Lattice investigation of the tetraquark candidates $a_0(980)$ and κ

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Introduction – Light scalar mesons

Do light scalar mesons have exotic (four-quark) components?

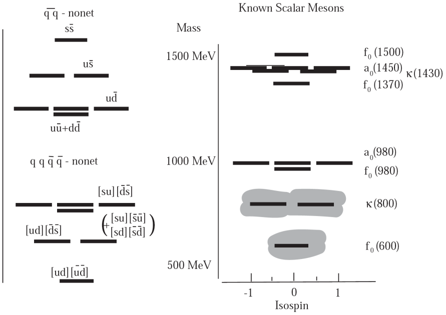


Figure: R. L. Jaffe, Physics Reports **409**, 1 (2005)

- Hints at four-quark bound states for $f_0(600)$, κ observed recently with lattice QCD

S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. F. Liu, N. Mathur, D. Mohler, Phys. Rev. **D 82**, 094507 (2010)

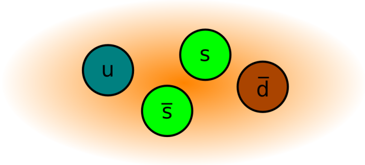
Introduction

The correct way (very challenging!)

- Problem: a_0 instable in QCD $\longrightarrow$ resonance not directly accessible

$$a_0 \longrightarrow \eta\pi, \bar{K}K$$

- Study volume dependence of energy levels
- Use extension of Lüscher's finite size method



Goal of this project

- Do we observe a candidate for a bound four-quark state?

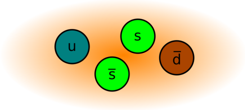
Interpolating operators

Interpolating operators

Diquark $(\bar{\mathbf{3}}_c, \bar{\mathbf{3}}_f)$ anti-diquark	$[q(\boldsymbol{x})\Gamma q(\boldsymbol{x})]_a \, [\bar{q}(\boldsymbol{x})\Gamma \bar{q}(\boldsymbol{x})]_a$
Mesonic molecule	$(q(\boldsymbol{x})\Gamma \bar{q}(\boldsymbol{x}))\,(q(\boldsymbol{x})\Gamma \bar{q}(\boldsymbol{x}))$
Two mesons	$(q(\boldsymbol{x})\Gamma \bar{q}(\boldsymbol{x}))\,(q(\boldsymbol{y})\Gamma \bar{q}(\boldsymbol{y}))$

Exploratory study: neglect disconnected contributions

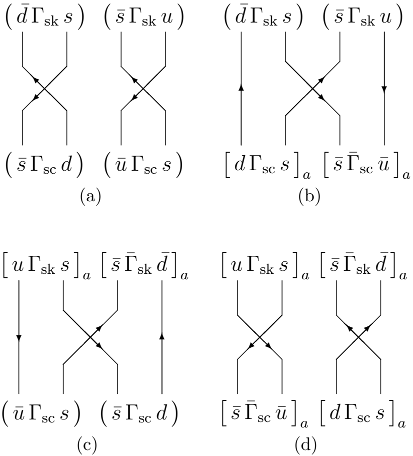
- Jacobi and APE smearing
- Valence quark content for $a_0(980)^+$: $\bar{d}s\bar{s}u$
  $\longrightarrow$ molecules: $\eta_s\pi$, $K\bar{K}$
- Valence quark content for κ: $\sum_{q=u,d,s} \bar{s}q\bar{q}u$
  $\longrightarrow$ molecule: $K\pi$



S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. F. Liu, N. Mathur, D. Mohler, Phys. Rev. **D 82**, 094507 (2010)

Spectroscopy on the lattice – Four-quark states

How to handle four-quark correlators?



Spectroscopy on the lattice – Four-quark states

How to handle four-quark correlators?

- More diagrams for $a_0(980)$ if one includes $\eta_s \pi$ molecule $(\bar{s}\gamma_5 s)(\bar{d}\gamma_5 u)$
- More diagrams for κ
- Possible extensions

Automatic code generation

- Start from analytic correlator
- Apply Wick's theorem
- Sort according to Dirac indices and write as traces
- Generate the corresponding C++ code

Extraction of energies

- Solve the generalized eigenvalue problem for $C_{ij}(t)$

$$C(t)\vec{u}^n(t) = \lambda^n(t,t_0)C(t_0)\vec{u}^n(t).$$

$$\Rightarrow \lambda^n(t) \longrightarrow e^{-E_n(t-t_0)} + \mathcal{O}(e^{-E_{n+1}(t-t_0)})$$

- Single particle state correction:
  - For a diagonal correlator $C_{ii}(t)$

$$\begin{aligned} C_{ii}(t) \longrightarrow \quad & |A_i^n|^2 \quad \left[e^{-E_n t} + e^{-E_n(T-t)}\right] \\ + \quad & |C_i^n|^2 \quad \left[e^{-m_1 t}e^{-m_2(T-t)} + e^{-m_2 t}e^{-m_1(T-t)}\right] \end{aligned}$$

  E_n: two-particle state energy, m_1, m_2: single-particle masses

  W. Detmold, K. Orginos, M. Savage, A. Walker-Loud, Phys. Rev. **D 78**, 054514 (2008)

  - Apply to generalized eigenvalues

$$\begin{aligned} \lambda^n(t) \approx \quad & a_n \quad \left[e^{-E_n t} + e^{-E_n(T-t)}\right] \\ + \quad & c_n \quad \left[e^{-m_1 t}e^{-m_2(T-t)} + e^{-m_2 t}e^{-m_1(T-t)}\right] \end{aligned}$$

S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. F. Liu, N. Mathur, D. Mohler, Phys. Rev. **D 82**, 094507 (2010)

- Twisted mass $N_f = 2 + 1 + 1$ fermions (ETMC)

ETMC, R. Baron *et al.*, JHEP **06**, 111 (2010)

- Mixed action approach (Osterwalder-Seiler)
  - Different regularization for valence and sea strange quarks

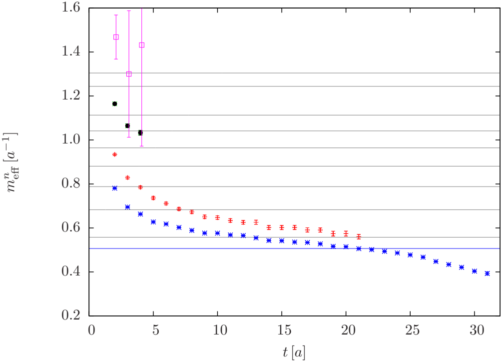
R. Frezzotti, G. Rossi, JHEP **10**, 070 (2004)

- Connected-only
- Jacobi and APE smearing

Simulation parameters

	$a\,[\mathrm{fm}]$	$L\,[\mathrm{fm}]$	$m_{\pi^+}\,[\mathrm{MeV}]$	N_{conf}
A30.32	0.086	2.8	280	1313
A40.24	0.086	2.0	330	1259
A40.20	0.086	1.7	340	500
A80.24	0.086	2.0	460	1225

Effective masses of $\lambda^n(t, t_0)$ for $a_0^+ = (\bar{d}s\bar{s}u)$
(Four-quark interpolators only, 6×6, A30.32)



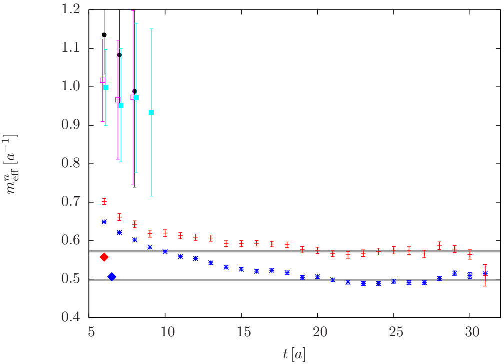
gray lines: $K\bar{K}$ free scattering states, blue line: $m_{\eta_s} + m_\pi$

$(a = 0.086\,\text{fm},\ (L/a)^3 \times (T/a) = 32^3 \times 64,\ m_{\pi^+} \approx 280\,\text{MeV}\ N_{\text{conf}} = 1313)$

No addidtional state observed

Results – a_0 four-quark analysis

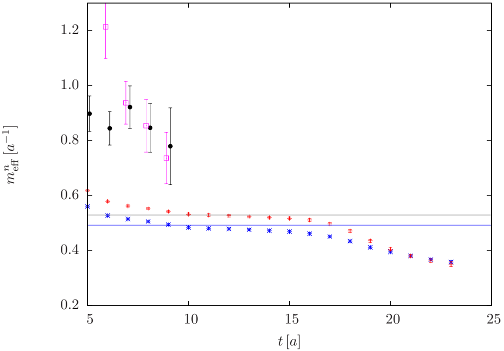
Effective masses of $\lambda^n(t, t_0)$ for $a_0^+ = (\bar{d}s\bar{s}u)$
(Four-quark interpolators only, 5×5, A30.32)
Corrected for single-kaon states



red diamond: $m_K + m_{\bar{K}}$, blue diamond: $m_{\eta_s} + m_\pi$, gray: fit results
$(a = 0.086\,\text{fm},\ (L/a)^3 \times (T/a) = 32^3 \times 64,\ m_{\pi^+} \approx 280\,\text{MeV}\ N_{\text{conf}} = 1313)$

Results – a_0 four-quark analysis

Effective masses of $\lambda^n(t, t_0)$ for $a_0^+ = (\bar{d}s\bar{s}u)$
$(4 \times 4$, A40.20)
Two-meson states included

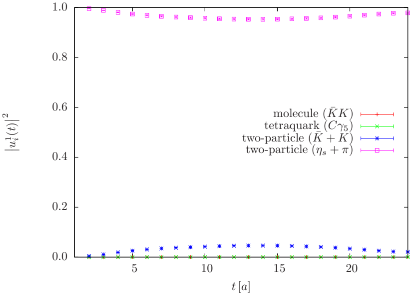


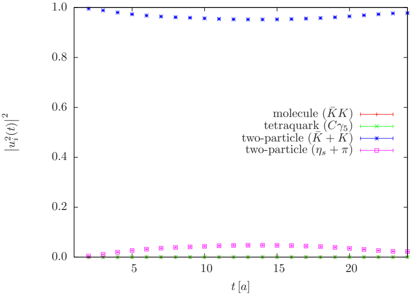
gray line: $m_K + m_{\bar{K}}$, blue line: $m_{\eta_s} + m_\pi$

$(a = 0.086\,\text{fm},\ (L/a)^3 \times (T/a) = 20^3 \times 48,\ m_{\pi^+} \approx 340\,\text{MeV}\ N_{\text{conf}} = 500)$

Results – a_0 four-quark analysis

Eigenvectors of $C_{ij}(t)$ (4×4, A40.20)
Two-meson states included

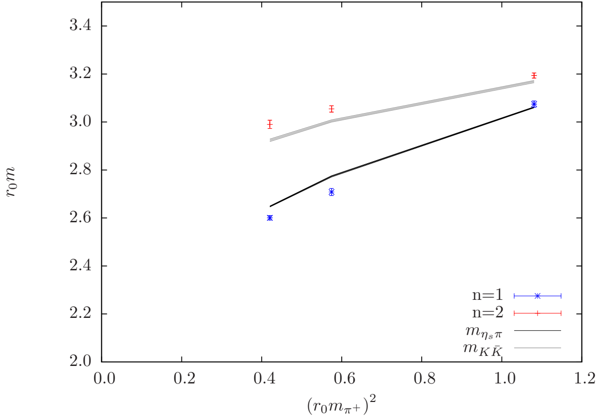




Correlator dominated by two-meson states!

Results – a_0 four-quark analysis

Summary of the four-quark analysis for $a_0^+ = (\bar{d}s\bar{s}u)$

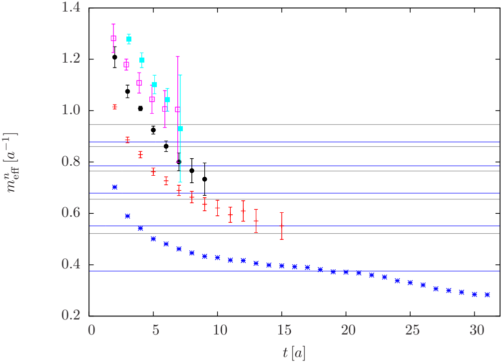


Conclusions

- Two states close to non-interacting scattering states
- No candidates for bound four-quark states observed

Results – κ four-quark analysis

Effective masses of $\lambda^n(t, t_0)$ for $\kappa = \sum_{q=u,d,s}(\bar{s}q\bar{q}u)$
(Four-quark interpolators only, 5×5, A30.32)



blue lines: $m_{K^0\pi^+}$, gray lines: $m_{K^+\pi^0}$

($a = 0.086\,\text{fm}$, $(L/a)^3 \times (T/a) = 32^3 \times 64$, $m_{\pi^+} \approx 280\,\text{MeV}$ $N_{\text{conf}} = 1313$)

No addidtional state observed

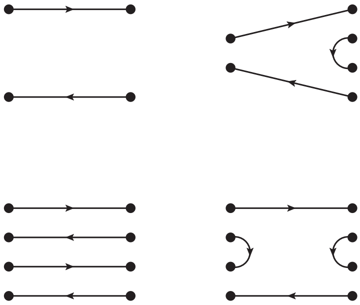
Summary

- Diquark anti-diquark, meson molecule and two-meson operators studied in the a_0 and κ channels
- We observe states close to the non-interacting scattering states
- No additional (bound) states observed for a_0 or κ
- κ result in contradiction to previous $N_f = 2$ calculation

  S. Prelovsek, T. Draper, C. B. Lang, M. Limmer, K. F. Liu, N. Mathur, D. Mohler, Phys. Rev. **D 82**, 094507 (2010)

Outlook

- Resonance does not need to correspond to an energy level in finite volume
- Calculate overlap of two- and four-quark states on the lattice



- Study volume dependence of energy levels
- Use extension of Lüscher's finite size method

Thank you for your attention.