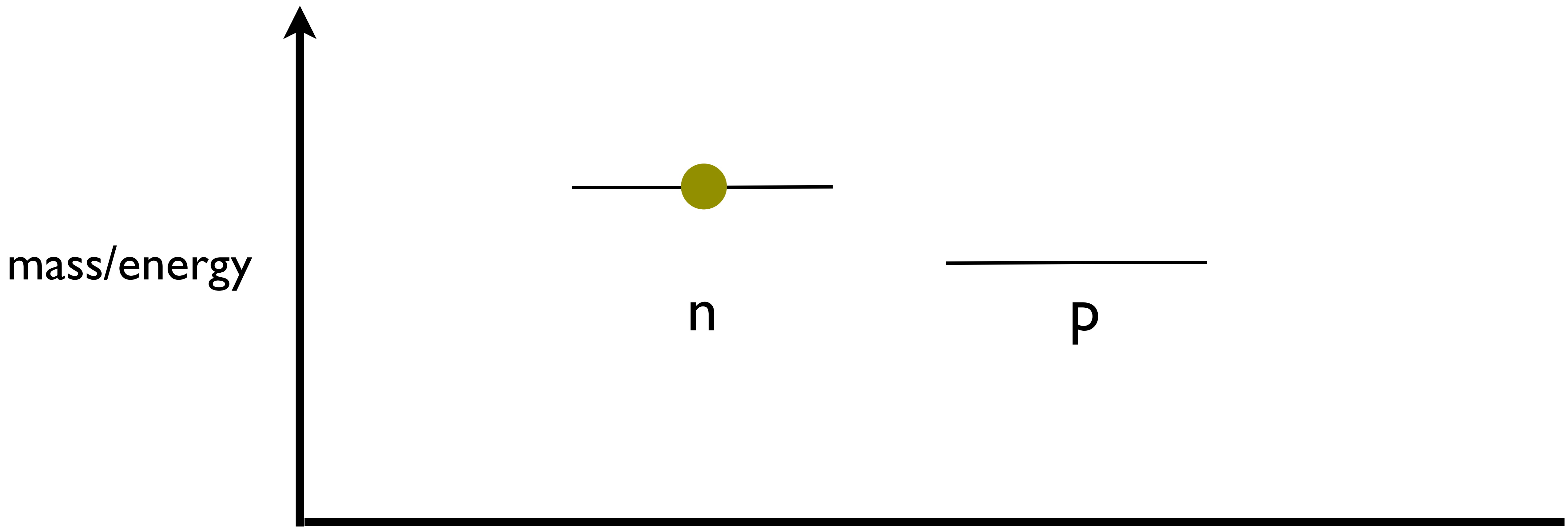
André Walker-Loud

Big Bang Nucleosynthesis

$$T \simeq 1 \text{ trillion K} \to 1 \text{ billion K}$$

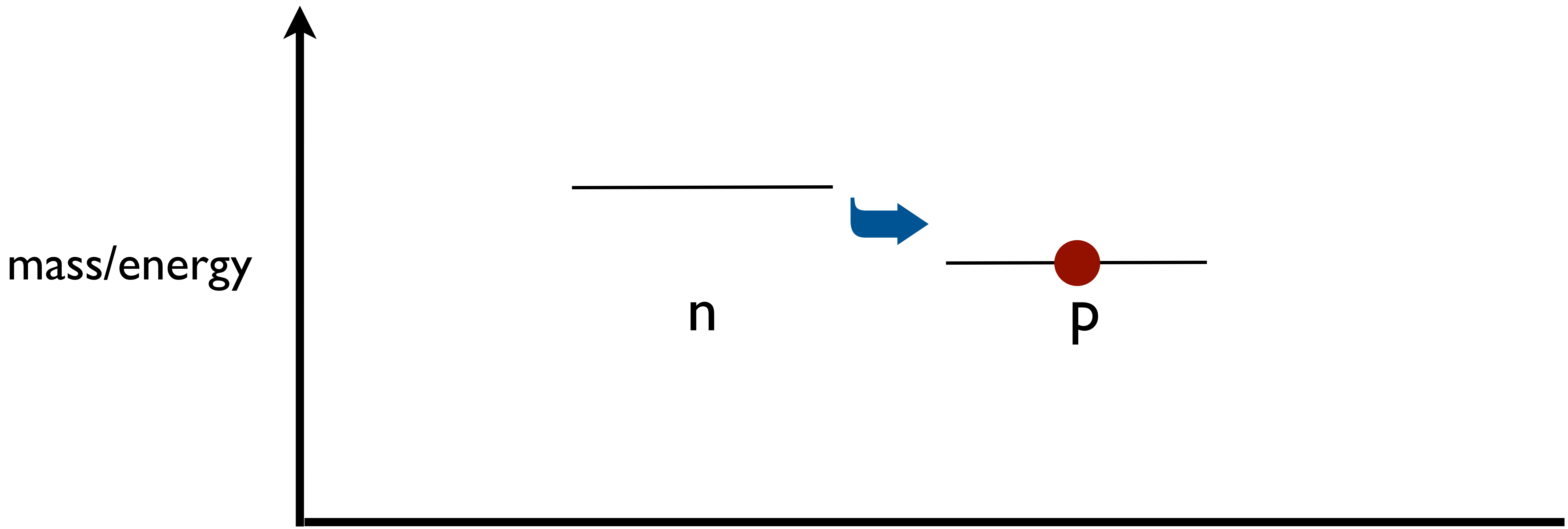
$$t \simeq 3 \times 10^{-5} s \to 3 \text{ min}$$

when systems cool, they settle into the lowest energy state



$$\tau_n \sim 10 \text{ min}$$

when systems cool, they settle into the lowest energy state

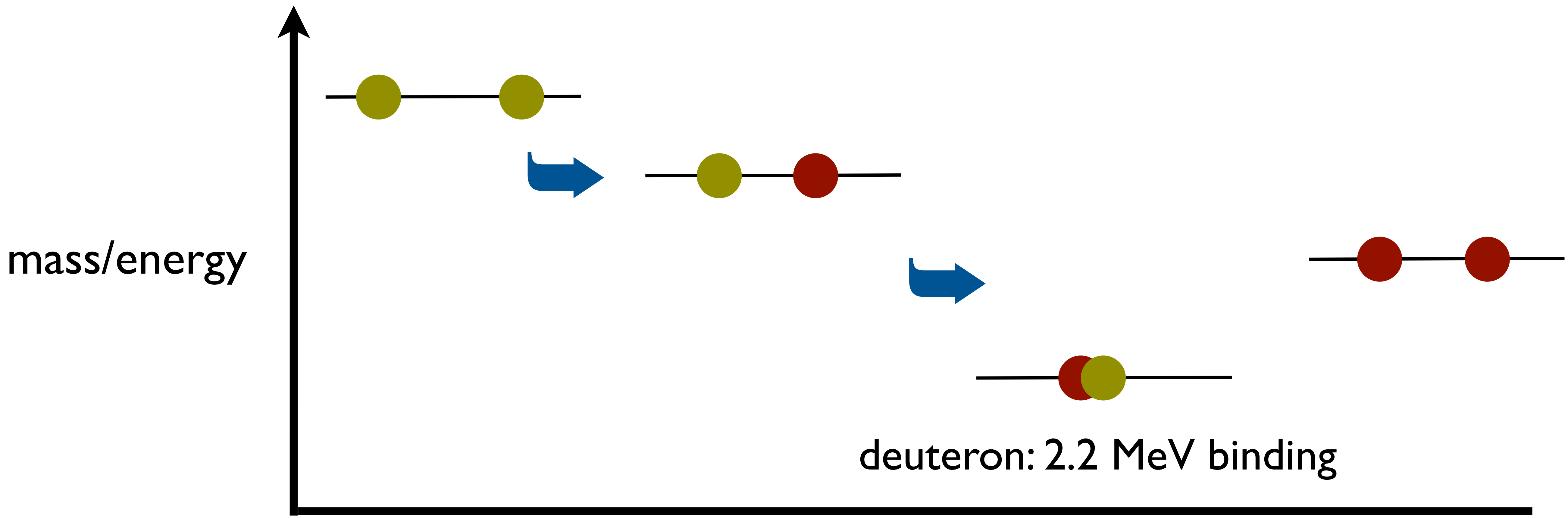


$$\tau_n \sim 10 \text{ min}$$

what prevented this from destroying all the neutrons?

if nothing else were to happen in the next few minutes, our universe would be full of only H

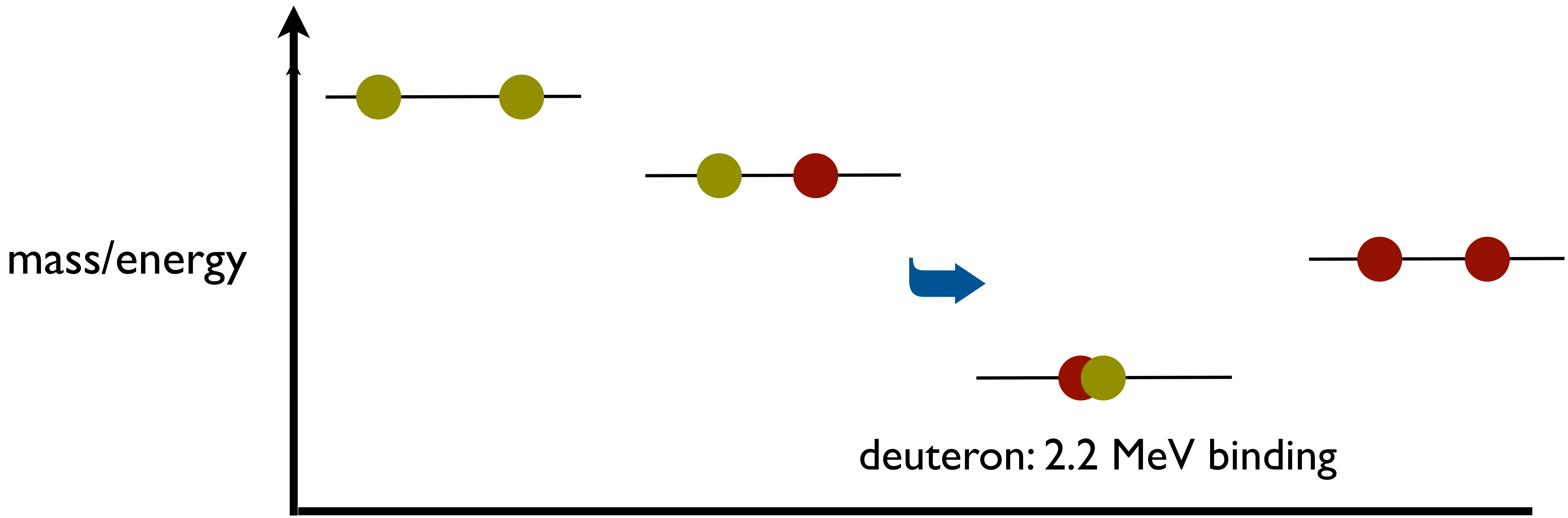
when systems cool, they settle into the lowest energy state

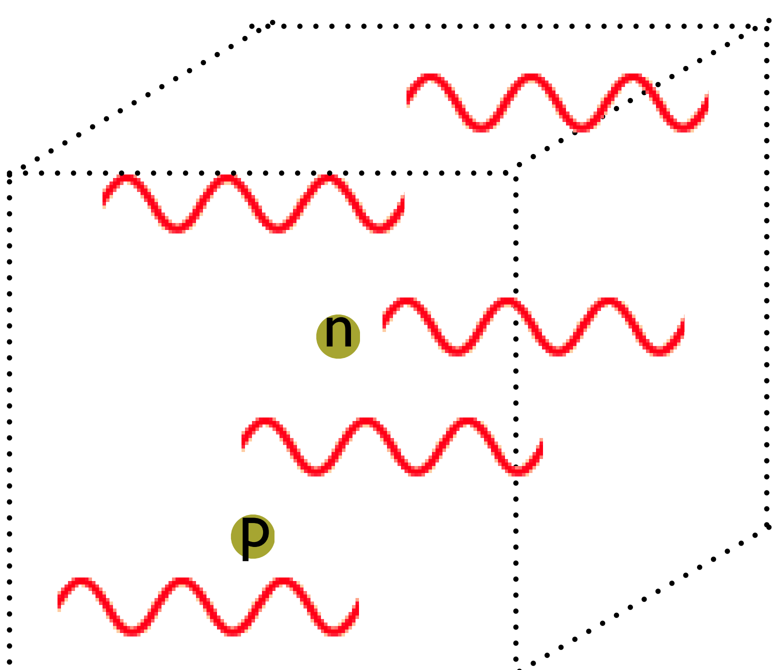


Answer: formation of nuclei

a system with protons and neutrons can collapse to a compact bound state, the deuteron: the attractive binding of a neutron and proton allows neutrons to survive when embedded in nuclei

The deuterium “bottleneck”



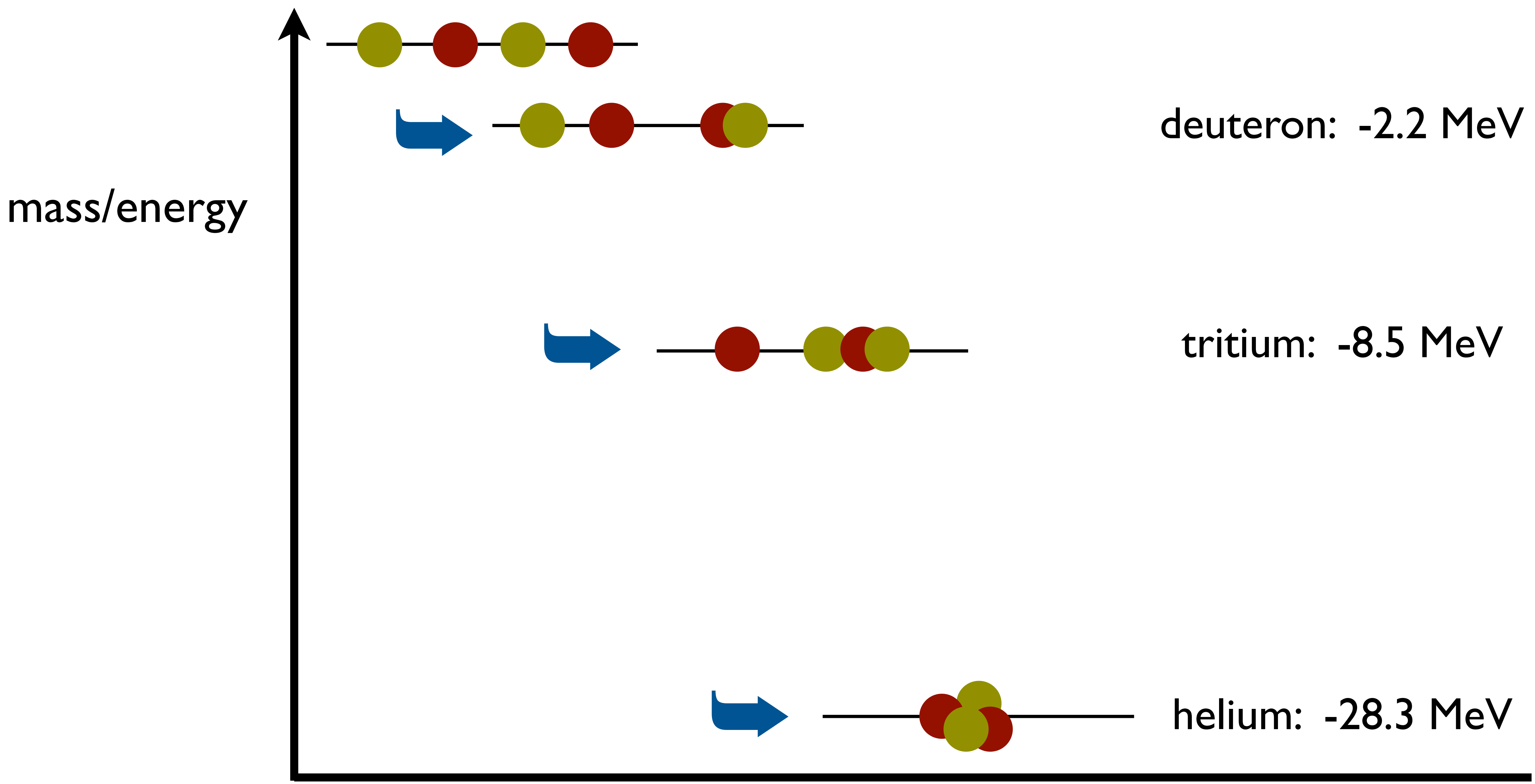


$$p + n \longleftrightarrow d + \gamma$$

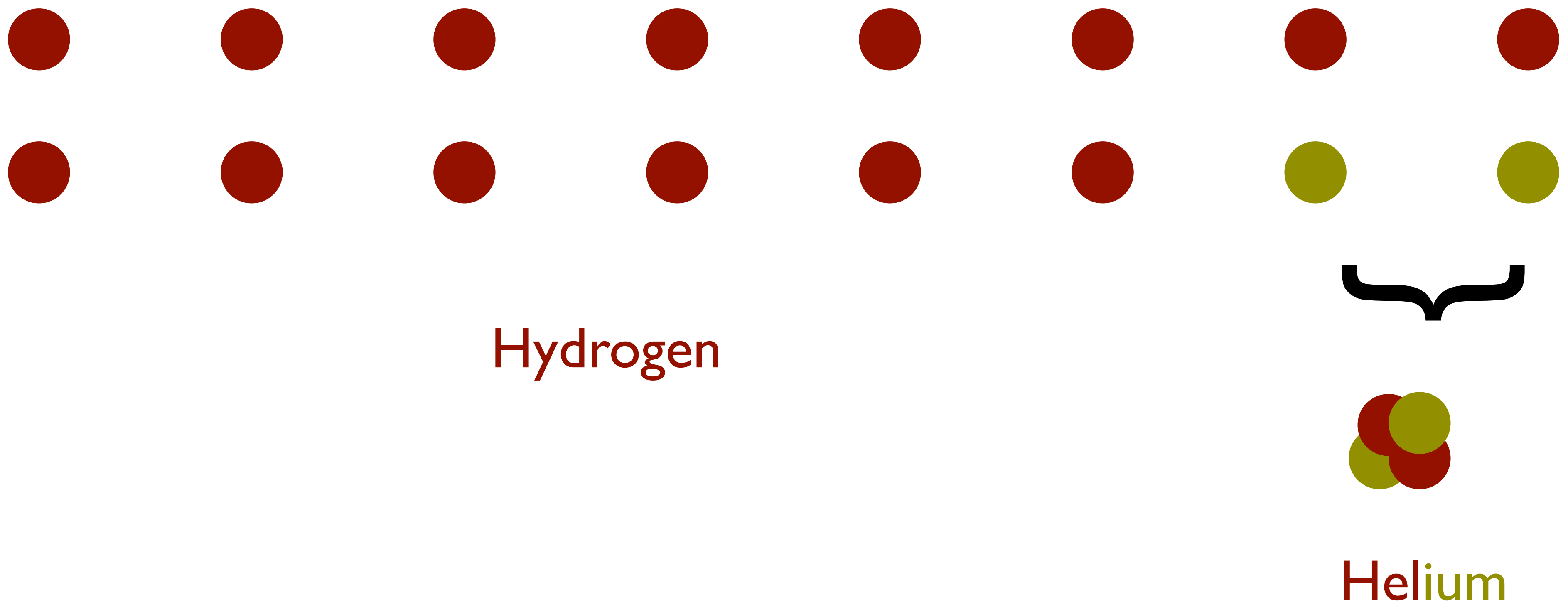
until T ≃ 100 keV (1 billion K), t ≃ 3 min

$$p + n \longrightarrow d + \gamma$$

The deuterium "bottleneck" is broken, neutrons flow into He



He stability: ↑,↓ protons and ↑,↓ neutrons can be packed together



The early universe contains 75% H and 25% He by mass fraction this picture very sensitive to binding energy of deuterium which is finely tuned (most nuclei have ~8 MeV binding per nucleon)!

$$B_d = 2.22 \texttt{ MeV}$$

What if

$B_d \ll 2.22$ MeV — more finely tuned

all neutrons decay - no helium

mostly hydrogen stars?

$B_d \gg 2.22$ MeV — natural scenario

all neutrons captured in deuterium and helium - no hydrogen

no stars like ours!

this picture very sensitive to binding energy of deuterium which is finely tuned (most nuclei have ~8 MeV binding per nucleon)!

$$B_d = 2.22\ \texttt{MeV}$$

What if

$B_d \ll 2.22\ \mathrm{MeV}$ more finely tuned

all neutrons decay - no helium

mostly hydrogen stars?

$B_d \gg 2.22\ \mathrm{MeV}$ natural scenario

all neutrons captured in deuterium and helium - no hydrogen

no stars like ours!

(also very sensitive to $m_n - m_p \propto \begin{cases} m_d - m_u \\ e^2/4\pi \end{cases}$)

we want to understand this from QCD

this picture most sensitive to neutron proton mass splitting

primordial ratio

$$\frac{X_n}{X_p} = e^{-(m_n - m_p)/kT}$$

$$m_n - m_p = 1.29333217(42)\ \text{MeV}$$

$$m_n - m_p = \delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}$$

this picture most sensitive to neutron proton mass splitting

primordial ratio

$$\frac{X_n}{X_p} = e^{-(m_n - m_p)/kT}$$

$$m_n - m_p = 1.29333217(42)\ \text{MeV}$$

$$m_n - m_p = \underbrace{\delta M^{\gamma}_{n-p} + \delta M^{m_d - m_u}_{n-p}}$$

this separation only
at LO in isospin breaking

$\langle N|(m_d - m_u)\bar{q}q|N\rangle$ needed to renormalize EM self-energy

OUTLINE

- Electromagnetic self-energy corrections

  - self-energy related to forward Compton scattering
  - in principle, allows for robust, model independent determination of self-energy through dispersion theory
  - two challenges in realizing this method

    requires subtracted dispersion integral

    → unknown subtraction function

    requires renormalization

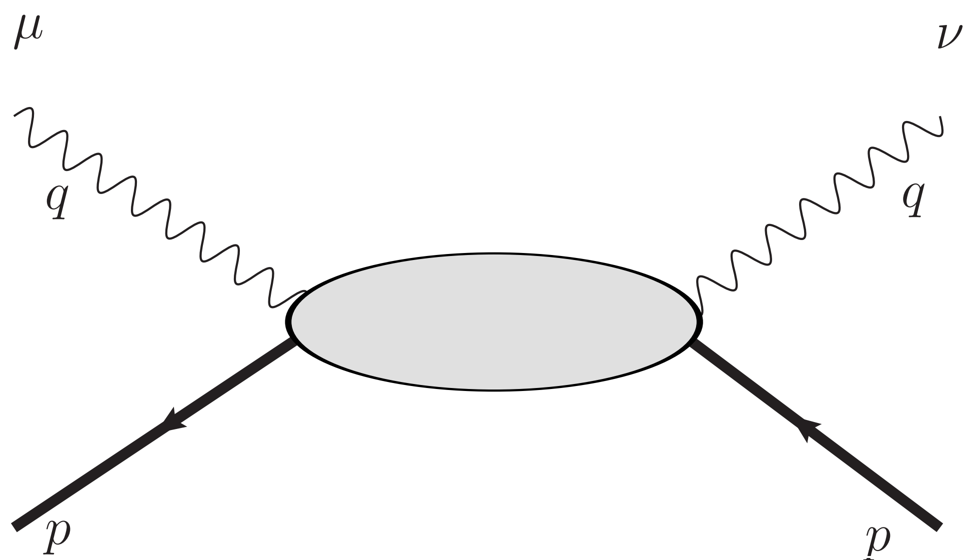
AWL, Carl Carlson, Jerry Miller: PRL 108 (2012)

Cottingham's Formula

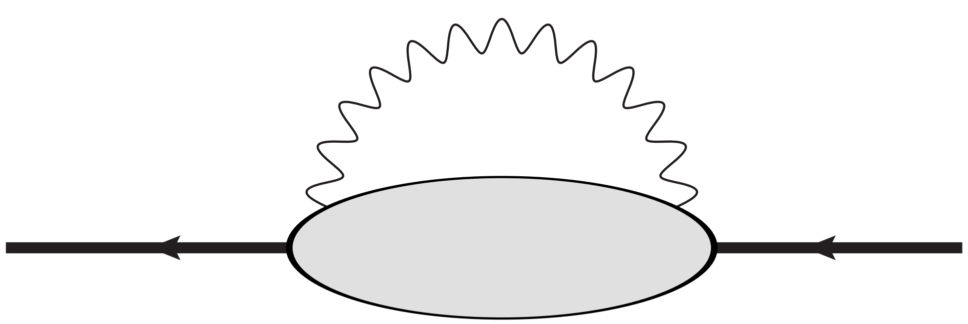
AWL, C.Carlson, G.Miller: PRL 108 (2012)

Cini, Ferrari, Gato: PRL 2 (1959)

Cottingham: Annals Phys 25 (1963)



$$T_{\mu\nu} = \frac{i}{2}\sum_\sigma \int d^4\xi \; e^{iq\cdot\xi} \langle p\sigma | T\left\{J_\mu(\xi) J_\nu(0)\right\} | p\sigma\rangle$$



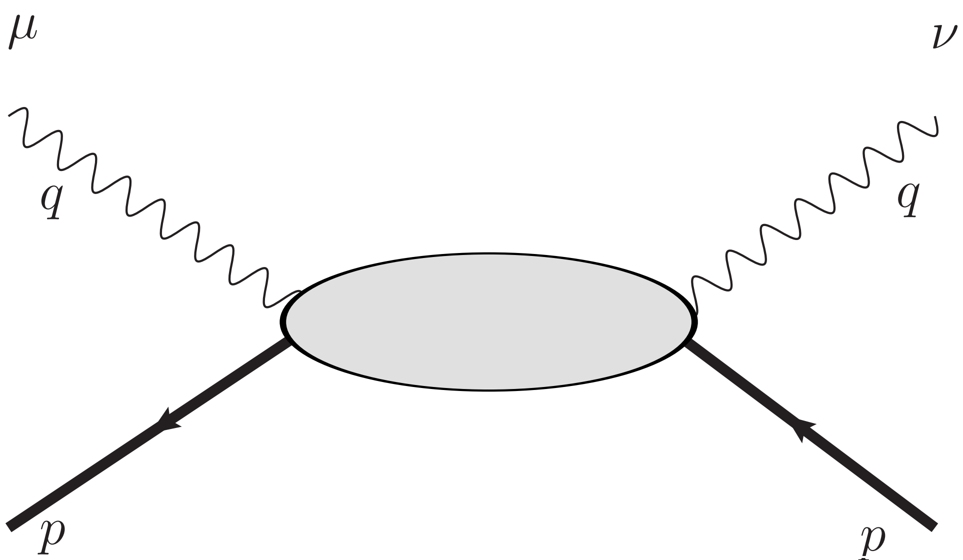
$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q \frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

$$\alpha = \frac{e^2}{4\pi}$$

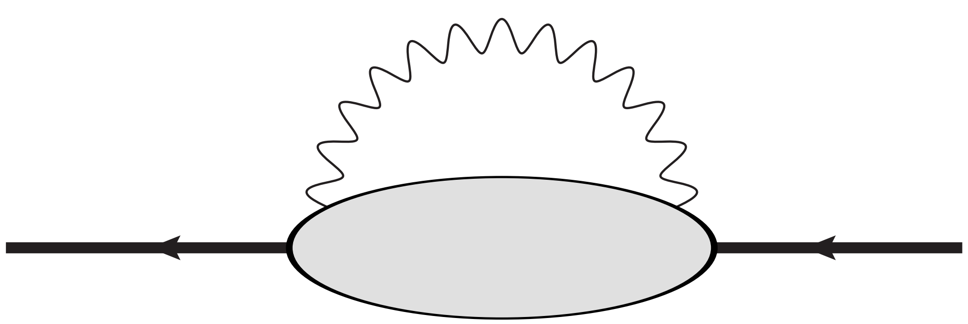
Cottingham's Formula

Cini, Ferrari, Gato: PRL 2 (1959)

Cottingham: Annals Phys 25 (1963)



$$T_{\mu\nu} = \frac{i}{2}\sum_{\sigma}\int d^4\xi\; e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$



$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q\,\frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

$$\alpha = \frac{e^2}{4\pi}$$

Integral diverges and must be renormalized

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$\delta M^\gamma = \frac{i}{2M}\frac{\alpha}{(2\pi)^3}\int_R d^4q \frac{T^\mu_\mu(p,q)}{q^2+i\epsilon}$$

- Wick rotate $q^0 \to i\nu$
- variable transform $Q^2 = \mathbf{q}^2 + \nu^2$

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

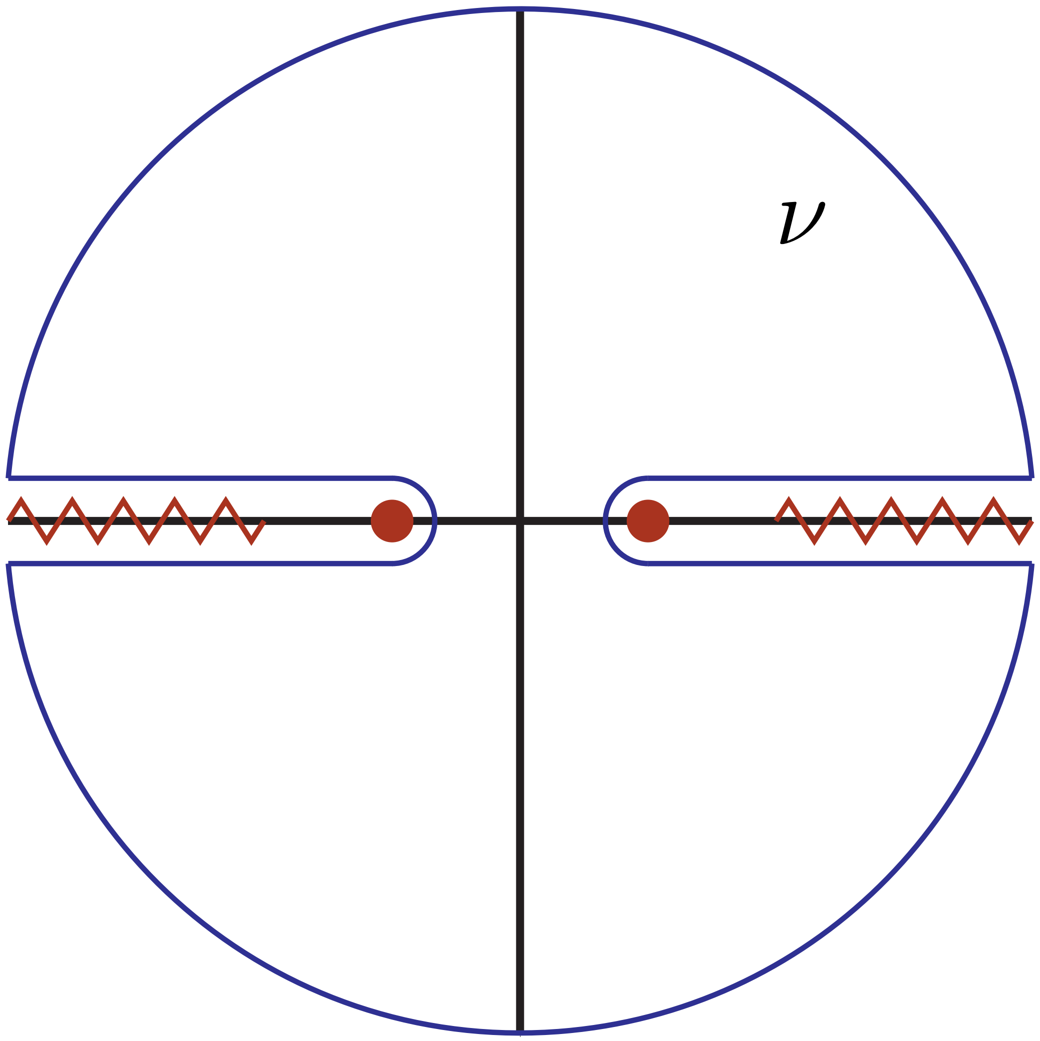
$$T^\mu_\mu = -3\,T_1(i\nu,Q^2) + \left(1-\frac{\nu^2}{Q^2}\right)T_2(i\nu,Q^2)\,, \quad (7a)$$

$$= -3Q^2\,t_1(i\nu,Q^2) + \left(1+2\frac{\nu^2}{Q^2}\right)Q^2 t_2(i\nu,Q^2)\,. \quad (7b)$$

use dispersion integrals to evaluate scalar functions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



$$T_i(\nu, Q^2) = \frac{1}{2\pi} \oint d\nu' \frac{T_i(\nu', Q^2)}{\nu' - \nu}$$

Crossing Symmetric

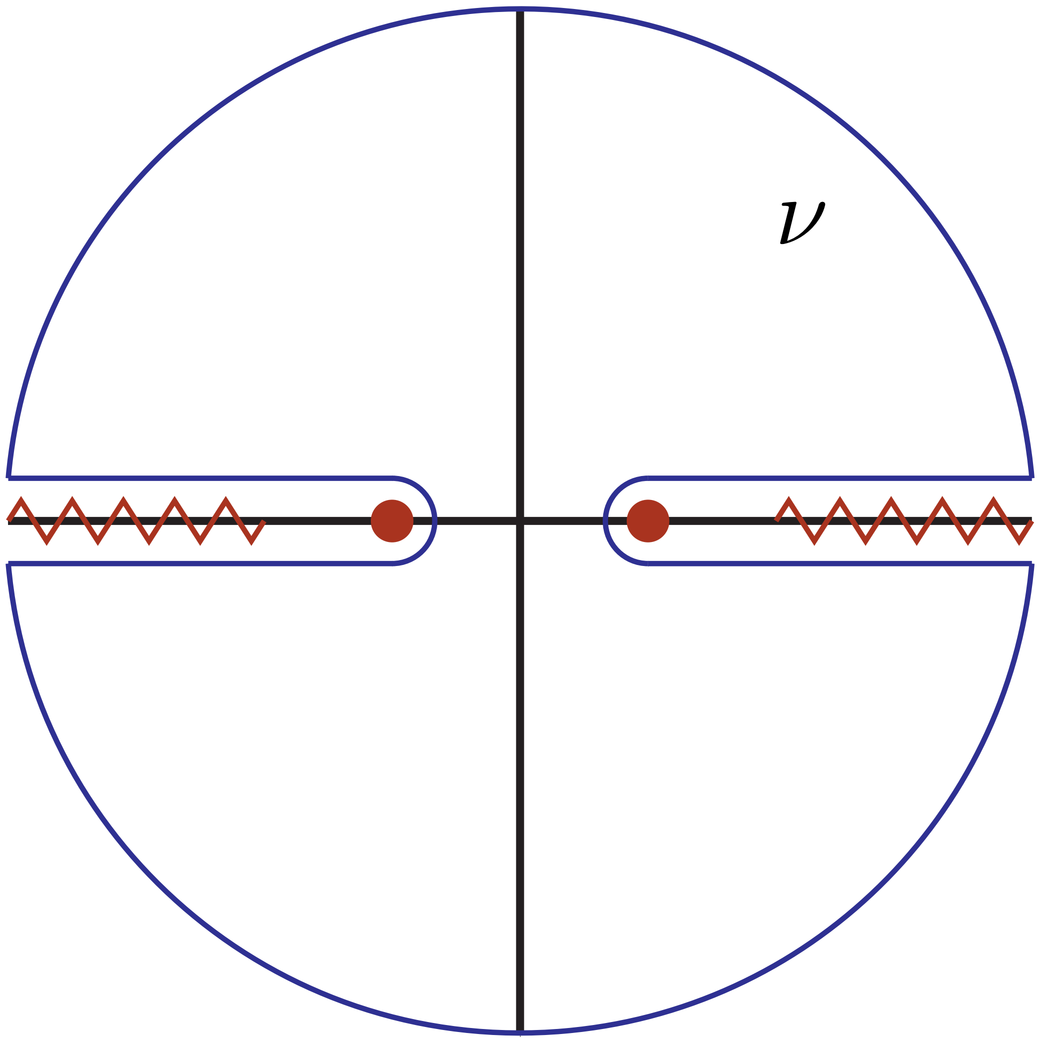
$$T_i(\nu, Q^2) = T_i(-\nu, Q^2)$$

$$T_i(\nu, Q^2) = \frac{1}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{(\nu')^2 - \nu^2} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2)$$

(provided contour and infinity vanishes)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



if contour at infinity does not vanish

subtracted dispersion integral

$$g(\nu) = \frac{T_i(\nu, Q^2)}{\nu^2}$$

introduces new pole at $\nu = 0$
which you need to subtract

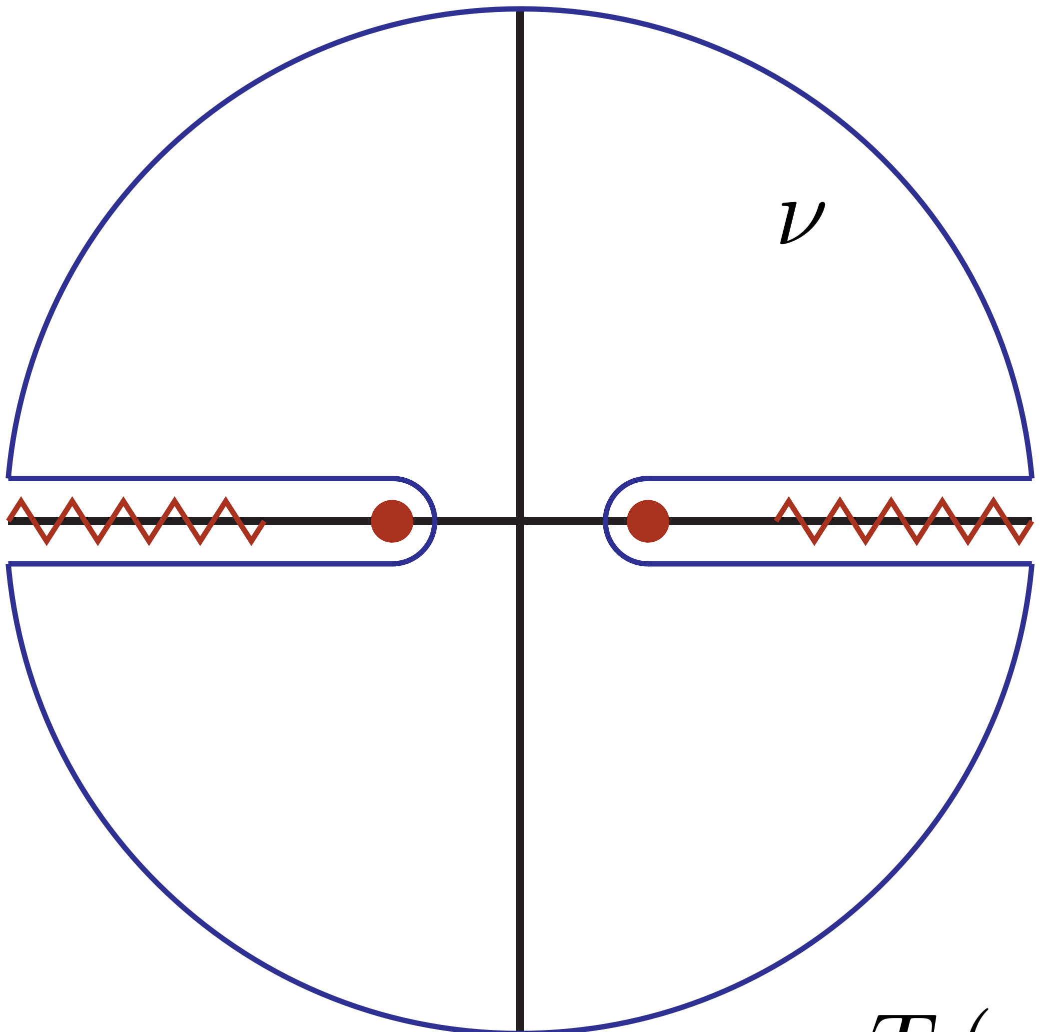
$$T_i(\nu, Q^2) = \frac{\nu^2}{2\pi} \int_{\nu_t}^{\infty} d\nu' \frac{2\nu'}{\nu'^2(\nu'^2 - \nu^2)} 2\mathrm{Im} T_i(\nu' + i\epsilon, Q^2) + T_i(0, Q^2)$$

measured experimentally

unknown function

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



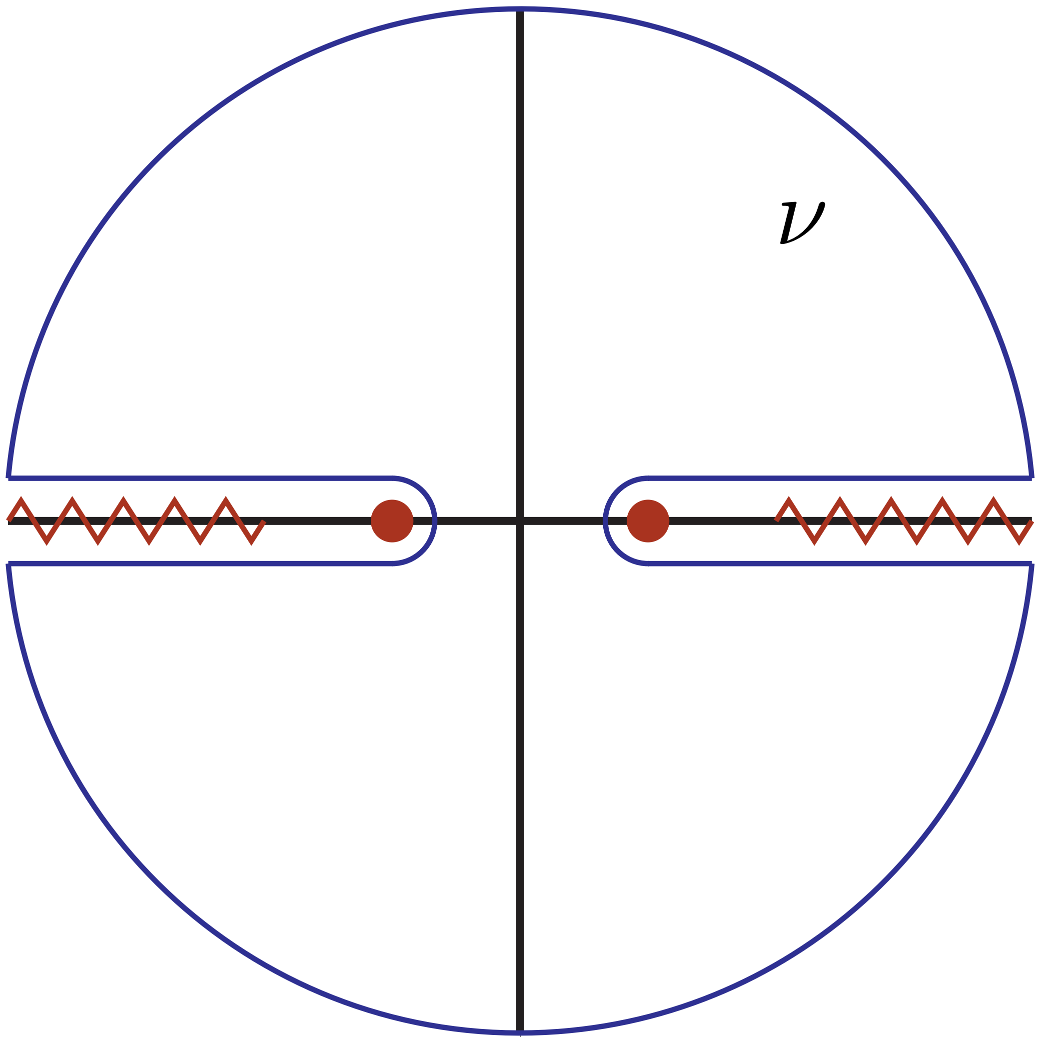
also - if there is a fixed pole in the function, one needs a subtracted dispersion integral

$$T_i(\nu, Q^2) = \frac{f(\nu, Q^2)}{(Q^2 + i\epsilon)^2 - 4M^2\nu^2} + g(Q^2)$$

this term also gives a problem at infinity

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)



It is known that

$$T_2(\nu, Q^2) \quad [t_2(\nu, Q^2)]$$

satisfies unsubtracted dispersion integral while

$$T_1(\nu, Q^2) \quad [t_1(\nu, Q^2)]$$

requires a subtraction

Regge behavior

H. Harari: PRL 17 (1966)

H.D. Abarbanel S. Nussinov: Phys.Rev. 158 (1967)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

J. Gasser and H. Leutwyler: Nucl Phys B94 (1975)

Claimed that the elastic contributions could be evaluated without subtracted dispersive integral

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T_\mu^\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T_\mu^\mu = -3\,T_1(i\nu,Q^2) + \left(1-\frac{\nu^2}{Q^2}\right) T_2(i\nu,Q^2)\,, \qquad (7a)$$

$$= -3Q^2\,t_1(i\nu,Q^2) + \left(1+2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu,Q^2)\,. \quad (7b)$$

$$\delta M^\gamma_{p-n} = 0.76(30)\ \text{MeV}$$

central value: from elastic contribution

uncertainty: estimates of inelastic structure contributions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2}\sum_{\sigma}\int d^4\xi\; e^{iq\cdot\xi}\langle p\sigma|T\left\{J_{\mu}(\xi)J_{\nu}(0)\right\}|p\sigma\rangle$$

Insert complete set of states:

isolate elastic contributions

$$1 = \sum_{\Gamma}|\Gamma\rangle\langle\Gamma|$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2} \sum_{\sigma} \int d^4\xi \; e^{iq\cdot\xi} \langle p\sigma | T\left\{ J_\mu(\xi) J_\nu(0) \right\} | p\sigma \rangle$$

Insert complete set of states:

isolate elastic contributions

$$1 = \sum_{\Gamma} |\Gamma\rangle\langle\Gamma|$$

$$\delta M^{el}_{unsub,a} = \frac{\alpha}{\pi} \int_0^{\Lambda^2} dQ \left\{ \left[G_E^2(Q^2) - 2\tau_{el} G_M^2(Q^2) \right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}}{1+\tau_{el}} - \frac{3}{2} G_M^2(Q^2) \frac{{\tau_{el}}^{3/2}}{1+\tau_{el}} \right\}, \quad (8a)$$

$$\delta M^{el}_{unsub,b} = \frac{\alpha}{\pi} \int_0^{\Lambda^2} dQ \left\{ \left[G_E^2(Q^2) - 2\tau_{el}\, G_M^2(Q^2) \right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2}}{1+\tau_{el}} + 3 G_M^2(Q^2) \frac{{\tau_{el}}^{3/2}}{1+\tau_{el}} \right\}, \quad (8b)$$

$$\delta M^{\gamma} = \frac{\alpha}{8\pi^2} \int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2 - \nu^2}}{Q^2} \frac{T^{\mu}_{\mu}}{M} + \delta M^{ct}(\Lambda)$$

$$T^{\mu}_{\mu} = -3\, T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2} \right) T_2(i\nu, Q^2)\,, \quad (7a)$$

$$= -3Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2} \right) Q^2 t_2(i\nu, Q^2)\,. \quad (7b)$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2}\sum_\sigma \int d^4\xi\ e^{iq\cdot\xi}\langle p\sigma|T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$

Insert complete set of states:

isolate elastic contributions

$$1 = \sum_\Gamma |\Gamma\rangle\langle\Gamma|$$

$$\delta M^{el}_{unsub,a} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ\left\{ \left[G_E^2(Q^2) - 2\tau_{el} G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}}{1+\tau_{el}} - \frac{3}{2}G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}}\right\}, \quad (8a)$$

$$\delta M^{el}_{unsub,b} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ\left\{ \left[G_E^2(Q^2) - 2\tau_{el}\, G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2}}{1+\tau_{el}} + 3G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}}\right\}, \quad (8b)$$

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T^\mu_\mu = -3\,T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \quad (7a)$$

$$= -3Q^2\,t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \quad (7b)$$

One must use a subtracted dispersive integral even for elastic terms

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$T_{\mu\nu} = \frac{i}{2}\sum_\sigma \int d^4\xi \; e^{iq\cdot\xi}\langle p\sigma | T\left\{J_\mu(\xi)J_\nu(0)\right\}|p\sigma\rangle$$

Insert complete set of states:

isolate elastic contributions

$$1 = \sum_\Gamma |\Gamma\rangle\langle\Gamma|$$

$$\delta M^{el}_{unsub,a} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ \left\{ \left[G_E^2(Q^2) - 2\tau_{el} G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}}}{1+\tau_{el}} - \frac{3}{2} G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}} \right\}, \quad (8a)$$

$$\delta M^{el}_{unsub,b} = \frac{\alpha}{\pi}\int_0^{\Lambda^2} dQ \left\{ \left[G_E^2(Q^2) - 2\tau_{el}\, G_M^2(Q^2)\right] \frac{(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2}}{1+\tau_{el}} + 3 G_M^2(Q^2)\frac{{\tau_{el}}^{3/2}}{1+\tau_{el}} \right\}, \quad (8b)$$

typically quoted as elastic Cottingham

$$\delta M^\gamma = \frac{\alpha}{8\pi^2}\int_0^{\Lambda^2} dQ^2 \int_{-Q}^{+Q} d\nu \frac{\sqrt{Q^2-\nu^2}}{Q^2}\frac{T^\mu_\mu}{M} + \delta M^{ct}(\Lambda)$$

$$T^\mu_\mu = -3\,T_1(i\nu, Q^2) + \left(1 - \frac{\nu^2}{Q^2}\right) T_2(i\nu, Q^2)\,, \quad (7a)$$

$$= -3Q^2\, t_1(i\nu, Q^2) + \left(1 + 2\frac{\nu^2}{Q^2}\right) Q^2 t_2(i\nu, Q^2)\,. \quad (7b)$$

One must use a subtracted dispersive integral even for elastic terms

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^{\gamma} = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta \tilde{M}^{ct}$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

perform once subtracted dispersion integral for $T_1(t_1)$
and unsubtracted dispersion integral for $T_2(t_2)$

$$\delta M^\gamma = \delta M^{el} + \delta M^{inel} + \delta M^{sub} + \delta \tilde{M}^{ct}$$

$$\delta M^{el} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - \tau_{el}{}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{inel} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu,Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu,Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\},$$

$$\tau_{el} = \frac{Q^2}{4M^2} \qquad \tau = \frac{\nu^2}{Q^2}$$

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M} \int_0^{\Lambda_0^2} dQ^2 \; T_1(0, Q^2)\,,$$

$$\delta \tilde{M}^{ct} = -\frac{3\alpha}{16\pi M} \int_{\Lambda_0^2}^{\Lambda_1^2} dQ^2 \sum_i C_{1,i} \langle \mathcal{O}^{i,0} \rangle\,,$$

OPE: operators and Wilson coeffic.
J.C. Collins: Nucl. Phys. B149 (1979)

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

elastic contribution: use well measured form factors

$$\delta M^{el} = \frac{\alpha}{\pi}\int_0^{\Lambda_0^2} dQ \left\{ \frac{3\sqrt{\tau_{el}} G_M^2}{2(1+\tau_{el})} + \frac{\left[G_E^2 - 2\tau_{el}\, G_M^2\right]}{1+\tau_{el}} \left[(1+\tau_{el})^{3/2} - {\tau_{el}}^{3/2} - \frac{3}{2}\sqrt{\tau_{el}} \right] \right\}$$

$$\delta M^{el}\Big|_{p-n} = 1.39(02)\ \mathrm{MeV}$$

- insensitive to value of Λ_0 since form factors fall as $1/Q^4$
- uncertainty from Monte Carlo evaluation of parameters describing form factors

central values: $\Lambda_0^2 = 2\ \mathrm{GeV}^2$

uncertainties: $1.5\ \mathrm{GeV}^2 \le \Lambda_0^2 \le 2.5\ \mathrm{GeV}^2$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

inelastic terms: use modern knowledge of structure functions to improve determination of inelastic contributions

$$\delta M^{inel} = \frac{\alpha}{\pi} \int_0^{\Lambda_0^2} \frac{dQ^2}{2Q} \int_{\nu_{th}}^{\infty} d\nu \left\{ \frac{3F_1(\nu, Q^2)}{M} \left[\frac{\tau^{3/2} - \tau\sqrt{1+\tau} + \sqrt{\tau}/2}{\tau} \right] + \frac{F_2(\nu, Q^2)}{\nu} \left[(1+\tau)^{3/2} - \tau^{3/2} - \frac{3}{2}\sqrt{\tau} \right] \right\},$$

$$\delta M^{inel}\Big|_{p-n} = 0.057(16)\ \mathrm{MeV}$$

- contributions from two regions:
  - resonance region
  - scaling region
- uncertainty dominated by choice of transition between two regions

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

renormalization: complicated story (no time)

quark mass operator renormalizes EM self-energy: can not cleanly separate these two contributions (but mixing is higher order in isospin breaking)

summary: (J.C. Collins) with Naive Dimensional Analysis and suitable renormalization (dim. reg.) one can show the contribution from the operator is numerically second order in isospin breaking

$$\delta \tilde{M}^{ct}_{p-n} = 3\alpha \ln\left(\frac{\Lambda_0^2}{\Lambda_1^2}\right) \frac{e_u^2 m_u - e_d^2 m_d}{8\pi M \delta} \langle p|\delta(\bar{u}u - \bar{d}d)|p\rangle$$

$$\left|\delta \tilde{M}^{ct}_{p-n}\right| < 0.02 \text{ MeV}$$

$$2\delta = m_d - m_u$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\ T_1(0,Q^2)\,,$$

- low energy: constrained by effective field theory

$$T_1(0,Q^2) = 2\kappa(2+\kappa) - Q^2\left\{\frac{2}{3}\left[(1+\kappa)^2 r_M^2 - r_E^2\right] + \frac{\kappa}{M^2} - 2M\frac{\beta_M}{\alpha}\right\} + \mathcal{O}(Q^4)\,,$$

most of these contributions come from Low Energy Theorems and are "elastic" (arising from a photon striking an on-shell nucleon)

K. Pachucki: Phys. Rev. A53 (1996); A. Pineda: Phys. Rev. C67 (2003); Phys. Rev. C71 (2005); R.J. Hill, G. Paz: PRL 107 (2011); C. Carlson, M. Vanderhaeghen: Phys.Rev.A84 (2011); arXiv1109.3779; M.. Birse, J. McGovern: arXiv:1206.3030

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2\; T_1(0,Q^2)\,,$$

- high energy: OPE (perturbative QCD) constrains

$$\lim_{Q^2\to\infty} T_1(0,Q^2) \propto \frac{1}{Q^2}$$

$$T_1(0,Q^2) \simeq 2G_M^2(Q^2) - 2F_1^2(Q^2) + Q^2 2M\frac{\beta_M}{\alpha}\left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

subtraction term: most challenging part - dealing with unknown subtraction function

$$\delta M_{el}^{sub} = -\frac{3\alpha}{16\pi M}\int_0^{\Lambda_0^2} dQ^2 \left[2G_M^2 - 2F_1^2\right], \qquad \delta M_{el}^{sub}\Big|_{p-n} = -0.62 \text{ MeV}$$

$$\delta M_{inel}^{sub} = -\frac{3\beta_M}{8\pi}\int_0^{\Lambda_0^2} dQ^2 Q^2 \left(\frac{m_0^2}{m_0^2+Q^2}\right)^2$$

$$\beta_M^{p-n} = -1.0 \pm 1.0 \times 10^{-4}\ \text{fm}^3$$

H.W. Griesshammer, J.A. McGovern, D.R. Phillips, G. Feldman: Prog.Nucl.Part.Phys. (2012)

taking $m_0^2 = 0.71\ \text{GeV}^2$

$$\delta M_{inel}^{sub}\Big|_{p-n} = 0.47 \pm 0.47 \text{ MeV}$$

adding it all up:

$$\delta M^\gamma \Big|_{p-n} = 1.30(03)(47)\ \mathrm{MeV}$$

AWL, C.Carlson, G.Miller:
PRL 108 (2012)

$$= 0.76(30)\ \mathrm{MeV}$$

J. Gasser and H. Leutwyler:
Nucl Phys B94 (1975)

We reduced the uncertainty from structure by an order of magnitude! But we uncovered an oversight that dominates the uncertainty :(

Cottingham's Formula

AWL, C.Carlson, G.Miller: PRL 108 (2012)

adding it all up:

$$\delta M^{\gamma}\Big|_{p-n} = 1.30(03)(47)\ \text{MeV}$$

AWL, C.Carlson, G.Miller: PRL 108 (2012)

$$= 0.76(30)\ \text{MeV}$$

J. Gasser and H. Leutwyler: Nucl Phys B94 (1975)

expectation from experiment + lattice QCD

$$\delta M^{\gamma}\Big|_{p-n} = -1.29333217(42) + 2.53(40)\ \text{MeV}$$

$$= 1.24(40)\ \text{MeV}$$

average of 3 independent lattice results

Conclusions

- attempt to improve the old determination of nucleon iso-vector EM self-energy uncovered an oversight
  - no avoiding the subtraction (dispersion integral)
  - modeling was necessary to control uncertainty subtraction function
  - a central value was found in much better agreement with expectations from lattice QCD + experiment (but in agreement within uncertainties with G&L)
  - comparison with independent determinations of iso-vector nucleon magnetic polarizability show the modeling is not crazy
- improvements will come from three areas
  - improved measurement of β_M^{p-n}
  - lattice QCD calculation of β_M^{p-n}
  - including EM effects with lattice QCD

Back Up

Entire discussion - intimately related to recent proton size puzzle

Sticking In Form Factor

start with relativistic Lagrangian for nucleons - at vertices, insert measured form factors (SIFF)

$$T_1(\nu, Q^2) = \frac{1}{M}\left[\frac{Q^4 G_M^2(Q^2)}{(Q^2 + i\epsilon)^2 - 4M^2\nu^2} - F_1^2(Q^2)\right]$$

C. Carlson, M.Vanderhaeghen: Phys.Rev.A84 (2011); arXiv1109.3779;

validated by low energy theorems for nucleon Compton scattering, and verified to $\mathcal{O}(Q^4)$ in heavy baryon $\chi\mathrm{PT}$

M.. Birse, J. McGovern: arXiv:1206.3030