Two-point correlator fits on HISQ

for f_π, f_K, f_D, and f_{D_s}

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Lattice 2012 @ Cairns

[1] speaker

Introduction

- Highly Improved Staggered Quark (HISQ) action
- Symanzik-tadpole improved gauge action
- 2+1+1 flavors of sea quarks (light+strange+charm)
- Charm quark dispersion relation is fixed by "Naik ε" at tree-level
- Variety of the HISQ ensembles (# ens. = 20)
- 1000 lattices in completed ensembles
- $10 \sim 12$ partial/un-quenched valence quark masses per lattice
- Full covariance least square fit
- Constrained fit (using Bayesian prior)
- Results of this work is used to chiral/continuum inter/extra-polation to get the physical values with a good control of systematic uncertainties (Doug Toussaint's talk)

Interpolating Operators

- use staggered bilinear operators defined at a time-slice with the pseudo-scalar spin and taste, i.e., $(\gamma_5 \otimes \xi_5)$
- Point operator:

$$O_P(\vec{x},t) = \sum_c \bar{\chi}^c(\vec{x},t)(-1)^{\vec{x}+t}\chi^c(\vec{x},t)\,, \tag{1}$$

  where χ^c is the staggered quark field with the color index c.
- Coulomb wall operator (after Coulomb gauge fixing):

$$O_W(\vec{x},t) = \sum_{\vec{x},\vec{y},c} \bar{\chi}^c(\vec{x},t)(-1)^{\vec{x}+t}\chi^c(\vec{y},t)\,. \tag{2}$$

  Note: Coulomb wall operator is less contaminated from excited states.
- Note: time-slice operator couples to both opposite parity states

Correlators

- Point source - Point sink

$$C_{PP}(t) = \frac{1}{V_S} \sum_{\vec{x}} \langle O_P(\vec{x},t) O_P^\dagger(\vec{0},0) \rangle \,, \tag{3}$$

  In practice, use the random wall source to get better statistics (will be denoted by $C_{RP}(t)$).

- Coulomb wall source - Point sink

$$C_{WP}(t) = \frac{1}{V_S} \sum_{\vec{x}} \langle O_P(\vec{x},t) O_W^\dagger(\vec{0},0) \rangle \tag{4}$$

- At large t, the correlators become

$$C_{RP}(t) = c_{RP} e^{-M_{PS} t} + \cdots \tag{5}$$

$$C_{WP}(t) = c_{WP} e^{-M_{PS} t} + \cdots \tag{6}$$

- do combined fits with common masses, so C_{WP} helps isolating the ground state and C_{RP} is used to get the decay constants.

Decay Constants from the Correlators

- Decay constant is defined (on the Euclidean) as

$$\langle 0|\bar{q}_a\gamma_\mu\gamma_5 q_b(p)|P_{ab}(p)\rangle = f_{P_{ab}} p_\mu \,, \tag{7}$$

where a and b are flavor indices.

- PCAC relation implies (when $\vec{p}=0$)

$$(m_a+m_b)\langle 0|\bar{q}_a\gamma_5 q_b(p)|P_{ab}(p)\rangle = f_{P_{ab}} M_{P_{ab}}^2 \,, \tag{8}$$

- Random-wall source propagator can be written as (at large t)

$$C_{RP}(t) \simeq \frac{4}{3V_S 2M_{P_{ab}}} |\langle 0|\bar{q}_a\gamma_5 q_b(p)|P_{ab}(p)\rangle|^2 e^{-M_{P_{ab}}t} = c_{RP} e^{-M_{P_{ab}}t} \tag{9}$$

- Decay constant can be given by

$$f_{P_{ab}} = (m_a+m_b)\sqrt{\frac{3V_S c_{RP}}{2M_{P_{ab}}^3}} \tag{10}$$

HISQ Lattices

a(fm)	flavors	am_l	am_s	volume	N_{mea}
0.15	2+1+1	0.013	0.065	$16^3 \times 48$	1020
0.15	2+1+1	0.0064	0.064	$24^3 \times 48$	1000
0.15	2+1+1	0.00235	0.064	$32^3 \times 48$	1000
0.12	2+1+1	0.0102	0.0509	$24^3 \times 64$	1040
0.12	2+1+1	0.00507	0.0507	$24^3 \times 64$	1020
0.12	2+1+1	0.00507	0.0507	$32^3 \times 64$	1000
0.12	2+1+1	0.00507	0.0507	$40^3 \times 64$	1030
0.12	2+1+1	0.00184	0.0507	$48^3 \times 64$	840
0.12	2+1+1	0.00507	0.0304	$32^3 \times 64$	1020
0.12	2+1+1	0.00507	0.022815	$32^3 \times 64$	1020
0.12	2+1+1	0.00507	0.012675	$32^3 \times 64$	1020
0.12	3+1	0.00507	0.00507	$32^3 \times 64$	1020
0.12	1+1+1+1	0.00507/0.012675	0.022815	$32^3 \times 64$	1020
0.12	2+1+1	0.0088725	0.022815	$32^3 \times 64$	1020
0.09	2+1+1	0.0074	0.037	$32^3 \times 96$	1011
0.09	2+1+1	0.00363	0.0363	$48^3 \times 96$	1000
0.09	2+1+1	0.0012	0.0363	$64^3 \times 96$	535
0.06	2+1+1	0.0048	0.024	$48^3 \times 144$	1000
0.06	2+1+1	0.0024	0.024	$64^3 \times 144$	601
0.06	2+1+1	0.00084	0.0231	$96^3 \times 192$	80

Green: $m_l = 0.2 m_s$ Blue: $m_l = 0.1 m_s$ Red: $m_l = 0.037 m_s$

Valence Quark Masses

- use 10 or 12 partial/un-quenched values per lattice for the chiral inter/extra-polation.
- sea strange/charm quark masses m_s/m_c are tuned to have nearly physical values (except on the ensembles that are intended to have unphysical small strange quark masses).
- $0.9m_c$, m_c for charm
- $0.8m_s$, m_s for strange
- $0.1m_s$, $0.15m_s$, $\cdots$, $0.6m_s$ for light
- $0.036m_s$, $0.07m_s$ for light, only on physical sea quark mass ensembles.

Fitting Forms

- opposite parity states have to be included with the alternating phase $(-1)^t$ except for degenerated quark masses.
- backward propagations $\sim e^{-M(N_T-t)}$ also have to be included due to the periodic boundary condition. (N_T = time extent of lattice)
- The correlators take the form, up to $(J+K)$-states,

$$C_{\mathrm{RP}}(t) = \sum_{j=0}^{J-1} A_j M_j^3 \left(e^{-M_j T} + e^{-M_j(N_T-t)}\right) + \sum_{k=0}^{K-1} (-1)^t A'_k M'^3_k \left(e^{-M'_k t} + e^{-M'_k(N_T-t)}\right)$$

$$C_{\mathrm{WP}}(t) = \sum_{j=0}^{J-1} B_j M_j^3 \left(e^{-M_j t} + e^{-M_j(N_T-t)}\right) + \sum_{k=0}^{K-1} (-1)^t B'_k M'^3_k \left(e^{-M'_k t} + e^{-M'_k(N_T-t)}\right)$$

- combined fit of the two propagators
- Note: contribution from the opposite parity states is suppressed when two valence quark masses are close. (vanishes when degenerated)
- $f_{P_{xy}} = (m_x + m_y)\sqrt{3 V_S A_0 / 2}$

Full Covariance Least Square Fit

- Correlators at different (Euclidean) times are correlated.
- Let i be a collective index for the source type and the Euclidean time, and τ be the Monte Carlo simulation time.
- The covariance matrix of the correlators can be estimated

$$\hat{C}_{ij} = \frac{1}{(N-1)}\left[\overline{C(i)C(j)} - \overline{C(i)}\ \overline{C(j)}\right] \tag{11}$$

where overbar $\bar{\bullet}$ denotes the sample average.

- minimize χ^2 (or T^2), given by

$$\chi^2 = \sum_{ij}\left[\overline{C(i)} - f(i;\alpha)\right]\hat{C}_{ij}^{-1}\left[\overline{C(j)} - f(j;\alpha)\right] + \chi^2_{\text{prior}}(\alpha)\,, \tag{12}$$

where $f(i;\alpha)$ denotes the theoretical curve with the parameters α. Here $f(i;\alpha) = C_{RP}(t;\alpha)$ or $C_{Wp}(t;\alpha)$ and α is a collective index for A_j, B_j, M_j and their primed counterparts.

Autocorrelation

- HISQ lattices are saved to disk for every five or six units (five for $a \geq 0.12$ fm, otherwise six) in the simulation time.
- can see the autocorrelation in the correlators.
- block successive configurations, and estimate the covariance matrix using block averages.

$$\hat{C}_{ij}^{[b]} = \frac{1}{(B-1)}\left[\overline{C^{[b]}(i)C^{[b]}(j)} - \overline{C^{[b]}(i)}\;\overline{C^{[b]}(j)}\right], \tag{13}$$

$$\overline{C^{[b]}(i)} \equiv \frac{1}{B}\sum_{s=0}^{B-1} C^{[b]}(i)_s\,, \qquad C^{[b]}(i)_s = \frac{1}{b}\sum_{\tau=sb}^{(s+1)b-1} C(i)_\tau\,, \tag{14}$$

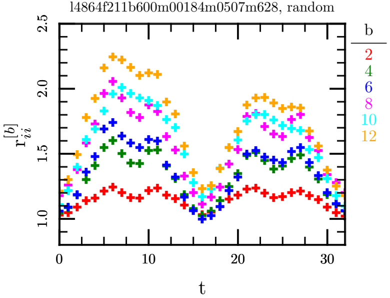
where b is the size of blocks, and B is no. of blocks.

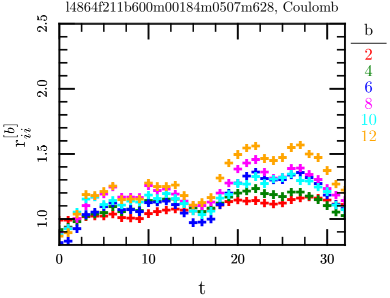
- use $b = 4$ in completed ensembles, otherwise $b < 4$.
- to see how the covariance matrix scales with the block size, define the covariance ratio as

$$r_{ij}^{[b]} \equiv \hat{C}_{ij}^{[b]} / \hat{C}_{ij} \tag{15}$$

Covariance Ratio

- The (co)variance ratios of the light-light correlator on the 0.12 fm physical sea quark mass ensemble;

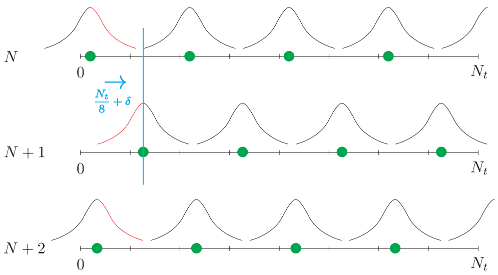




- The autocorrelation of the correlators is strongly dependent on the Euclidean time and the source type.
- can see apparent structures; two peaks for the random wall source correlator, and so on.
- It originates from how we put the sources on the lattice (next slide).

Source Position

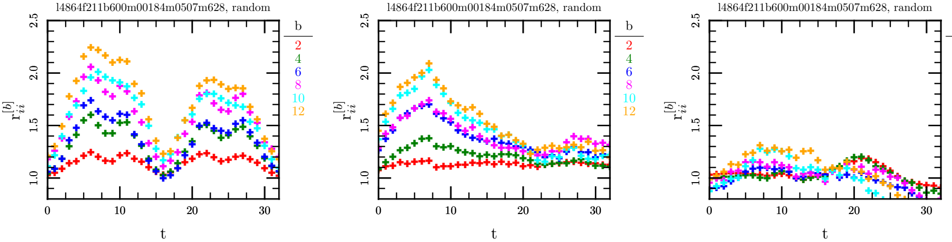
- measured four times per lattice, separated by $N_T/4$, and averaged over four sources and over forward/backward propagations.
- moves by $N_t/8+\delta$ ($\delta = 1$ or 2) through the consecutive lattices.



- The peak positions coincide with the "moving distance," $(N_T/8+\delta)$, and $(N_T/8+\delta)+N_T/4$.
- Multiple measurements can introduce additional autocorrelations.

Covariance Ratio - revisited

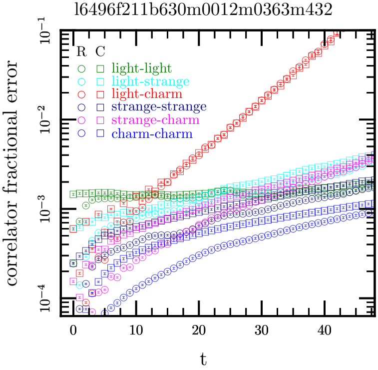
- The (co)variance ratios of the light-light correlator on the 0.12 fm physical sea quark mass ensemble for the random-wall source

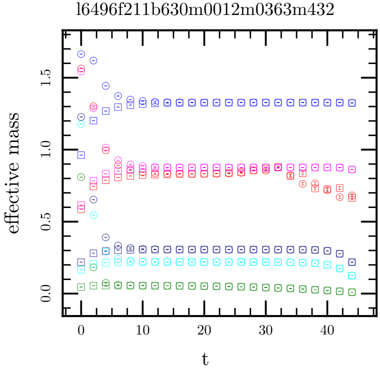


- still averaged over forward/backward
- LEFT: four sources averaged (the same as before)
- MIDDLE: one source, moves back and forth by $N_T/8+\delta$ (= 9 in this case)
- RIGHT: one source, moves back and forth by $(N_T/8+\delta)+N_T/4$ (= 25)
- autocorrelation depends on no. and positions of the sources.

Looking at Correlators

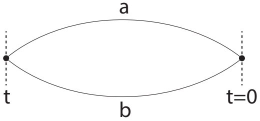
- Fractional errors (left) and effective masses (right)
- on the 0.09 fm physical sea quark mass ensemble

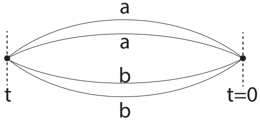




- light - charm: fractional error grows much as t increases, excited contamination comes in earlier as t decreases.
- light - light: fractional error is almost constant in t.
- $m_{\text{eff}} \equiv \frac{1}{2} \log \left[C(t)/C(t+2) \right]$.

Fractional Errors (FE)

$E[C(t)] =$ 

, $\sigma^2[C(t)] \sim$ 

- For a pseudo-scalar meson, $O_{ab} = \bar{q}_a \gamma_5 q_b$ with flavors a and b, at large t (ignoring disconnected diagrams when $a = b$)

$$E[C(t)] = \langle O_{ab}(t) O^{\dagger}_{ab}(0) \rangle \approx A e^{-M_{P_{ab}} t} \tag{16}$$

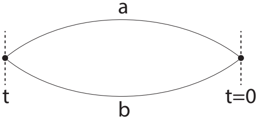
$$\begin{aligned} \sigma^2[C(t)] &\simeq \langle O_{ab}(t) O^{\dagger}_{ab}(t) O^{\dagger}_{ab}(0) O_{ab}(0) \rangle - \Big(E[C(t)] \Big)^2 \\ &\approx B e^{-M_{\text{lowest}} t} - A^2 e^{-2M_{P_{ab}} t} \\ &\approx B' e^{-M_{\text{lowest}} t} \qquad \text{if } M_{\text{lowest}} \lesssim 2M_{P_{ab}} , \end{aligned} \tag{17}$$

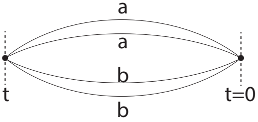
where M_{lowest} is the lowest state mass consisting of four (anit-)quarks $q_a, q_b, \bar{q}_a$, and $\bar{q}_b$. [Lepage '90]

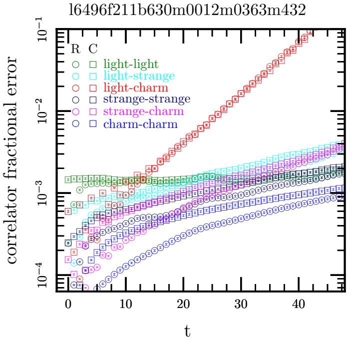
- Fractional error is asymptotically given as

$$\text{FE(t)} \ = \sigma[C(t)] / E[C(t)] \approx e^{[M_{P_{ab}} - \frac{1}{2} M_{\text{lowest}}] t} \tag{18}$$

Fractional Errors (FE), II

$E[C(t)] =$ 

, $\sigma^2[C(t)] \sim$ 



- For light-charm (D meson),

$$M_{P_{ab}} = M_D\,, \quad M_{\text{lowest}} = M_{\eta_c} + M_\pi$$

$$\text{FE} \approx e^{[M_D - \frac{1}{2}(M_{\eta_c}+M_\pi)]t} \approx e^{300 MeV \times t}$$

On a 0.09 fm lattice,

$$\log_{10}[FE(t)] \approx 0.06t + constant \tag{19}$$

- For light-light (pion),

$$M_{P_{ab}} = M_\pi\,, \quad M_{\text{lowest}} = 2M_\pi$$

$$\log_{10}[\text{FE}(t)] \approx \text{constant} \tag{20}$$

Looking at PDG

		light - light/strange			light/strange - charm	
		$I^G(J^{PC})$	M (MeV)		$I^G(J^{PC})$	M (MeV)
gro.	$\pi^\pm$	$1^-(0^-)$	139.6	$D^\pm$	$\frac{1}{2}(0^-)$	1869.5(4)
	π^0	$1^-(0^{-+})$	134.8	D^0	$\frac{1}{2}(0^-)$	1864.9(2)
	$K^\pm$	$\frac{1}{2}(0^-)$	493.7	$D_s^\pm$	$0(0^-)$	1969(1)
	K^0	$\frac{1}{2}(0^-)$	497.6			
alt.				$D_0^*(2400)^\pm$	$\frac{1}{2}(0^+)?$	2403(38)
	$a_0(980)$	$1^-(0^{++})$	980(20)	$D_0^*(2400)^0$	$\frac{1}{2}(0^+)$	2318(29)
	$K_0^*(800)$	$\frac{1}{2}(0^+)?$	676(40)	$D_{s0}^*(2317)^\pm$	$0(0^+)?$	2318(1)
	$K_0^*(1430)$	$\frac{1}{2}(0^+)$	1425(50)			
exc.	$\pi(1300)$	$1^-(0^{-+})$	1300(100)	$D(2550)^0$	$\frac{1}{2}(0^-)$	2539(8)
	$K(1460)$	$\frac{1}{2}(0^-)$	$\sim$1460			

- Very roughly, may find

	light - light/strange	light/strange - charm
$M_0' - M_0$	$M_{K^*(800)} - M_K \approx 200$ MeV	$M_{D^*} - M_D \approx 400$ MeV
$M_1' - M_0$	$M_{a_0} - M_\pi \approx 800$ MeV	
$M_1 - M_0$	$M_{\pi(1300)} - M_\pi \approx 1100$ MeV	$M_{D(2550)} - M_D \approx 700$ MeV

- PDG says ? needs confirmation

Fitting Strategy

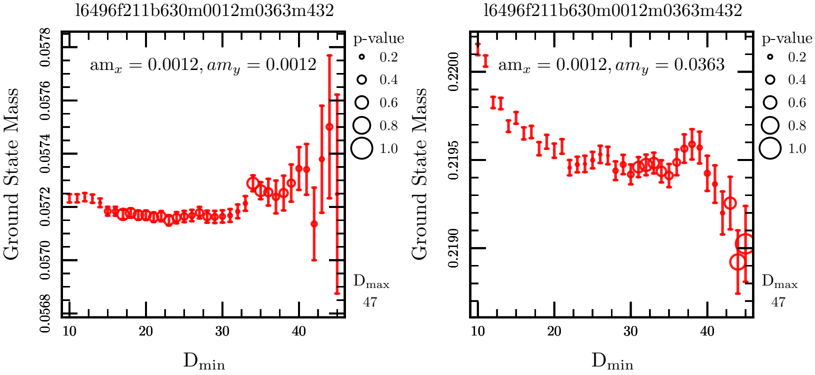
- For light - light/strange, use the $1+0$ fit form with a large enough minimum distance.
- For light/strange - charm, use the $2+1$ fit form with a small (but not too small) minimum distance, and with the alternate/excited states constrained by priors.

	light - light/strange			light/strange - charm		
No. of states	J+K = 1+0			J+K = 2+1		
Bayesian priors	No			$M_1 - M_0 = 700 \pm 140$ MeV $M_0' - M_0 = 400 \pm 200$ MeV		
Fitting Ranges	a (fm)	D_{min}	D_{max}	a (fm)	D_{min}	D_{max}
	0.15	16	23	0.15	8	23
	0.12	20	31	0.12	10	31
	0.09	30	47	0.09	15	47
	0.06	40	71	0.06	20	71

- check if changes are consistent/stable under variations of D_{min}, D_{max}, priors, fit forms, and so on.

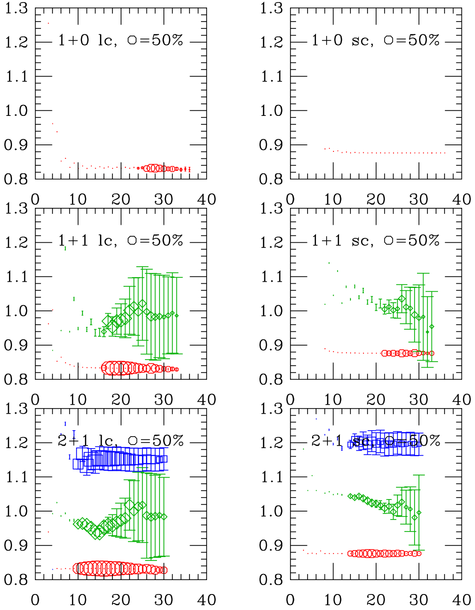
light - light/strange fits

- on the 0.09 fm physical sea quark mass ensemble



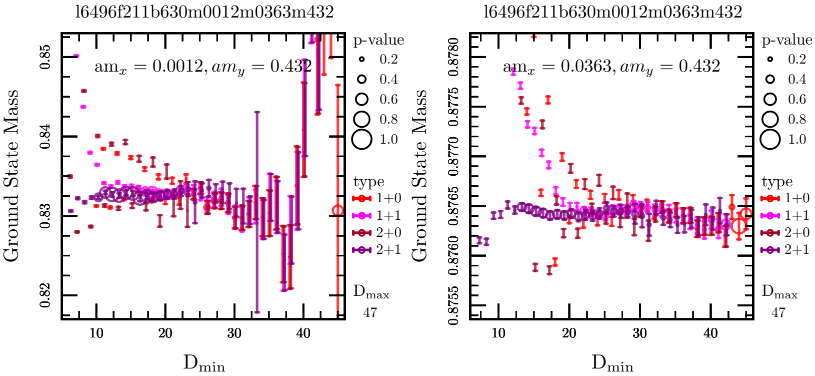
- light - light (left) and light - strange (right)
- The size of symbols is proportional to p-value.
- used the 1+0 fit form
- choose $D_{\min} = 30$

Light/strange - charm fits



- The ground, alternate, and excited state masses vs. D_{min}
- On the 0.09 fm, physical sea quark mass ensemble
- For light-charm (left) and strange-charm (right)
- The size of symbols is proportional to p-value
- At large D_{min}'s, alternate/exited state errors are the prior widths.
- 2+1 fit form works very well
- choose the 2+1 fit form with $D_{\text{min}} = 15$.

Light/strange - charm fits, II



- light - charm (left) and strange - charm (right)
- minimum # states with large D_{min} does not work.
- 2+1 fit form shows the widest "flat window" with smaller errors, so is the best choice.
- choose the 2+1 fit form with $D_{min} = 15$.

Systematic Uncertainties in Fits

- decay constants with finitesize, isospin, and (some) E&M corrections (Doug Toussaint's talk for detail)
- on the 0.09 fm physical sea quark mass ensemble
- To estimate the excited contamination, vary D_{min}, D_{max}, priors, and # of states.

"Fpi scale"	central	excited
f_K (MeV)	155.00(21)	$\lesssim \pm$ 0.2
f_D (MeV)	210.00(98)	$\lesssim \pm$ 0.5
f_{D_s} (MeV)	246.56(26)	$\lesssim \pm$ 0.5
f_{D_s}/f_D	1.174(5)	$\lesssim \pm$ 0.04

Conclusions

- performed two point correlator fits on the variety of HISQ ensembles, which allows us to calculate f_π, f_K, f_D, and f_{D_s} with a good control of systematic uncertainties. (Doug Toussaint's talk)
- learned that additional autocorrelations can be introduced when multiple measurements (per lattice) are used.
- found systematic uncertainties from the correlator fits are small but not negligible.

Thank you!